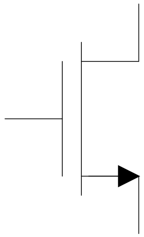
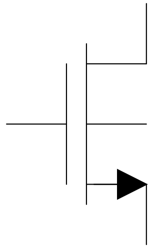
EE 330
Lecture 24

- Small Signal Analysis
  - SS Models for MOSFET
  - SS Models for BJT

Small Signal Model of MOSFET



3-terminal device



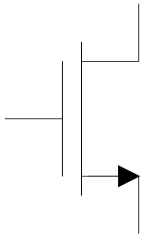
4-terminal device

MOSFET is actually a 4-terminal device but for many applications acceptable predictions of performance can be obtained by treating it as a 3-terminal device by neglecting the bulk terminal

In this course, we have been treating it as a 3-terminal device and in this lecture will develop the small-signal model by treating it as a 3-terminal device

When treated as a 4-terminal device, the bulk voltage introduces one additional term to the small signal model which is often either negligibly small or has no effect on circuit performance (will develop 4-terminal ss model later)

Small Signal Model of MOSFET

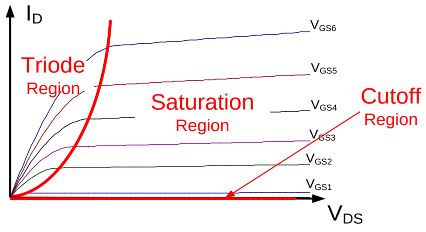


Large Signal Model

$$I_G = 0$$

3-terminal device

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \frac{W}{L}\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \frac{W}{2L}\left(V_{GS} - V_T\right)^2\left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$



MOSFET is usually operated in saturation region in linear applications where a small-signal model is needed so will develop the small-signal model in the saturation region

Small Signal Model of MOSFET

$$I_1 = f_1(V_1, V_2) \Longleftrightarrow I_G = 0$$

$$I_2 = f_2(V_1, V_2) \Longleftrightarrow I_D = \mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$$I_G = f_1(V_{GS}, V_{DS})$$

$$I_D = f_2(V_{GS}, V_{DS})$$

Small-signal model:

$$y_{ij} = \left. \frac{\partial f_i(V_1, V_2)}{\partial V_j} \right|_{\vec{V} = \vec{V}_Q}$$

$$y_{11} = \left. \frac{\partial I_G}{\partial V_{GS}} \right|_{\vec{V} = \vec{V}_Q} \qquad y_{12} = \left. \frac{\partial I_G}{\partial V_{DS}} \right|_{\vec{V} = \vec{V}_Q}$$

$$y_{21} = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{\vec{V} = \vec{V}_Q} \qquad y_{22} = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{\vec{V} = \vec{V}_Q}$$

Small Signal Model of MOSFET

$$I_G = 0$$

$$I_D = \mu C_{OX} \frac{W}{2L}\left(V_{GS} - V_T\right)^2\left(1 + \lambda V_{DS}\right)$$

Small-signal model:

$$y_{11} = \left.\frac{\partial I_G}{\partial V_{GS}}\right|_{\vec{V} = \vec{V}_Q} = ?$$

$$y_{12} = \left.\frac{\partial I_G}{\partial V_{DS}}\right|_{\vec{V} = \vec{V}_Q} = ?$$

$$y_{21} = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{\vec{V} = \vec{V}_Q} = ?$$

$$y_{22} = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{\vec{V} = \vec{V}_Q} = ?$$

Recall: termed the y-parameter model

Small Signal Model of MOSFET

$$I_1 = f_1(V_1, V_2) \quad \Longleftrightarrow \quad I_G = 0$$

$$I_2 = f_2(V_1, V_2) \quad \Longleftrightarrow \quad I_D = \mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

Small-signal model:

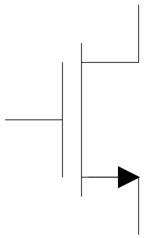
$$y_{11} = \left.\frac{\partial I_G}{\partial V_{GS}}\right|_{\vec{V} = \vec{V}_Q} = 0 \qquad\qquad y_{12} = \left.\frac{\partial I_G}{\partial V_{DS}}\right|_{\vec{V} = \vec{V}_Q} = 0$$

$$y_{21} = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{\vec{V} = \vec{V}_Q} = \left. 2\mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)(1 + \lambda V_{DS})\right|_{\vec{V} = \vec{V}_Q} = \frac{2I_D}{(V_{GSQ} - V_T)}$$

$$y_{21} \cong \mu C_{OX} \frac{W}{L}(V_{GSQ} - V_T)$$

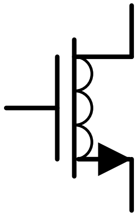
$$y_{22} = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{\vec{V} = \vec{V}_Q} = \left.\mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2 \lambda\right|_{\vec{V} = \vec{V}_Q} \cong \lambda I_{DQ}$$

Small Signal Model of MOSFET



$$I_G = 0$$

$$I_D = \mu C_{OX} \frac{W}{2L} \left(V_{GS} - V_T\right)^2 \left(1 + \lambda V_{DS}\right)$$



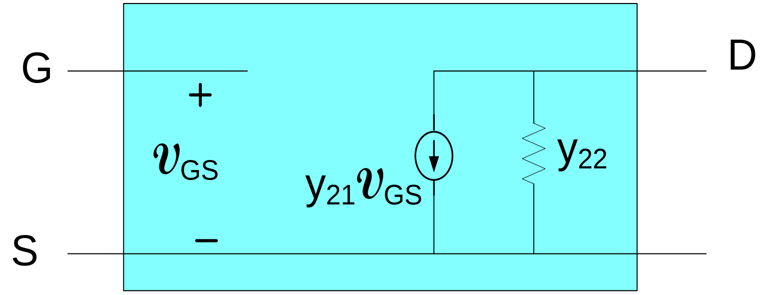
$$y_{12} = 0$$

$$y_{11} = 0$$

$$y_{21} \cong \frac{2I_{DQ}}{V_{GSQ} - V_T}$$

$$y_{22} \cong \lambda I_{DQ}$$

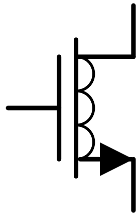
$$i_G = y_{11} v_{GS} + y_{12} v_{DS}$$
$$i_D = y_{21} v_{GS} + y_{22} v_{DS}$$



An equivalent circuit

(y-parameter model)

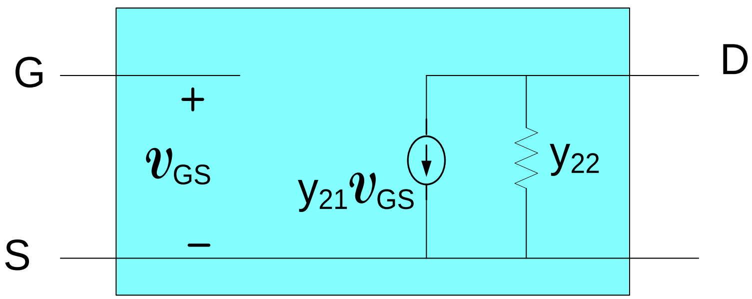
Small-Signal Model of MOSFET



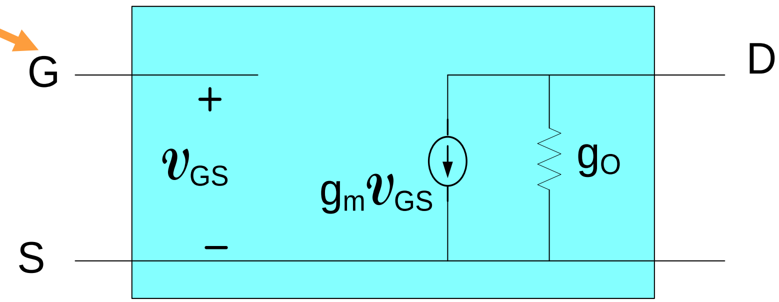
by convention, $y_{21}=g_m$, $y_{22}=g_0$

$$\therefore \quad y_{21} \cong g_m = \mu C_{OX}\frac{W}{L}(V_{GSQ} - V_T)$$

$$y_{22} = g_o \cong \lambda I_{DQ}$$



(y-parameter model)



