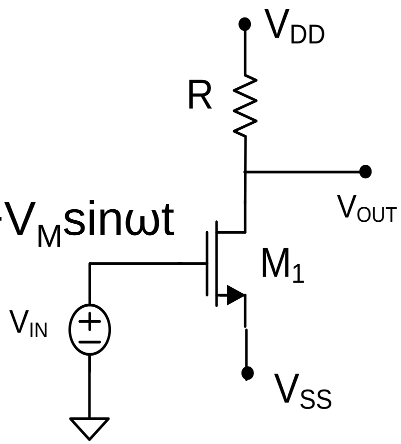
EE 330
Lecture 22

- Small Signal Analysis
- Small Signal Modelling

Small signal analysis example

$V_{IN}=V_{INQ}+V_M \sin\omega t$



$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$A_V \cong -\frac{\mu C_{OX} W}{L}\left[V_{INQ}-V_{SS}-V_T\right]R$$

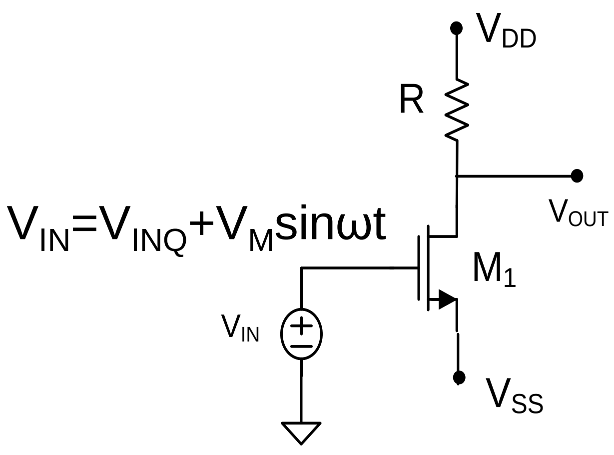
But – this expression gives little insight into how large the gain is !

Can the gain be made arbitrarily large by simply making R large?

Observe increasing R with W,L, and V_{SS} fixed will change Q-point

Difficult to answer this question with the information provided !

Small signal analysis example



$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$A_V \cong -\frac{\mu C_{OX} W}{L}\left[V_{INQ}-V_{SS}-V_T\right]R$$

But recall:

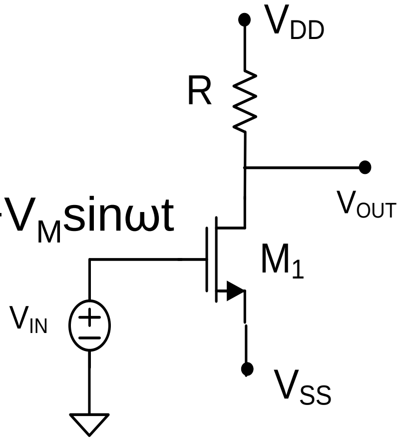
$$I_{DQ} = \frac{\mu C_{OX} W}{2L}\left(V_{INQ}-V_{SS}-V_T\right)^2$$

Thus, substituting from the expression for I_{DQ} we obtain

$$A_v = -\frac{2I_{DQ}R}{\left[V_{INQ}-V_{SS}-V_T\right]} = -\frac{2I_{DQ}R}{\left[V_{GSQ}-V_T\right]}$$

Small signal analysis example

$V_{IN}=V_{INQ}+V_M \sin\omega t$



$$A_v = -\frac{2I_{DQ}R}{\left[V_{GSQ}-V_T\right]}$$

- Small signal voltage gain is twice the Quiescent voltage across R divided by V_{GSQ} - V_T
- Making $I_{DQ}R$ too big or too small will limit signal swing
- Can make $|A_V|$ large by making V_{GSQ} - V_T small

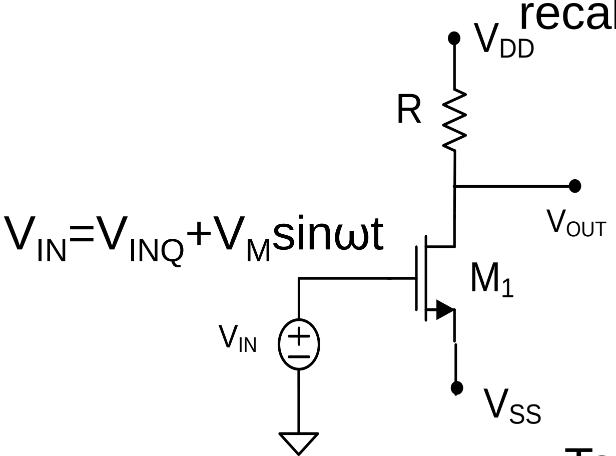
- This analysis which required linearization of a nonlinear output voltage is quite tedious.

- This approach becomes unwieldy for even slightly more complicated circuits

- A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements

Small signal analysis example

(Consider what was neglected in the previous analysis)



recall

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

However, there are invariably small errors in this analsis

$$V_{OUT} = V_{OUTQ} + A_V V_M \sin\omega t + \varepsilon(t)$$

To see the effects of the approximations consider again

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(V_M \sin(\omega t) + \left[V_{GSQ} - V_T\right]\right)^2$$

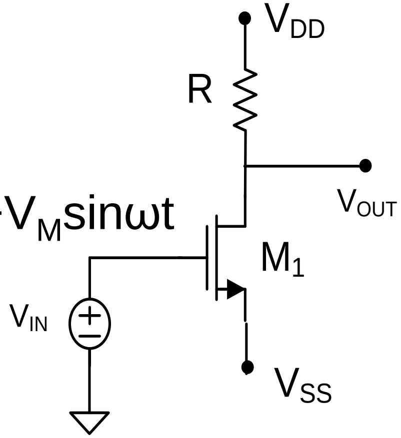
$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(V_M^2 \sin(\omega t)^2 + 2\left[V_{GSQ} - V_T\right]V_M \sin\omega t + \left[V_{GSQ} - V_T\right]^2\right)$$

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(V_M^2\left[\frac{1-\cos 2\omega t}{2}\right] + 2\left[V_{GSQ} - V_T\right]V_M \sin\omega t + \left[V_{GSQ} - V_T\right]^2\right)$$

$$V_{OUT} = \left\{V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(\frac{V_M^2}{2} + \left[V_{GSQ} - V_T\right]^2\right)\right\} - \left\{\left(\frac{\mu C_{OX} W}{L}\left[V_{GST} - V_T\right]R\right)V_M \sin\omega t\right\} + \boxed{\left\{\left(\frac{\mu C_{OX} RW}{4L} V_M^2\right)\cos 2\omega t\right\}}$$

Note presence of second harmonic distortion term !

Small signal analysis example



Nonlinear distortion term

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$V_{OUT} = V_{OUTQ} + A_V V_M \sin\omega t + \varepsilon(t)$$

$$V_{OUT} = \left\{ V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(\frac{V_M^2}{2} + \left[V_{GSQ} - V_T\right]^2\right)\right\} - \left\{\left(\frac{\mu C_{OX} W}{L}\left[V_{GSQ} - V_T\right]R\right)V_M \sin\omega t\right\} + \left\{\left(\frac{\mu C_{OX} RW}{4L}V_M^2\right)\cos 2\omega t\right\}$$

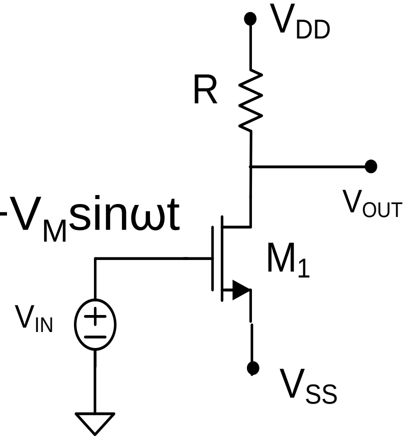
$$V_{OUTDC} = V_{OUTQ} - \frac{\mu C_{OX} RW}{4L}V_M^2$$

$$A_V = -\frac{\mu C_{OX} W}{L}\left[V_{GSQ} - V_T\right]R$$

$$A_2 = \frac{\mu C_{OX} RW}{4L}V_M$$

$$V_{OUT} = V_{OUTDC} + \left\{A_V V_M \sin\omega t\right\} + \left\{A_2 V_M \cos 2\omega t\right\}$$

Small signal analysis example



$V_{IN}=V_{INQ}+V_M\sin\omega t$

Nonlinear distortion term

$$V_{OUT} = V_{OUTDC} + \left\{A_V V_M \sin\omega t\right\} + \left\{A_2 V_M \cos 2\omega t\right\}$$

$$A_V = -\frac{\mu C_{OX} W}{L}\left[V_{GSQ}-V_T\right]R \qquad A_2 = \frac{\mu C_{OX} RW}{4L} V_M$$

Total Harmonic Distortion:

Recall, if $x(t) = \sum_{k=0}^{\infty} b_k \sin(k\omega T + \phi_k)$ then $\mathrm{THD} = \frac{\sqrt{\sum_{k=2}^{\infty} b_k^2}}{|b_1|}$

