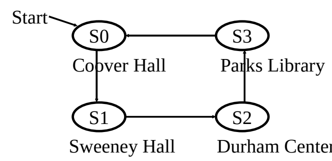
Sequential Circuit and State Machine

- Combinational circuits
  - output is simply dependent on the current input
- Sequential circuits
  - output may depend on the input sequence

- The effect of the input sequence can be memorized as a state of the system
- So a sequential circuit is also called a State Machine
- Memory elements (usually D flop-flips) are used to store the state
- System state changes with input
- A different input sequence produces different final state and different output sequence

State Transition Diagram (or State Diagram)

- Example:
  - A very simple machine to remember which building I am at
  - The only input is the clock signal
  - The state machine is represented as a state transition diagram (or called state diagram) below
  - One step (i.e., transition) can be taken whenever there is a clock signal



State Transition Table (State Table)

- States can be coded as binary combinations of variables
- Let N be total number of states, each state can be represented by $n=\log_2 N$ bits
- n bits can represent up to 2^n states
- This is called the state assignment

State Assignment

	X	Y
S0	0	0
S1	0	1
S2	1	0
S3	1	1

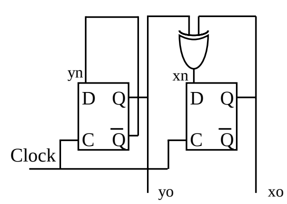
- A truth table will then give the next state
- This is called a state transition table (or called state table)
- xn and yn can be specified in terms xo and yo

Current State xo yo	Next State xn yn
0 0	0 1
0 1	1 0
1 0	1 1
1 1	0 0

$xn = xo \oplus yo$

$yn = yo'$

The Resulting Sequential Circuit / State Machine



Current State xo yo	Next State xn yn
0 0	0 1
0 1	1 0
1 0	1 1
1 1	0 0

$xn = xo \oplus yo$

$yn = yo'$

Counter state machine

- A counter counts
- Number of elements in counter determines how many different states we need
- For example, an eight-state counter can count eight steps

Current X Y Z	Next X Y Z
0 0 0	0 0 1
0 0 1	0 1 0
0 1 0	0 1 1
0 1 1	1 0 0
1 0 0	1 0 1
1 0 1	1 1 0
1 1 0	1 1 1
1 1 1	0 0 0

X=

Y=

Z=

Another counter

- Counter need not have number of states that is equal to a power of 2
- Here is a five state counter
- Is it simpler?

Current X Y Z	Next X Y Z
0 0 0	0 0 1
0 0 1	0 1 0
0 1 0	0 1 1
0 1 1	1 0 0
1 0 0	0 0 0

X=

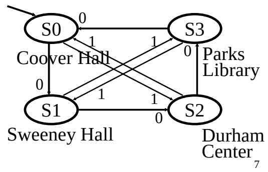
Y=

Z=

State Machine with Explicit Inputs

- In a state transition diagram, state may change with time
- A clock signal represents passage of time
- Each time a clock arrives, state changes to next state
- Clock is an implicit input
- There may or may not be other explicit inputs

- For the previous example, let say we also have an explicit input i
- For the state transition diagram shown, i can be 0 or 1
- Next state depends on current state and the value of input i
- When the next state depends upon the inputs, the inputs are examined at the clock edges



State Transition Table with Explicit Inputs

- State transition table will have two sets of inputs
- Current state variable and explicit input variables
- Total number of row in table is $2^{(n+m)}$
  - n is number of variables representing states
  - m is number of input variables

State Assignment

	X	Y
S0	0	0
S1	0	1
S2	1	0
S3	1	1

Current xo	yo	Input i	Next State xn	yn
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	1
1	0	1	0	0
1	1	0	0	0
1	1	1	0	1