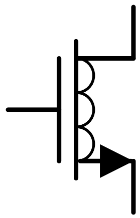
$$i_G = 0$$

$$i_D = g_m v_{GS} + g_o v_{DS}$$

Note: g_o vanishes when $\lambda=0$

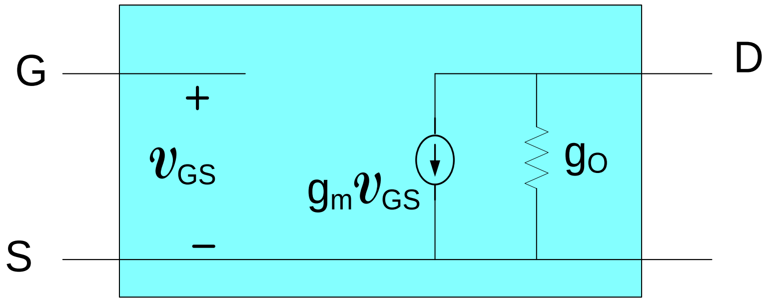
still y-parameter model
but use “g” parameter notation

Small-Signal Model of MOSFET



$$g_m = \mu C_{OX} \frac{W}{L}\left(V_{GSQ} - V_T\right)$$

$$g_o \cong \lambda I_{DQ}$$



Alternate equivalent expressions for g_m:

$$I_{DQ} = \mu C_{OX} \frac{W}{2L}\left(V_{GSQ} - V_T\right)^2 \left(1 + \lambda V_{DSQ}\right) \cong \mu C_{OX} \frac{W}{2L}\left(V_{GSQ} - V_T\right)^2$$

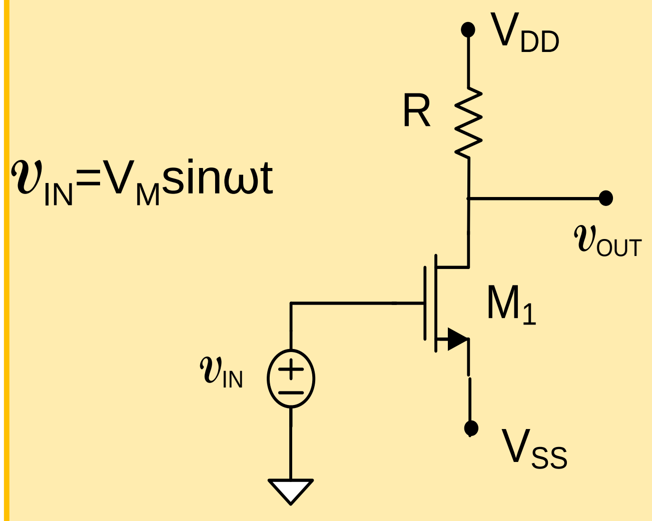
$$g_m = \mu C_{OX} \frac{W}{L}\left(V_{GSQ} - V_T\right)$$

$$g_m = \sqrt{2\mu C_{OX} \frac{W}{L}} \bullet \sqrt{I_{DQ}}$$

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T}$$

Consider again:

Small-signal analysis example

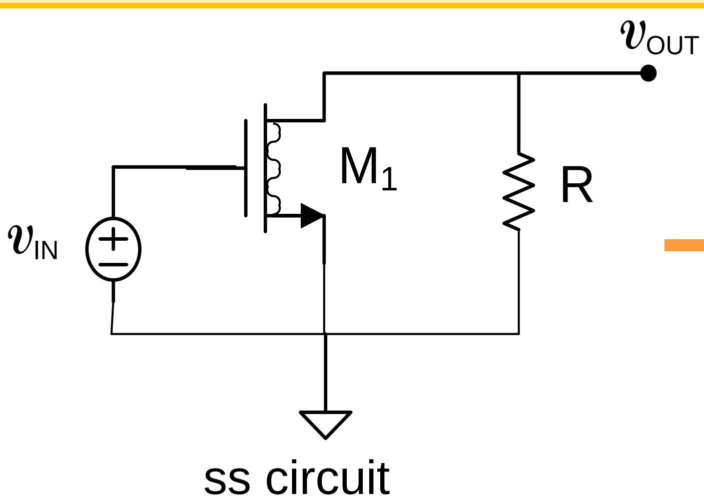


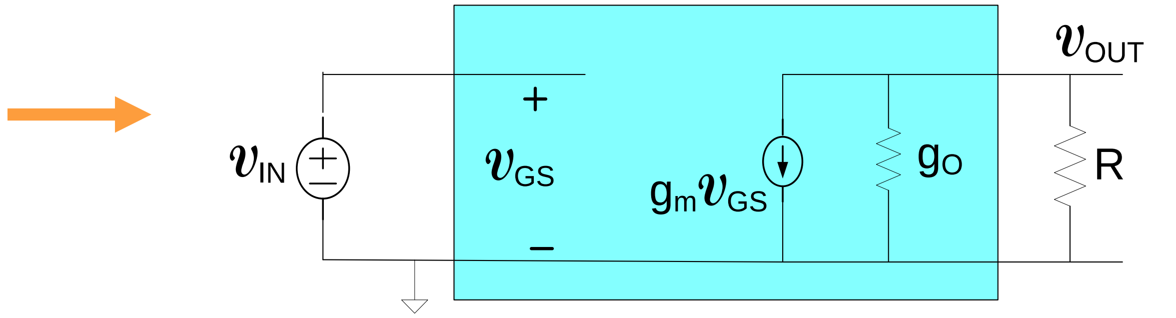
$$A_v = \frac{2I_{DQ}R}{[V_{SS} + V_T]}$$

Derived for λ=0 (equivalently g_0=0)

$$I_D = \mu C_{OX}\frac{W}{2L}(V_{GS} - V_T)^2$$

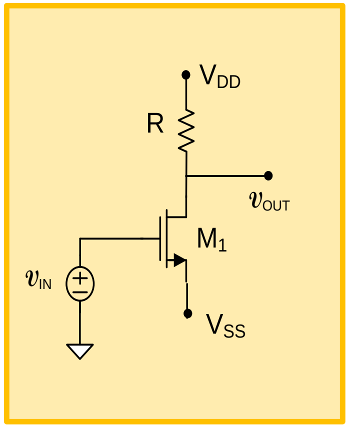
Recall the derivation was very tedious and time consuming!

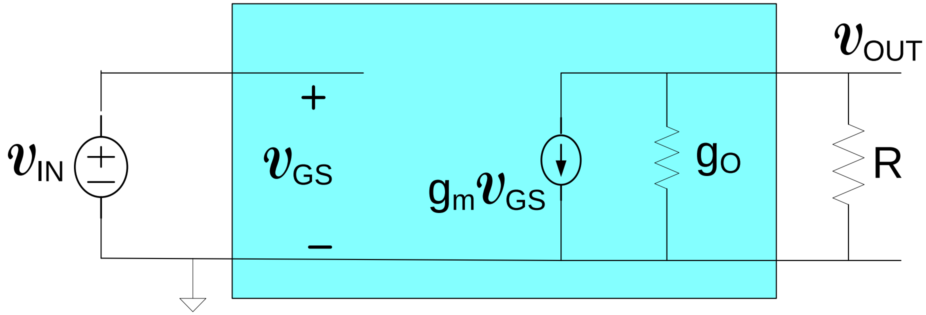




Consider again:

Small-signal analysis example

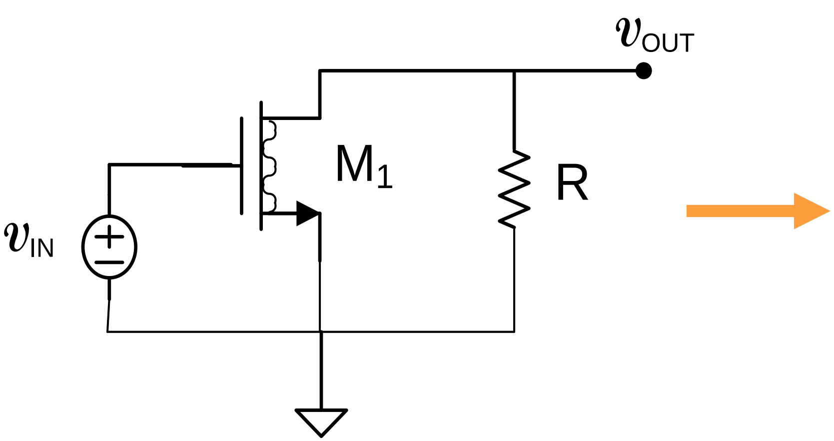




$$A_V = \frac{V_{OUT}}{V_{IN}} = -\frac{g_m}{g_O + 1/R}$$

This gain is expressed in terms of small-signal model parameters

For λ=0, $g_O = \lambda I_{DQ} = 0$



$$A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R$$

but

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T}$$

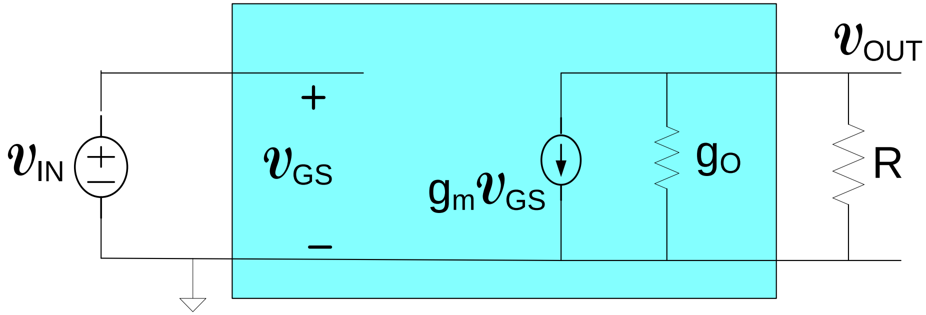
$V_{GSQ} = -V_{SS}$

thus

$$A_v = \frac{2I_{DQ} R}{[V_{SS} + V_T]}$$

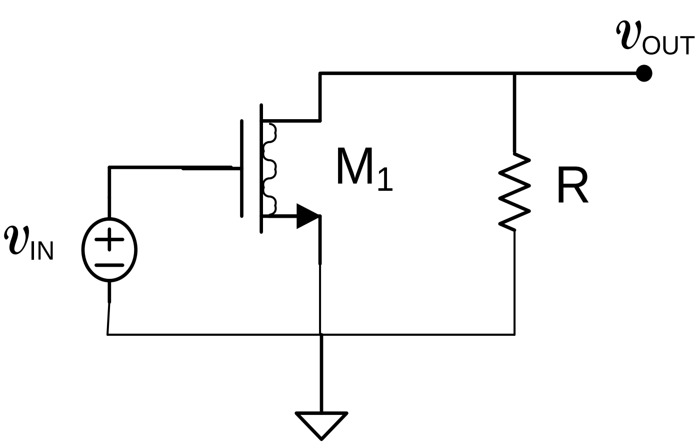
Consider again:

Small-signal analysis example



$$A_V = \frac{V_{OUT}}{V_{IN}} = -\frac{g_m}{g_O + 1/R}$$

For λ=0, $g_O = \lambda I_{DQ} = 0$

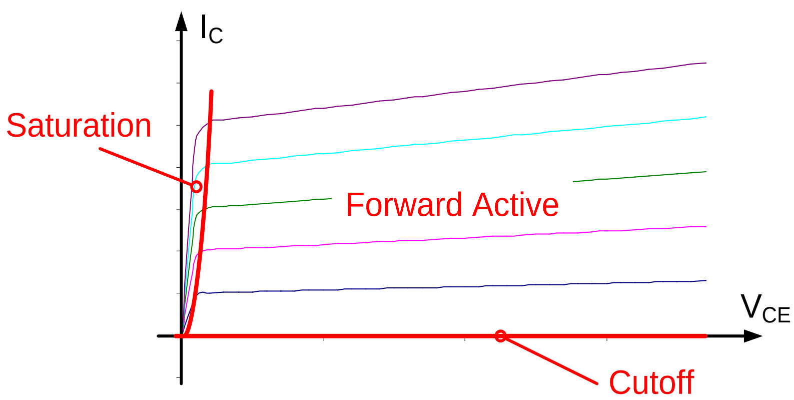


$$A_v = \frac{2I_{DQ}R}{[V_{SS} + V_T]}$$

- Same expression as derived before !
- More accurate gain can be obtained if λ effects are included and does not significantly increase complexity of small-signal analysis

Small Signal Model of BJT

3-terminal device



Forward Active Model:

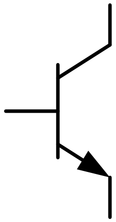
$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

$$\mathbf{I_B} = \frac{\mathbf{J_S A_E}}{\boldsymbol{\beta}} \mathbf{e}^{\frac{\mathbf{V_{BE}}}{\mathbf{V_t}}}$$

- Usually operated in Forward Active Region when small-signal model is needed
- Will develop small-signal model in Forward Active Region

Small Signal Model of BJT

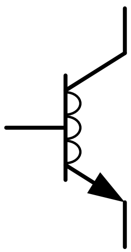
Nonlinear model:



$$I_1 = f_1(V_1, V_2) \quad \Longleftrightarrow \quad I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$I_2 = f_2(V_1, V_2) \quad \Longleftrightarrow \quad I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

Small-signal model:



$$i_B = y_{11} v_{BE} + y_{12} v_{CE}$$

$$i_C = y_{21} v_{BE} + y_{22} v_{CE}$$

$$y_{ij} = \left. \frac{\partial f_i(V_1, V_2)}{\partial V_j} \right|_{\vec{V} = \vec{V}_Q}$$

y-parameter model

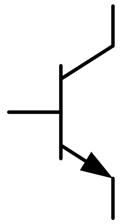
$$y_{11} = g_\pi = \left. \frac{\partial I_B}{\partial V_{BE}} \right|_{\vec{V} = \vec{V}_Q} \qquad y_{12} = \left. \frac{\partial I_B}{\partial V_{CE}} \right|_{\vec{V} = \vec{V}_Q}$$

$$y_{21} = g_m = \left. \frac{\partial I_C}{\partial V_{BE}} \right|_{\vec{V} = \vec{V}_Q} \qquad y_{22} = g_o = \left. \frac{\partial I_C}{\partial V_{CE}} \right|_{\vec{V} = \vec{V}_Q}$$

Note: g_m, g_π and g_o used for notational consistency with legacy terminology

Small Signal Model of BJT

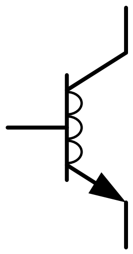
Nonlinear model:



$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

Small-signal model:



$$i_B = y_{11} v_{BE} + y_{12} v_{CE}$$

$$i_C = y_{21} v_{BE} + y_{22} v_{CE}$$

$$y_{ij} = \left. \frac{\partial f_i(V_1, V_2)}{\partial V_j} \right|_{\vec{V} = \vec{V}_Q}$$

$$y_{11} = g_\pi = \left. \frac{\partial I_B}{\partial V_{BE}} \right|_{\vec{V} = \vec{V}_Q} = ?$$

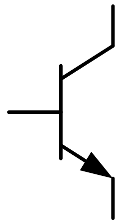
$$y_{12} = \left. \frac{\partial I_B}{\partial V_{CE}} \right|_{\vec{V} = \vec{V}_Q} = ?$$

$$y_{21} = g_m = \left. \frac{\partial I_C}{\partial V_{BE}} \right|_{\vec{V} = \vec{V}_Q} = ?$$

$$y_{22} = g_o = \left. \frac{\partial I_C}{\partial V_{CE}} \right|_{\vec{V} = \vec{V}_Q} = ?$$

Small Signal Model of BJT

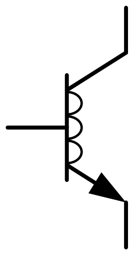
Nonlinear model:



$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

Small-signal model:



$$i_B = y_{11} v_{BE} + y_{12} v_{CE}$$

$$i_C = y_{21} v_{BE} + y_{22} v_{CE}$$

$$y_{11} = g_\pi = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{\vec{V}=\vec{V}_Q} = \left.\frac{1}{V_t}\frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}\right|_{\vec{V}=\vec{V}_Q} = \frac{I_{BQ}}{V_t} \cong \frac{I_{CQ}}{\beta V_t}$$

