

EE 330

Lecture 13

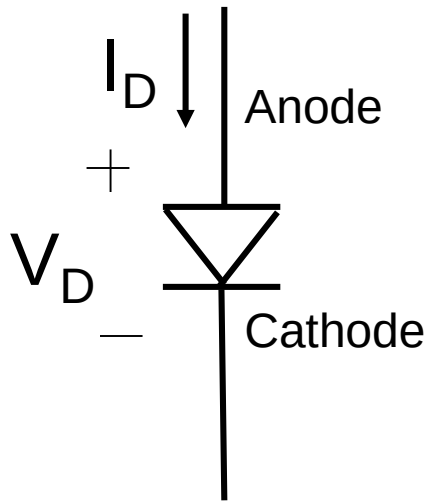
Devices in Semiconductor Processes

- Diodes
- Capacitors
- MOSFETs

pn Junctions

- As forward bias increases, depletion region thins and current starts to flow
- Current grows very rapidly as forward bias increases

Simple Diode Model:

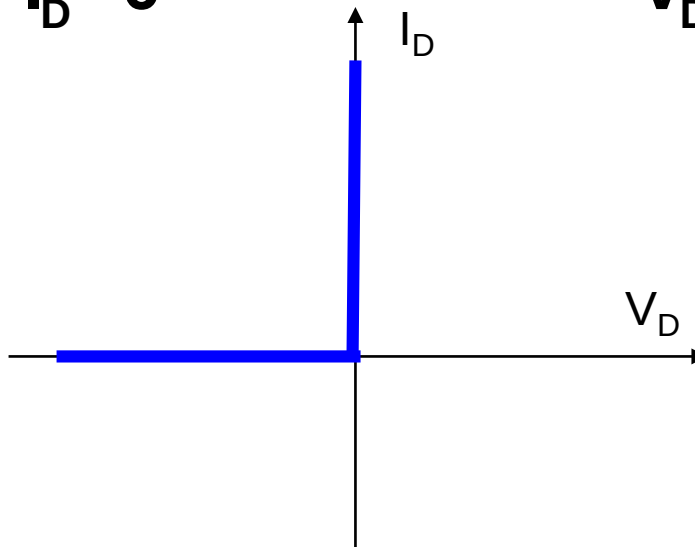


$$V_D = 0$$

$$I_D = 0$$

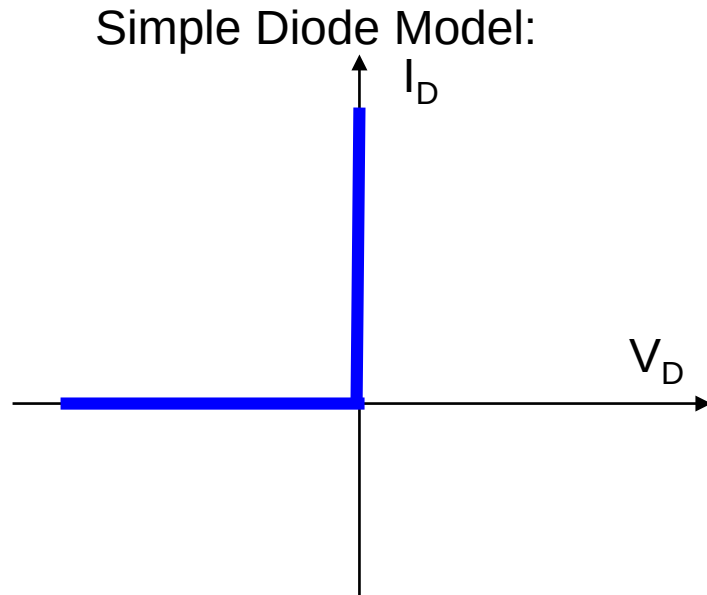
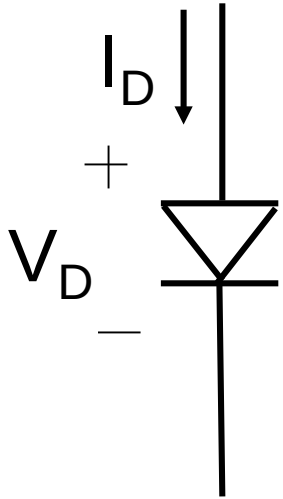
$$I_D > 0$$

$$V_D < 0$$



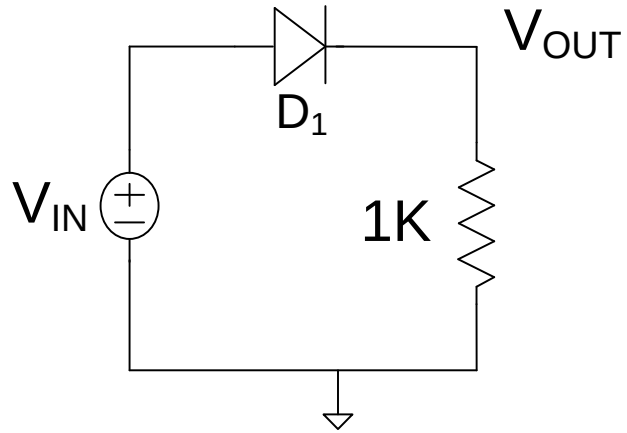
Simple model often referred to as the “Ideal” diode model

pn Junctions

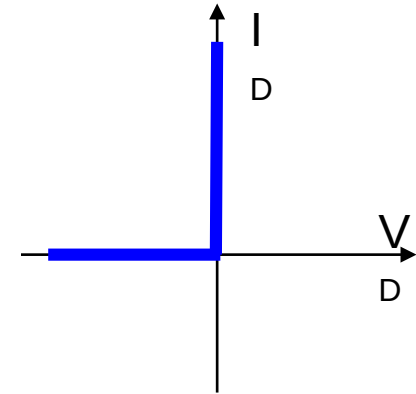


pn junction serves as a rectifier passing current in one direction and blocking it in the other direction

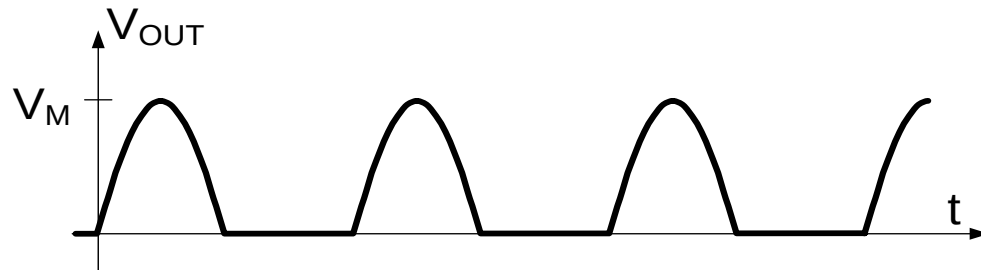
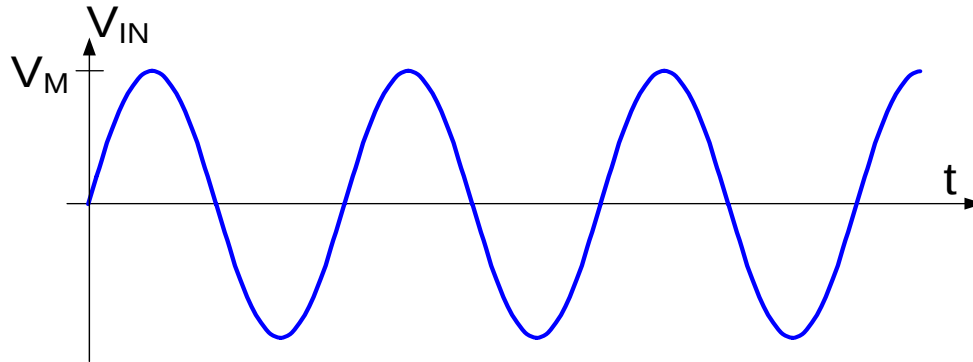
Rectifier Application:



Simple Diode Model:



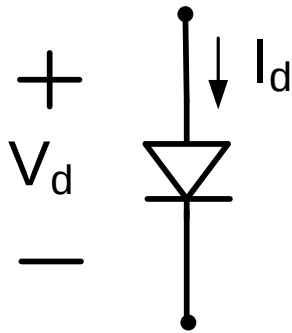
$$V_{IN} = V_M \sin \omega t$$



I-V characteristics of pn junction

(signal or rectifier diode)

Improved Diode Model:



I_S in the 10fA to 100fA range

I_S proportional to junction area

$$V_t = \frac{kT}{q}$$

$$k = 1.380\,64852 \times 10^{-23} \text{ J K}^{-1}$$

$$q = -1.60217662 \times 10^{-19} \text{ C}$$

$$k/q = 8.62 \times 10^{-5} \text{ V K}^{-1}$$

Diode Equation

$$I_D = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

What is V_t at room temp?

V_t is about 26mV at room temp

Diode equation due to William Shockley, inventor of BJT

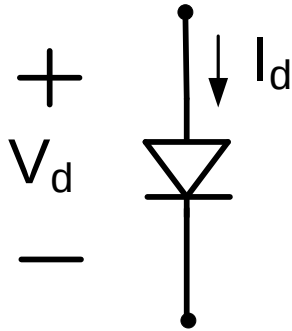
In 1919, [William Henry Eccles](#) coined the term **diode**

In 1940, Russell Ohl “stumbled upon” the p-n junction diode

I-V characteristics of pn junction

(signal or rectifier diode)

Improved Diode Model:

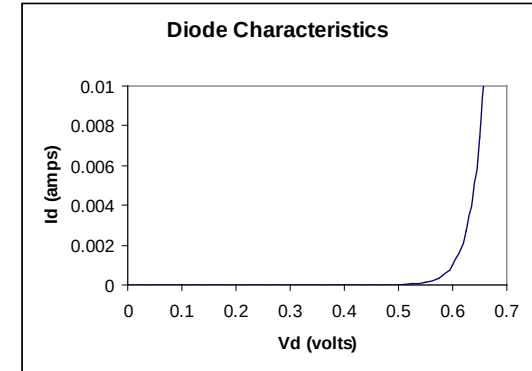


Diode Equation $I_D = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$

Simplification of Diode Equation:

Under reverse bias ($V_d < 0$), $I_D \cong -I_S$

Under forward bias ($V_d > 0$), $I_D = I_S e^{\frac{V_d}{V_t}}$



I_S in 10fA -100fA range (for signal diodes)

$$V_t = \frac{kT}{q}$$

$$k = 1.380\,6504(24) \times 10^{-23} \text{ JK}^{-1}$$

$$q = -1.602176487(40) \times 10^{-19} \text{ C}$$

$$k/q = 8.62 \times 10^{-5} \text{ VK}^{-1}$$

V_t is about 26mV at room temp

Simplification essentially identical model except for V_d very close to 0

Diode Equation or forward bias simplification is unwieldy to work with analytically

I-V characteristics of pn junction

(signal or rectifier diode)

Improved Diode Model:

Diode Equation

$$I_D = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

I_S often in the 10fA to 100fA range
 I_S proportional to junction area

V_t is about 26mV at room temp

Simplification of Diode Equation:

Under reverse bias, $I_D \cong -I_S$

Under forward bias, $I_D = I_S e^{\frac{V_d}{V_t}}$

How much error is introduced using the simplification for $V_d > 0.5V$?

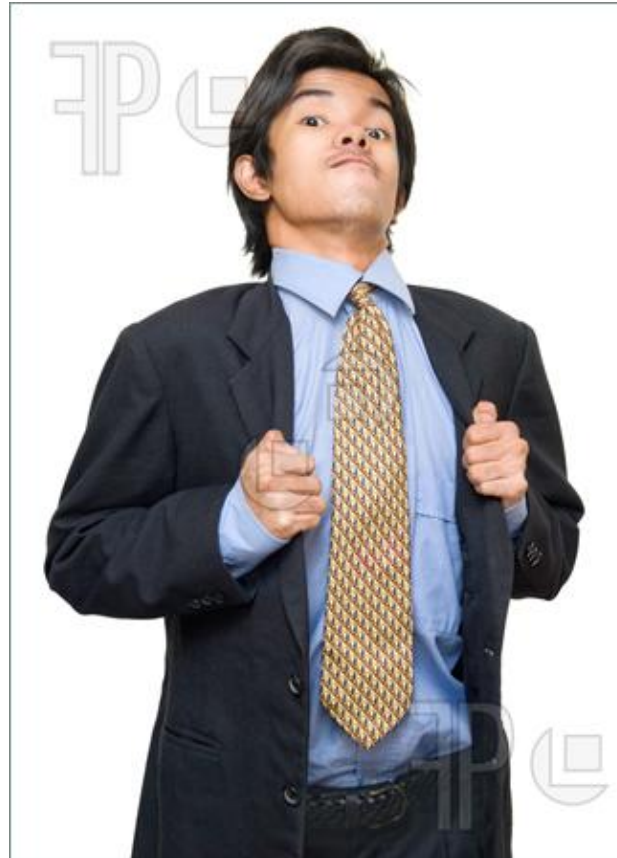
$$\varepsilon = \frac{I_S \left(e^{\frac{V_d}{V_t}} - 1 \right) - I_S e^{\frac{V_d}{V_t}}}{I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)} \quad \varepsilon < \frac{1}{e^{\frac{0.5}{0.026}}} = 4.4 \bullet 10^{-9}$$

How much error is introduced using the simplification for $V_d < -0.5V$?

$$\varepsilon < e^{\frac{-0.5}{0.026}} = 4.4 \bullet 10^{-9}$$

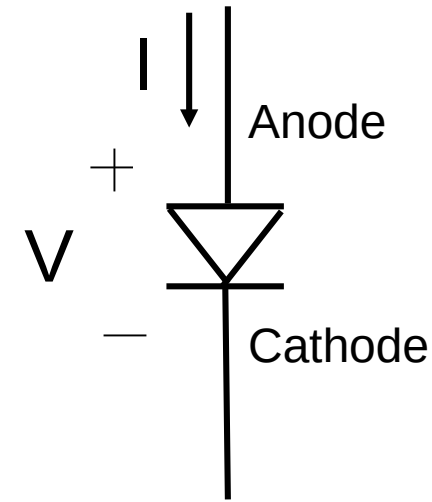
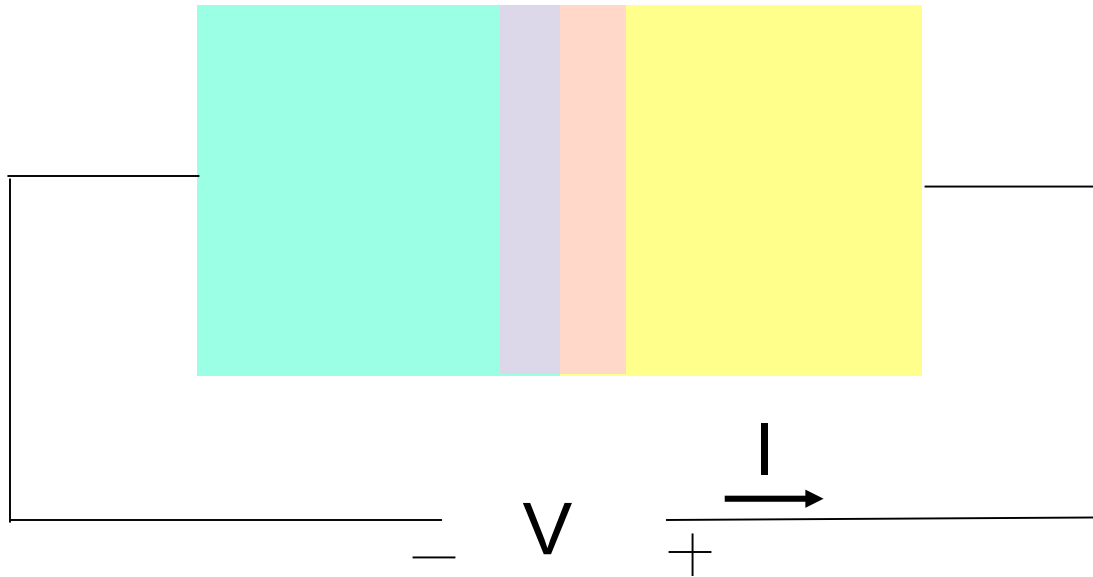
Simplification almost never introduces any significant error