$xn = xo'\,yo'\,i + xo'\,yo\,i' + xo'\,yo\,i + xo\,yo'\,i'$

$= xo'\,i + xo'\,yo + xo\,yo'\,i'$

$yn = xo'\,yo'\,i' + xo'\,yo\,i + xo\,yo'\,i' + xo\,yo\,i$

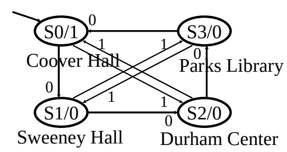
$= yo'\,i' + yo\,i$

Output of state machine

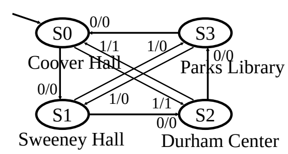
- Output of a state machine may depend on state, or state & input:
  - Mealy machine: Output depends on both current state and current input (i.e., depends on transition)
  - Moore machine: Output depends on current state
- Thus we have two different circuits to implement
  - 1. Decides what is the next state
  - 2. Decides what is the output
- Both circuits are combinational
- States are remembered by memory elements
  - Usually D flips-flops are used to remember states

State Transition Diagram with Outputs

- Moore Machine:
  (For example, output 1 whenever in Coover)

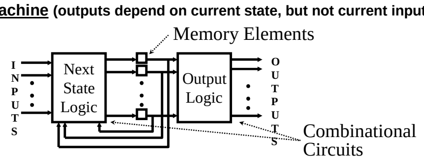


- Mealy Machine:
  (For example, output 1 whenever walking between Coover and Durham)

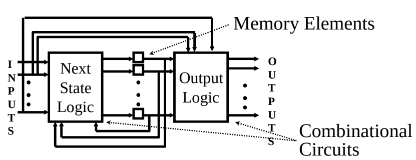


Overall structure of a State machine

Moore machine (outputs depend on current state, but not current inputs)



Mealy machine (outputs depend on both current state and current inputs)



Steps in designing a state machine

- Start writing a state transition diagram
  - It has an initial state
  - It has other states to keep track of various activities
  - It has some transitions
- Generate a state transition table and a output table
- Write state transition table and output table in binary
  - Needs state assignment, i.e., the code used for each state
  - State assignment is a complex process
  - For the time being assume straightforward combinations
- Derive canonical sum-of-product expressions
  - You can simplify the expressions

Determining number of states

- Identify how many different things we need to keep track of
- This is critical to know
- Otherwise the number of states (and their meaning) may get out of hand very quickly
- This is different from what is the output of interest (in each state we may have some outputs)
- For example, if we are to process a sequence of input bits, depending on interest, the number of states may be different
  - If we need to know how many 1's there are, we need states corresponding to the count
  - If we need to know if we have even or odd number of 1's, we may need only two states

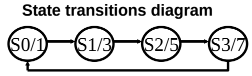
Example

- Design a state machine that will repeatedly display in binary values 1, 3, 5, and 7
- Solutions:
  - How many states we need?
  - What is the state transition diagram?
  - What is the output in each state?
  - What is the next state logic?
  - Construct the truth tables with state variables
  - Derive the next state logic and output logic
  - Draw the circuits

Example (contd.)

- We need four states: S0, S1, S2, S3

State transitions diagram



State transition table

Current State	Next State
S0	S1
S1	S2
S2	S3
S3	S0

Implementation level state transition table

Current X Y	Next X Y
0 0	0 1
0 1	1 0
1 0	1 1
1 1	0 0

Output table

Current State	Out put
S0	1
S1	3
S2	5
S3	7

Implementation level output tables

Current X Y	Output L2 L1 L0
0 0	0 0 1
0 1	0 1 1
1 0	1 0 1
1 1	1 1 1

$X = X'Y+XY'$
$Y = X'Y'+XY' = Y'$

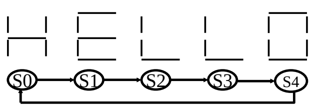
$L2 = XY'+XY = X$
$L1 = X'Y+XY = Y$
$L0 = X'Y'+X'Y+XY'+XY = X'+X = 1$

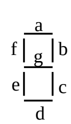
Another Example for State Machine

- Design a state machine to display the characters in the string HELLO using a seven segment display
- How many states do we need?
  - Five, one for each character
  - In state S0 (000) we display H
  - In state S1 (001) we display E
  - In state S2 (010) we display L
  - In state S3 (011) we display L
  - In state S4 (100) we display O
- State transitions are

  S0 -> S1
  S1 -> S2
  S2 -> S3
  S3 -> S4
  S4 -> S0

Example (contd.)