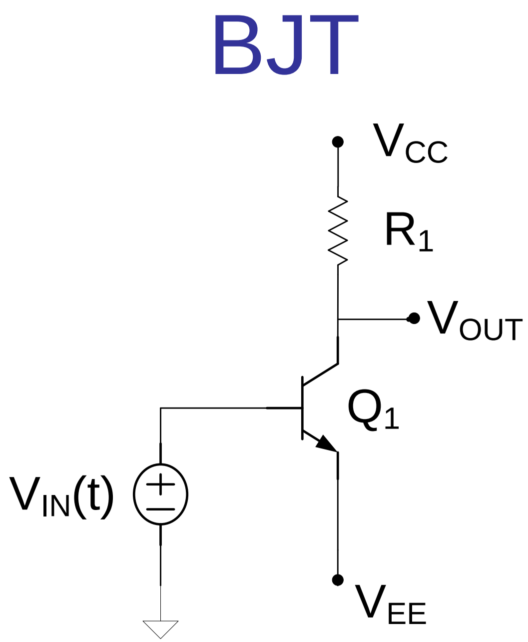
Thus, for this amplifier, as long as M_1 stays in the saturation region

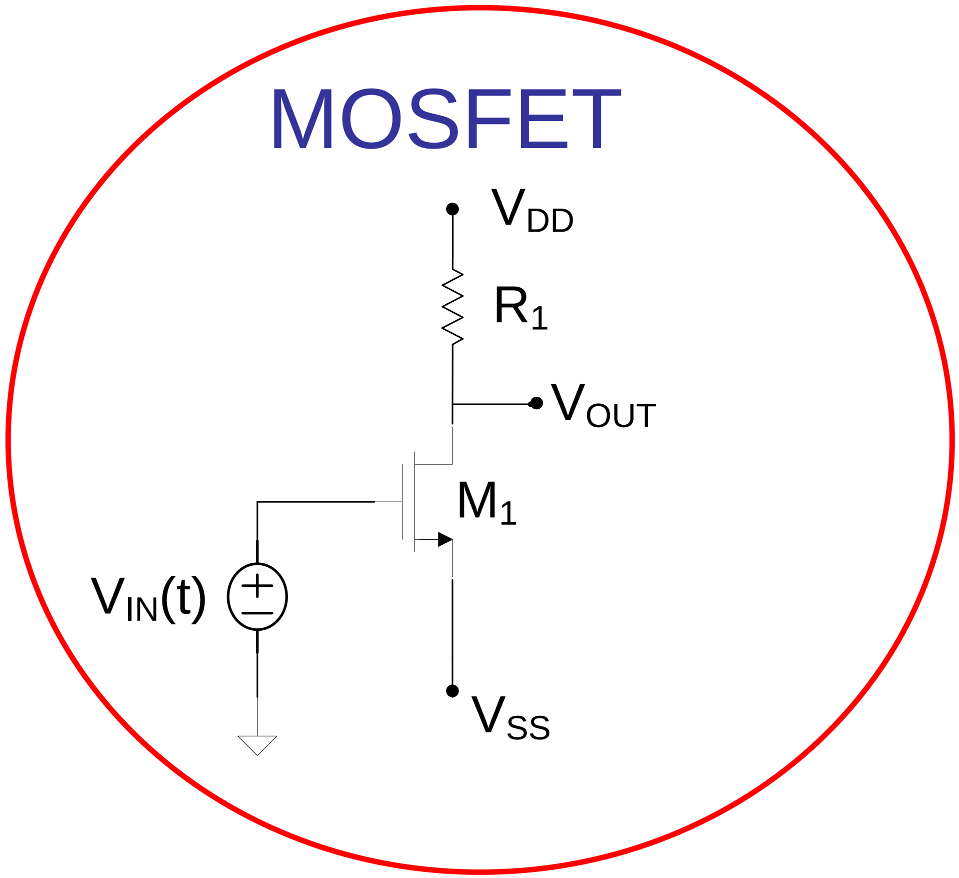
$$\mathrm{THD} = \frac{\sqrt{(A_2V_M)^2}}{|A_V V_M|} = \frac{A_2}{|A_V|} = \frac{\frac{\mu C_{OX}W}{4L}RV_M}{\frac{\mu C_{OX}W}{L}R(V_{GSQ}-V_T)} = \frac{V_M}{4(V_{GSQ}-V_T)}$$

Distortion will be small for $V_M \ll (V_{GSQ}-V_T)$

Distortion will be much worse (larger and more harmonic terms) if M_1 leaves saturation region.

Consider the following MOSFET and BJT Circuits

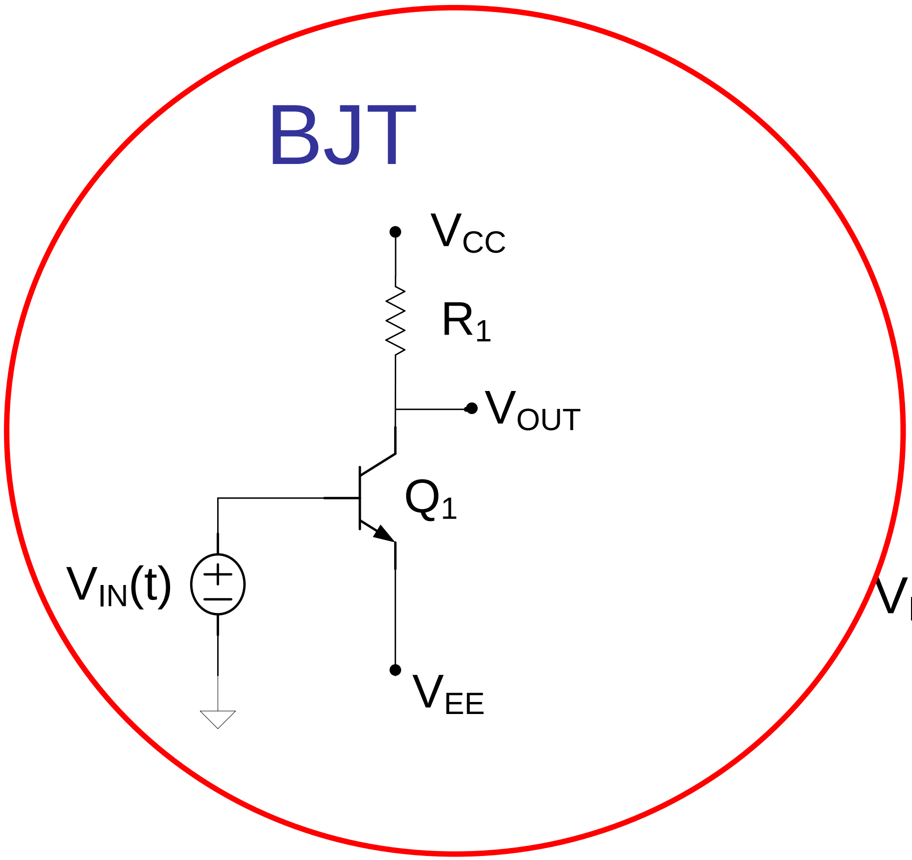


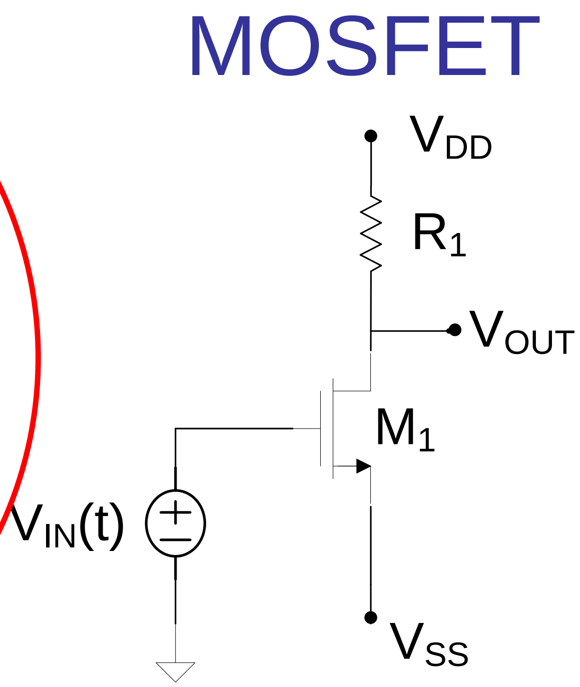


- Analysis was very time consuming
- Issue of operation of circuit was obscured in the details of the analysis