Will you impress your colleagues or your boss if you use the more exact diode equation when $V_d < -0.5V$ or $V_d > +0.5V$?



Will your colleagues or your boss be unimpressed if you use the more exact diode equation when $V_d < -0.5V$ or $V_d > +0.5V$?

pn Junctions



Diode Equation:
(good enough for most applications)

$$I = \begin{cases} J_s A e^{\frac{V}{nV_T}} & V > 0 \\ 0 & V < 0 \end{cases}$$

Note: $I_s = J_s A$

J_s = Sat Current Density (in the $1\text{aA}/\text{u}^2$ to $1\text{fA}/\text{u}^2$ range)

A = Junction Cross Section Area

$V_T = kT/q$ ($k/q = 1.381 \times 10^{-23} \text{V} \cdot \text{C} / ^\circ\text{K} / 1.6 \times 10^{-19} \text{C} = 8.62 \times 10^{-5} \text{V} / ^\circ\text{K}$)

n is approximately 1

pn Junctions

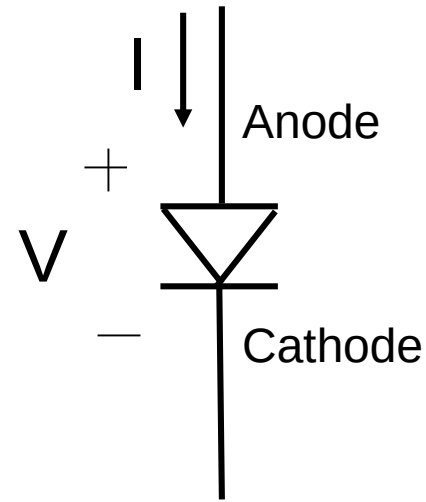
Diode Equation:
$$I = \begin{cases} J_s A e^{\frac{V}{nV_T}} & V > 0 \\ 0 & V < 0 \end{cases}$$

J_s is strongly temperature dependent

With $n=1$, for $V>0$,

$$I(T) = \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_D}{V_t}}$$

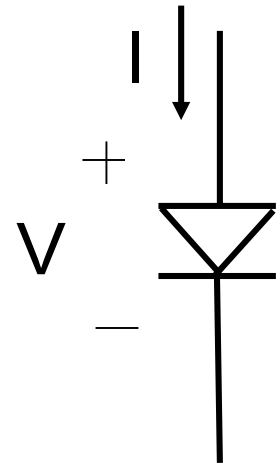
Typical values for key parameters: $J_{SX}=0.5\text{A}/\mu^2$, $V_{G0}=1.17\text{V}$, $m=2.3$



pn Junctions

Example:

$$I(T) = \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_D}{V_t}}$$



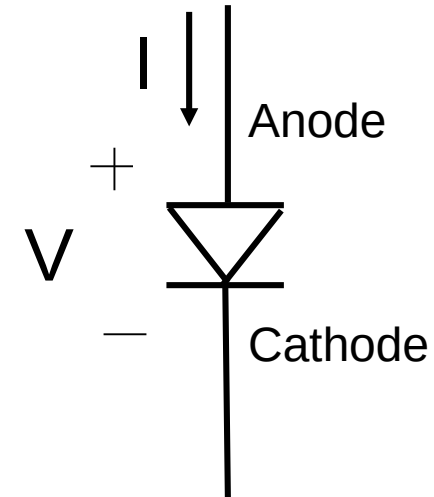
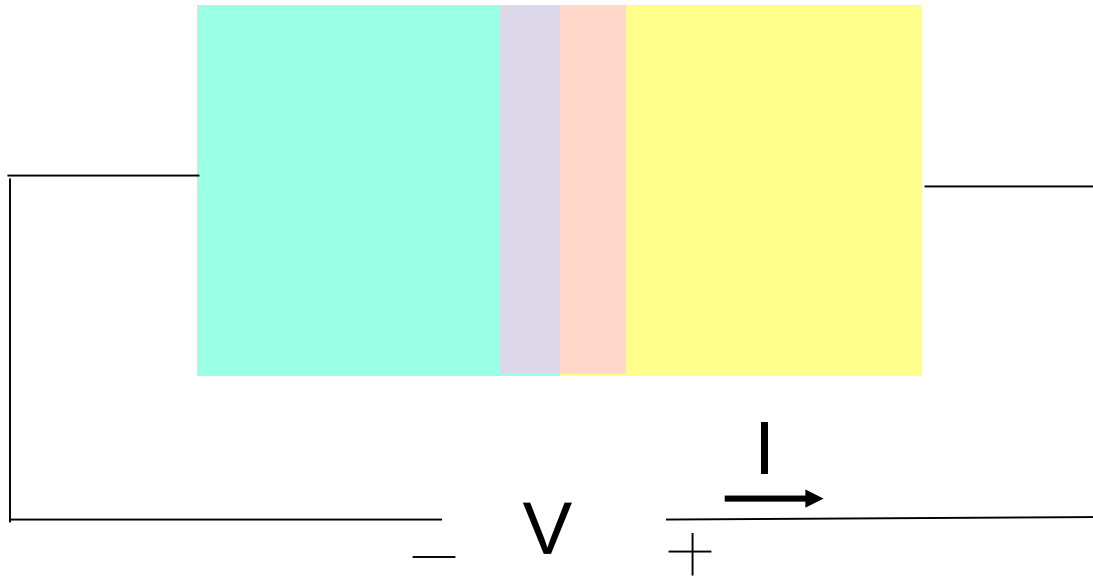
What percent change in I_S will occur for a 1°C change in temperature at room temperature?

$$\frac{\Delta I_S}{I_S} = \frac{\left(J_{SX} \left[T_2^m e^{\frac{-V_{G0}}{V_t(T_2)}} \right] \right) - \left(J_{SX} \left[T_1^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right)}{\left(J_{SX} \left[T_1^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right)} = \frac{\left(\left[T_2^m e^{\frac{-V_{G0}}{V_t(T_2)}} \right] \right) - \left(\left[T_1^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right)}{\left(\left[T_1^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right)}$$

$$\frac{\Delta I_S}{I_S} = \frac{(1.240 \times 10^{-15}) - (1.025 \times 10^{-15})}{(1.025 \times 10^{-15})} 100\% = 21\%$$

