EE 330
Lecture 21

- Small Signal Modelling
- Operating Points for Amplifier Applications
- Amplification with Transistor Circuits

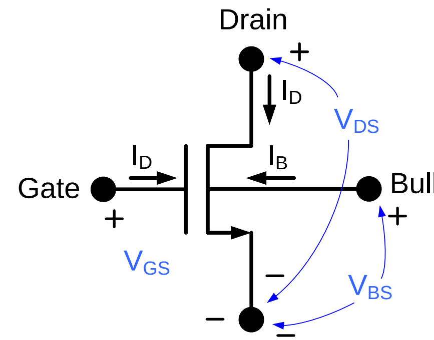
Small-Signal Models

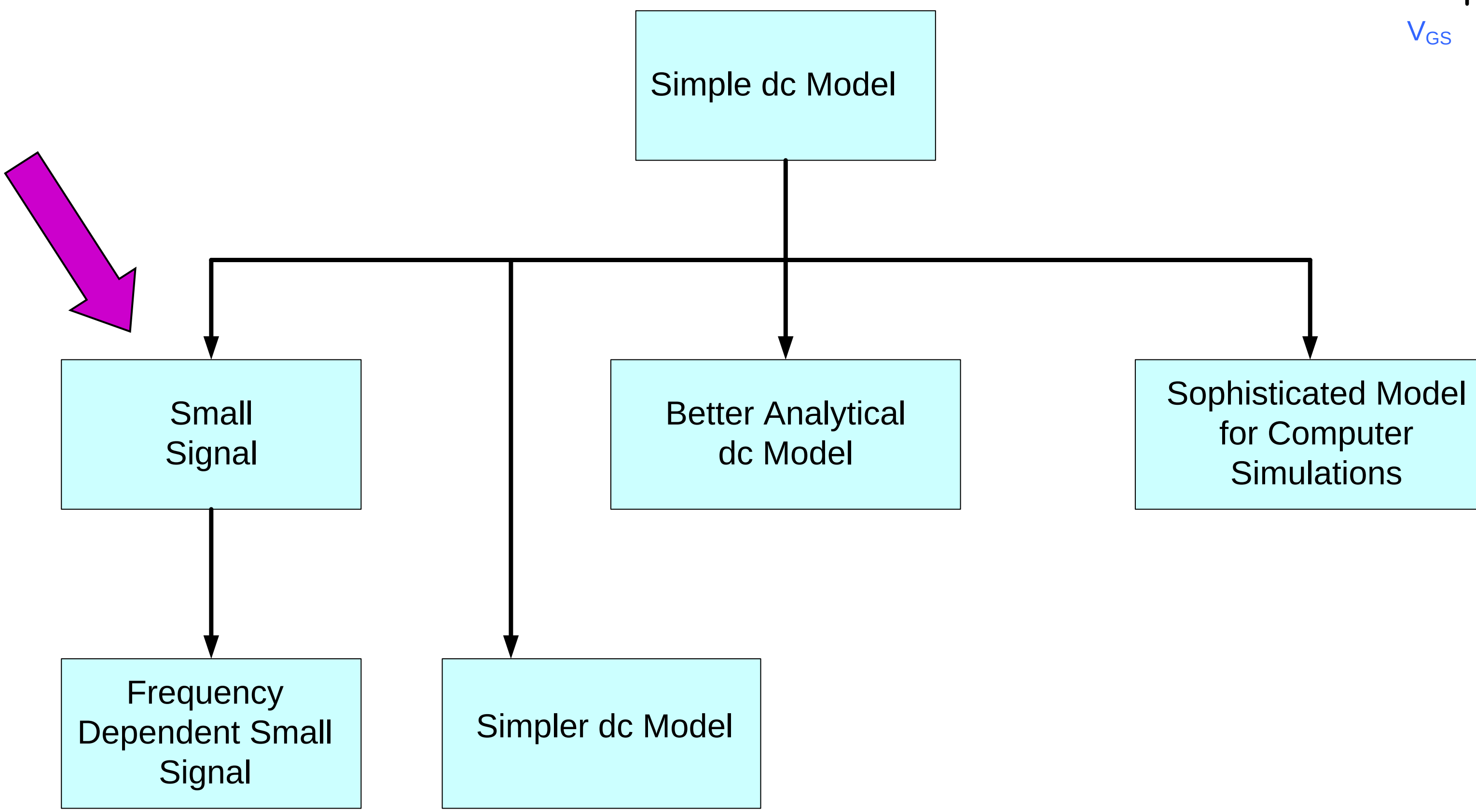
- MOSFET
- BJT
- Diode (of limited use)

Modeling of the MOSFET

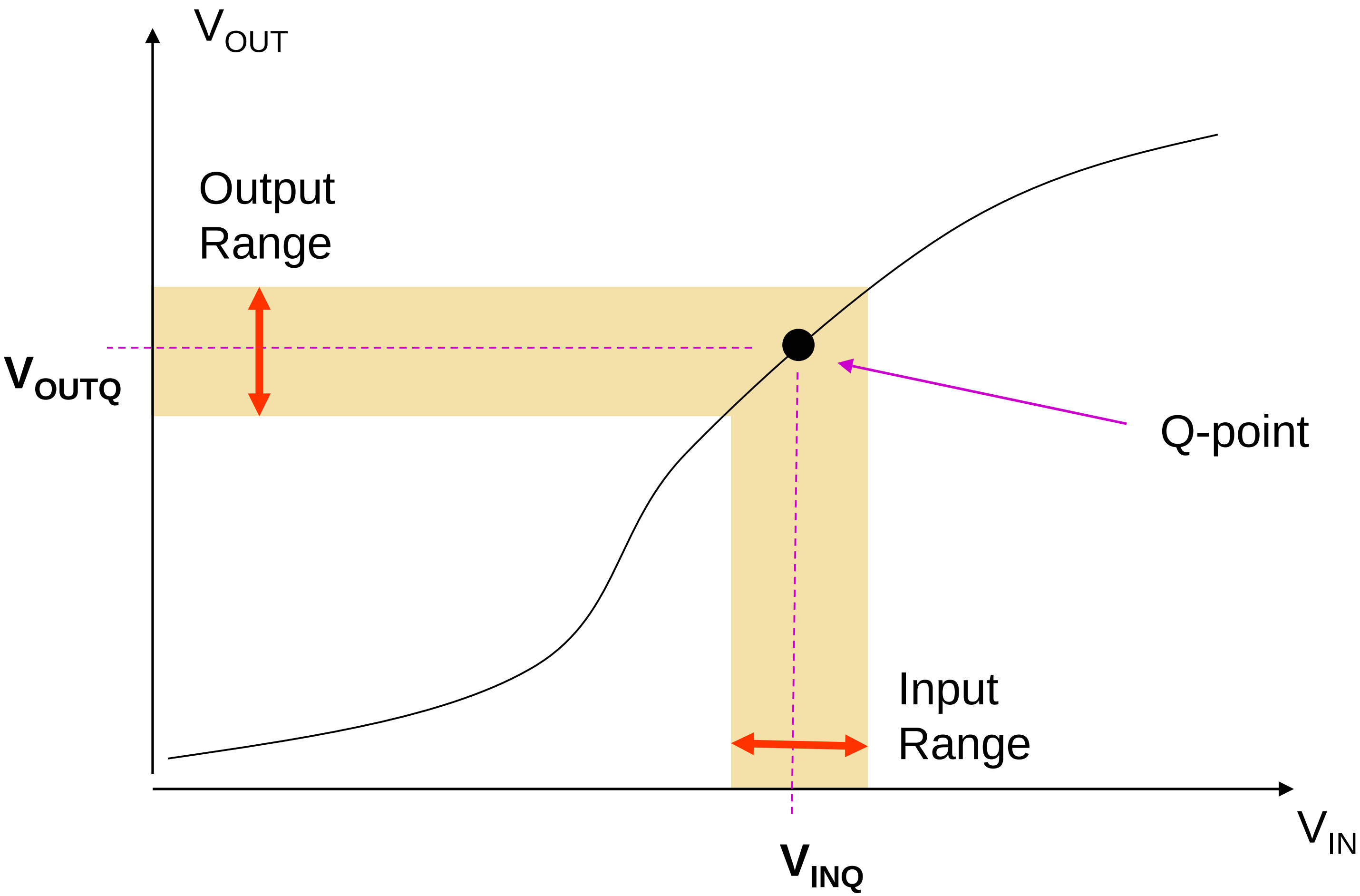
Goal: Obtain a mathematical relationship between the port variables of a device.

$$\left.\begin{aligned} I_D &= f_1(V_{GS}, V_{DS}, V_{BS}) \\ I_G &= f_2(V_{GS}, V_{DS}, V_{BS}) \\ I_B &= f_3(V_{GS}, V_{DS}, V_{BS}) \end{aligned}\right\}$$