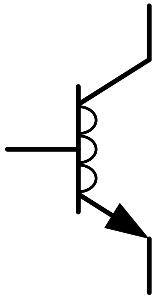
$$y_{21} = g_m = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{\vec{V}=\vec{V}_Q} = \left.\frac{1}{V_t} J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)\right|_{\vec{V}=\vec{V}_Q} = \frac{I_{CQ}}{V_t}$$

$$y_{12} = \left.\frac{\partial I_B}{\partial V_{CE}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

$$y_{22} = g_o = \left.\frac{\partial I_C}{\partial V_{CE}}\right|_{\vec{V}=\vec{V}_Q} = \left.\frac{J_S A_E e^{\frac{V_{BE}}{V_t}}}{V_{AF}}\right|_{\vec{V}=\vec{V}_Q} \cong \frac{I_{CQ}}{V_{AF}}$$

Note: usually prefer to express in terms of I_{CQ}

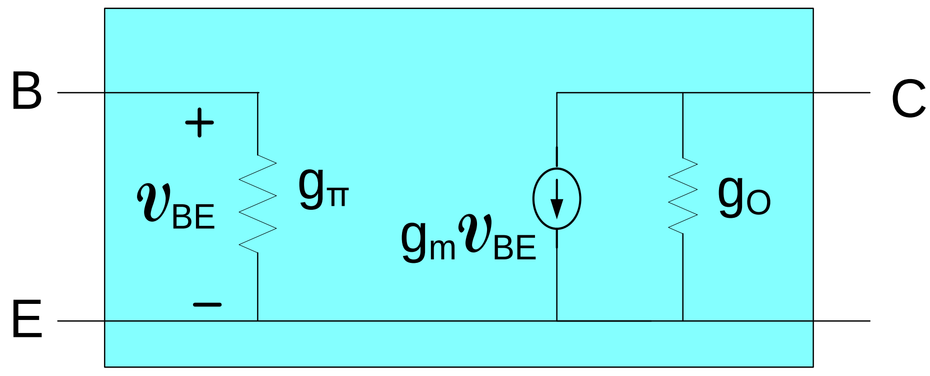
Small Signal Model of BJT



$$i_B = y_{11} v_{BE} + y_{12} v_{CE}$$
$$i_C = y_{21} v_{BE} + y_{22} v_{CE}$$

$$i_B = g_\pi v_{BE}$$
$$i_C = g_m v_{BE} + g_o v_{CE}$$

$$g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_m = \frac{I_{CQ}}{V_t} \qquad g_o = \frac{I_{CQ}}{V_{AF}}$$

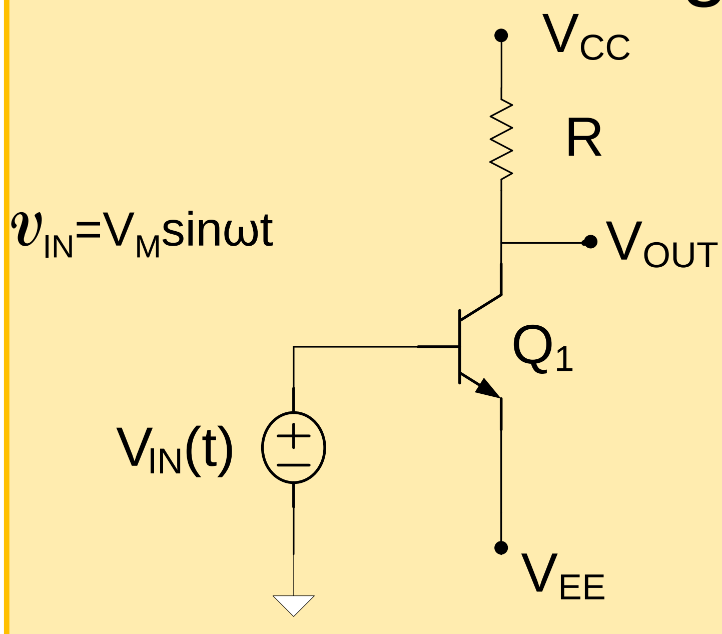


An equivalent circuit

y-parameter model using “g” parameter notation

Consider again:

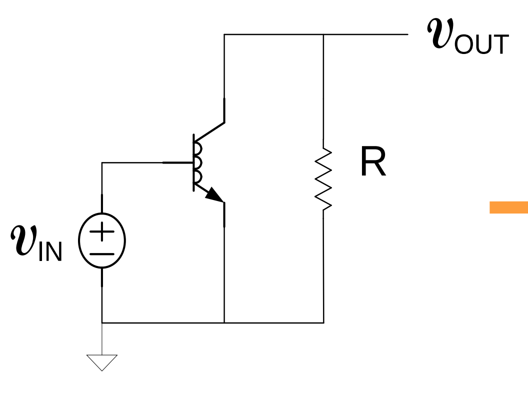
Small signal analysis example



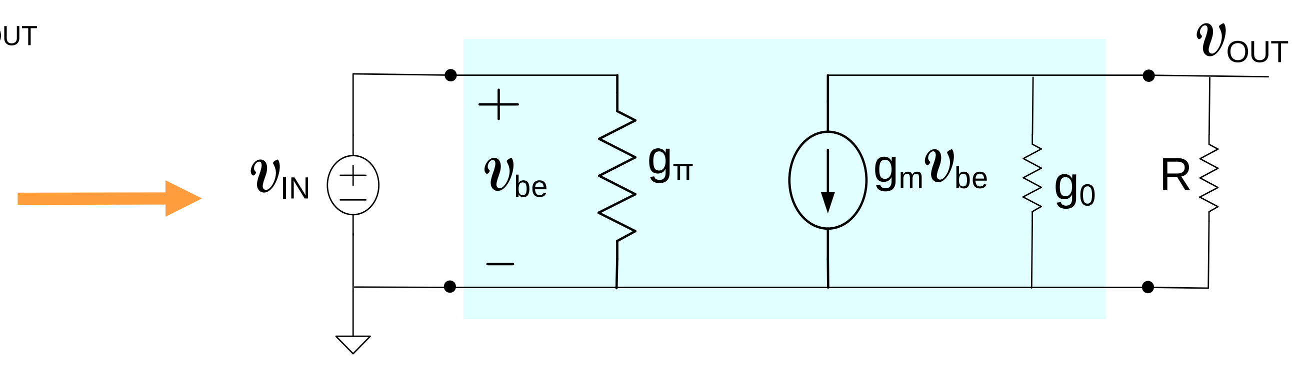
$$A_{VB} = -\frac{I_{CQ}R}{V_t}$$

Derived for $V_{AF}=0$ (equivalently $g_o=0$)

Recall the derivation was very tedious and time consuming!

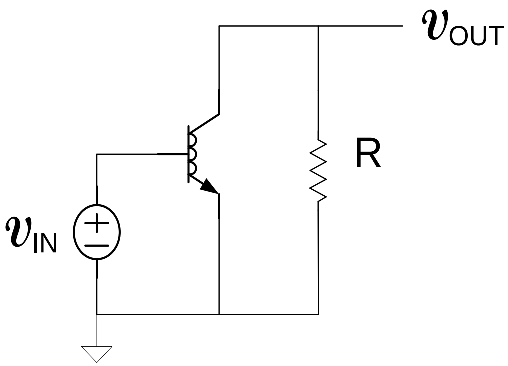


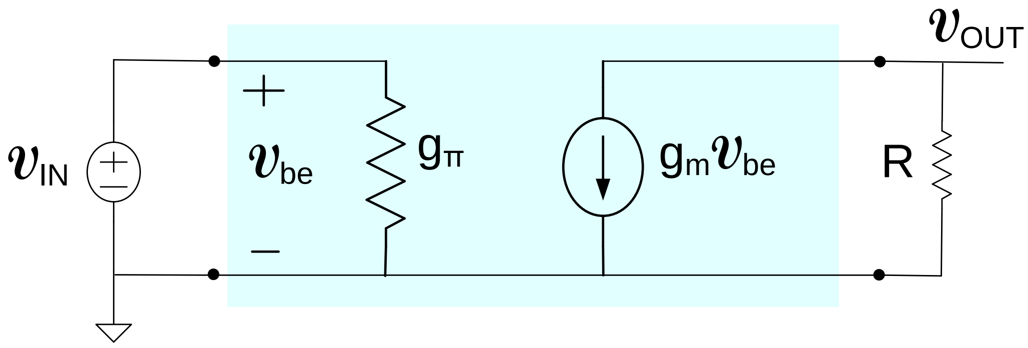
ss circuit



Neglect V_{AF} effects (i.e. $V_{AF}=\infty$) to be consistent with earlier analysis

$$g_o = \frac{I_{CQ}}{V_{AF}} \underset{V_{AF}=\infty}{=} 0$$





$$\left.\begin{aligned} v_{OUT} &= -g_m R v_{BE} \\ v_{IN} &= v_{BE} \end{aligned}\right\}$$