- Attempts to measure I_S in out laboratories can result in large errors !
- Most circuits whose performance depends upon precise value for I_S are not practical

pn Junctions

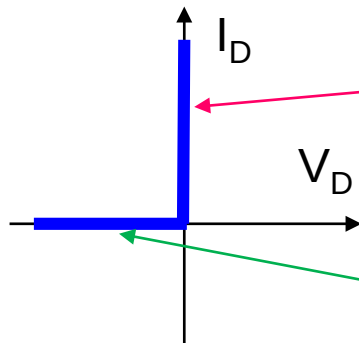


Diode Equation:
(good enough for most applications)

$$I = \begin{cases} J_S A e^{\frac{V}{nV_T}} & V > 0 \\ 0 & V < 0 \end{cases}$$

$$I_S = J_S A$$

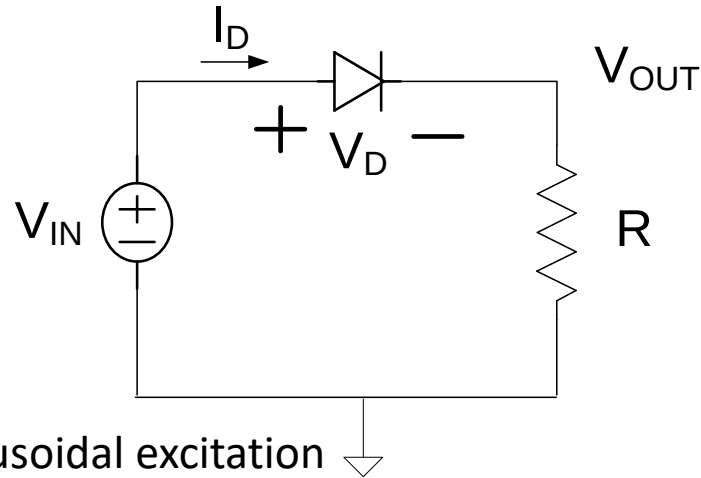
Simple Diode Model:



Often termed the "conducting" or "ON" state

Often termed the "nonconducting" or "OFF" state

Consider again the basic rectifier circuit



- Previously considered sinusoidal excitation
- Previously gave “qualitative” analysis
- **Rigorous analysis method is essential**

$$V_{OUT} = ?$$

Analysis of Nonlinear Circuits

(Circuits with one or more nonlinear devices)

What analysis tools or methods can be used?

KCL ?

Nodal Analysis ?

KVL?

Mesh Analysis ?

~~Superposition?~~

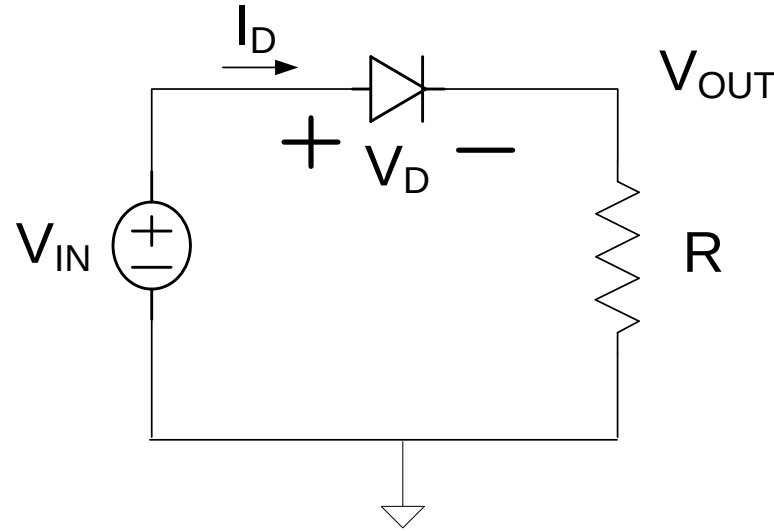
Two-Port Subcircuits ?

~~Voltage Divider ?~~

~~Current Divider?~~

~~Thevenin and Norton Equivalent Circuits?~~

Consider again the basic rectifier circuit



$$V_{IN} = V_D + I_D R$$

$$V_{OUT} = I_D R$$

$$I_D = I_S \left(e^{\frac{V_D}{V_t}} - 1 \right)$$

}

$$V_{OUT} = I_S R \left(e^{\frac{V_{IN} - V_{OUT}}{V_t}} - 1 \right)$$

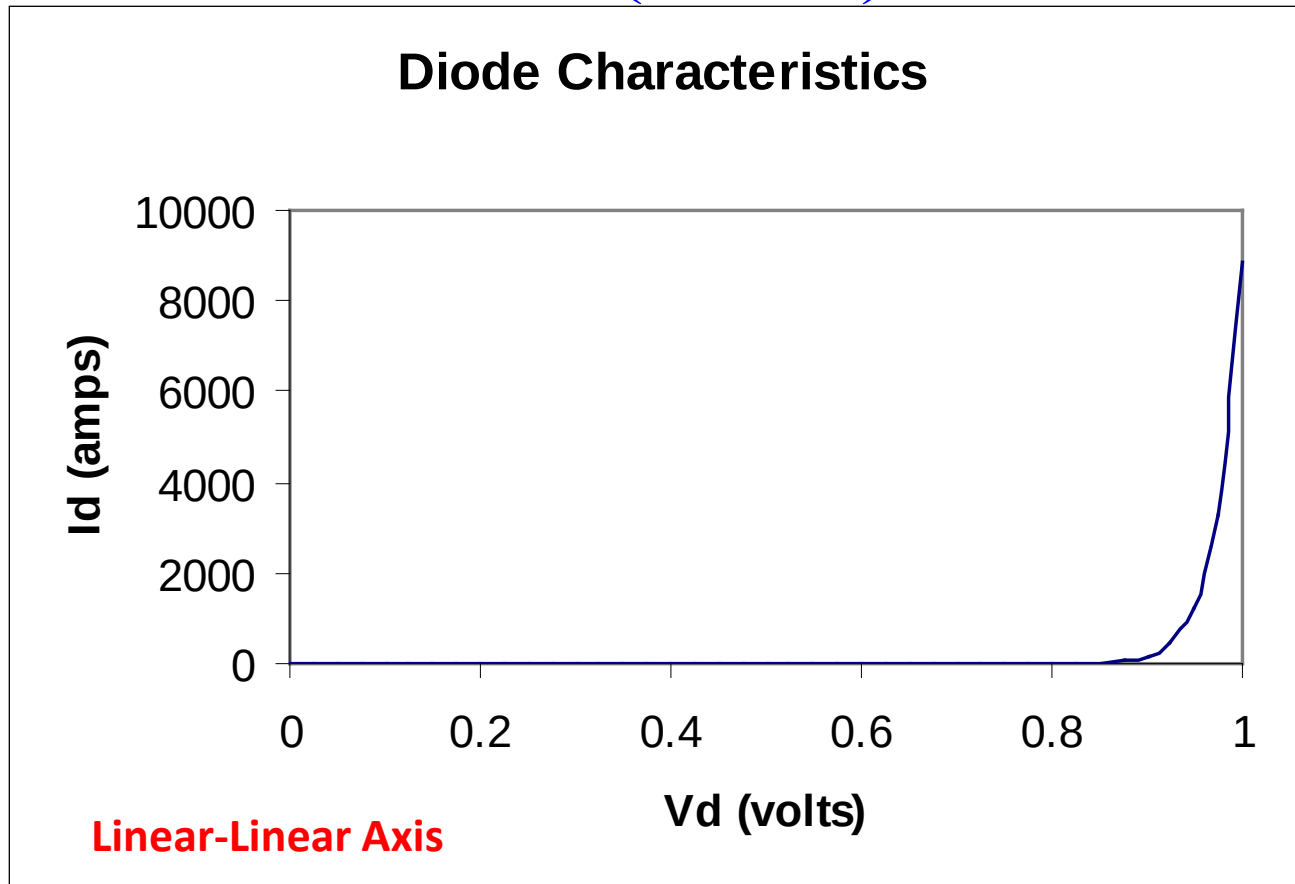
Even the simplest diode circuit does not have a closed-form solution when diode equation is used to model the diode !!