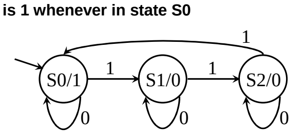
- Next State and Output logic tables are

Cur State Xc Yc Zc	Next State Xn Yn Zn
0 0 0	0 0 1
0 0 1	0 1 0
0 1 0	0 1 1
0 1 1	1 0 0
1 0 0	0 0 0

State Xc Yc Zc	Output a b c d e f g
0 0 0	0 1 1 0 1 1 1
0 0 1	1 0 0 1 1 1 1
0 1 0	0 0 0 1 1 1 0
0 1 1	0 0 0 1 1 1 0
1 0 0	1 1 1 1 1 1 0

To Detect if # of 1's in Input is Divisible by 3

- Design a state machine with 1 bit of input and 1 bit of output
- The output bit will be 1 whenever the number of bits in input sequence is divisible by 3
- How many states do we need?
- What are the meaning of the states?
  - In state S0 (00), remainder = 0 (i.e., divisible by 3)
  - In state S1 (01), remainder = 1
  - In state S2 (10), remainder = 2
- Choose to design a Moore machine
  - Output is 1 whenever in state S0



State machines as sequence detector

- State machine by nature are ideally suited to track state and detect specific sequence of events
- For example, we may design specific machines to track certain pattern in an input sequence
- Examples:
  - to count 1's in a sequence and produce an output if a specific situation occurs like 3rd one, or every 2nd one, or nth one
  - to generate an output or stop if a specific pattern in the sequence (such as 011 or 0101 or 1111) is observed
- In each of these cases, it is to create a relationship between input and output sequence
- We will review input and output relations for such operations

Example input/output sequences

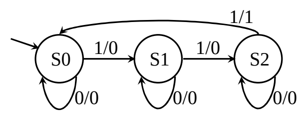
- n-th one detector, n=2
  - Input: 0 0 1 0 0 1 1 1 0 1 1 0 0 1 0 1 0 1 0 1 1 1 0 0 0 1
  - Output: 0 0 0 0 0 1 0 1 0 0 1 0 0 0 0 1 0 0 0 1 0 1 0 0 0 0
- n-th one detector, n=3
  - Input: 0 0 1 0 0 1 1 1 0 1 1 0 0 1 0 1 0 1 0 1 1 1 0 0 0 1
  - Output: 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0
- 011 pattern detector
  - Input: 0 0 1 0 0 1 1 1 0 1 1 0 0 1 0 1 0 1 0 1 1 1 0 0 0 1
  - Output: 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
- 1010 pattern detector
  - Input: 0 0 1 0 0 1 1 1 0 1 1 0 0 1 0 1 0 1 0 1 1 1 0 0 0 1
  - Output: 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0

How to design sequence detector

- Our goal is to be able to identify minimum number of states
- It is very easy to miss that goal (in terms of number of states)
- Sometimes CAD tools may identify redundant states
- We first discuss the number of possible states to track
- For example in sequence detection, for 011,
  - we need states representing we have not seen the first zero, we have seen only the first 0, we have seen 01, and finally we have seen 011
  - So a four state system will work
- 1010 has a pattern that also repeats part of the sequence
  - So we need states that represent starting state, received first 1, first 10, first 101, and finally 1010 (a total of five state)
  - However after we see 1010, we have already seen 10 pattern for the next output (i.e., if we have 101010 repeating)

3-rd One Detector

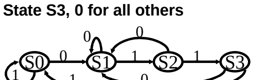
- Use a Mealy machine design
- 3 states are enough
- Have a similar structure to the <u>Moore machine</u> to detect if # of 1's in Input is Divisible by 3



- If Moore machine design is used, 4 states is needed

Design of a sequence detector for 011

- Four states and state transitions are shown in the figure
- Output: 1 for State S3, 0 for all others



Current State	Input	Next State
S0	0	S1
S0	1	S0
S1	0	S1
S1	1	S2
S2	0	S1
S2	1	S3
S3	0	S1
S3	1	S0