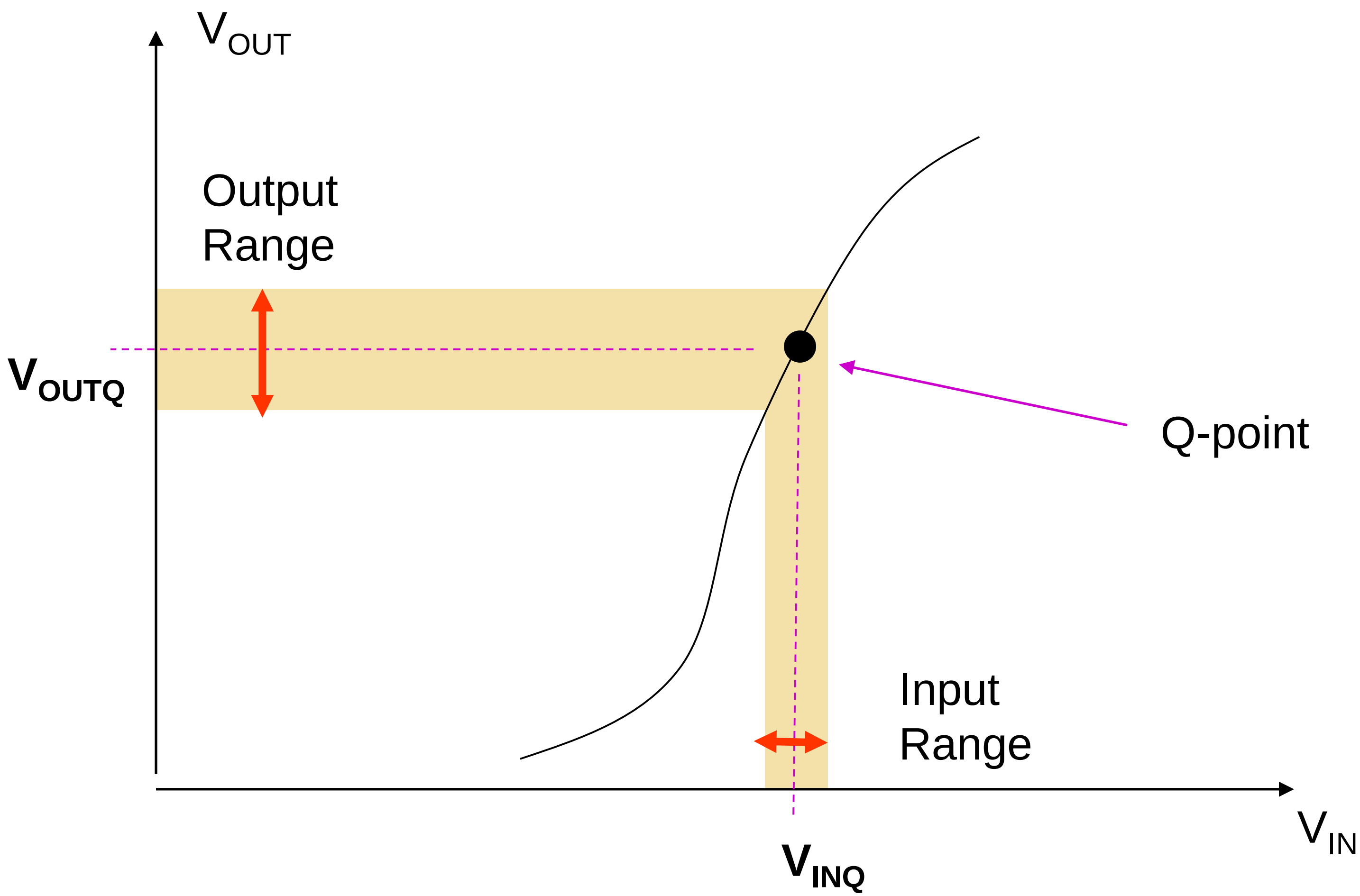


Small-Signal Operation



Throughout the small input range, the “distant” nonlinearities do not affect performance

Small-Signal Operation

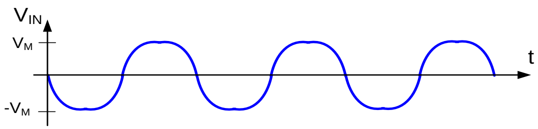


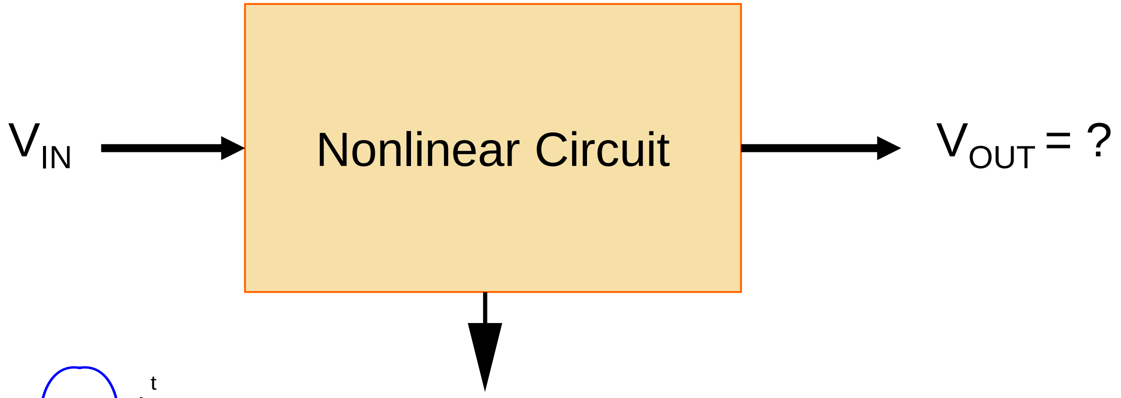
- If slope is steep, output range can be much larger than input range
- The slope can be viewed as the voltage gain of the circuit
- Nonlinear circuit behaves as a linear circuit near Q-point with small-signal inputs

Small signal operation of nonlinear circuits

$V_{IN}=V_M \sin\omega t$

V_M is small





- **Small signal concepts often apply when building amplifiers**
- **If small signal concepts do not apply, usually the amplifier will not perform well**
- **Small signal operation is usually synonymous with "locally linear"**
- **Small signal operation is relative to an "operating point"**

Operating Point of Electronic Circuits

Often interested in circuits where a small signal input is to be amplified

The electrical port variables where the small signal goes to 0 is termed the Operating Point, the Bias Point, the Quiescent Point, or simply the Q-Point

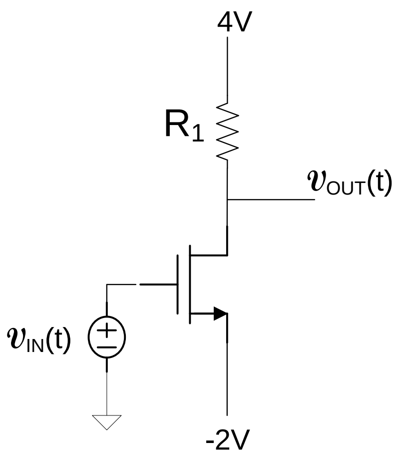
By setting the small signal to 0, it means replacing small voltage inputs with short circuits and small current inputs with open circuits

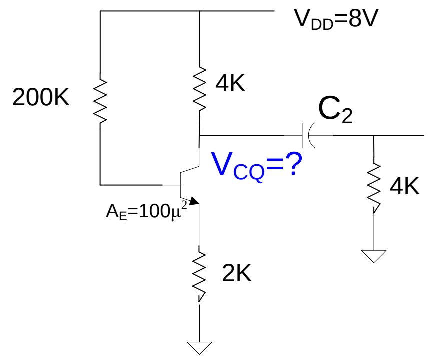
When analyzing small-signal amplifiers, it is necessary to obtain the Q-point

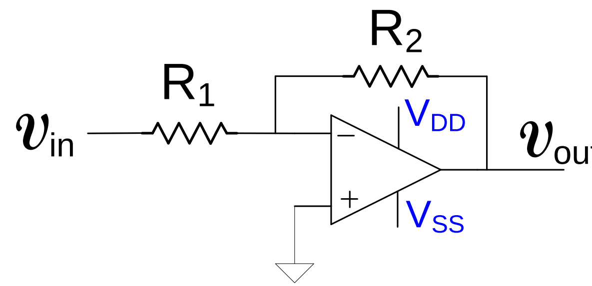
When designing small-signal amplifiers, establishing of the desired Q-point is termed "biasing"

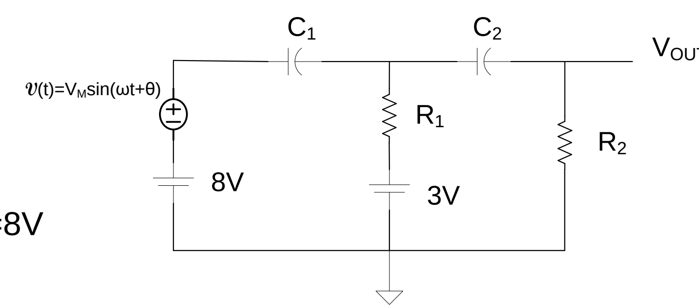
- Capacitors become open circuits (and inductors short circuits) when determining Q-points
- Simplified dc models of the MOSFET (saturation region) or BJT (forward active region) are usually adequate for determining the Q-point in practical amplifier circuits
- DC voltage and current sources remain when determining Q-points
- Small-signal voltage and current sources are set to 0 when determining Q-points

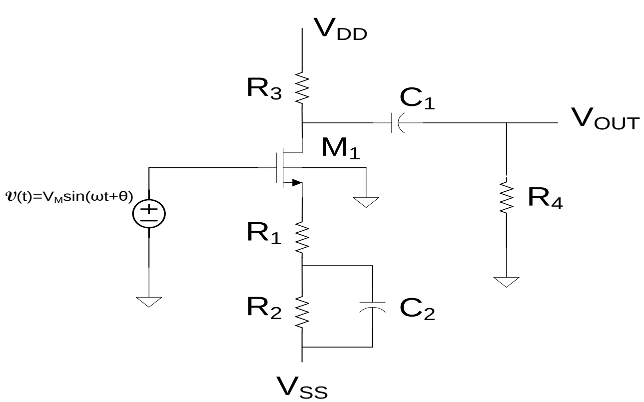
Operating Point of Electronic Circuits

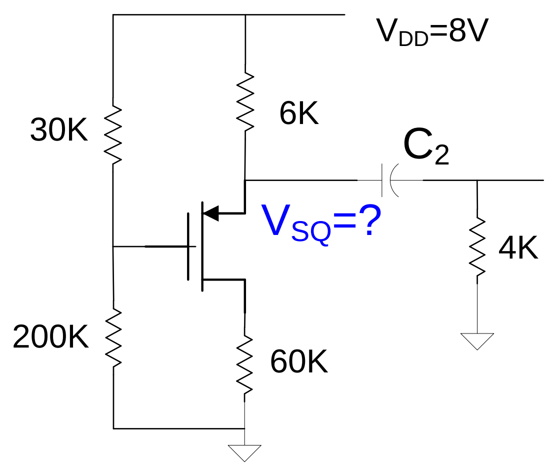








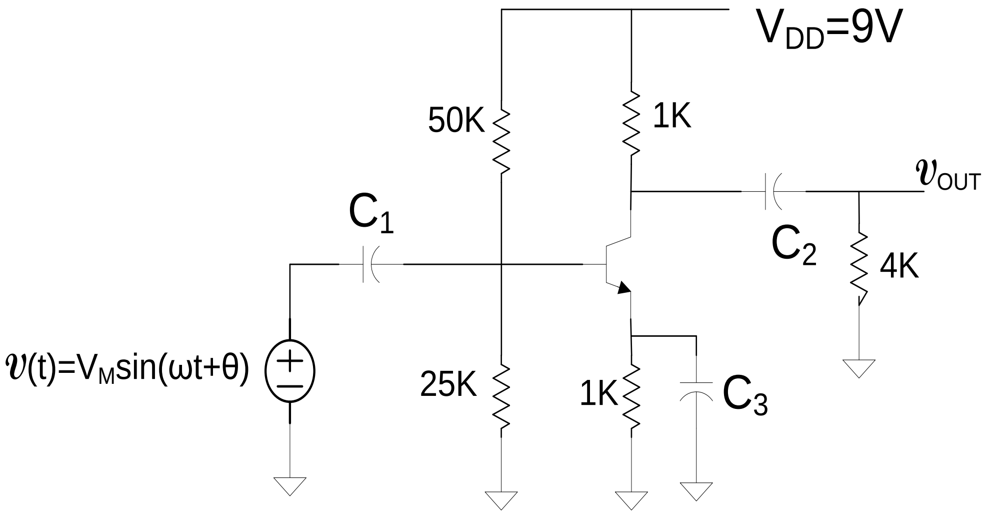




Operating Point Analysis of MOS and Bipolar Devices

Example:

Determine V_{OUTQ} and V_{CQ}

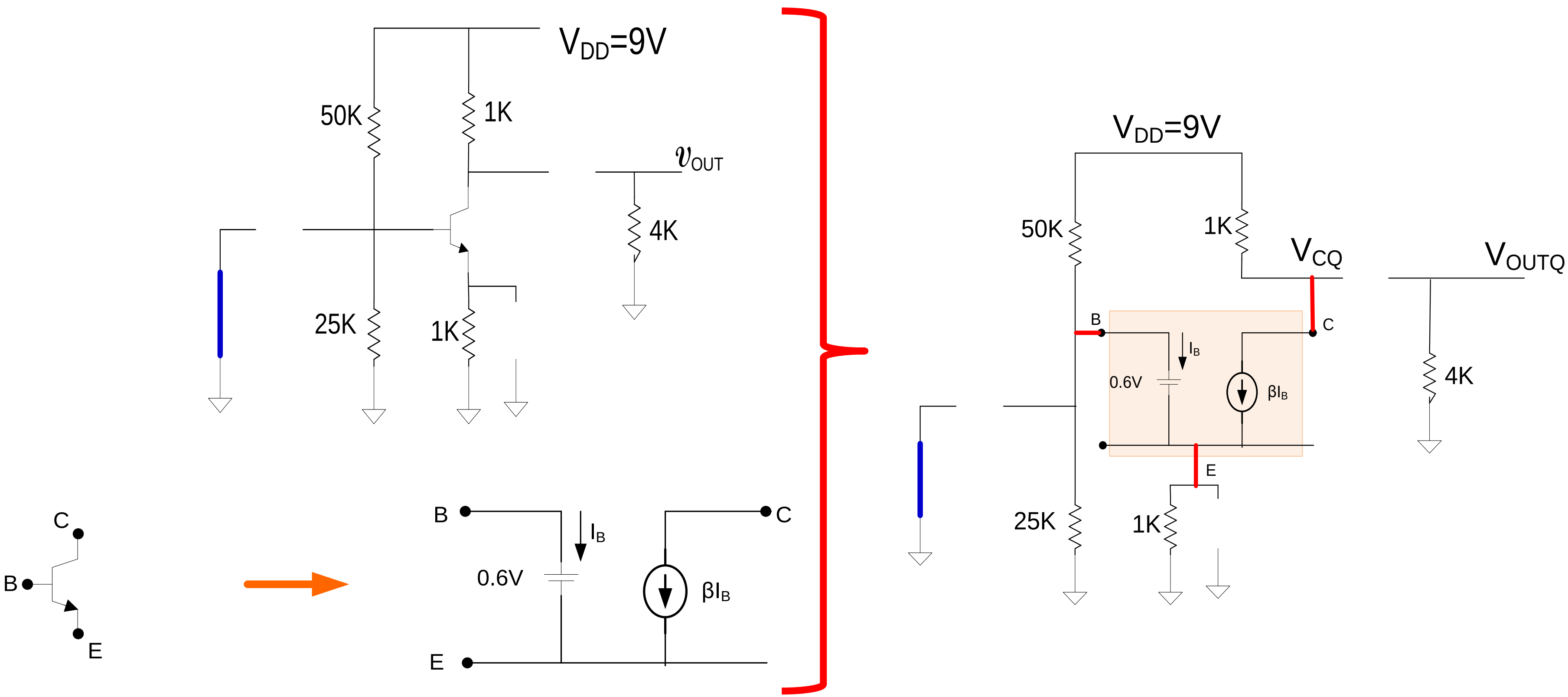


Will formally go through the process in this example, will go into more detail about finding the operating point later

Operating Point Analysis of MOS and Bipolar Devices

Example:

Determine V_{OUTQ} and V_{CQ}

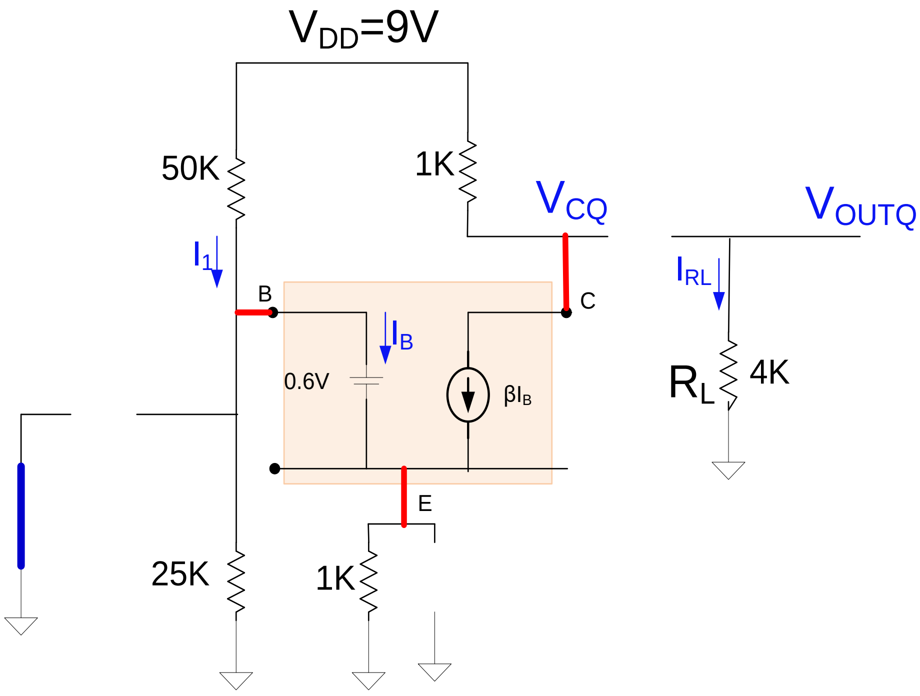


Operating Point Analysis of MOS and Bipolar Devices

Example:

Determine V_{OUTQ} and V_{CQ}

Assume β=100



Assume $I_B << I_1$ (must verify)

$$V_{BQ} = \frac{9V}{3} = 3V$$

$$V_{EQ} = 3V - 0.6V = 2.4V$$

$$I_{EQ} = I_{CQ} = \frac{2.4V}{1K} = 2.4mA$$

$$V_{CQ} = 9V - I_{CQ} \bullet 1K = 9V - 2.4V = 6.6V$$

$$V_{OUTQ} = I_{RL} \bullet 4K = 0V$$

V_{CQ}=6.6V

V_{OUTQ}=0V

Amplification with Transistors

From Wikipedia: (approx. 2010)

Generally, an **amplifier** or simply **amp**, is any device that changes, usually increases, the amplitude of a signal. The "signal" is usually voltage or current.

From Wikipedia: (Oct. 2015)

An **amplifier**, **electronic amplifier** or (informally) **amp** is an electronic device that increases the power of a signal.

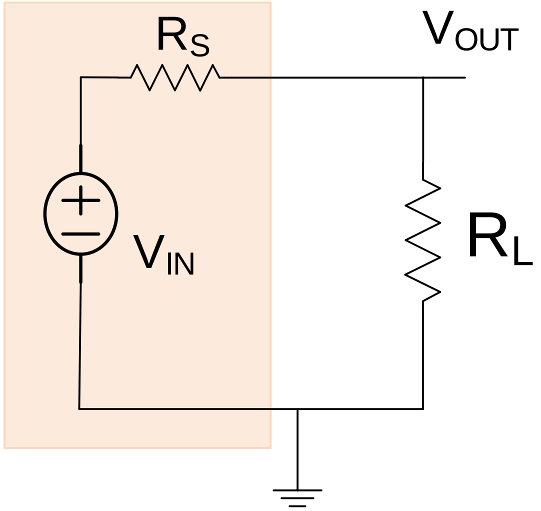
From Wikipedia: (Feb. 2017)

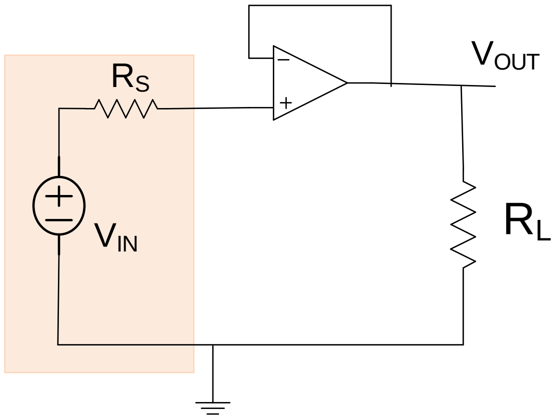
An **amplifier**, **electronic amplifier** or (informally) **amp** is an electronic device that increases the power of a signal (a time varying voltage or current.

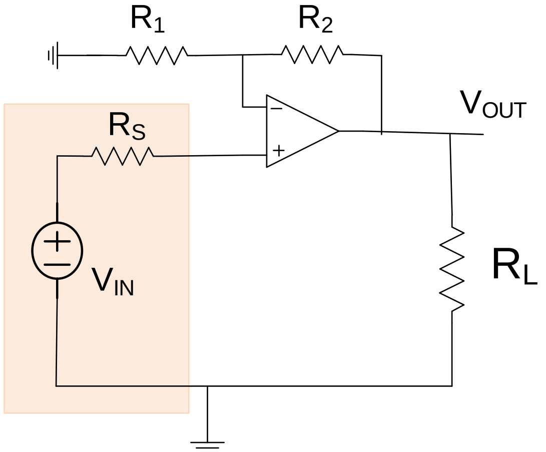
What is the "power" of a signal?

Does Wikipedia have such a basic concept right?