Due to the nonlinear nature of the diode equation

Simplifications are essential if analytical results are to be obtained

Lets study the diode equation a little further

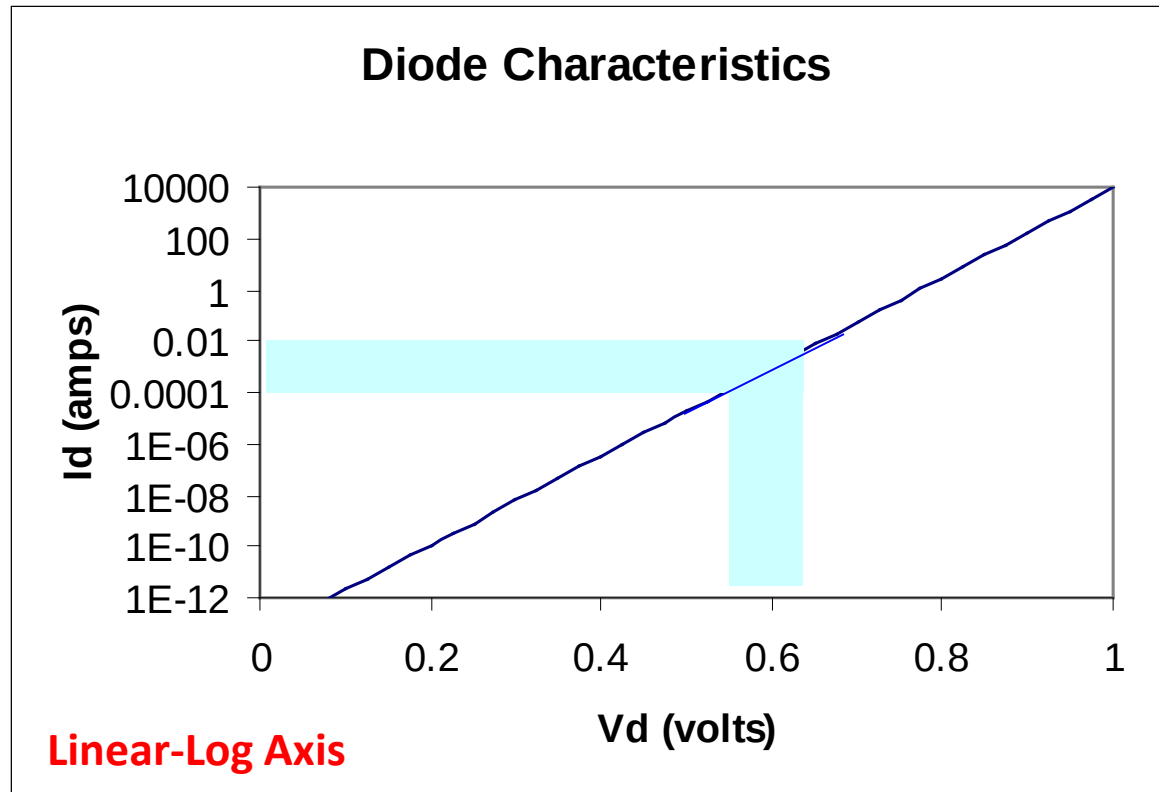
$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



Power Dissipation Becomes Destructive if $V_d > 0.85V$ (actually less)

Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

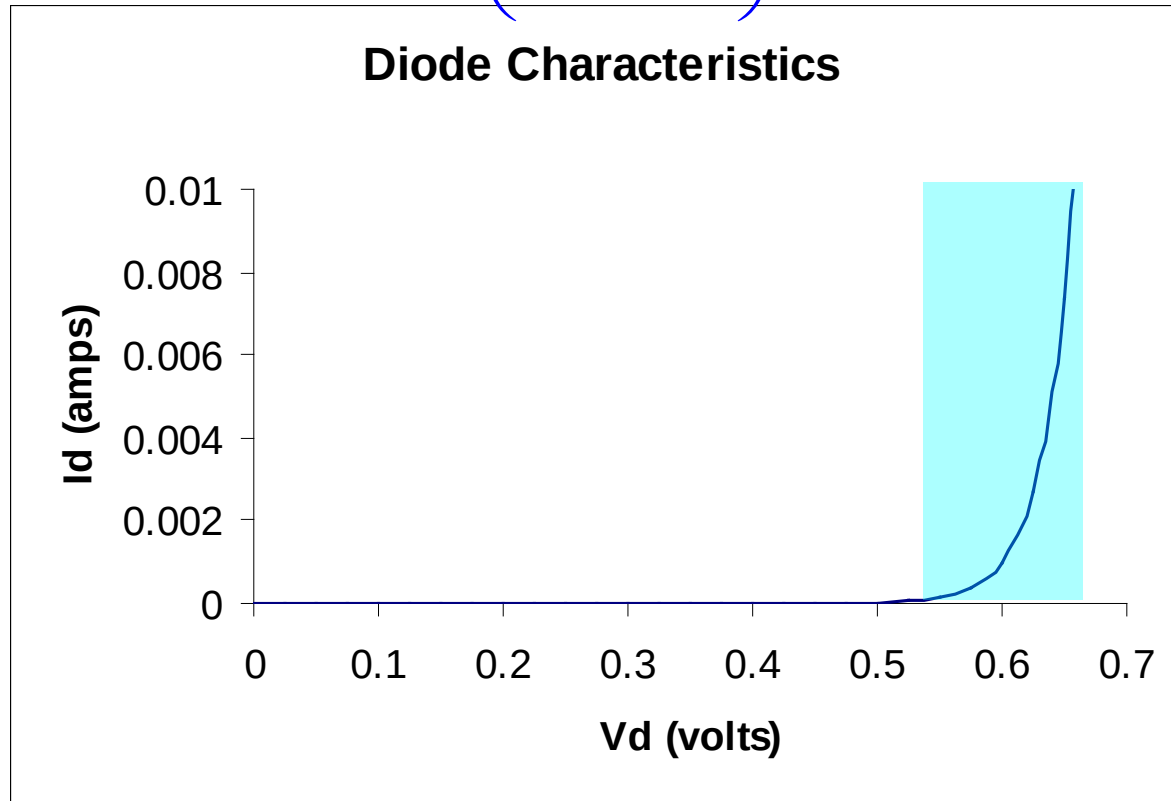


For two decades of current change, V_d is close to 0.6V

This is the most useful conducting current range for many applications

Lets study the diode equation a little further

$$I_d = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



For two decades of current change, V_d is close to 0.6V

This is the most useful current range when conducting for many applications

Lets study the diode equation a little further

$$I_d = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

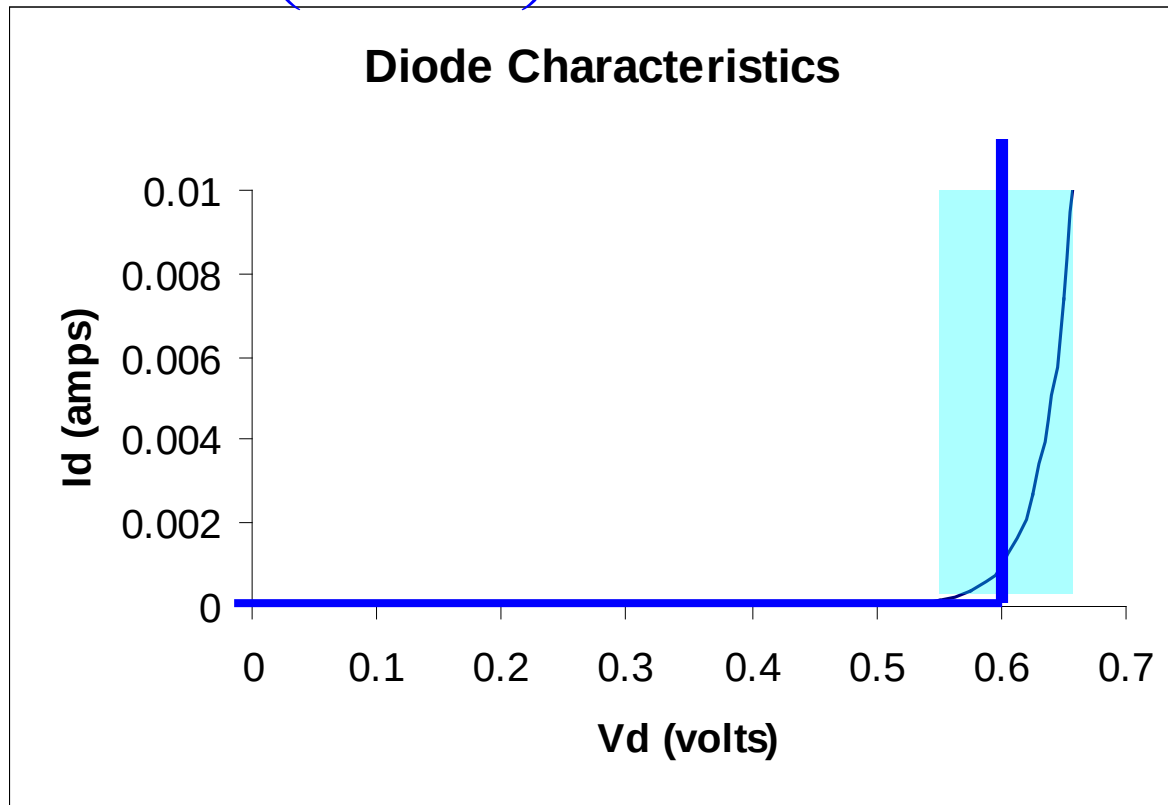


$$I_d = 0$$

$$V_d < 0.6V$$

$$V_d = 0.6V$$

$$I_d > 0$$

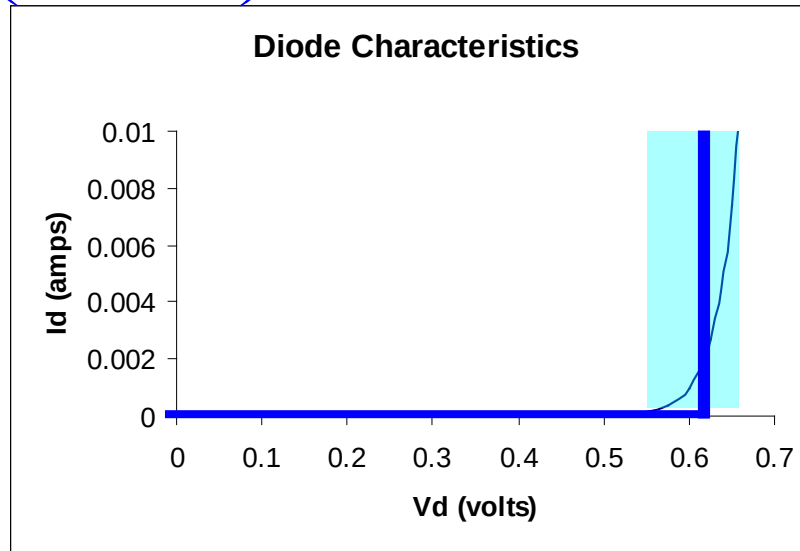


Widely Used Piecewise Linear Model

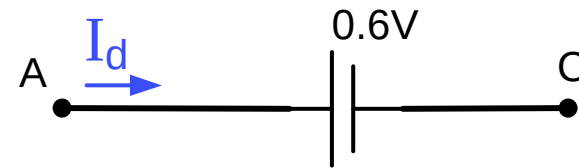
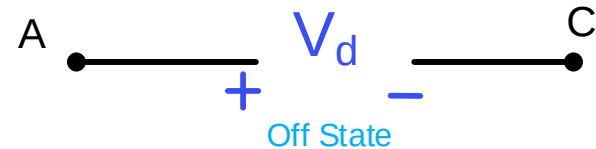
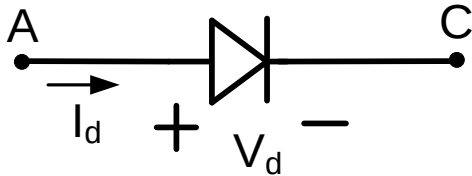
Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