One of the most widely used amplifier architectures

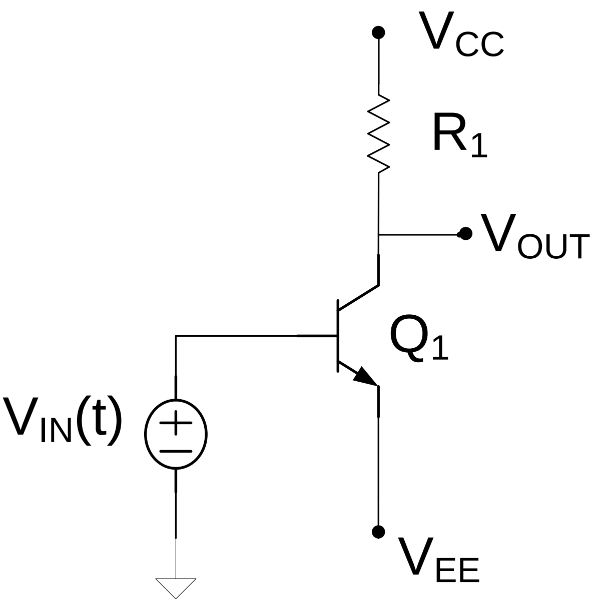
Consider the following MOSFET and BJT Circuits





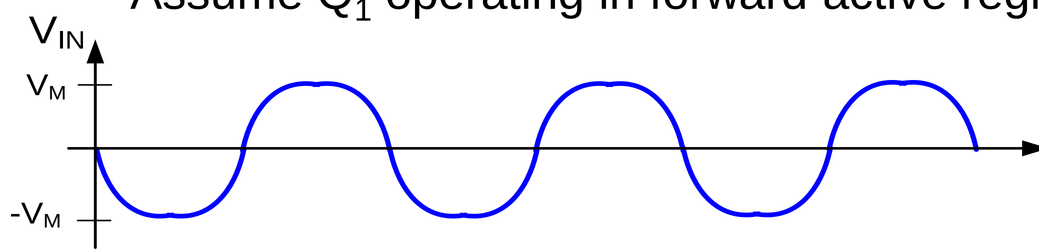
One of the most widely used amplifier architectures

Small signal analysis using nonlinear models



By selecting appropriate value of V_{SS}, M_1 will operate in the forward active region

Assume Q_1 operating in forward active region



$$V_{OUT} = V_{CC} - I_C R_1$$

$$I_C = J_S A_E e^{\frac{V_{IN} - V_{EE}}{V_t}}$$

$V_{IN} = V_{INQ} + V_M \sin\omega t$

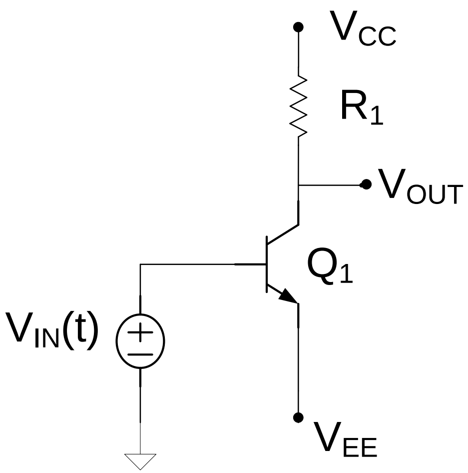
V_M is small

$$\mathbf{I_{CQ} = J_S A_E e^{\frac{V_{INQ} - V_{EE}}{V_t}} = J_S A_E e^{\frac{V_{beQ}}{V_t}}}$$

$$V_{OUTQ} = V_{CC} - J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}}$$

$$V_{OUT} = V_{CC} - J_S A_E R_1 e^{\frac{V_M \sin(\omega t) + V_{beQ}}{V_t}}$$

Small signal analysis using nonlinear models



$$I_{CQ} = J_S A_E e^{\frac{V_{beQ}}{V_t}}$$

$$V_{OUT} = V_{CC} - J_S A_E R_1 e^{\frac{V_M \sin(\omega t) + V_{beQ}}{V_t}}$$

$$V_{OUT} = V_{CC} - J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}} e^{\frac{V_M \sin(\omega t)}{V_t}}$$

Recall that if x is small $e^{\varepsilon} \cong 1+\varepsilon$ (truncated Taylor's series)

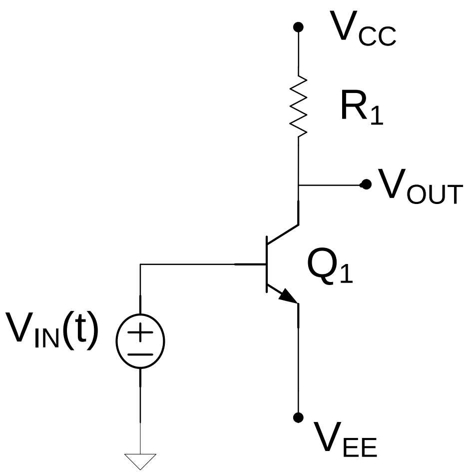
$V_{IN}=V_{INQ}+V_M \sin\omega t$

V_M is small

$$\therefore\ V_{OUT} \cong V_{CC} - J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}} \left(1+\frac{V_M \sin(\omega t)}{V_t}\right)$$

$$V_{OUT} \cong \left[V_{CC} - J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}}\right] - \frac{J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}}}{V_t} V_M \sin(\omega t)$$

Small signal analysis using nonlinear models



$$V_{OUT} \cong \left[V_{CC} - \boxed{J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}}} \right] - \frac{\boxed{J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}}}}{V_t} V_M \sin(\omega t)$$

$$\mathbf{I_{CQ} = J_S A_E e^{\frac{V_{beQ}}{V_t}}}$$

$V_{IN}=V_{INQ}+V_M\sin\omega t$

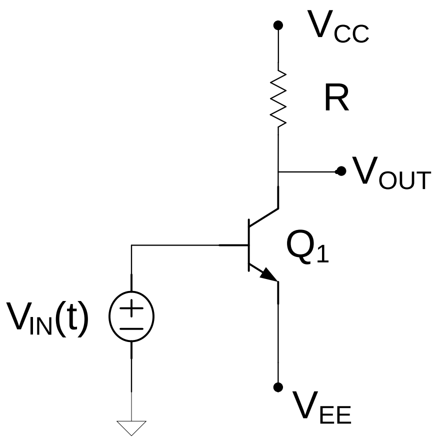
V_M is small

$$V_{OUT} \cong \underbrace{\left[V_{CC} - I_{CQ} R_1 \right]}_{\text{Quiescent Output}} - \boxed{\left(\frac{I_{CQ} R_1}{V_t} \right)} V_M \sin(\omega t)$$

ss Voltage Gain

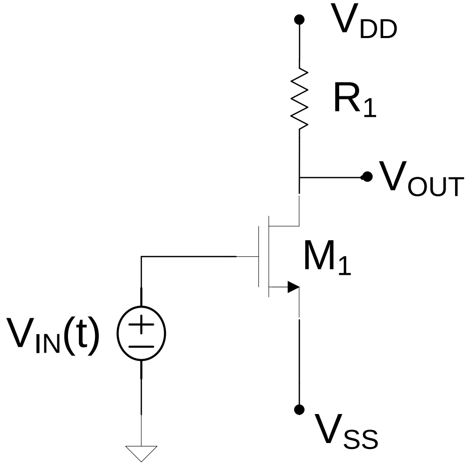
Comparison of Gains for MOSFET and BJT Circuits