Signal and Power Levels



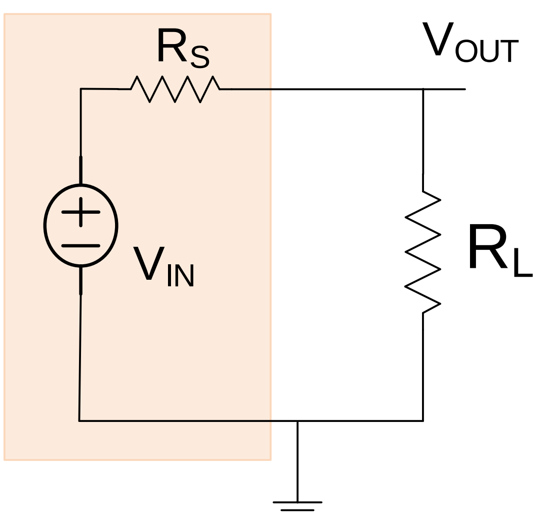


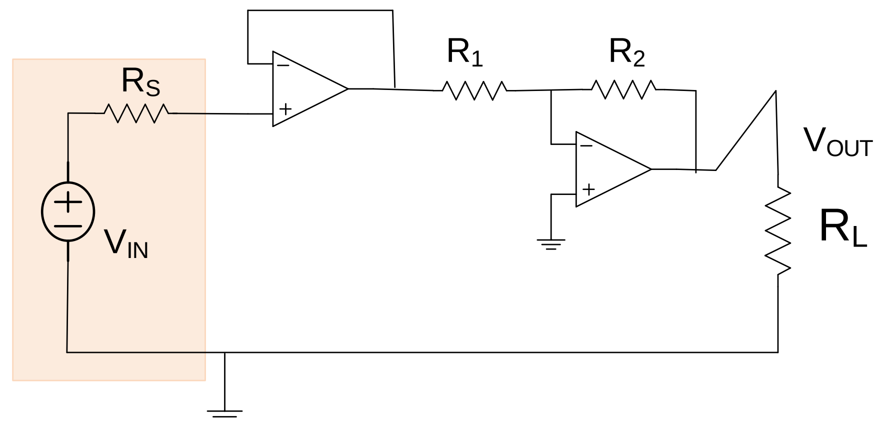


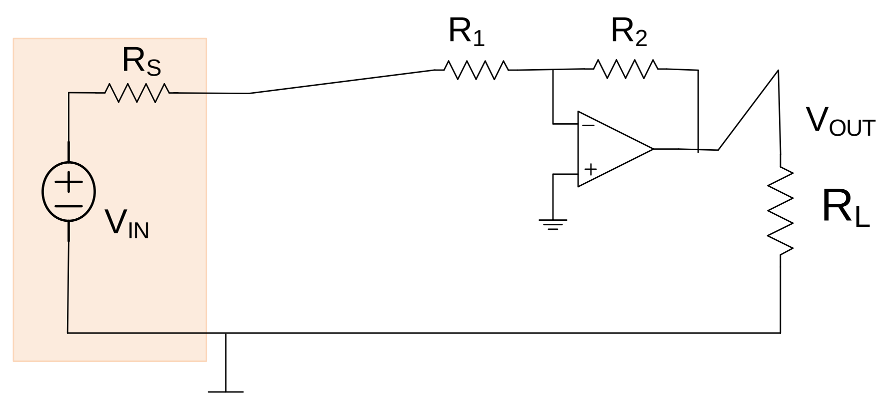
$P_{RL} < P_{VIN}$

$V_{OUT} < V_{IN}$

Signal and Power Levels







$P_{RL} < P_{VIN}$

$V_{OUT} < V_{IN}$

Amplification with Transistors

From Wikipedia: (Feb. 2017)

An **amplifier**, **electronic amplifier** or (informally) **amp** is an electronic device that increases the power of a signal (a time varying voltage or current.

- It is difficult to increase the voltage or current very much with passive RC circuits
- Voltage and current levels can be increased a lot with transformers but not practical in integrated circuits
- Power levels can not be increased with passive elements (R, L, C, and Transformers)

- Often an amplifier is defined to be a circuit that **can** increase power levels (be careful with Wikipedia and WWW even when some of the most basic concepts are discussed)
- Transistors can be used to increase not only signal levels but power levels to a load
- In transistor circuits, power that is delivered in the signal path is supplied by a biasing network
- Signals that are amplified are often not time varying

Amplification with Transistors

Usually the gain of an amplifier is larger than 1

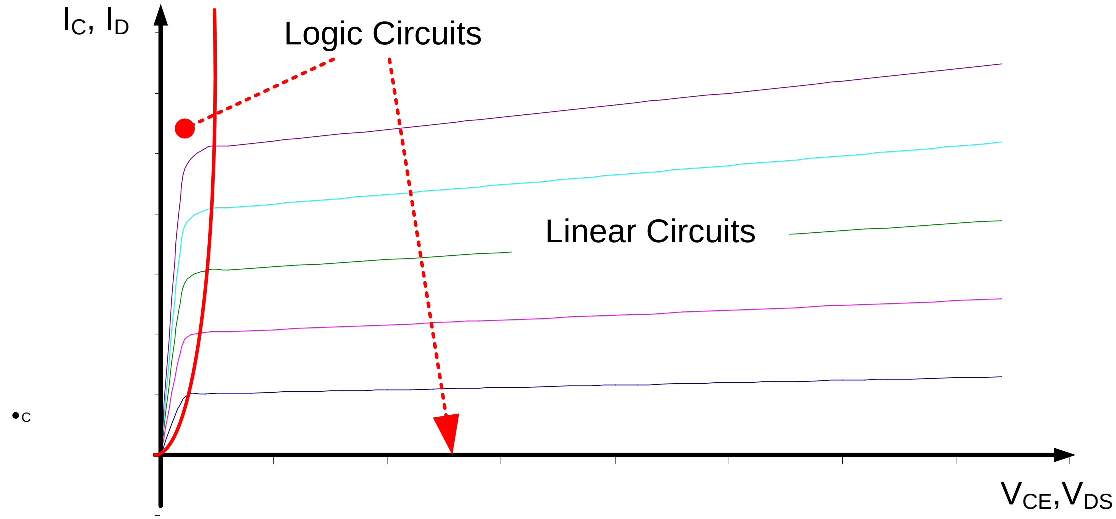
$$V_{OUT} = A_V V_{IN}$$

Often the power dissipated by R_L is larger than the power supplied by V_{IN}

An amplifier can be thought of as a dependent source that was discussed in EE 201

Input and output variables can be either V or I or mixed

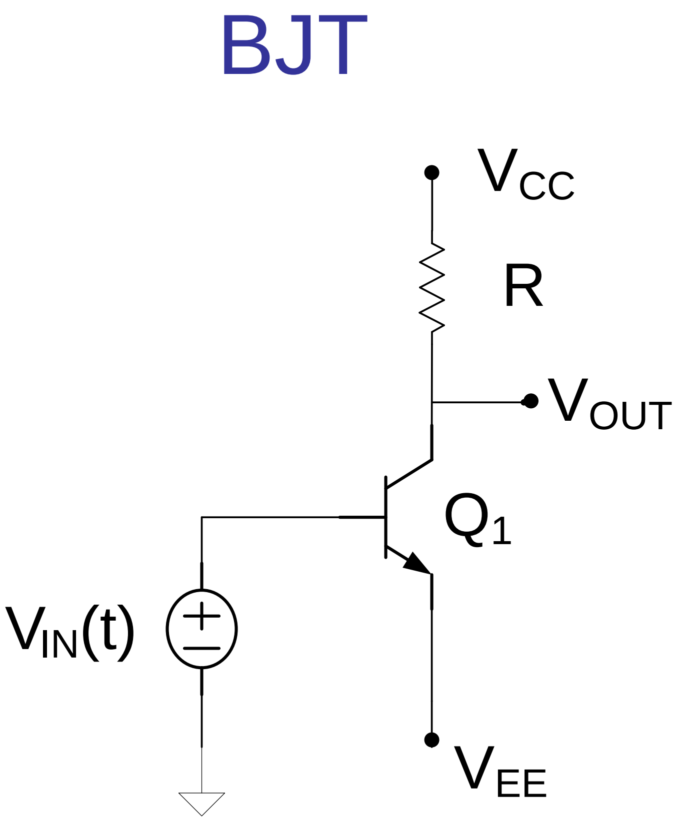
Applications of Devices as Amplifiers

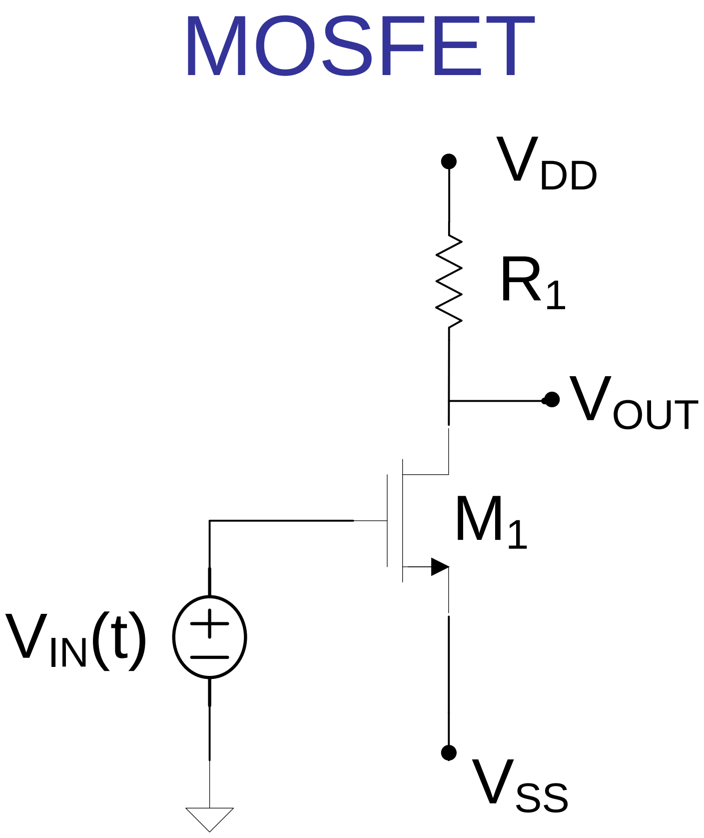


Typical Regions of Operation by Circuit Function

	MOS	Bipolar
Logic Circuits	Triode and Cutoff	Saturation and Cutoff
Linear Circuits	Saturation	Forward Active