$$\begin{aligned} I_d &= 0 & V_d < 0.6V \\ V_d &= 0.6V & I_d > 0 \end{aligned}$$

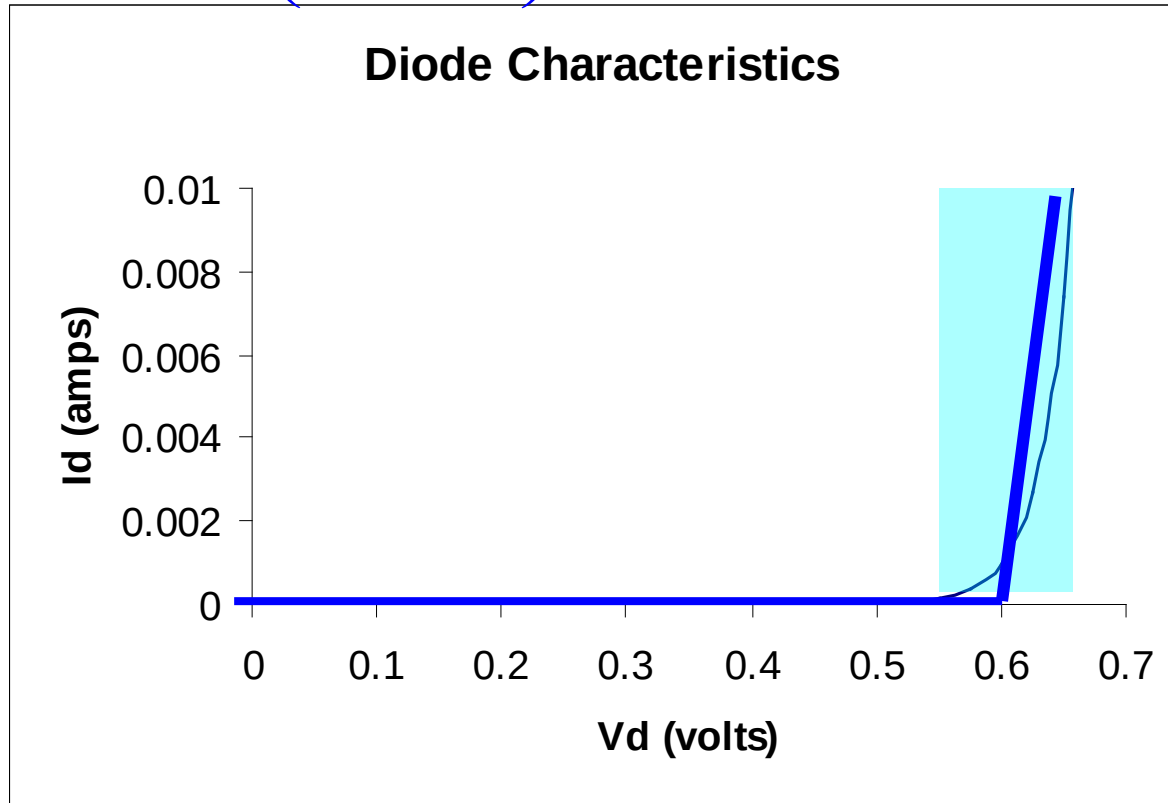


Equivalent Circuit



Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



Better model in “ON” state though often not needed

Includes Diode “ON” resistance

Lets study the diode equation a little further

$$I_d = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

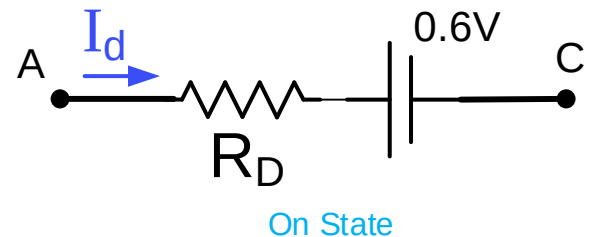
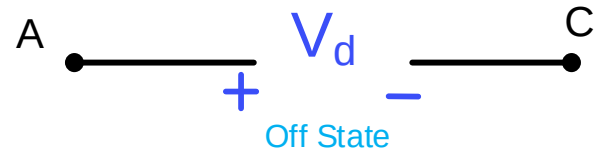
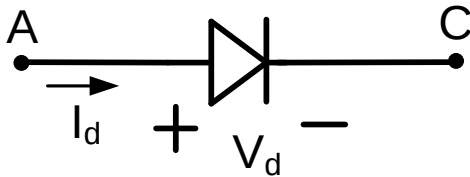
Piecewise Linear Model with Diode Resistance

$$I_d = 0 \quad \text{if } V_d < 0.6V$$

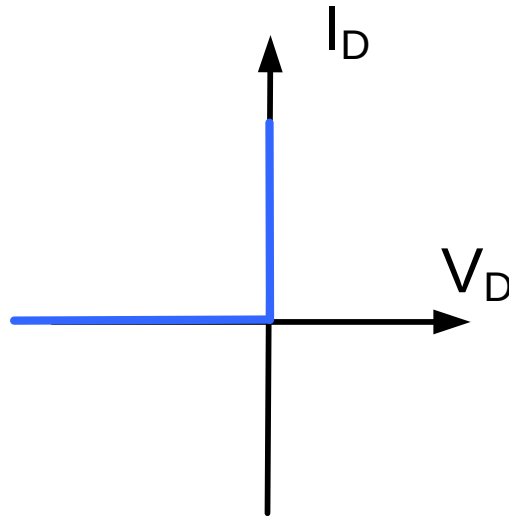
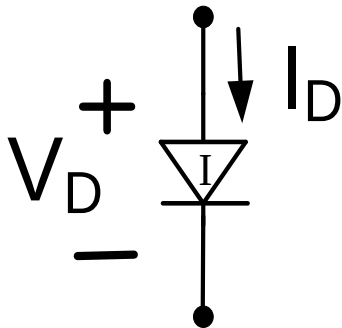
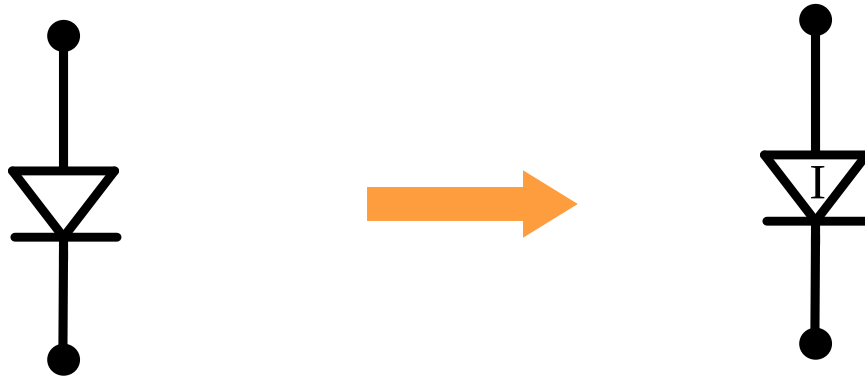
$$V_d = 0.6V + I_d R_D \quad \text{if } I_d > 0$$

(R_D is rather small: often in the 20Ω to 100Ω range):

Equivalent Circuit

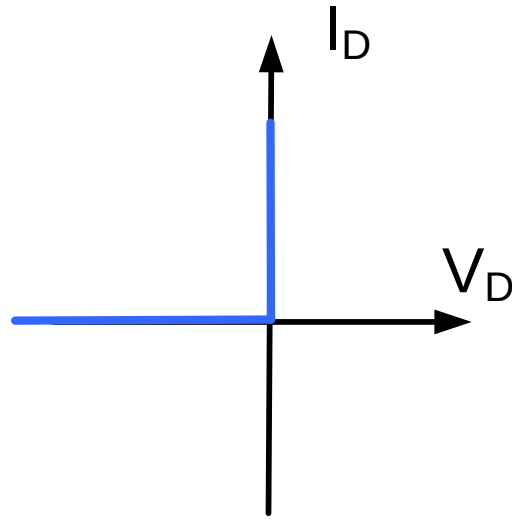


The Ideal Diode



$$\begin{aligned} I_D &= 0 & \text{if } V_D \leq 0 \\ V_D &= 0 & \text{if } I_D > 0 \end{aligned}$$

The Ideal Diode

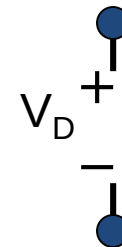
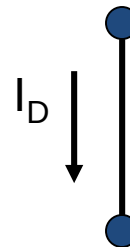


$$\begin{aligned} I_D &= 0 & \text{if } V_D &\leq 0 & \text{"OFF"} \\ V_D &= 0 & \text{if } I_D &> 0 & \text{"ON"} \end{aligned}$$



"ON"

"OFF"

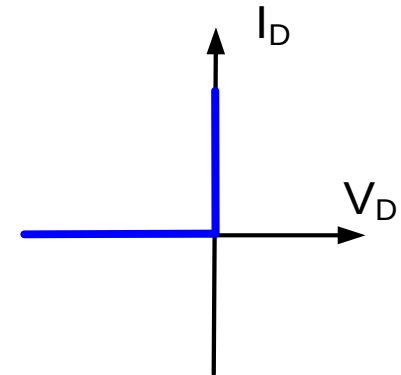
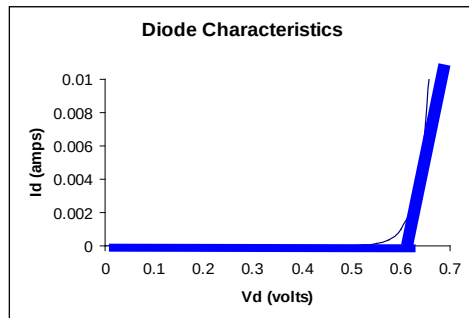
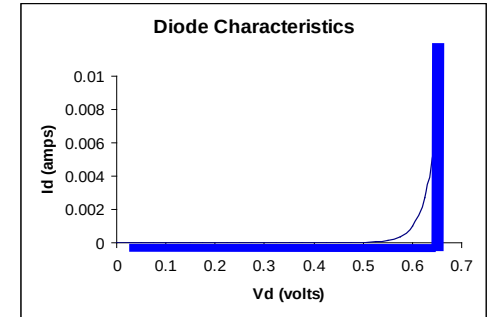
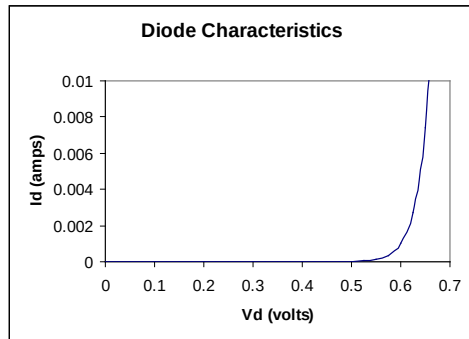


Valid for

$$I_D > 0$$

$$V_D \leq 0$$

Diode Models



Which model should be used?