BJT



$$A_{VB} = -\frac{I_{CQ}R}{V_t}$$

MOSFET



$$A_{VM} = -\frac{2I_{DQ}R_1}{V_{GSQ} - V_T}$$

If $I_{DQ}R_1 = I_{CQ}R = 2V$, $V_{GSQ} - V_T = 1V$, $V_t = 25mV$

$$A_{VB} = -\frac{I_{CQ}R}{V_t} = -\frac{2V}{25mV} = -80$$

$$A_{VM} = -\frac{2I_{DQ}R_1}{V_{GSQ} - V_T} = -\frac{4V}{1V} = -4$$

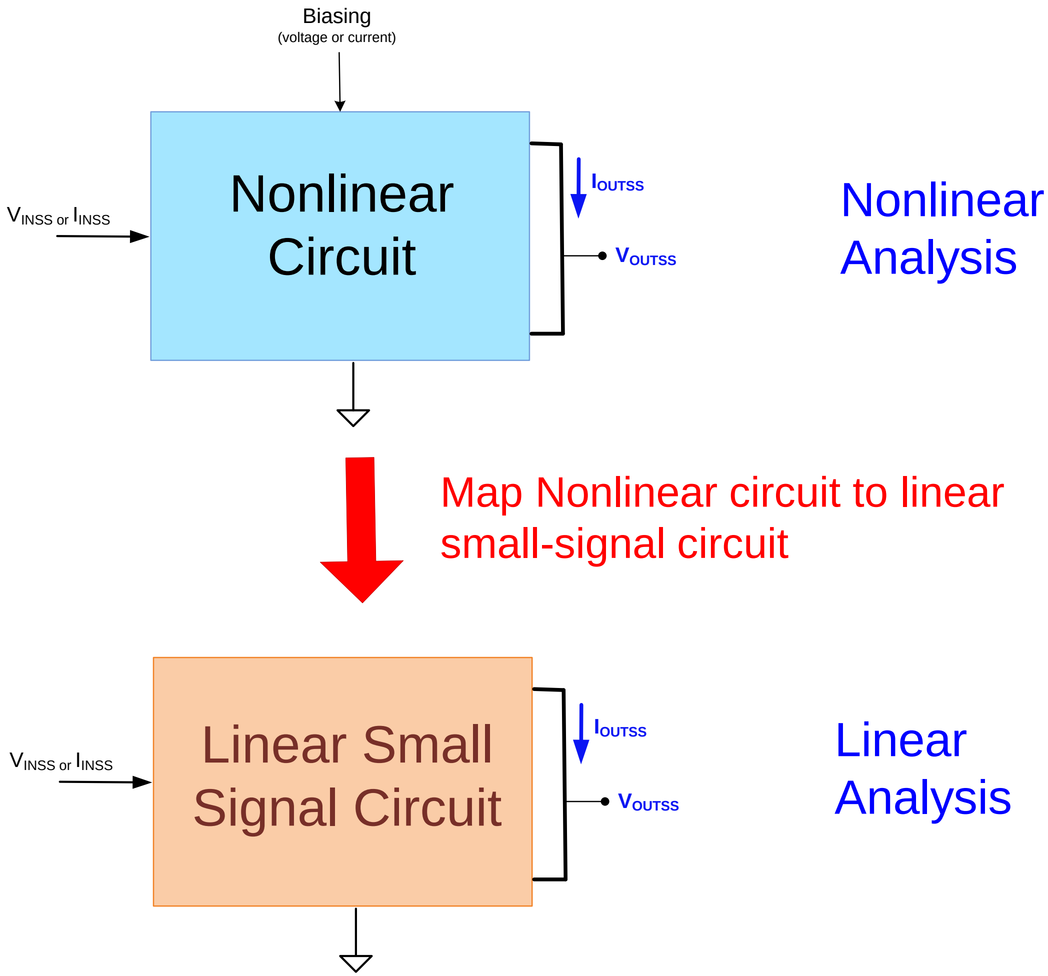
Observe $A_{VB} >> A_{VM}$

Due to exponential-law rather than square-law model

Operation with Small-Signal Inputs

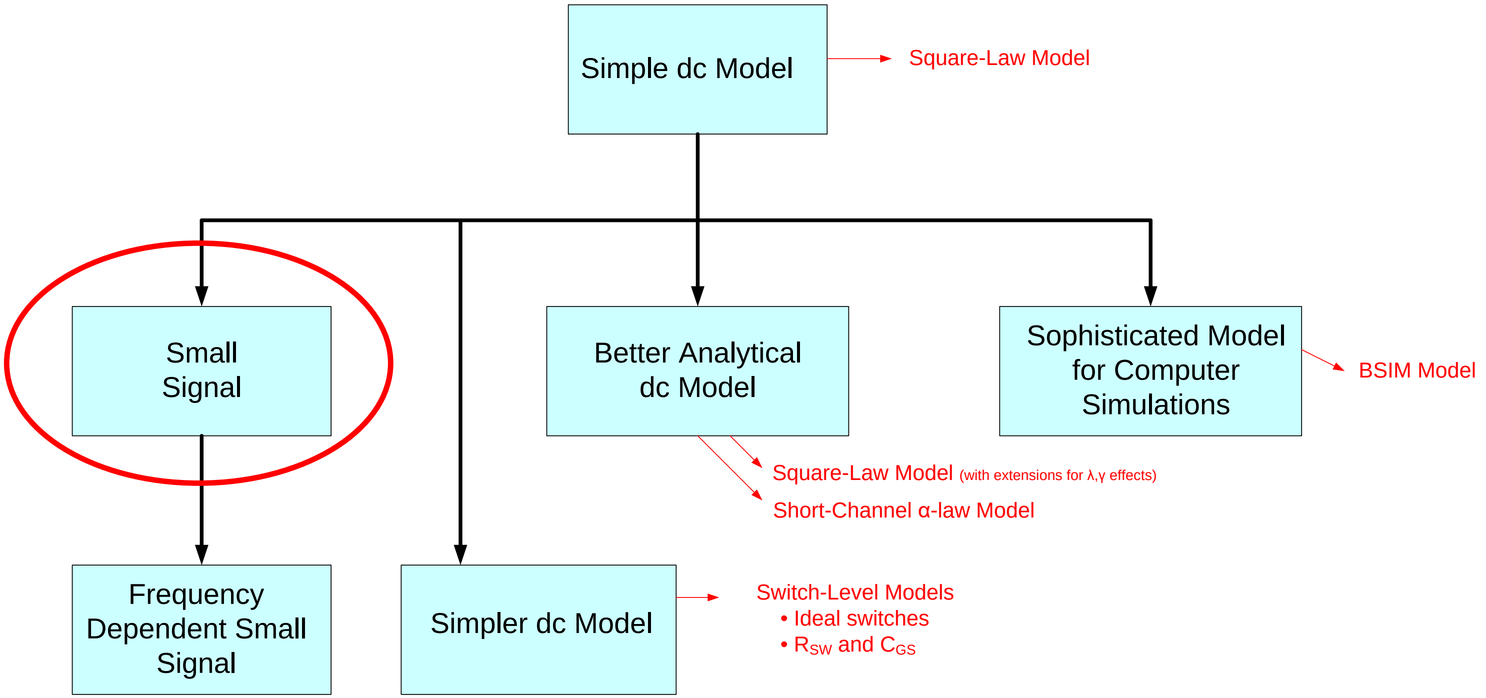
- Analysis procedure for these simple circuits was very tedious
- This approach will be unmanageable for even modestly more complicated circuits
- **Faster analysis method is needed !**

Small-Signal Analysis



- Will commit next several lectures to developing this approach
- Analysis will be MUCH simpler, faster, and provide significantly more insight
- Applicable to many fields of engineering

Small-Signal Analysis



Operation with Small-Signal Inputs

Why was this analysis so tedious?

Because of the nonlinearity in the device models

What was the key technique in the analysis that was used to obtain a simple expression for the output (and that related linearly to the input)?

$$V_{OUT} = V_{CC} - J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}} e^{\frac{V_M \sin(\omega t)}{V_t}}$$

$$V_{OUT} \cong \left[V_{CC} - I_{CQ} R_1\right] - \left(\frac{I_{CQ} R_1}{V_t}\right) V_M \sin(\omega t)$$

Linearization of the nonlinear output expression at the operating point

Operation with Small-Signal Inputs

$$\mathbf{I_{CQ} = J_S A_E e^{\frac{V_{beQ}}{V_t}}}$$

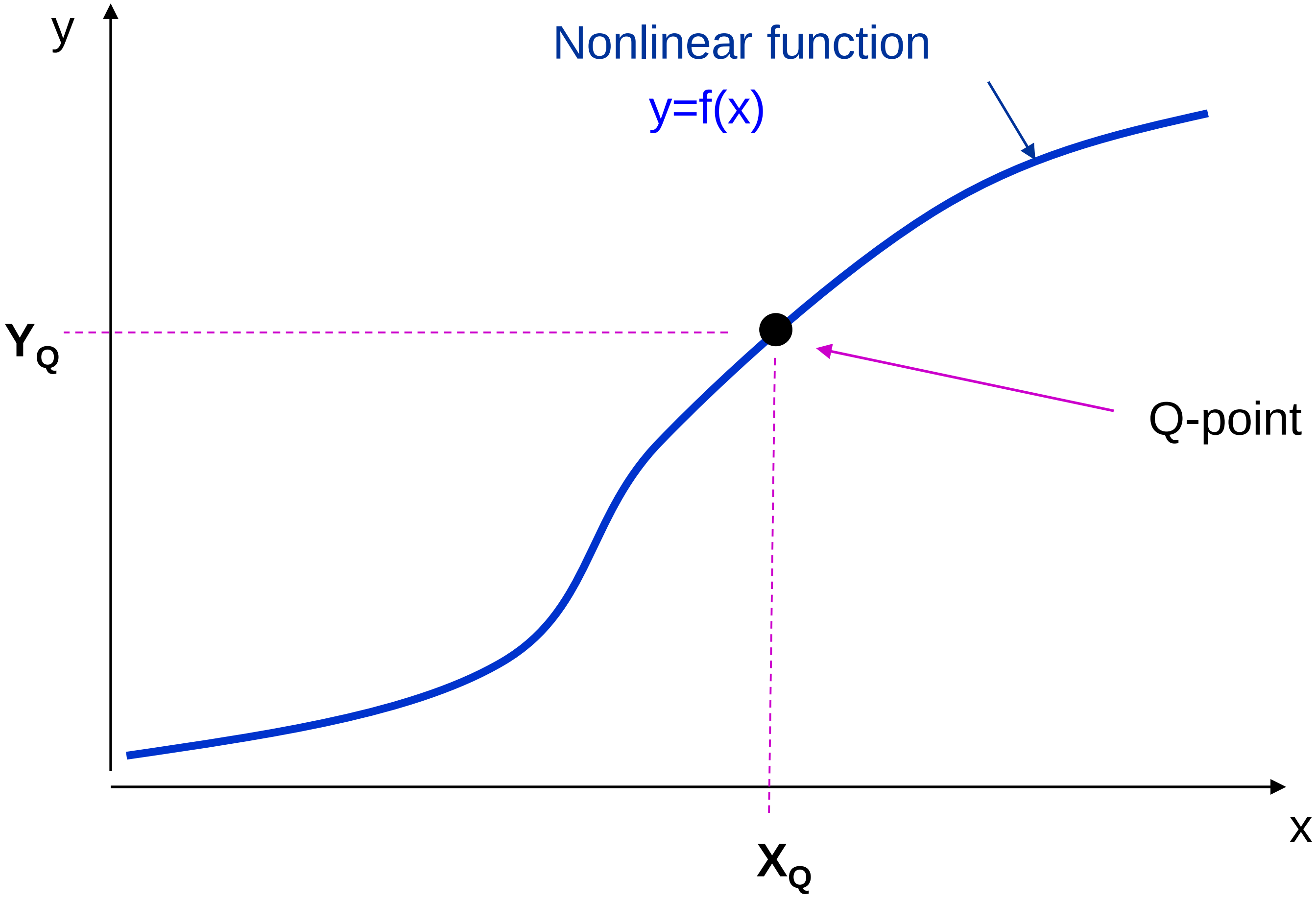
$$V_{OUT} \cong \underbrace{\boxed{\left[V_{CC} - I_{CQ}R_1\right]}}_{\textit{Quiescent Output}} - \boxed{\left(\frac{I_{CQ}R_1}{V_t}\right)} V_M \sin(\omega t)$$