Consider the following MOSFET and BJT Circuits





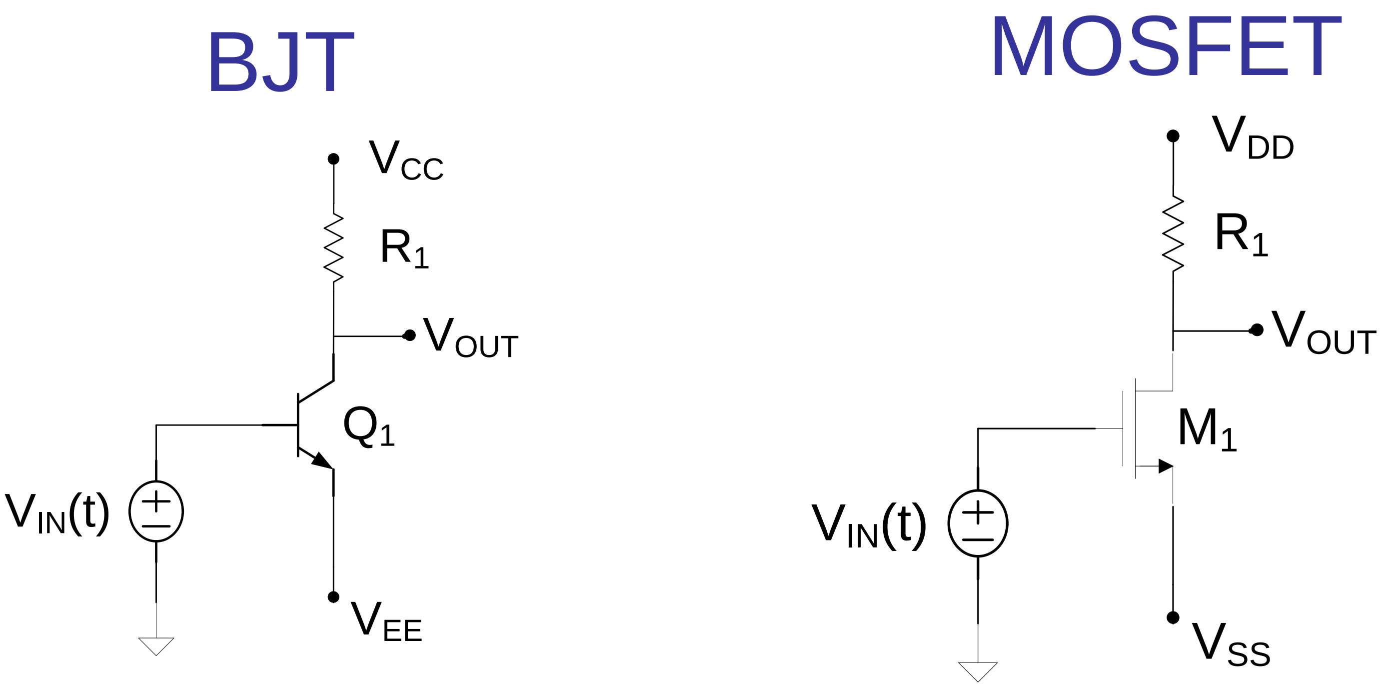
Assume BJT operating in FA region, MOSFET operating in Saturation

Assume same quiescent output voltage and same resistor R_1

Note architecture is same for BJT and MOSFET circuits

One of the most widely used amplifier architectures

Consider the following MOSFET and BJT Circuits



- MOS and BJT Architectures often Identical
- Circuit are Highly Nonlinear
- Nonlinear Analysis Methods Must be used to analyze these and almost any other nonlinear circuit

Methods of Analysis of Nonlinear Circuits

KCL and KVL apply to both linear and nonlinear circuits

Superposition, voltage divider and current divider equations, Thevenin and Norton equivalence apply only to linear circuits!

Some other analysis techniques that have been developed may apply only to linear circuits as well

Methods of Analysis of Nonlinear Circuits

Will consider <u>three</u> different analysis requirements and techniques for some particularly common classes of nonlinear circuits

1. Circuits with continuously differential devices

   Interested in obtaining transfer characteristics of these circuits or outputs for given input signals

2. Circuits with piecewise continuous devices

   Interested in obtaining transfer characteristics of these circuits or outputs for a given input signals

3. Circuits with small-signal inputs that vary around some operating point

   Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Other types of nonlinearities may exist and other types of analysis may be required but we will not attempt to categorize these scenarios in this course

1. Nonlinear circuits with continuously differential devices

Analysis Strategy:

Use KVL and KCL for analysis

Represent nonlinear models for devices either mathematically or graphically

Solve the resultant set of nonlinear and linear equations for the variables of interest

2. Circuits with piecewise continuous devices

e.g. $$f(x) = \begin{cases} f_1(x) & x < x_1 \quad \text{region 1} \\ f_2(x) & x > x_1 \quad \text{region 2} \end{cases}$$

Analysis Strategy:

Guess region of operation

Solve resultant circuit using the previous method

Verify region of operation is valid

Repeat the previous 3 steps as often as necessary until region of operation is verified

- It helps to guess right the first time but a wrong guess will not result in an incorrect solution because a wrong guess can not be verified

- Piecewise models generally result in a simplification of the analysis of nonlinear circuits

3. Circuits with small-signal inputs that vary around some operating point

Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Analysis Strategy:

Use methods from previous class of nonlinear circuits

More Practical Analysis Strategy:

Determine the operating point (using method 1 or 2 discussed above after all small signal independent inputs are set to 0)

Develop small signal (linear) model for all devices in the region of interest (around the operating point or “Q-point”)

Create small signal equivalent circuit by replacing all devices with small-signal equivalent

Solve the resultant small-signal (linear) circuit

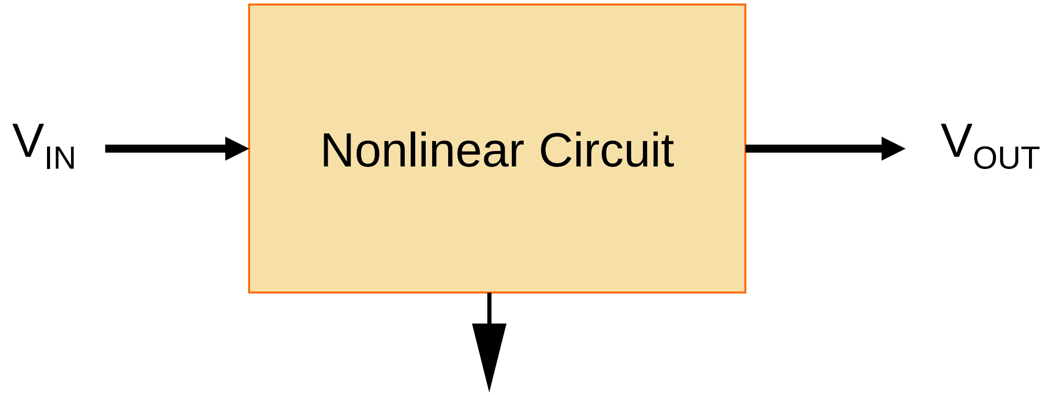
Can use KCL, DVL, and other linear analysis tools such as superposition, voltage and current divider equations, Thevenin and Norton equivalence

Determine boundary of region where small signal analysis is valid

Small signal operation of nonlinear circuits

$V_{IN}=V_M\sin\omega t$

V_M is small



If V_M is sufficiently small, then any nonlinear circuit operating at a region where there are no abrupt nonlinearities will have a nearly sinusoidal output and the variance of the magnitude of this output with V_M will be nearly linear (could be viewed as “locally linear”)

This is termed the “small signal” operation of the nonlinear circuit

When operating with “small signals”, the nonlinear circuit performs linearly with respect to these small signals thus other properties of linear networks such as superposition apply provided the sum of all superimposed signals remains sufficiently small

Other types of “small signals”, e.g. square waves, triangular waves, or even arbitrary waveforms often are used as inputs as well but the performance of the nonlinear network also behaves linearly for these inputs

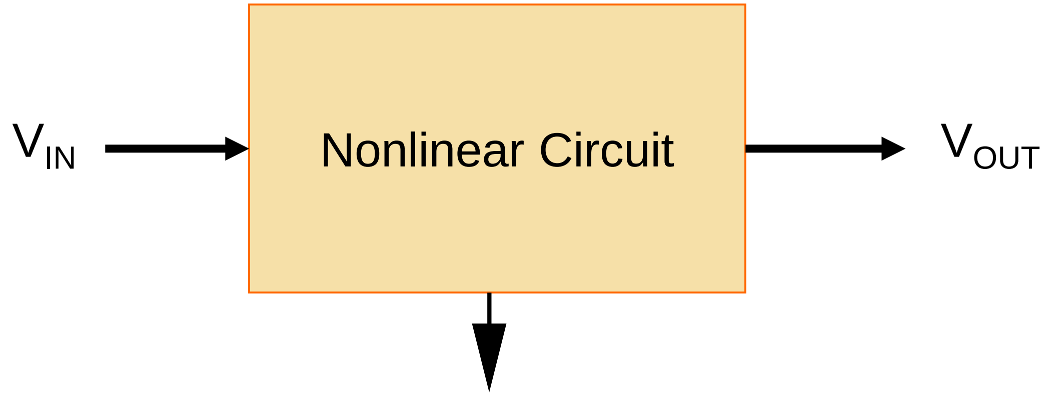
Many useful electronic systems require the processing of these small signals

Practical methods of analyzing and designing circuits that operate with small signal inputs are really important

Small signal operation of nonlinear circuits

$V_{IN}=V_M \sin\omega t$

V_M is small



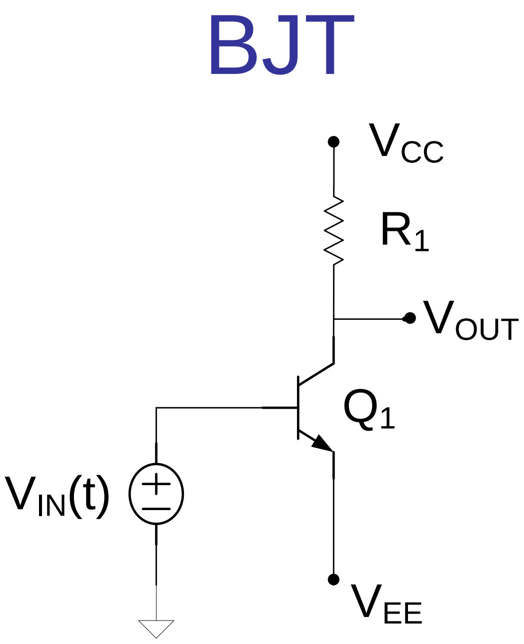
Practical methods of analyzing and designing circuits that operate with small signal inputs are really important