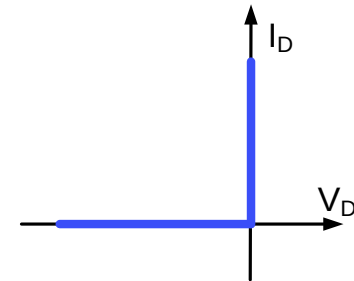
The simplest model that will give acceptable results in the analysis of a circuit

Diode Model Summary

Piecewise Linear Models

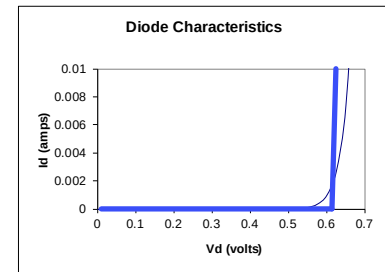
$$I_d = 0 \quad \text{if } V_d < 0$$

$$V_d = 0 \quad \text{if } I_d > 0$$



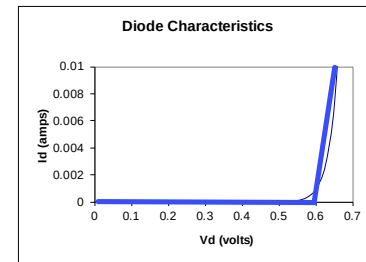
$$I_d = 0 \quad \text{if } V_d < 0.6V$$

$$V_d = 0.6V \quad \text{if } I_d > 0$$



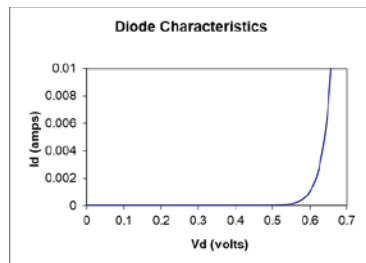
$$I_d = 0 \quad \text{if } V_d < 0.6$$

$$V_d = 0.6 + I_d R_d \quad \text{if } I_d > 0$$



Diode Equation

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



Diode Model Summary

Piecewise Linear Models

$$I_d = 0 \quad \text{if } V_d < 0$$

$$V_d = 0 \quad \text{if } I_d > 0$$

$$I_d = 0 \quad \text{if } V_d < 0.6V$$

$$V_d = 0.6V \quad \text{if } I_d > 0$$

$$I_d = 0 \quad \text{if } V_d < 0.6$$

$$V_d = 0.6 + I_d R_d \quad \text{if } I_d > 0$$

Diode Equation

$$I_d = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

When is the ideal model adequate?

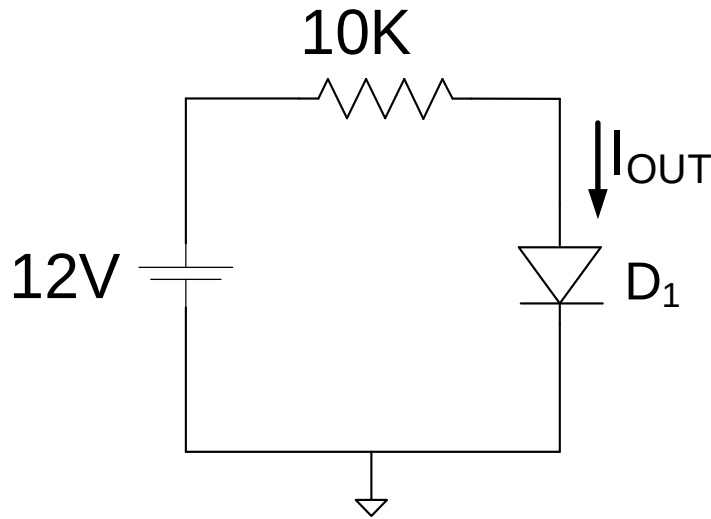
When it doesn't make much difference whether $V_d = 0V$ or $V_d = 0.6V$

When is the second piecewise-linear model adequate?

When it doesn't make much difference whether $V_d = 0.6V$ or $V_d = 0.7V$

Example:

Determine I_{OUT} for the following circuit



Solution:

Strategy:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)
3. Analyze circuit with model
4. Validate state of guess in step 2 (verify the “if” condition in model)

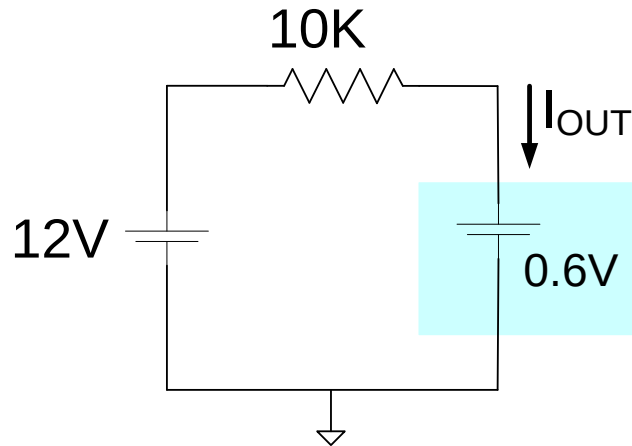
Select
Model

5. Assume PWL with $V_D=0.7V$
6. Guess state of diode (ON)
7. Analyze circuit with model
8. Validate state of guess in step 6 (verify the “if” condition in model)
9. Show difference between results using these two models is small
10. If difference is not small, must use a different model

Validate
Model

Solution:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)



3. Analyze circuit with model

$$I_{OUT} = \frac{12V - 0.6V}{10K} = 1.14mA$$

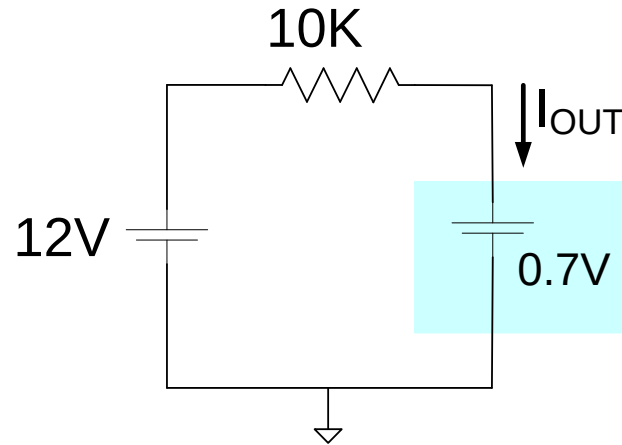
4. Validate state of guess in step 2

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 1.14mA > 0$$

Solution:

5. Assume PWL model with $V_D=0.7V$, $R_D=0$
6. Guess state of diode (ON)



7. Analyze circuit with model

$$I_{OUT} = \frac{12V - 0.7V}{10K} = 1.13mA$$

8. Validate state of guess in step 6

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 1.13mA > 0$$

Solution:

9. Show difference between results using these two models is small

$$I_{\text{OUT}} = 1.14 \text{mA} \text{ and } I_{\text{OUT}} = 1.13 \text{ mA} \quad \text{are close}$$

Thus, can conclude

$$I_{\text{OUT}} \cong 1.14 \text{mA}$$

End of Lecture 13