ss Voltage Gain

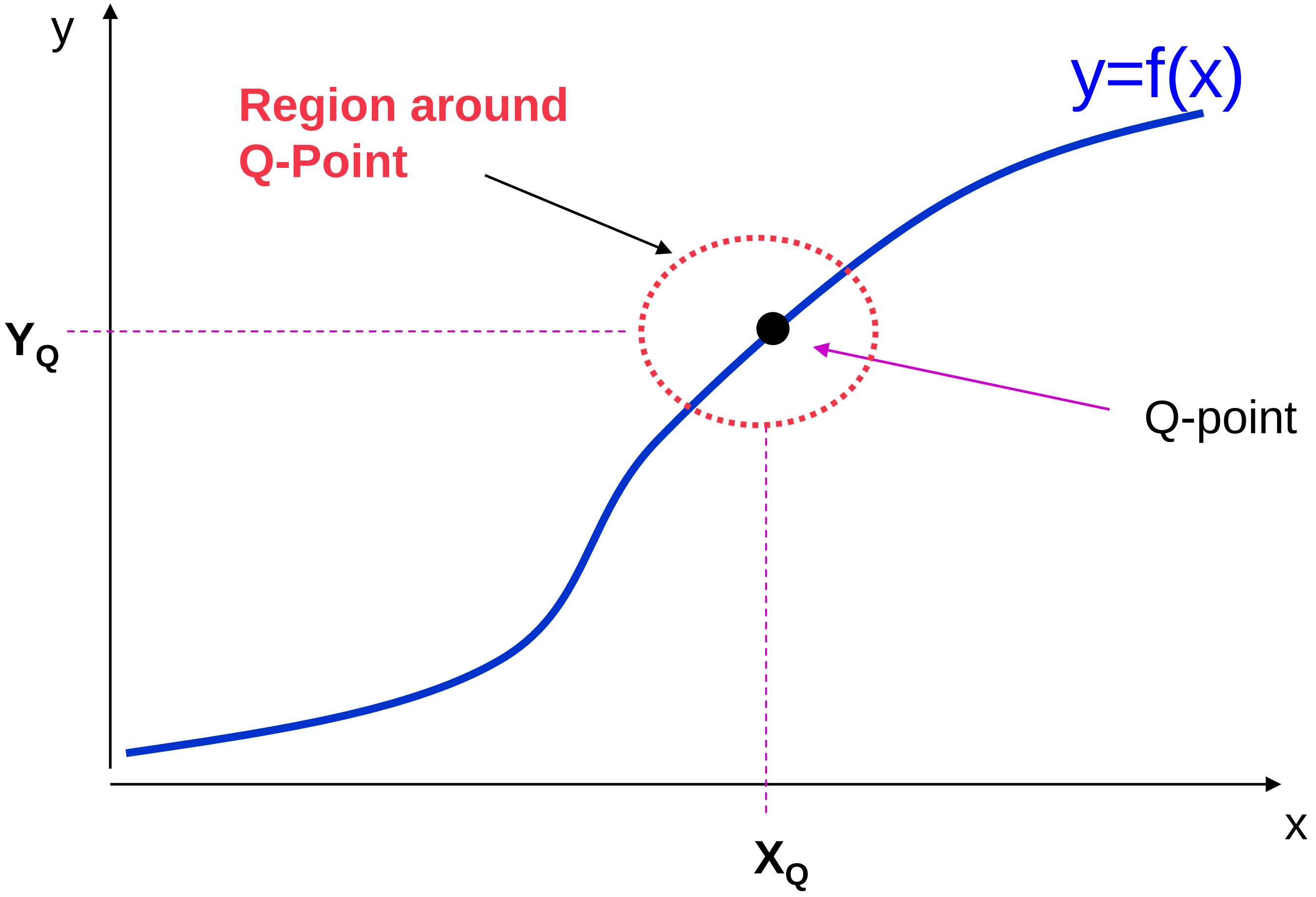
Small-signal analysis strategy

1. Obtain Quiescent Output (Q-point)
2. Linearize circuit at Q-point instead of linearize the nonlinear solution (this will be done by linearizing each component in the circuit)
1. Analyze linear “small-signal” circuit
2. Add quiescent and small-signal outputs to obtain good approximation to actual output