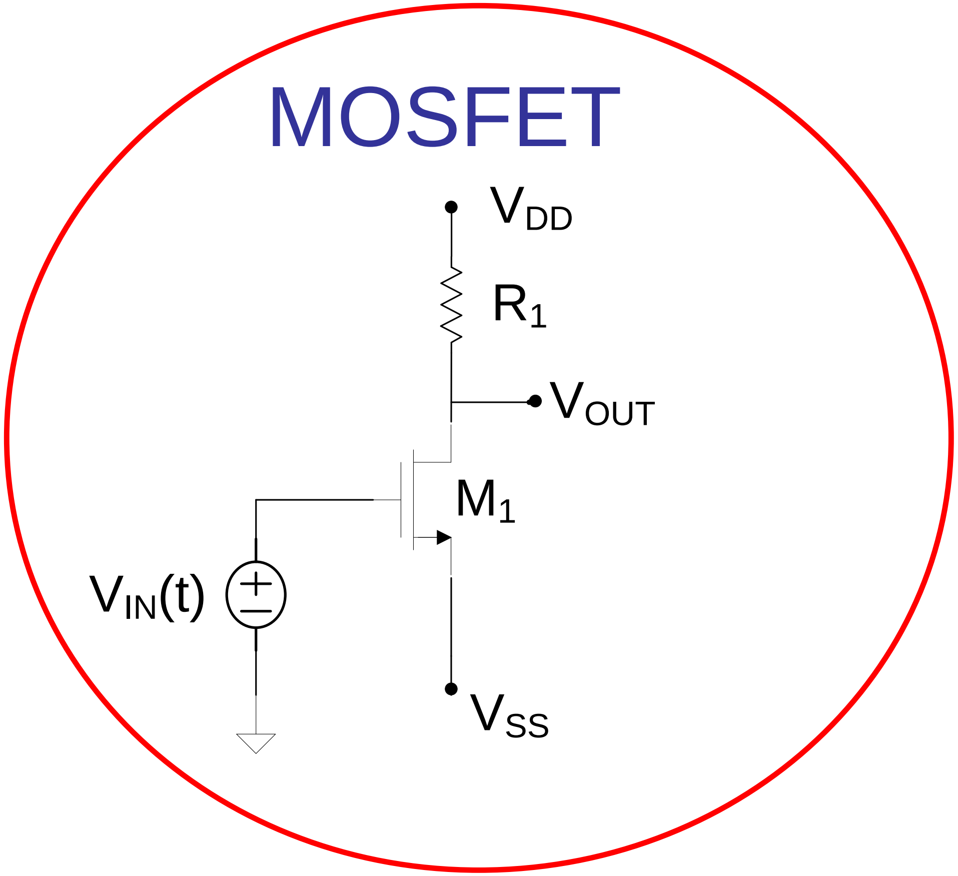
Two key questions:

How small must the input signals be to obtain locally-linear operation of a nonlinear circuit?

How can these locally-linear (alt small signal) circuits be analyzed and designed?

Consider the following MOSFET and BJT Circuits

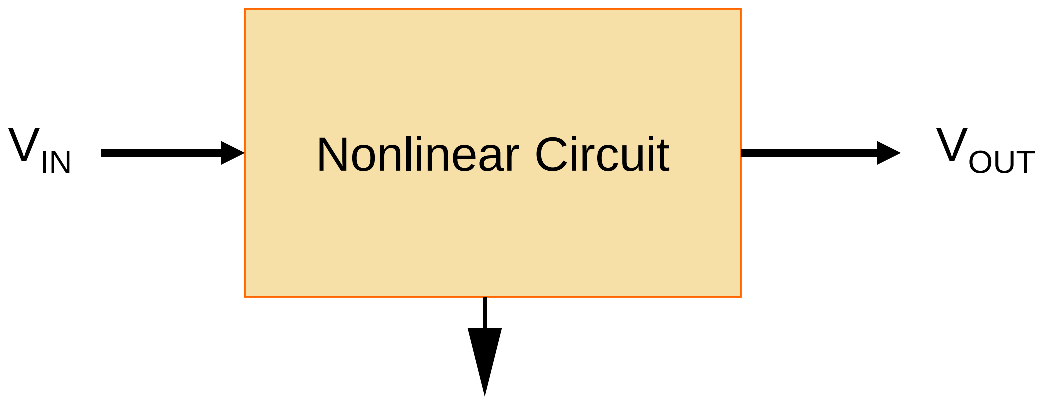




One of the most widely used amplifier architectures

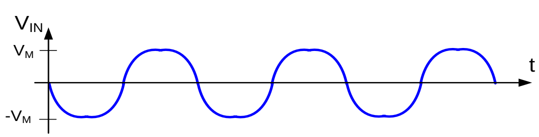
Small signal operation of nonlinear circuits

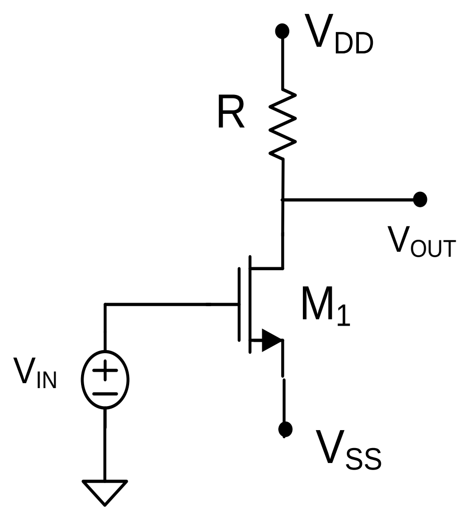
$V_{IN}=V_M\sin\omega t$



V_M is small

Example of circuit that is widely used in locally-linear mode of operation



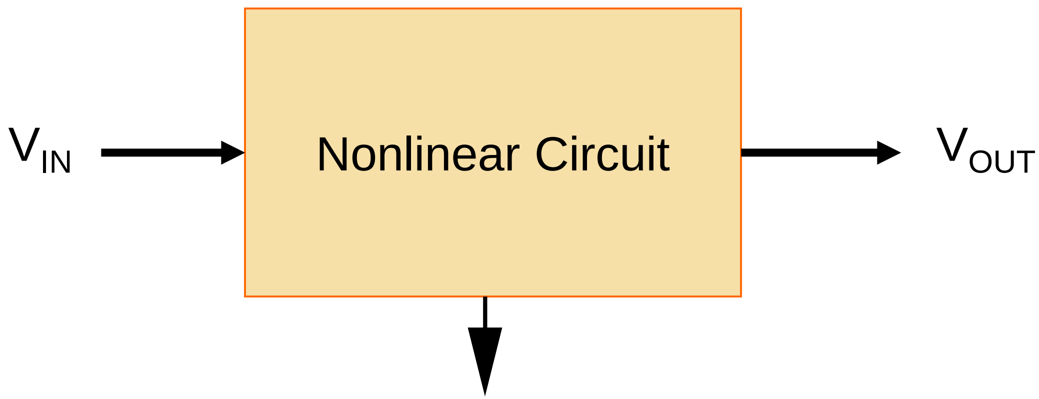


Two methods of analyzing locally-linear circuits will be considered, one of these is by far the most practical

Small signal operation of nonlinear circuits

$V_{IN}=V_M \sin\omega t$

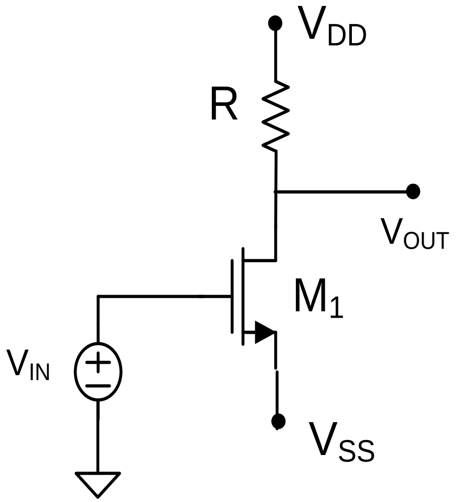
V_M is small



Two methods of analyzing locally-linear circuits for small-signal excitaions will be considered, one of these is by far the most practical

1. Analysis using nonlinear models
2. Small signal analysis using locally-linearized models

Small signal analysis using nonlinear models

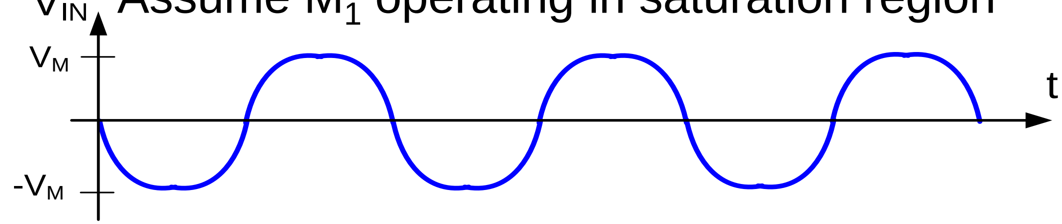


$V_{IN}=V_M\sin\omega t$

V_M is small

By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region

Assume M_1 operating in saturation region



Termed Load Line

$$V_{OUT}=V_{DD}-I_DR$$

$$I_D=\frac{\mu C_{OX}W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2$$

$$\mathbf{I_{DQ}=\frac{\mu C_{OX}W}{2L}\left(V_{SS}+V_T\right)^2}$$

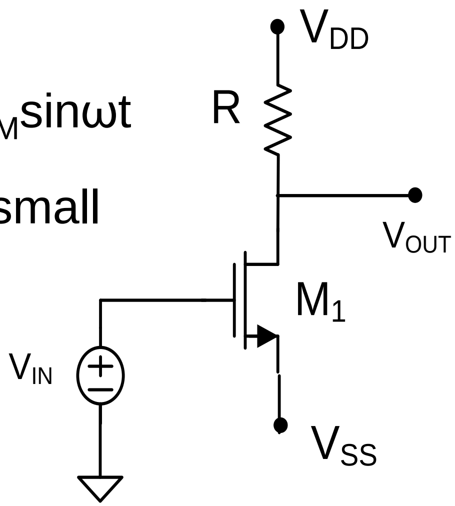
$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2R$$

$$\mathbf{V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_M\sin\omega t-\left[V_{SS}+V_T\right]\right)^2R}$$

Small signal analysis example

$V_{IN}=V_M \sin\omega t$

V_M is small



$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_M\sin\omega t-[V_{SS}+V_T]\right)^2R$$

$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2\left(1-\frac{V_M\sin\omega t}{[V_{SS}+V_T]}\right)^2R$$

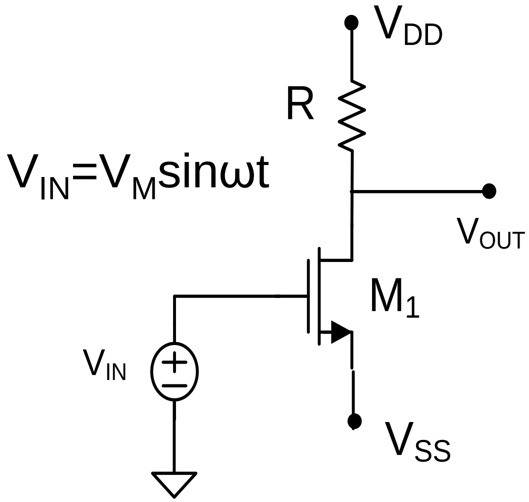
Recall that if x is small $(1+x)^2\cong 1+2x$

$$V_{OUT}\cong V_{DD}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2\left(1-\frac{2V_M\sin\omega t}{[V_{SS}+V_T]}\right)R$$

$$V_{OUT}\cong\left\{V_{DD}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2R\right\}+\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2\left(\frac{2V_M\sin\omega t}{[V_{SS}+V_T]}\right)R$$

$$V_{OUT}\cong\left\{V_{DD}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2R\right\}+\left\{\frac{\mu C_{OX}W}{L}[V_{SS}+V_T]R\right\}V_M\sin\omega t$$