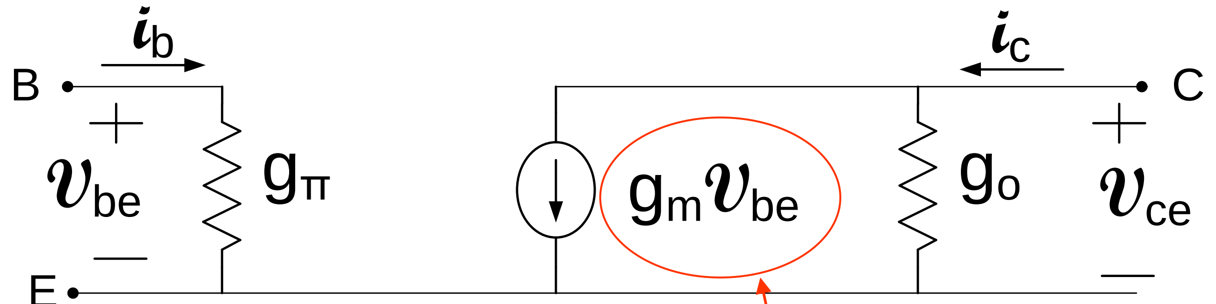
$$A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R$$

$$g_m = \frac{I_{CQ}}{V_t}$$

$$A_V = -\frac{I_{CQ} R}{V_t}$$

Note this is identical to what was obtained with the direct nonlinear analysis

Small Signal BJT Model – alternate representation



$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Observe :

$$g_\pi v_{be} = i_b$$

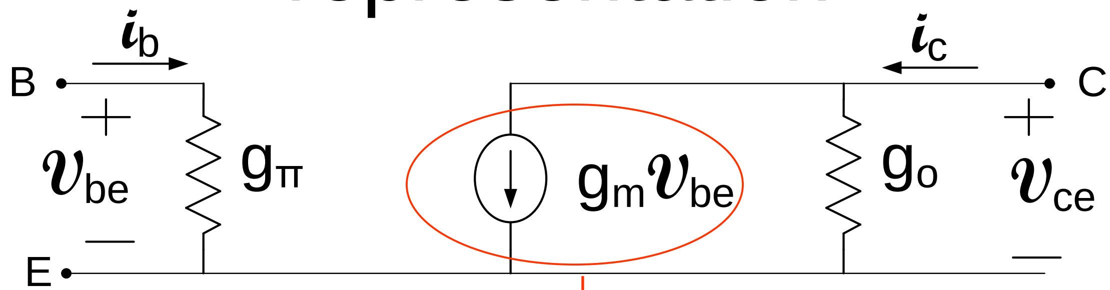
$$g_m v_{be} = i_b \frac{g_m}{g_\pi}$$

$$\frac{g_m}{g_\pi} = \frac{\left[\dfrac{I_Q}{V_t}\right]}{\left[\dfrac{I_Q}{\beta V_t}\right]} = \beta$$

$$g_m v_{be} = \beta\, i_b$$

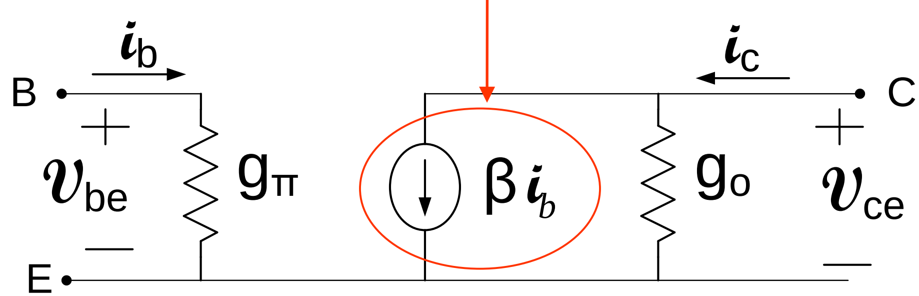
Can replace the voltage dependent current source with a current dependent current source

Small Signal BJT Model – alternate representation



$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

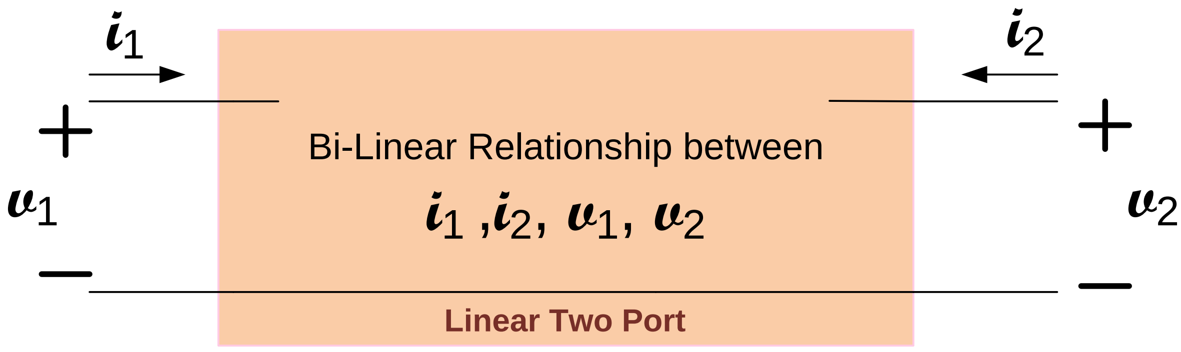
Alternate equivalent small signal model



$$g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

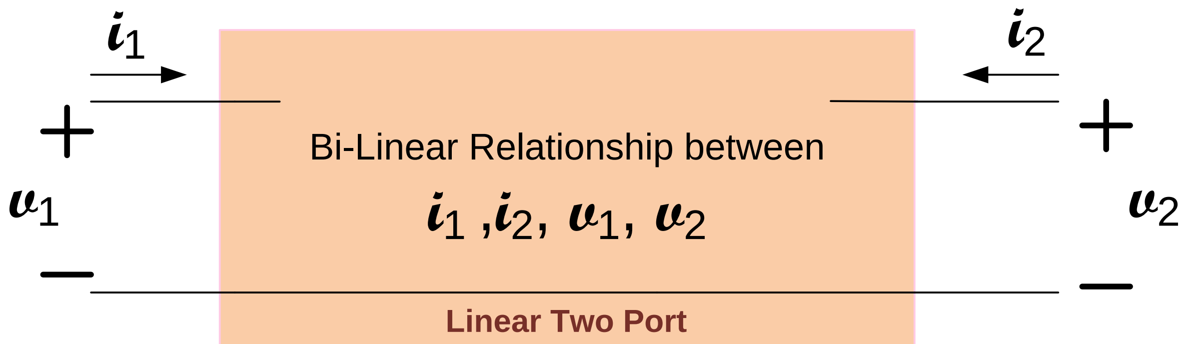
Small-Signal Model Representations

(3-terminal network – also relevant with 4-terminal networks)



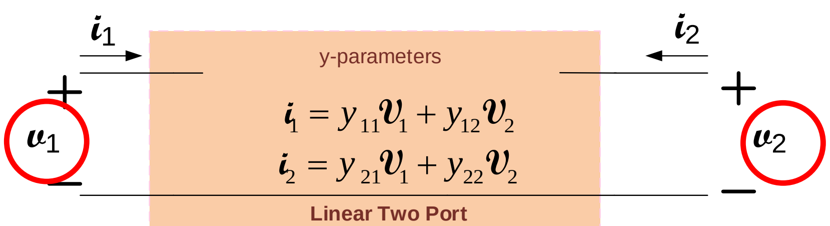
- Have developed small-signal models for the MOSFET and BJT
- Models have been based upon arbitrary assumption that v_1, v_2 are independent variables
- Models are y-parameter models expressed in terms of "g" parameters
- Have already seen some alternatives for "parameter" definitions in these models
- Alternative representations are sometimes used

Small-Signal Model Representations

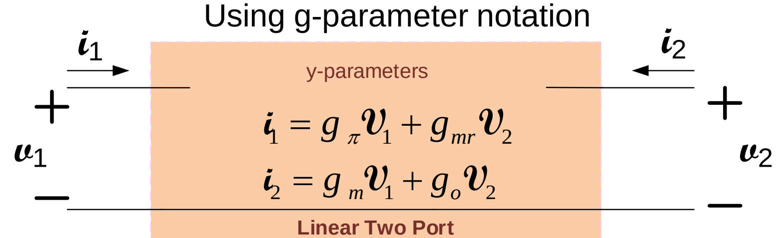


The good, the bad, and the unnecessary !!