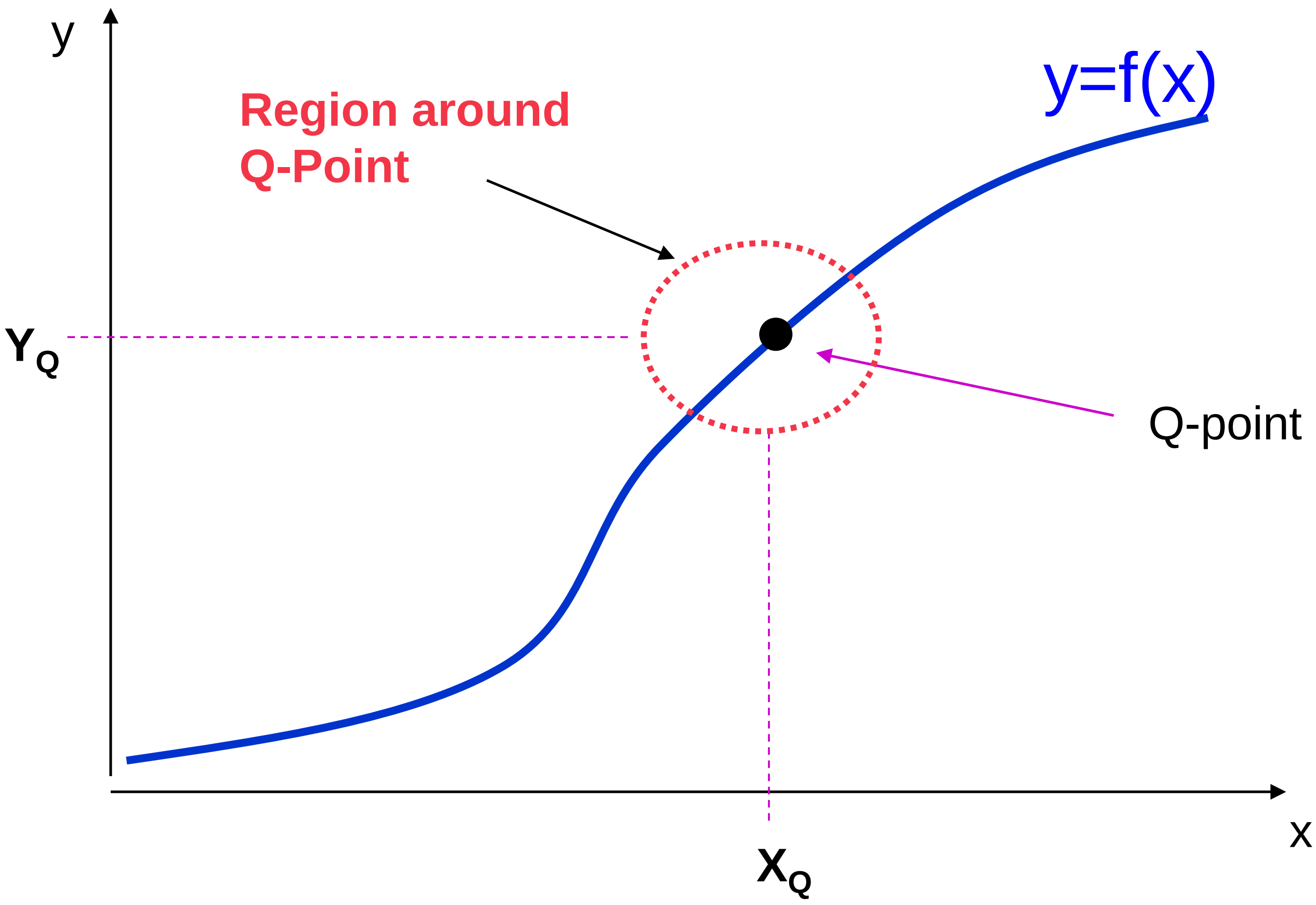
Small-Signal Principle



Small-Signal Principle

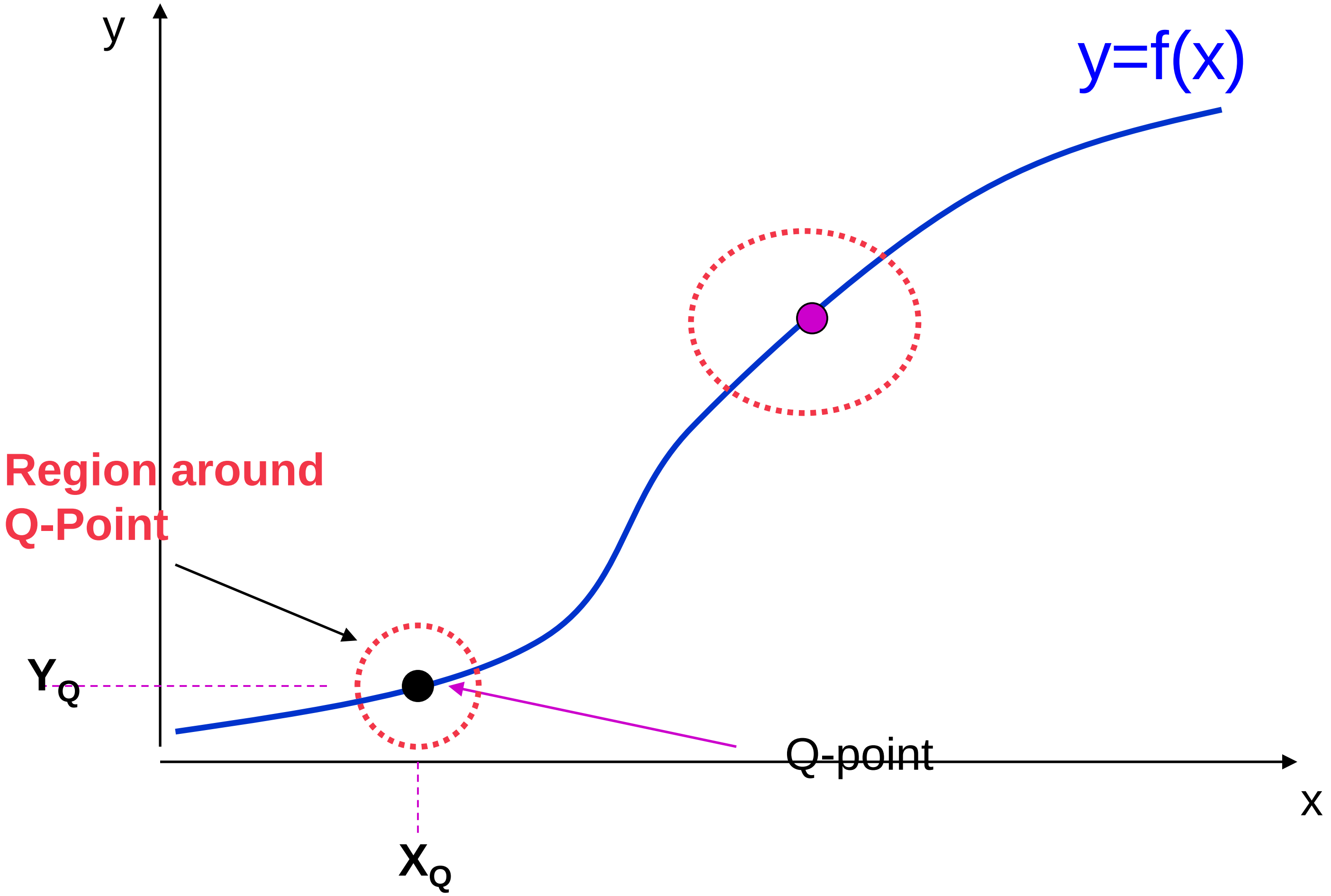


Small-Signal Principle



Relationship is nearly linear in a small enough region around Q-point
Region of linearity is often quite large
Linear relationship may be different for different Q-points

Small-Signal Principle



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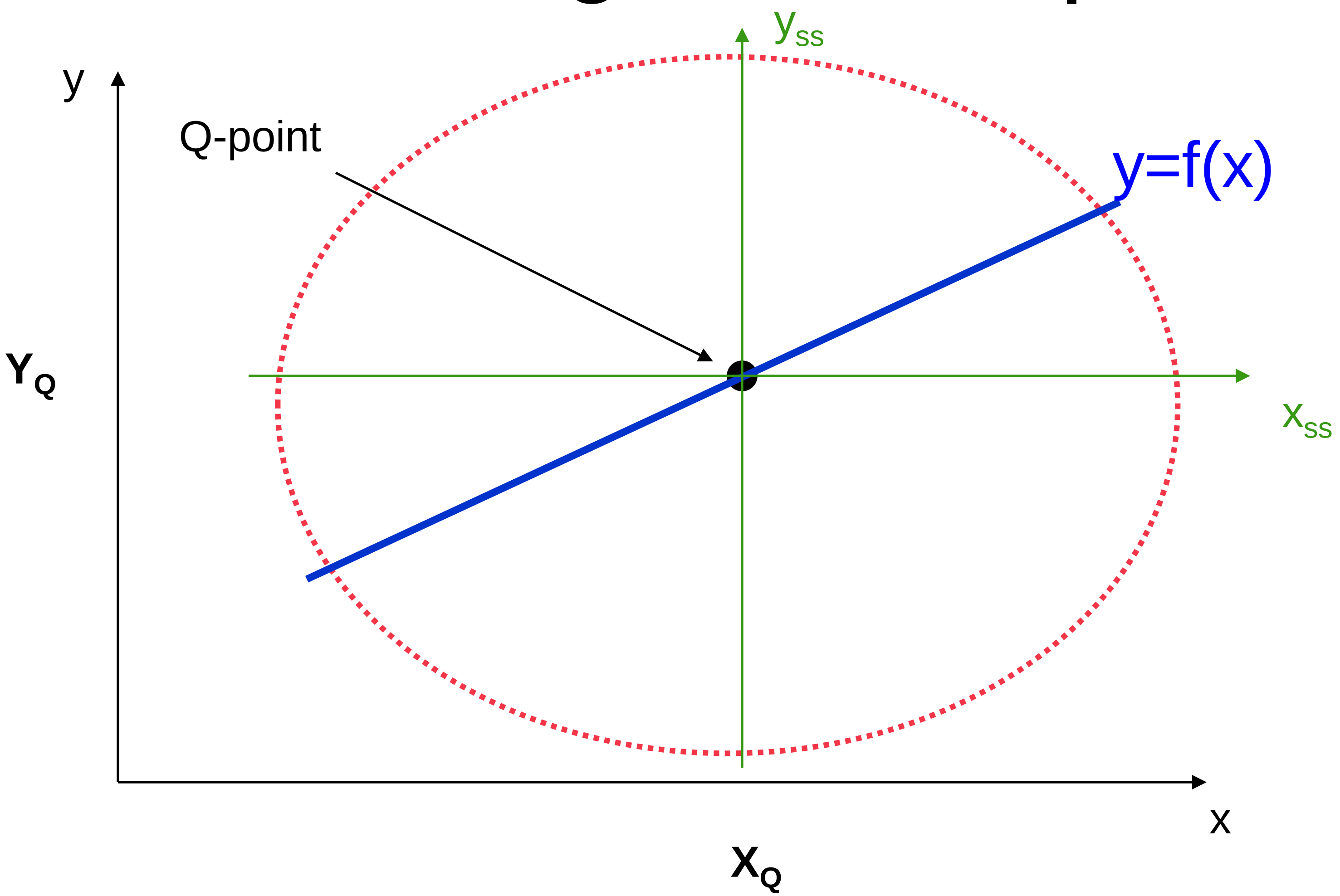
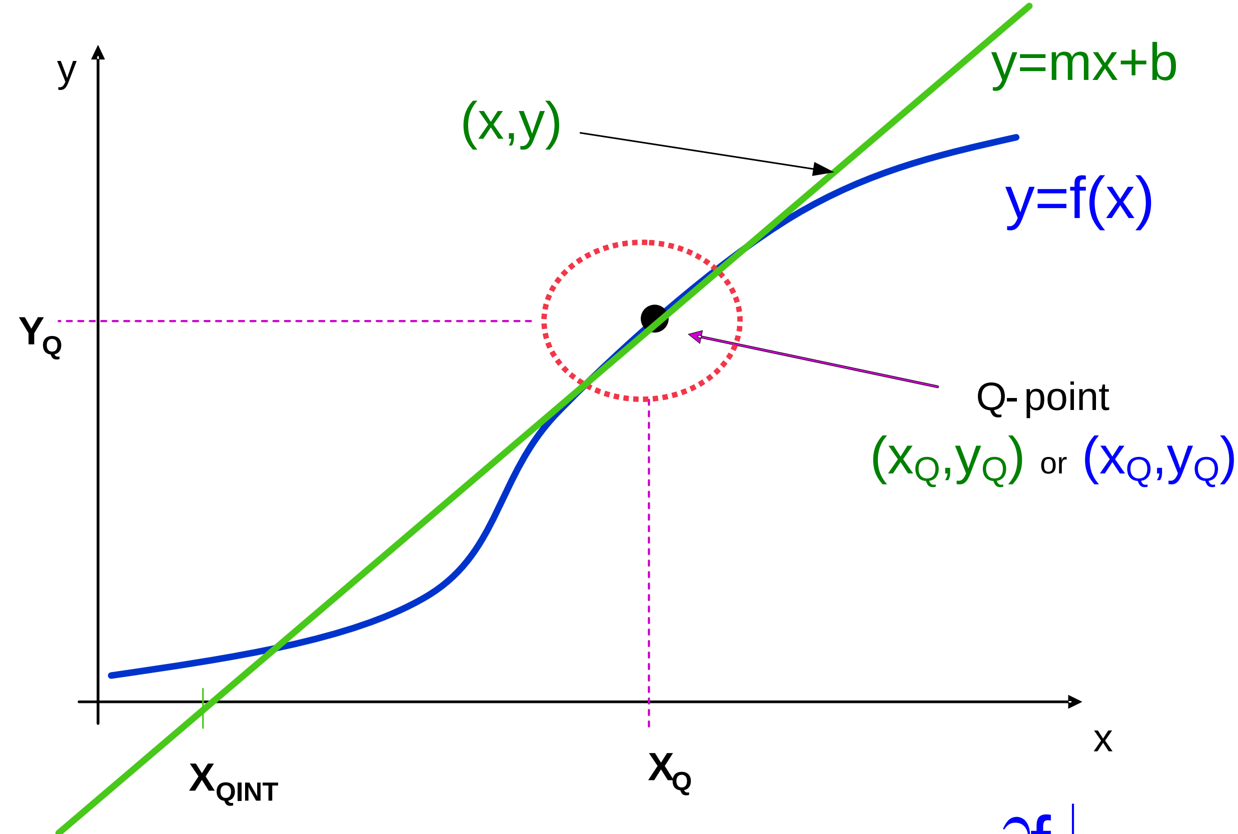
Small-Signal Principle



