EE 330 HW 14 Solutions Spring 2019 – Abdullah Al-Obaidi

Problem 1:

$W_{min} = L_{min} = 0.2\ \mu m$

$$R_{pu} = \frac{L_p}{u_p C_{ox} W_p (V_{DD} - |V_{tp}|)} = \frac{0.2\mu}{116\mu * 0.2\mu * (1.8 - 0.1)} = 5.071\ k\Omega$$

$$R_{pd} = \frac{L_n}{u_n C_{ox} W_n (V_{DD} - V_{tn})} = \frac{0.2\mu}{350\mu * 0.2\mu * (1.8 - 1.8 * 0.25)} = 2.116\ k\Omega$$

$C_L = 20$ fF

$t_{HL} = R_{pd} * C_L = 2.116\ k * 20\ f = 42.32\ ps$

$t_{LH} = R_{pu} * C_L = 5.071\ k * 20\ f = 101.42\ ps$

$$V_{trip} = \frac{V_{tn} + (V_{DD} + V_{tp})\sqrt{\frac{\mu_p}{\mu_n} * \frac{W_2}{W_1} * \frac{L_1}{L_2}}}{1 + \sqrt{\frac{\mu_p}{\mu_n} * \frac{W_2}{W_1} * \frac{L_1}{L_2}}} = \frac{0.45 + (1.8 - 0.1)\sqrt{\frac{1}{3} * 1}}{1 + \sqrt{\frac{1}{3} * 1}} = 0.908\ V$$

Problem 2:

a)

$$V_{trip} = \frac{V_{tn} + (V_{DD} + V_{tp})\sqrt{\frac{\mu_p}{\mu_n} * \frac{W_2}{W_1} * \frac{L_1}{L_2}}}{1 + \sqrt{\frac{\mu_p}{\mu_n} * \frac{W_2}{W_1} * \frac{L_1}{L_2}}} = \frac{V_{DD}}{2} = 0.9\ V$$

Group $\frac{W_2}{W_1} * \frac{L_1}{L_2}$ together as one term and solve

$\frac{W_2}{W_1} * \frac{L_1}{L_2} \approx 0.95$

b)

$$t_{HL} = t_{LH} \rightarrow R_{pu} = R_{pd} \rightarrow \frac{L_p}{u_p C_{ox} W_p (V_{DD} - |V_{tp}|)} = \frac{L_n}{u_n C_{ox} W_n (V_{DD} - V_{tn})}$$

I will set $L_n = L_p$ then I can simplify to:

$$\frac{W_n}{W_p} = \frac{V_{DD} - |V_{tp}|}{V_{DD} - V_{tn}} * \frac{u_p C_{ox}}{u_n C_{ox}} = \frac{1.8 - 0.1}{1.8 - 0.45} * \frac{1}{3} \approx 0.42$$

Problem 3:

a) $\frac{u_n C_{ox}}{u_p C_{ox}} = 3$

W$_n$ = 0.2 µm , L$_n$ = 0.2 µm

W$_p$ = 3*k*W$_{min}$ = 1.2 µm, L$_p$ = 0.2 µm

b) fastest t_{LH} is when both NMOS are on, slowest is only when one of them is on

$$R_{pd} = \frac{L_n}{u_n C_{ox} W_n (V_{DD} - V_{tn})} = \frac{0.2\mu}{350\mu * 0.2\mu * (2 - 0.5)} = 1.905\ k\Omega$$

$$C_L = C_{ox} * W * L = 2 * \ 8f * (0.2 * 0.2 + 1.2 * 0.2) = 4.48\ fF$$

$$t_{HLslow} = R_{pu} * C_L = 1.905\ k * 4.48\ f = 8.53\ ps$$

$$t_{HLfast} = \frac{t_{LHslow}}{2} = 4.27\ ns$$

Problem 4:

$\frac{u_n C_{ox}}{u_p C_{ox}} = 3$

$W_n = kW_{min} = 4 * 0.2 = 0.8\ \mu m$, $L_n = 0.2\ \mu m$

$W_p = 3W_{min} = 3 * 0.2 = 0.6\ \mu m$, $L_p = 0.2\ \mu m$

Problem 5:

a) $M_1\ off\ and\ M_2\ in\ saturation\ \rightarrow\ V_H = V_{DD} - V_{tn} = 4\ V$