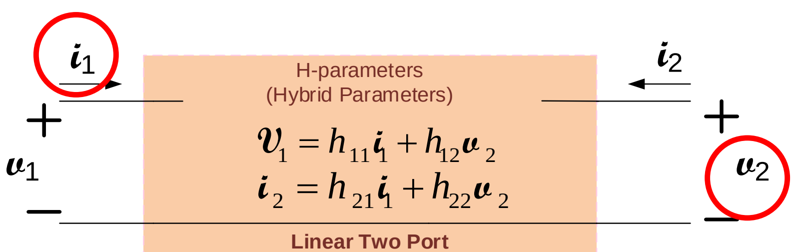
what we have developed:



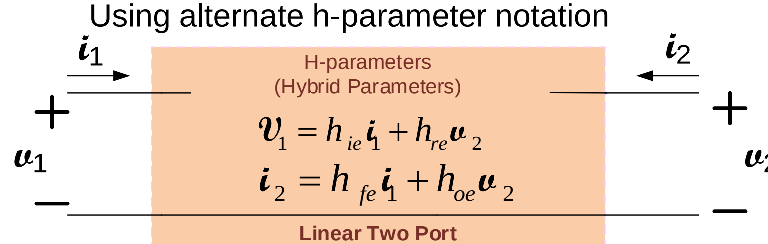
Using g-parameter notation



The hybrid parameters:

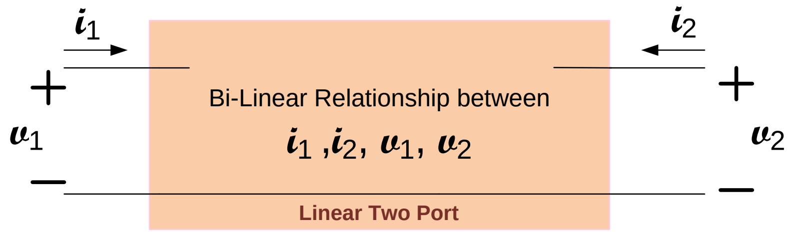


Using alternate h-parameter notation

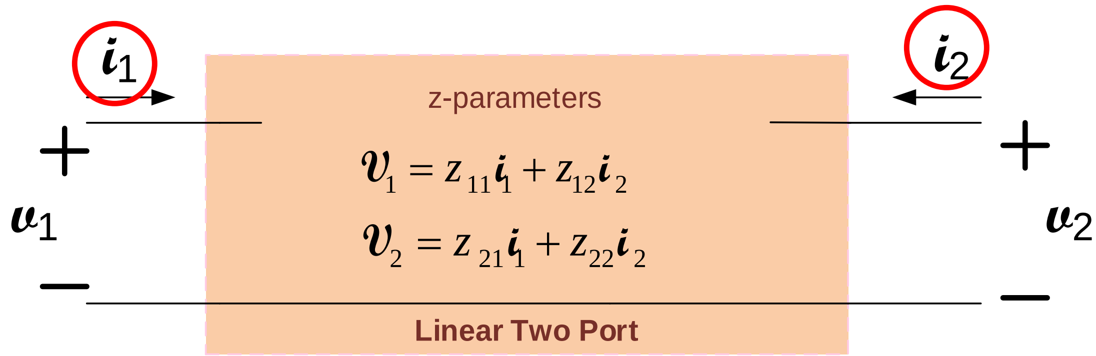


Independent parameters

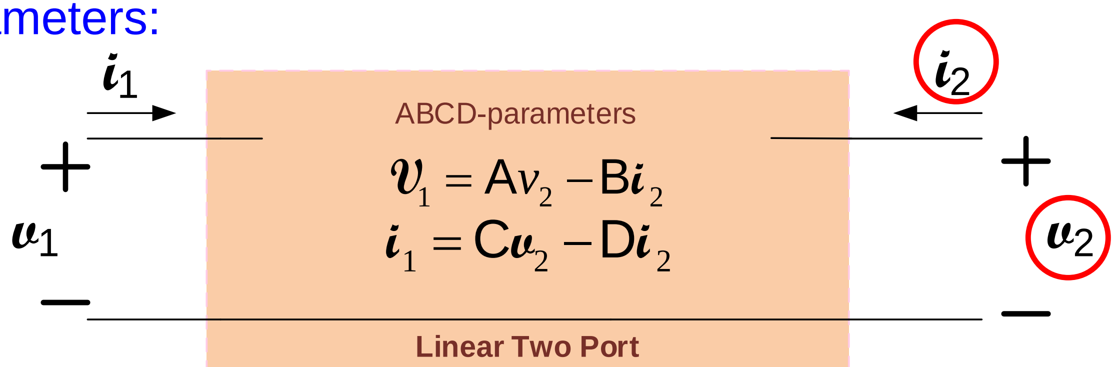
Small-Signal Model Representations



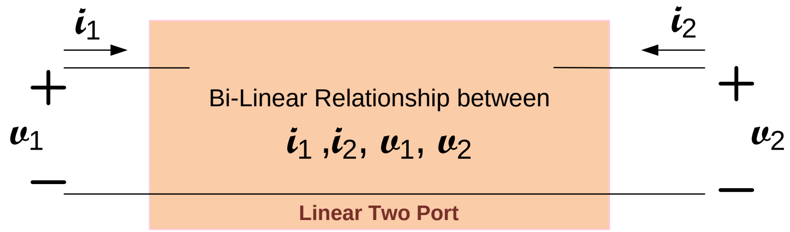
The z-parameters



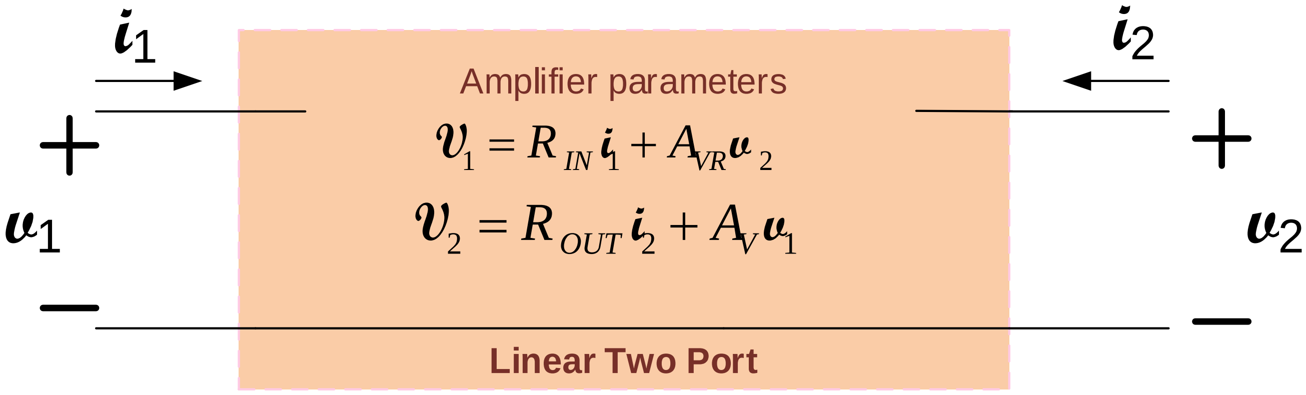
The ABCD parameters:



Small-Signal Model Representations

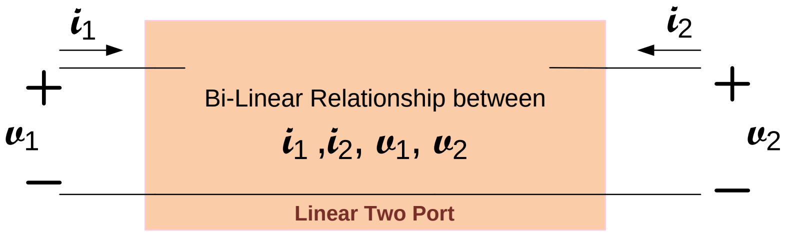


Amplifier parameters

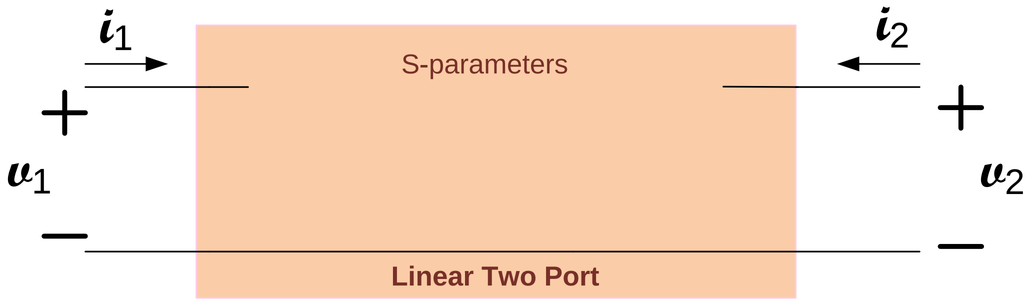


- Alternate two-port characterization but not expressed in terms of independent and dependent parameters
- Widely used notation when designing amplifiers

Small-Signal Model Representations

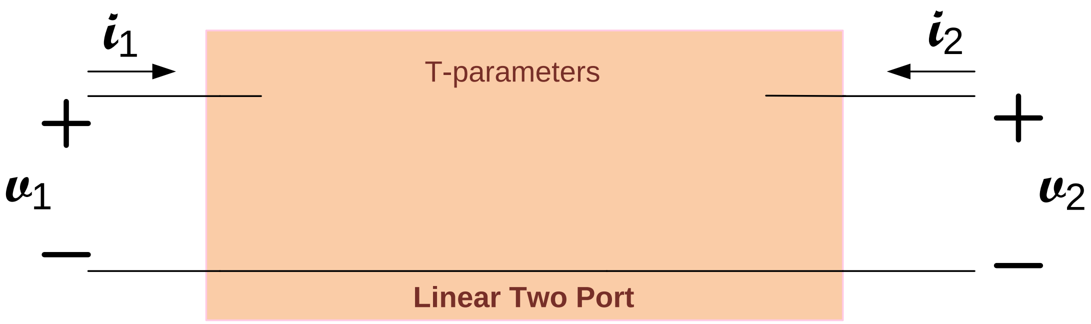


The S-parameters



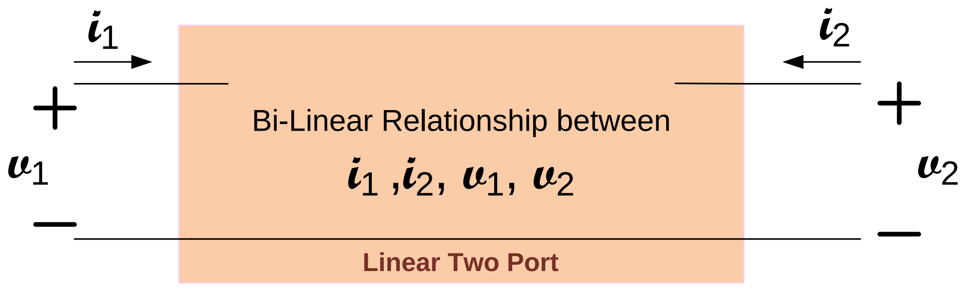
(embedded with source and load impedances)

The T parameters:

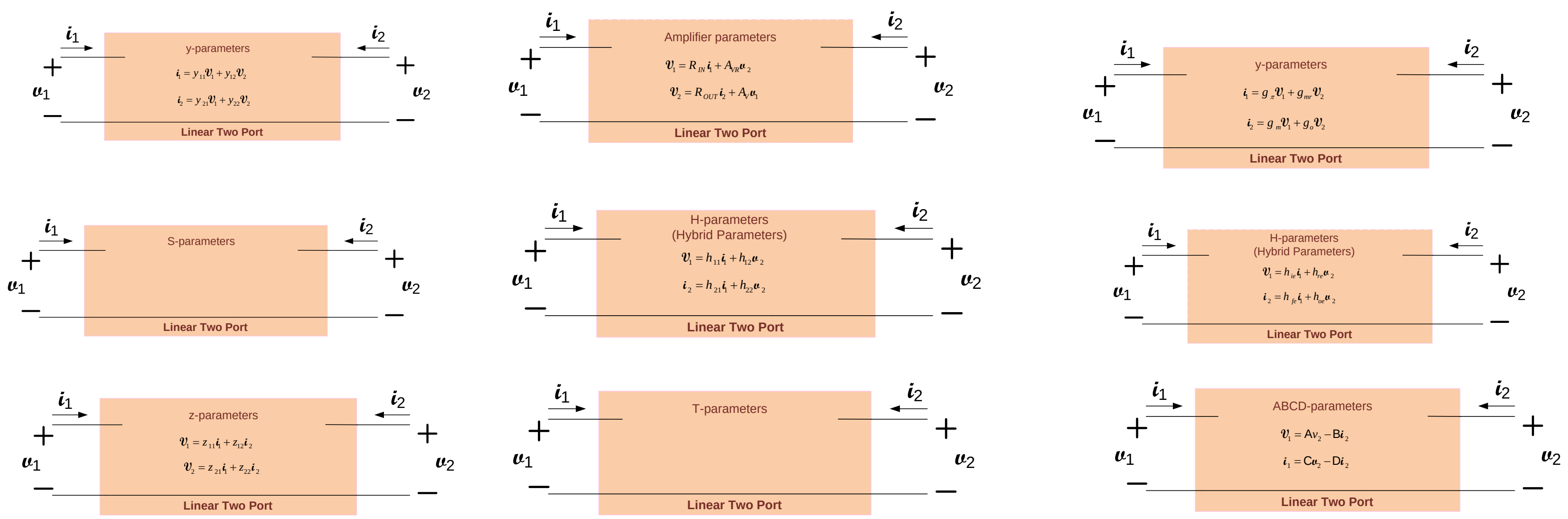


(embedded with source and load impedances)

Small-Signal Model Representations

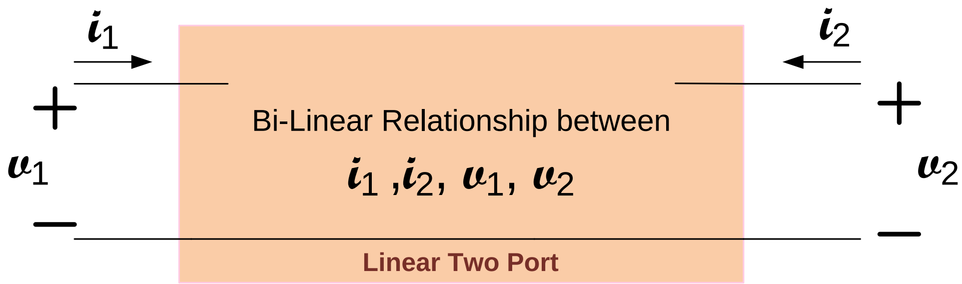


The good, the bad, and the **unnecessary** !!



- Equivalent circuits often given for each representation
- All provide identical characterization
- Easy to move from any one to another

Small-Signal Model Representations



The good, the bad, and the **unnecessary** !!