Device Behaves Linearly in Neighborhood of Q-Point
Can be characterized in terms of a small-signal coordinate system

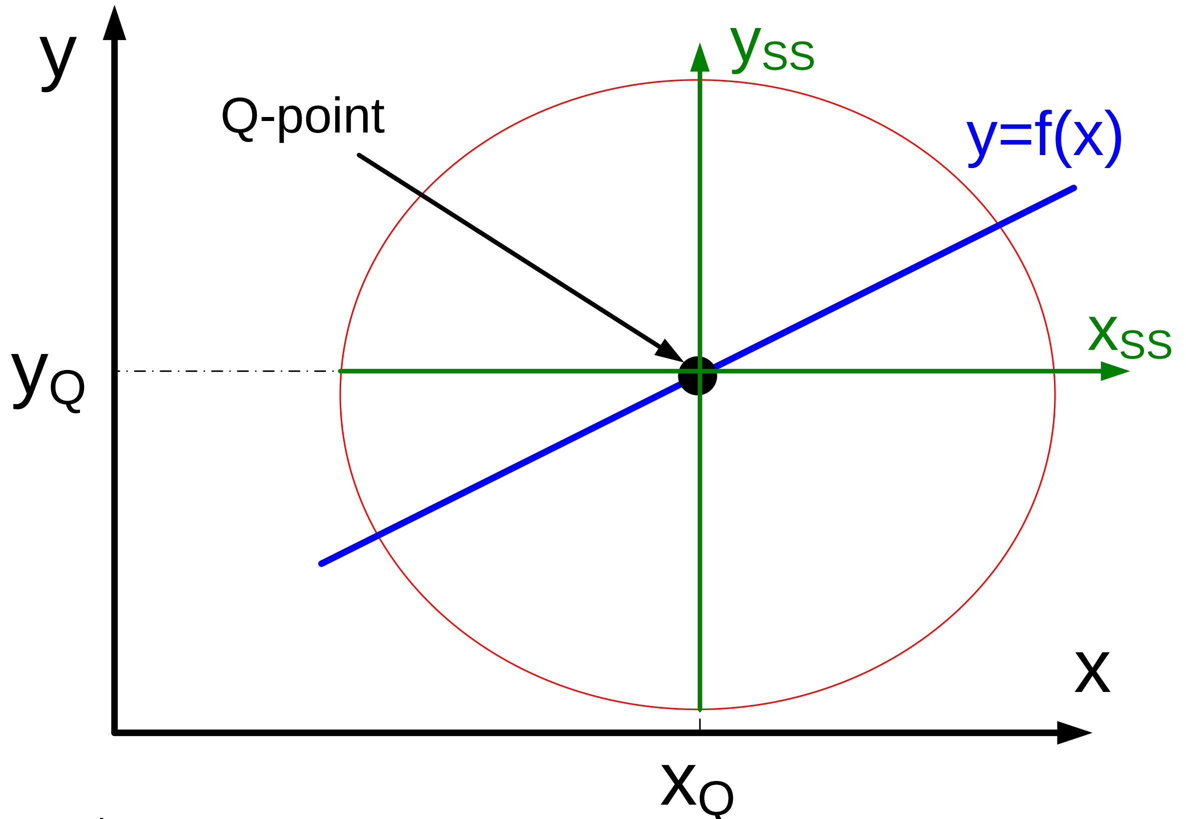
Small-Signal Principle



$$\frac{y-y_Q}{x-x_Q} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} \quad \Longrightarrow \quad y-y_Q = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q}\left(x-x_Q\right)$$

$$y = \left[\left.\frac{\partial f}{\partial x}\right|_{x=x_Q}\right]x + \left[y_Q - x_Q\left.\frac{\partial f}{\partial x}\right|_{x=x_Q}\right] \qquad m = \left[\left.\frac{\partial f}{\partial x}\right|_{x=x_Q}\right] \qquad b = \left[y_Q - x_Q\left.\frac{\partial f}{\partial x}\right|_{x=x_Q}\right]$$

Small-Signal Principle



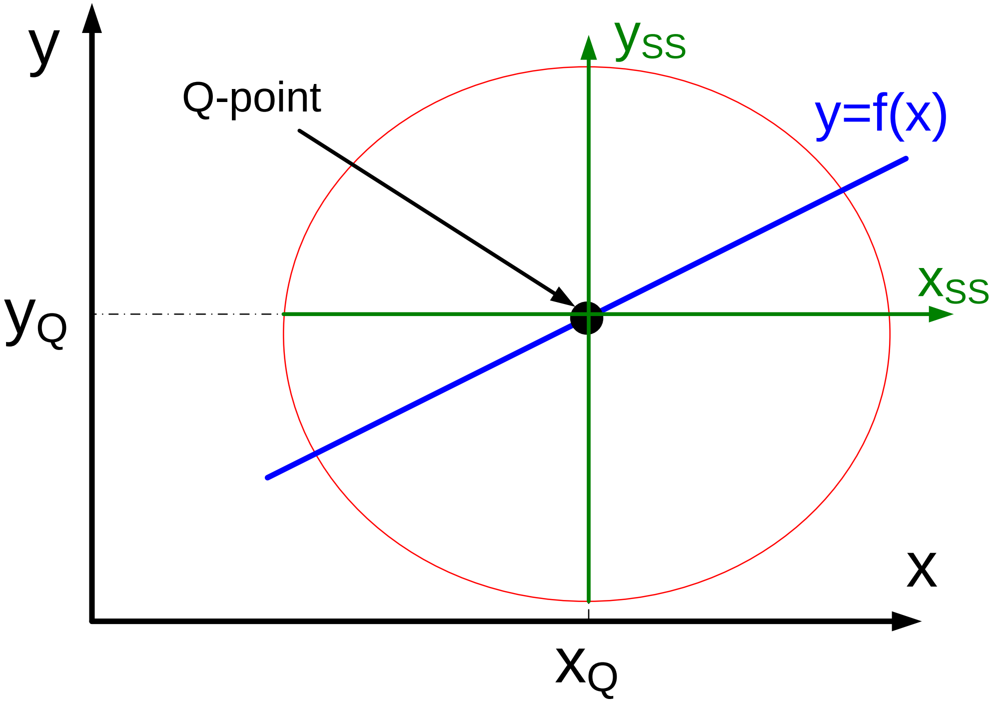
Changing coordinate systems:

$y_{SS}=y-y_Q$

$x_{SS}=x-x_Q$

$$y-y_Q=\left.\frac{\partial f}{\partial x}\right|_{x=x_Q}(x-x_Q) \longrightarrow y_{SS}=\left.\frac{\partial f}{\partial x}\right|_{x=x_Q}x_{SS}$$