$M_1\ in\ triode\ and\ M_2\ in\ saturation\ \rightarrow V_L = 0.62\ V$

Works as invertor

b) $M_1\ off\ and\ M_2\ in\ saturation\ \rightarrow\ V_H = V_{DD} - V_{tn} = 4\ V$

$M_1\ in\ triode\ and\ M_2\ in\ saturation\ \rightarrow no\ solution\ for\ V_L$

$can't\ work\ as\ invertor$

c) $M_1\ off\ and\ M_2\ in\ saturation\ \rightarrow\ V_H = V_{DD} - V_{tp} = 4\ V$

$M_1\ in\ triode\ and\ M_2\ in\ saturation\ \rightarrow V_L = 0.734\ V$

Works as invertor

Problem 6:

$C_{in1} = \frac{8+3}{4} C_{ref} = \frac{11}{4} C_{ref}, A_1 = 8(3W_{min}L_{min} + 8W_{min}L_{min}) = 3.52\ \mu^2$

$C_{in2} = \frac{4+3}{4} C_{ref} = \frac{7}{4} C_{ref},$

$A_2 = 8(3W_{min}L_{min} + 4W_{min}L_{min}) + 2(6W_{min}L_{min} + W_{min}L_{min}) + 4W_{min}L_{min} = 2.96\ \mu^2$

$C_{in3} = \frac{2+3}{4} C_{ref} = \frac{5}{4} C_{ref},$

$$A_3 = 8(3W_{min}L_{min} + 2W_{min}L_{min}) + 4(12W_{min}L_{min} + W_{min}L_{min}) + 4W_{min}L_{min} = 3.84\ \mu^2$$

$C_{in4} = \frac{2+3}{4} C_{ref} = \frac{5}{4} C_{ref},$

$$A_3 = 8(3W_{min}L_{min} + 2W_{min}L_{min}) + 4(6W_{min}L_{min} + W_{min}L_{min}) + 2(3W_{min}L_{min} + 2W_{min}L_{min} = 3.12\ \mu^2$$

Problem 7:

$$t_{prop1} = \frac{800}{4} t_{ref} = 200\ t_{ref}$$

$$t_{prop2} = \frac{8}{1} t_{ref} + \frac{64}{8} t_{ref} + \frac{800}{64 * 4} t_{ref} = 19.125\ t_{ref}$$

$$t_{prop1} = t_{ref} + 64 t_{ref} + \frac{800}{64 * 4} t_{ref} = 68.125\ t_{ref}$$

Problem 8:

a) $t_{propDtoH} = \frac{5}{5} t_{ref} + \frac{1.5+1+14}{5} t_{ref} + \frac{500}{4*1.5} t_{ref} = 87.63\ t_{ref} = 1.75\ nsec$

b) $t_{propDtoH} = \frac{2.5}{1.25} t_{ref} + \frac{1.5+1+1}{2.5} t_{ref} + \frac{500}{0.8*1.5} t_{ref} = 420.07 t_{ref} = 8.4\ nsec$

Problem 9:

a) For 3-input NAND with overdrive of 4:

$W_n = 3 * 4 * W_{min} = 2.4\ \mu m$, $L_n = L_{min} = 0.2\ \mu m$

$W_p = 3 * 4 * W_{min} = 2.4\ \mu m$, $L_p = L_{min} = 0.2\ \mu m$

For 2-input NOR with overdrive of 6:

$W_n = 1 * 6 * W_{min} = 1.2\ \mu m$, $L_n = L_{min} = 0.2\ \mu m$

$W_p = 6 * 6 * W_{min} = 7.2\ \mu m$, $L_p = L_{min} = 0.2\ \mu m$

b)

$$t_{propBtoG} = \frac{(10.5 + 2 + 4)}{6} t_{ref} + \frac{4}{10.5} t_{ref} = 3.13\ t_{ref}$$

$$t_{propBtoGinvertor} = \frac{(10.5 + 2 + 4)}{1} t_{ref} + \frac{4}{10.5} t_{ref} = 16.58\ t_{ref}$$

The 3-input NAND with overdrive of 4 is $\frac{16.58}{3.13} =$ 5.3 times faster than the minimum-sized reference invertor.

Problem 10:

Total effort of inverter chain is F = 500p/4f = 125000 C_{ref}

Optimum overdrive ratio per stage is e

$p = F^{1/N} \rightarrow e = 125000^{1/N} \rightarrow N \approx 11.7\ stages$ but pick 12 as it's the closest even integer

Actual overdrive ratio of each stage is $\sqrt[12]{125000} = 2.66$