Small signal analysis example

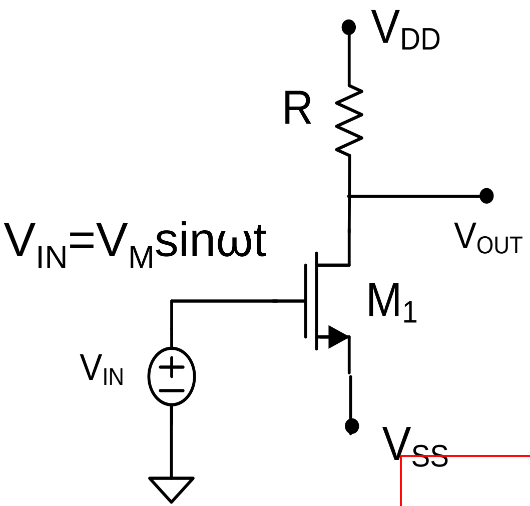


By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region

Assume M_1 operating in saturation region

$$V_{OUT} \cong \left\{ V_{DD} - \frac{\mu C_{OX} W}{2L} \left[V_{SS} + V_T \right]^2 R \right\} + \left\{ \frac{\mu C_{OX} W}{L} \left[V_{SS} + V_T \right] R \right\} V_M \sin \omega t$$

Small signal analysis example



Assume M_1 operating in saturation region

$$V_{OUT} \cong \underbrace{\left\{V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS}+V_T\right]^2 R\right\}}_{\textit{Quiescent Output}} + \underbrace{\left\{\frac{\mu C_{OX} W}{L}\left[V_{SS}+V_T\right]R\right\}}_{\textit{ss Voltage Gain}} V_M \sin\omega t$$

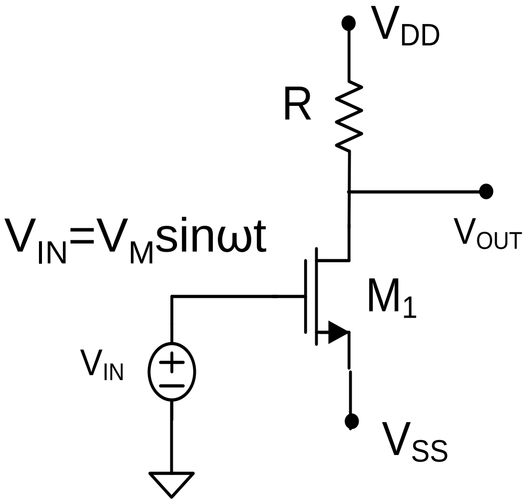
$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS}+V_T\right]R$$

$$V_{OUTQ} = \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS}+V_T\right]^2 R\right\}$$

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

Note the ss voltage gain is negative since $V_{SS}+V_T<0$!

Small signal analysis example



Assume M_1 operating in saturation region

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

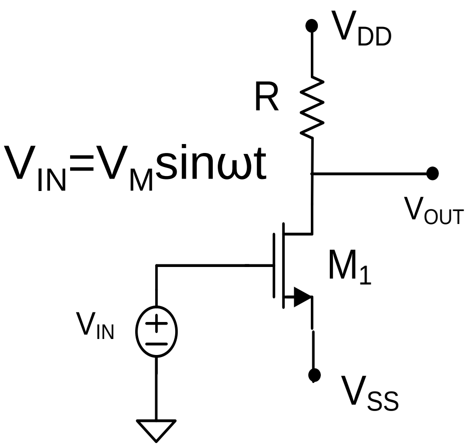
$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R$$

$$V_{OUTQ} = \left\{ V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS} + V_T\right]^2 R \right\}$$

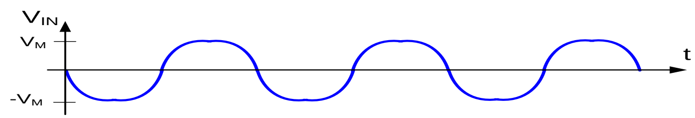
But – this expression gives little insight into how large the gain is !

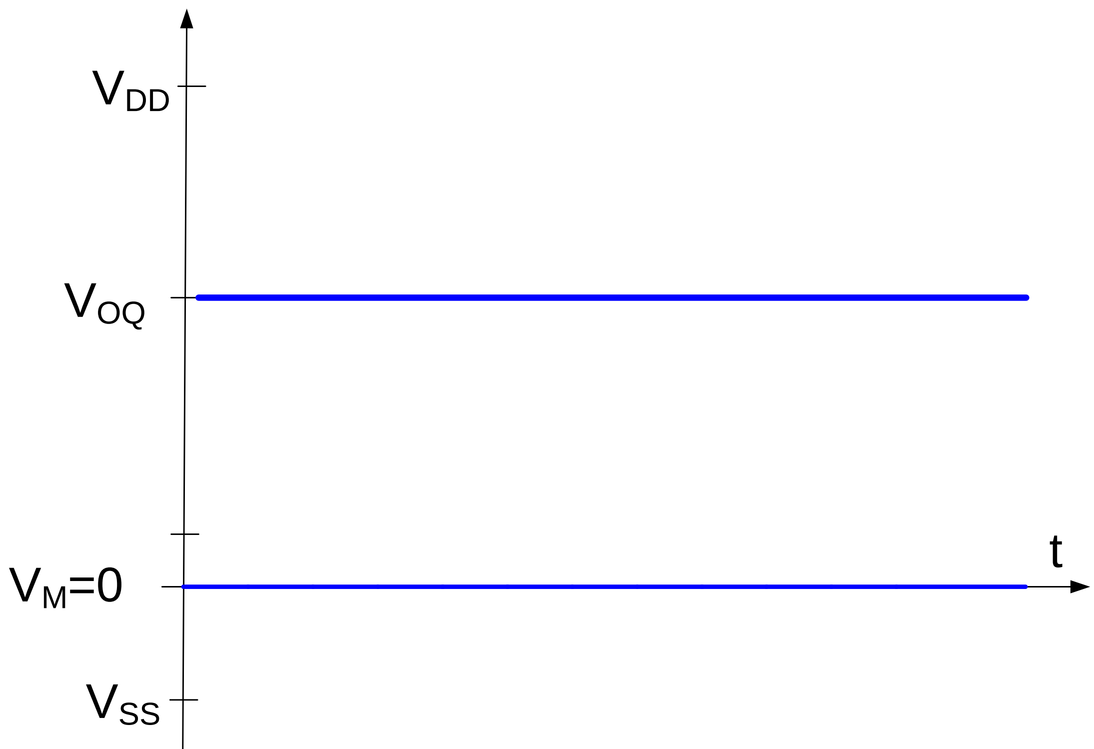
And the analysis for even this very simple circuit was messy!

Small signal analysis example

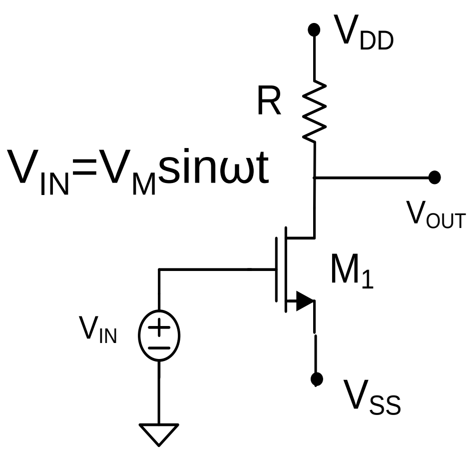


$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin \omega t$$



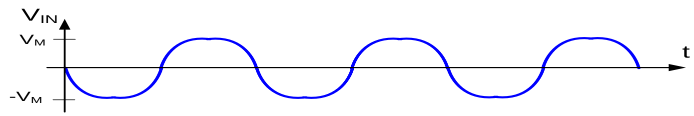


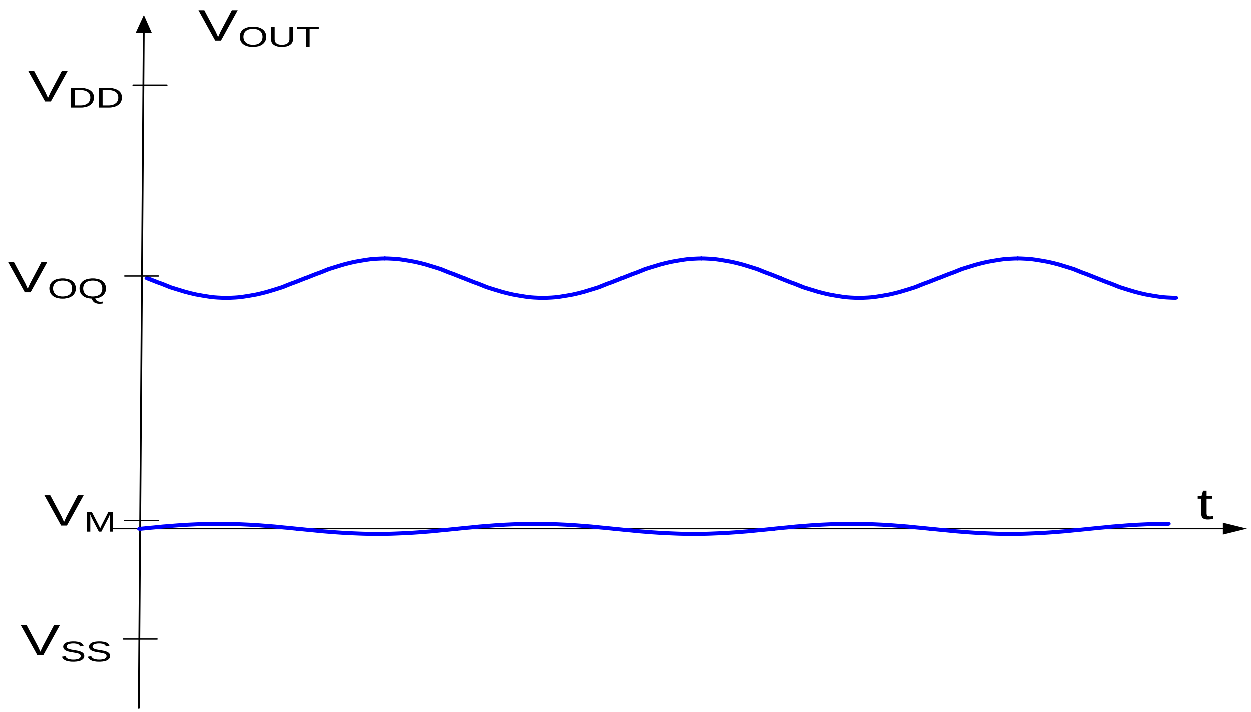
Small signal analysis example



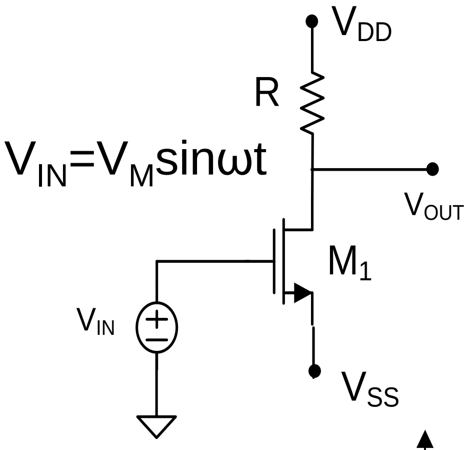
$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R$$