Small-Signal Principle



Small-Signal Model:

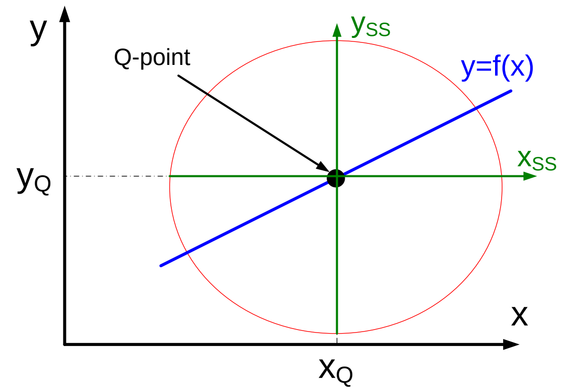
$$y_{ss} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} x_{ss}$$

- *Linearized model for the nonlinear function y=f(x)*
- *Valid in the region of the Q-point*
- *Will show the small signal model is simply Taylor's series expansion of f(x) at the Q-point truncated after first-order terms*

Small-Signal Principle

Observe:

$$y - y_Q = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} (x - x_Q) \quad \longrightarrow \quad y_{SS} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} x_{SS}$$



$$y_Q = f(x_Q)$$

$$y = f(x_Q) + \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} (x - x_Q)$$

Recall Taylors Series Expansion of nonlinear function f at expansion point x_0

$$y=f(x_0)+\sum_{k=1}^{\infty}\left(\frac{1}{k!}\left.\frac{df}{dx}\right|_{x=x_0}(x-x_0)^k\right)$$

Small-Signal Model:

Truncating after first-order terms (and defining "o" as "Q"):

$$y \cong f(x_Q) + \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} (x - x_Q) \quad \longleftrightarrow \quad y_{SS} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} x_{SS}$$

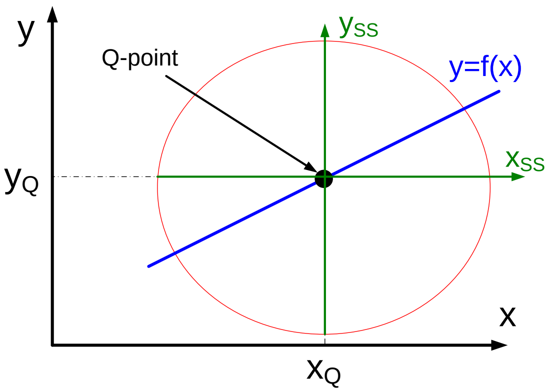
Mathematically, linearized model is simply Taylor's series expansion of the nonlinear function f at the Q-point truncated after first-order terms with notation $x_Q = x_0$

Small-Signal Principle

$$y = \boxed{f(x_Q)} + \boxed{\left.\frac{\partial f}{\partial x}\right|_{x=x_Q}} x_{SS}$$

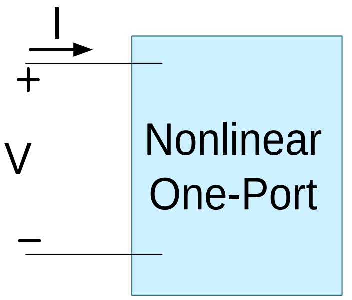
Quiescent Output

ss Gain

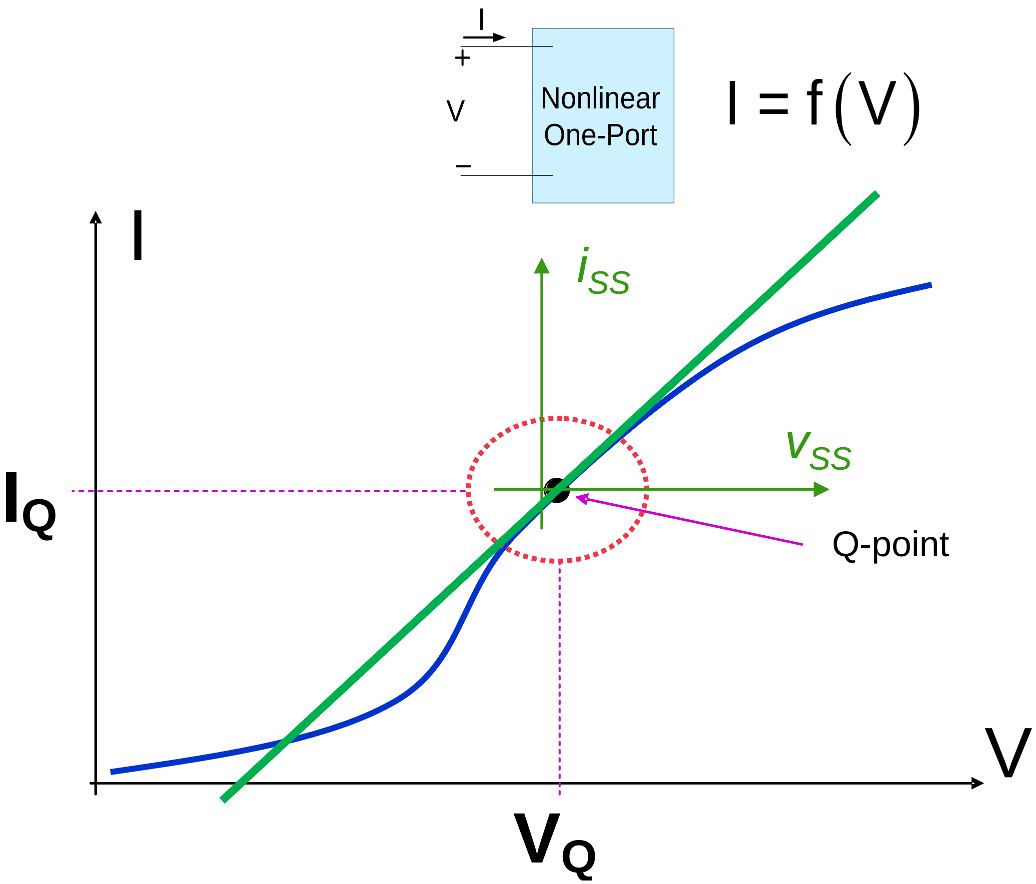


How can a **circuit** be linearized at an operating point as an alternative to linearizing a nonlinear function at an operating point?

Consider arbitrary nonlinear one-port network



Arbitrary Nonlinear One-Port



$$I = f(V)$$

$$i_{SS} = \left.\frac{\partial I}{\partial V}\right|_{V=V_Q} v_{SS}$$

$$i_{SS} \overset{\text{def}}{=} i$$

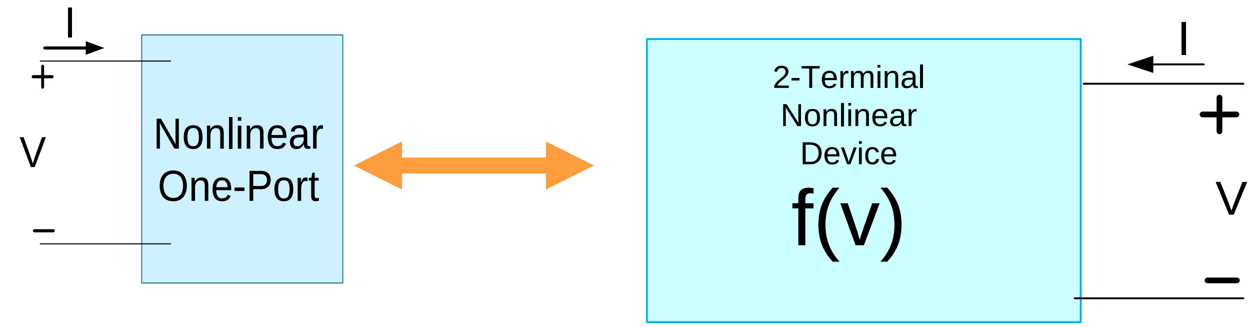
$$v_{SS} \overset{\text{def}}{=} v$$

$$y \overset{\text{defn}}{=} \left.\frac{\partial I}{\partial V}\right|_{V=V_Q}$$

Linear model of the nonlinear device at the Q-point

$$\boxed{i = y\, v}$$

Arbitrary Nonlinear One-Port

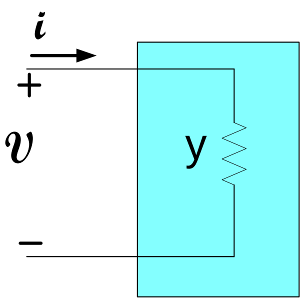


$$y = \left. \frac{\partial I}{\partial V} \right|_{V=V_Q}$$

Linear small-signal model:

$$\boxed{i = y\,v}$$

<u>A</u> Small Signal Equivalent Circuit:



- *The small-signal model of this 2-terminal electrical network is a resistor of value 1/y or a conductor of value y*
- ***One small-signal parameter** characterizes this one-port but it is dependent on Q-point*
- *This applies to **ANY** nonlinear one-port that is differentiable at a Q-point (e.g. a diode)*

End of Lecture 22