State Assignment

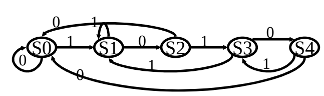
Current State	Binary
S0	0 0
S1	0 1
S2	1 0
S3	1 1

Output of states

Current State	Output
S0	0
S1	0
S2	0
S3	1

Design of a sequence detector for 1010

- Four states and state transitions are shown in the figure
- Output: 1 for State S4, 0 for all others



Current State	Input	Next State
S0	0	S0
S0	1	S1
S1	0	S2
S1	1	S1
S2	0	S0
S2	1	S3
S3	0	S4
S3	1	S1
S4	0	S0
S4	1	S3

State Assignment

Current State	Binary
S0	0 0 0
S1	0 0 1
S2	0 1 0
S3	0 1 1
S4	1 0 0

Output of states

Current State	Output
S0	0
S1	0
S2	0
S3	0
S4	1

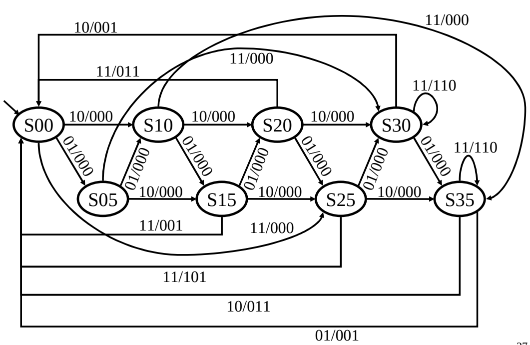
Another example: a complex vending machine

- **Vending Machine**
  - **Collect money, deliver product and change**
- **Vending machine may get three inputs, n, d, q**
  - **Inputs are nickel (5c), dime (10c), and quarter (25c)**
  - **Only one coin input at a time**
  - **Product cost is 40c**
  - **Does not accept more than 50c (blocks the coin slot)**
  - **Returns 5c or 10c back**
  - **Exact change appreciated**
- **How many states?**
- **What are the output signals?**

Design of Complex Vending Machine

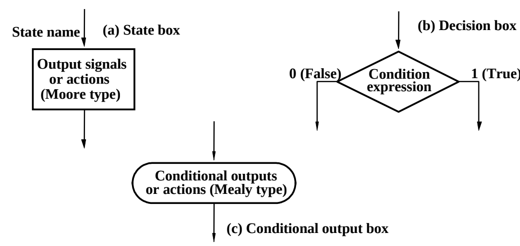
- **We are designing a Mealy state machine (i.e., output depends on both current state and inputs).**
- **Suppose we ask the machine to directly return the coin if it cannot accept an input coin.**
- **The following two-bit code is used:**
  - **00 -- no coin, 01 -- nickel, 10 -- dime, and 11 -- quarter**
- **Inputs: I_1 I_2 which represent the coin inserted**
- **Outputs: C_1 C_2 P where C_1 C_2 represent the coin returned and P indicates whether to deliver product**
- **States: S00, S05, S10, S15, S20, S25, S30, S35**
  - **3 bits are enough to encode the states**
  - **Notice the names (they need not be S0, S1….)**
- **State assignment: S00 – 000, S05 – 001, S10 – 010, S15 – 011, S20 – 100, S25 – 101, S30 – 110, S35 – 111**

State Diagram for Vending Machine

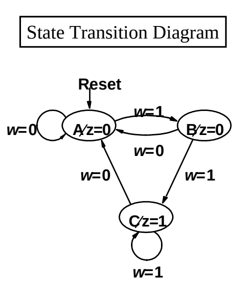


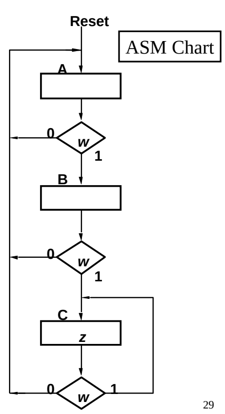
Algorithmic State Machine (ASM) Charts

- **Another way to represent a state machine**
- **State diagrams are useful when the machine has only a few inputs and outputs**
- **ASM charts may be more convenient for larger machines**



Example: Moore Machine





Example: Mealy Machine