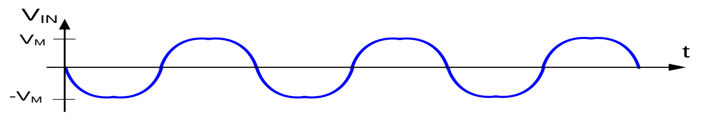


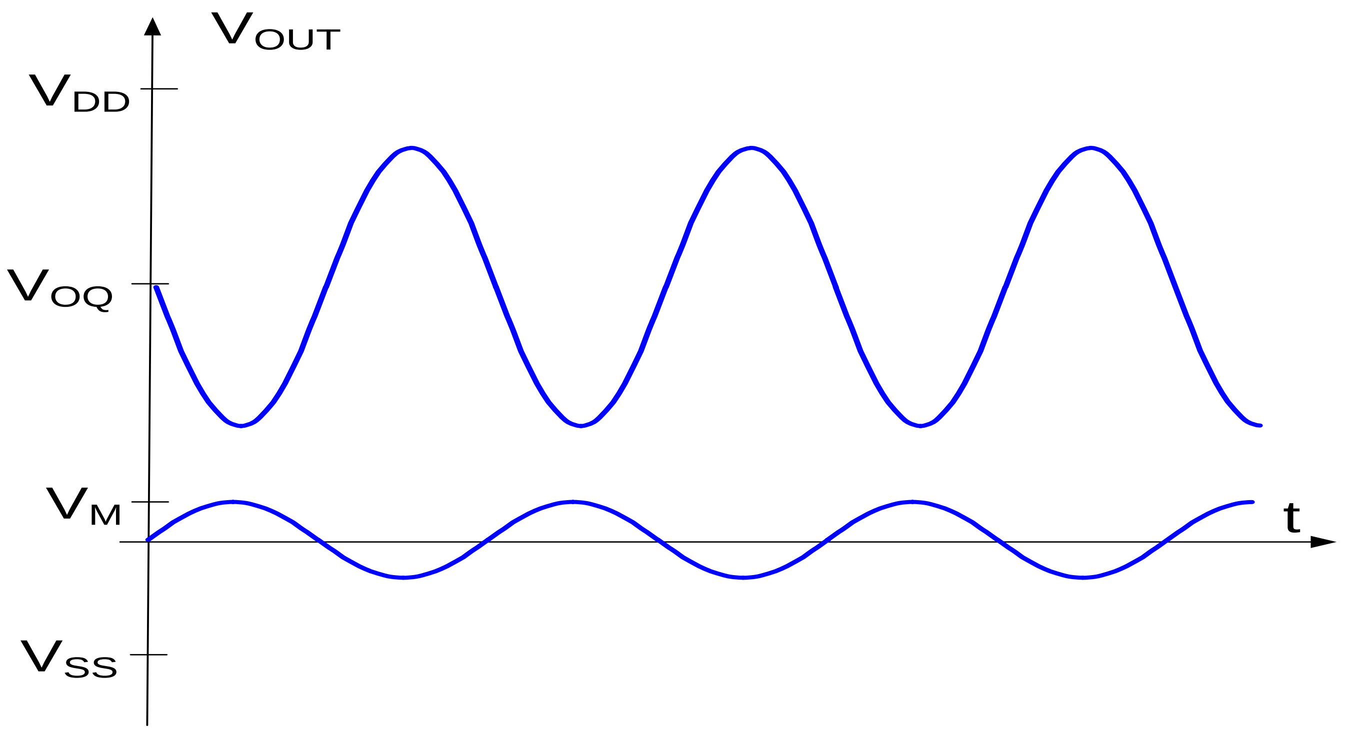
Small signal analysis example



$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R$$

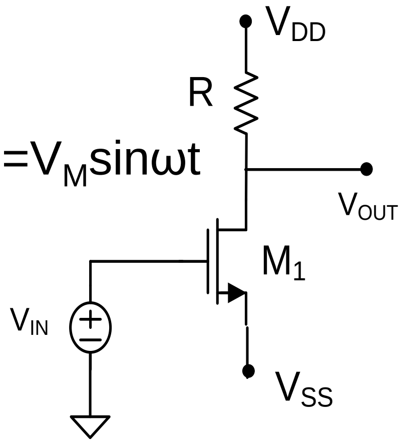


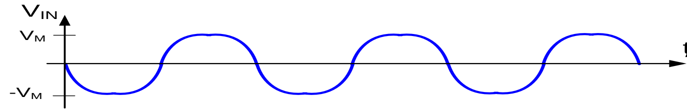


Small signal analysis example

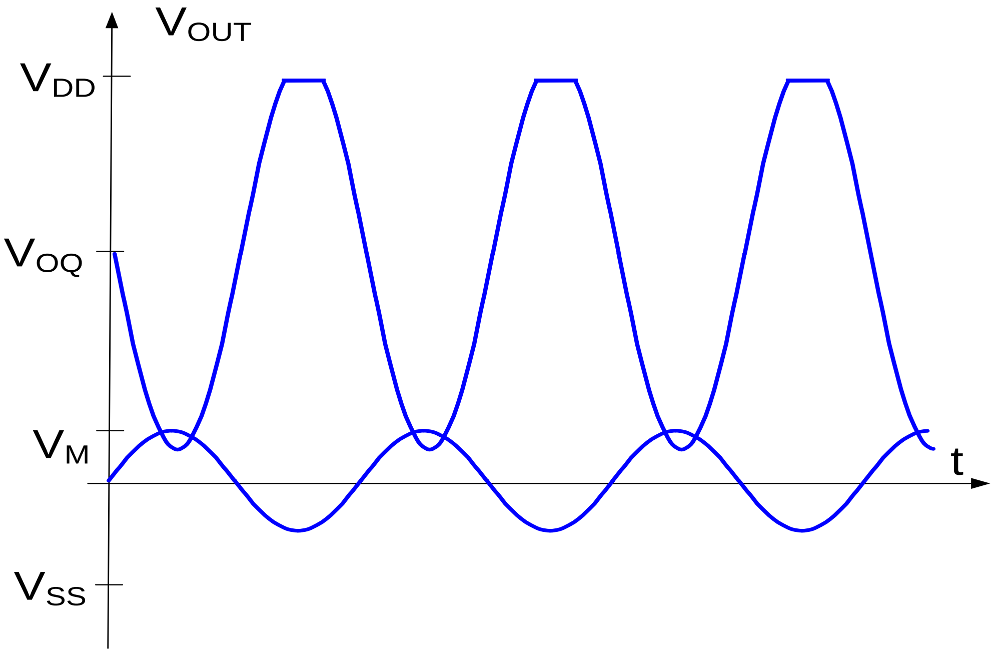
$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$V_{IN}=V_M\sin\omega t$



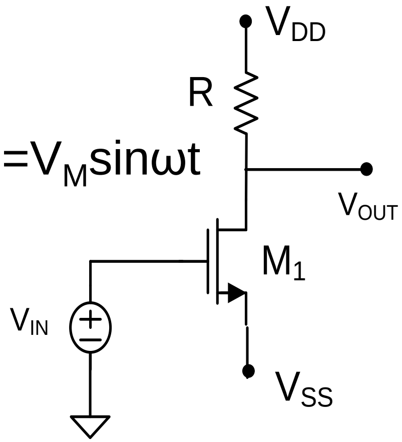


$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R$$



Serious Distortion occurs if signal is too large or Q-point non-optimal
Here “clipping” occurs for high V_{OUT}

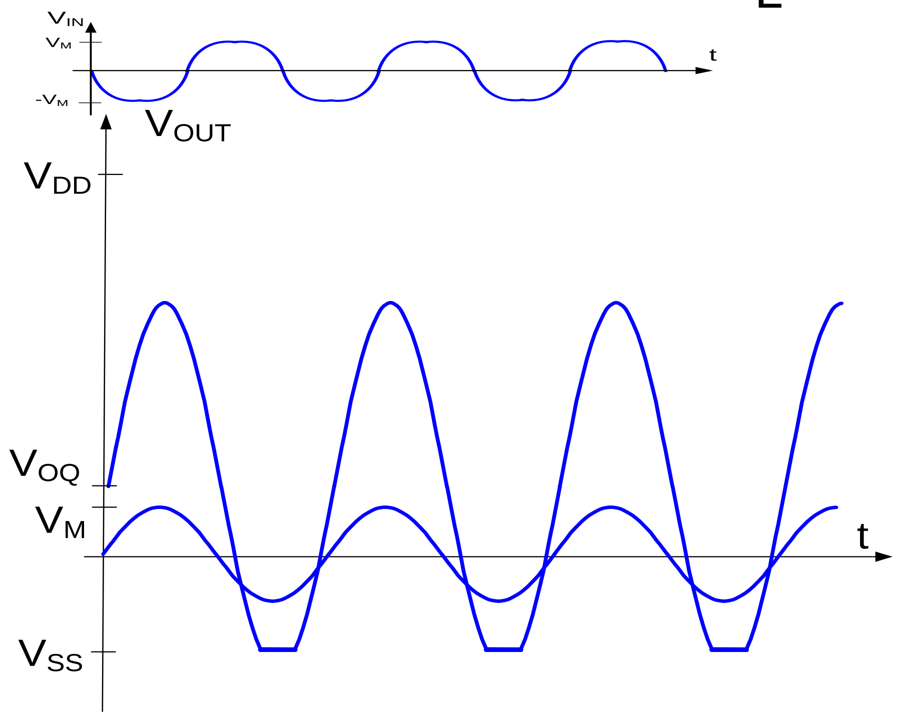
Small signal analysis example



$V_{IN}=V_M \sin\omega t$

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin \omega t$$

$$A_v = \frac{\mu C_{ox} W}{L}\left[V_{SS} + V_T\right] R$$



Serious Distortion occurs if signal is too large or Q-point non-optimal
Here “clipping” occurs for low V_{OUT}

End of Lecture 21