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Conversions Between S, Z, Y, h, $ABCD$, and T Parameters which are Valid for Complex Source and Load Impedances

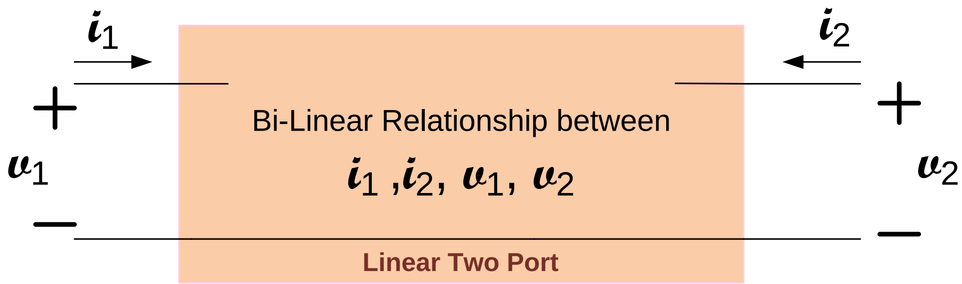
Conversions **between** S, Z, Y, H, ABCD, and T parameters which are valid for complex source and load impedances

DA **Frickey** - IEEE Transactions on microwave theory and …, 1994 - ieeexplore.ieee.org

This paper provides tables which contain the conversion between the various common two-port parameters, Z, Y, H, ABCD, S, and T. The conversions are valid for complex normalizing impedances. An example is provided which verifies the conversions to and from S

As of Mar 6, 2018

Small-Signal Model Representations



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Dean A. Frickey, *Member, IEEE*

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DA **Frickey** - IEEE Transactions on Microwave Theory and …, 1994 - osti.gov
Conversions between S, Z, Y, h, ABCD, and T parameters which are valid for complex source and load impedances This paper provides tables which contain the **conversion between** the various common two-port parameters, Z, Y, h, ABCD, S, and T. The ...
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Comments on" **Conversions between** S, Z, Y, h, ABCD, and T parameters which are valid for complex source and load impedances"[with reply] nist.gov [PDF]
…, DF Williams, DA **Frickey** - Microwave Theory and …, 1995 - ieeexplore.ieee.org
In his recent paper,'**Frickey** presents formulas for **conversions between** various network matrices. Four of these matrices (Z, Y, h, and ABCD) relate voltages and currents at the ports; the other two (S and 7 ') relate wave quantities. These relationships depend on the ...
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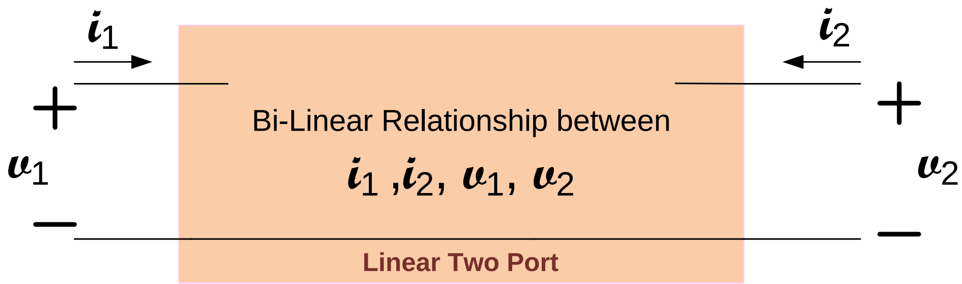
Small-Signal Model Representations



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Dean A. Frickey, *Member, IEEE*

Conversions between S, Z, Y, H, ABCD, and T parameters which are valid for complex source and load impedances
DA **Frickey** - ... theory and techniques, IEEE Transactions on, 1994 - ieeexplore.ieee.org
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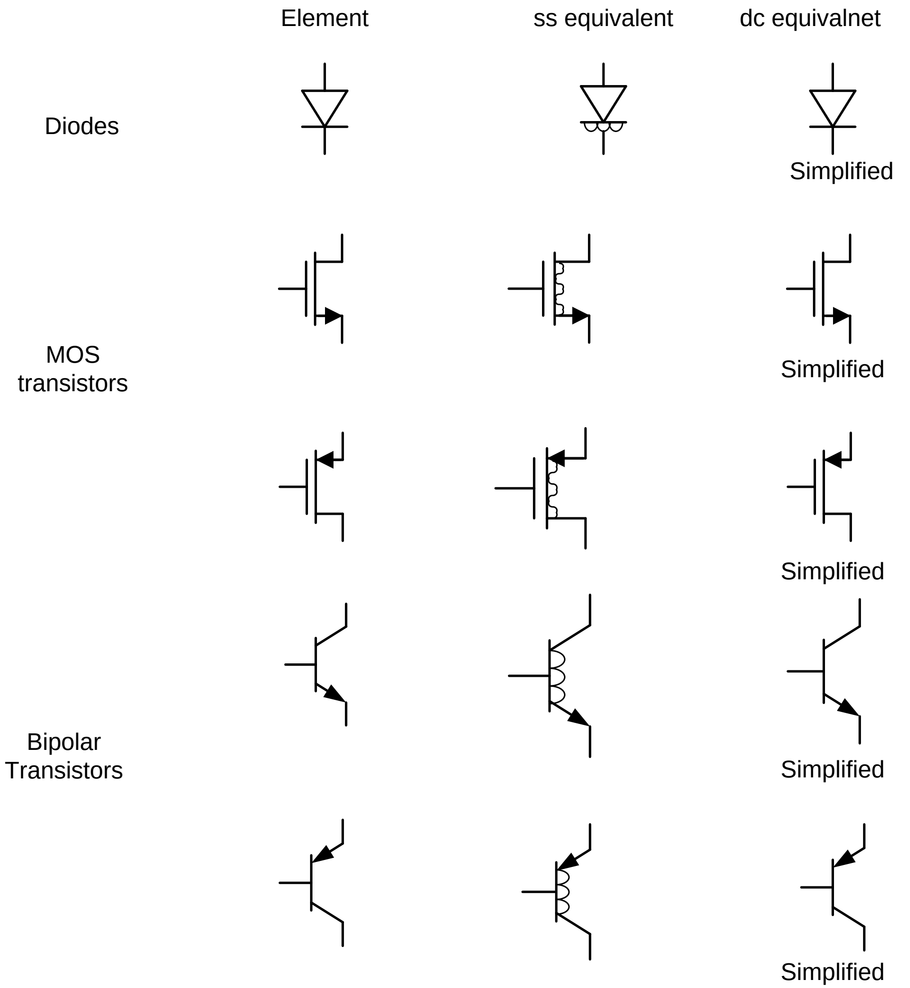
Active Device Model Summary

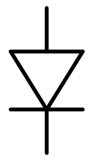


What are the simplified dc equivalent models?

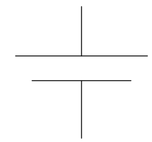
Active Device Model Summary

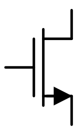
What are the simplified dc equivalent models?

dc equivalent



Simplified





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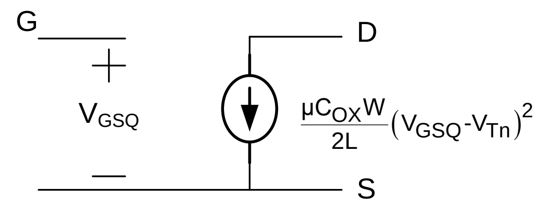

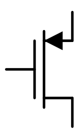

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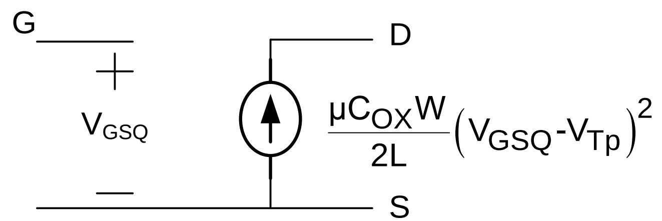

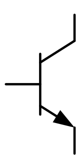

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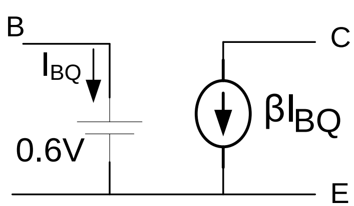

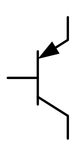

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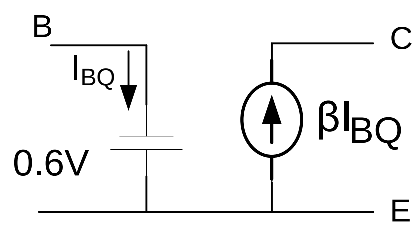

End of Lecture 24