EE 330
Lecture 25

- Small Signal Analysis Example Circuits
- Graphical Small Signal Analysis
- Model Extensions and Simplifications

Small-signal Operation of Nonlinear Circuits

- Small-signal principles
- Example Circuit
- Small-Signal Models
- Small-Signal Analysis of Nonlinear Circuits

Recall:

Alternative Approach to small-signal analysis of nonlinear networks

1. **Linearize nonlinear devices**

   (have small-signal model for key devices!)

2. **Replace all devices with small-signal equivalent**

3. **Solve linear small-signal network**

- Remember that the small-signal model is operating point dependent!
- Thus need Q-point to obtain values for small signal parameters
- Expressions for circuit characteristics such as gain can be expressed in terms of small-signal parameters or nonlinear device model parameters and Q-points
- Expressions for circuit characteristics such as gain in terms of small-signal parameters often give little insight into performance or design

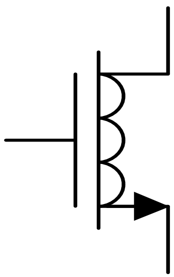
Recall:

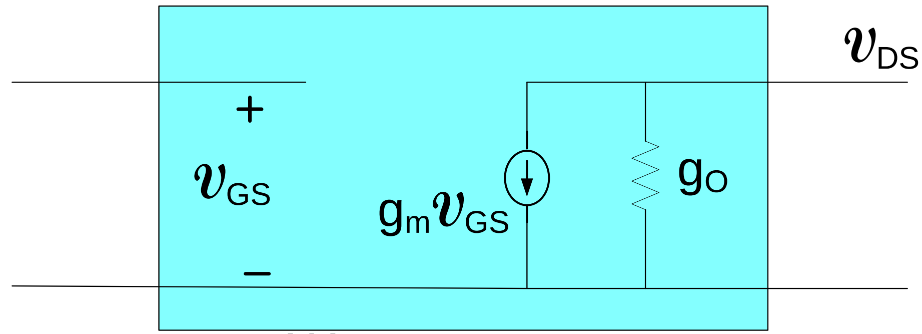
Q-point Computation

- Open all caps and short all inductors
- Assume correct region for all nonlinear devices
  - Diodes on, MOS in saturation, BJT in forward active
- Write down device models and KCL's
  - One KCL at each ono-trivial node in nodal analysis
  - For hand calculation, use simplified device models
- Solve the simultaneous equations
  - Produce Q values for all node voltages and branch currents
- Check correctness of assumptions in 2^{nd} step

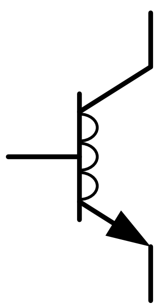
Recall:

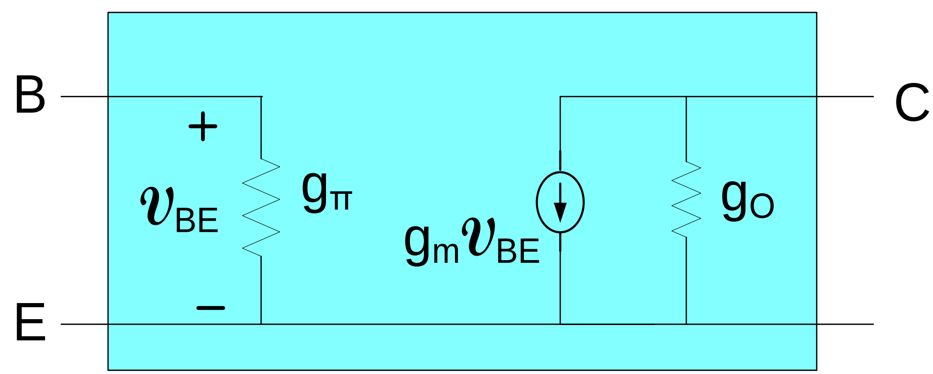
Small Signal Model for Active Devices





$$g_m = \mu C_{OX}\frac{W}{L}\left(V_{GSQ} - V_T\right) \qquad g_o \cong \lambda I_{DQ}$$

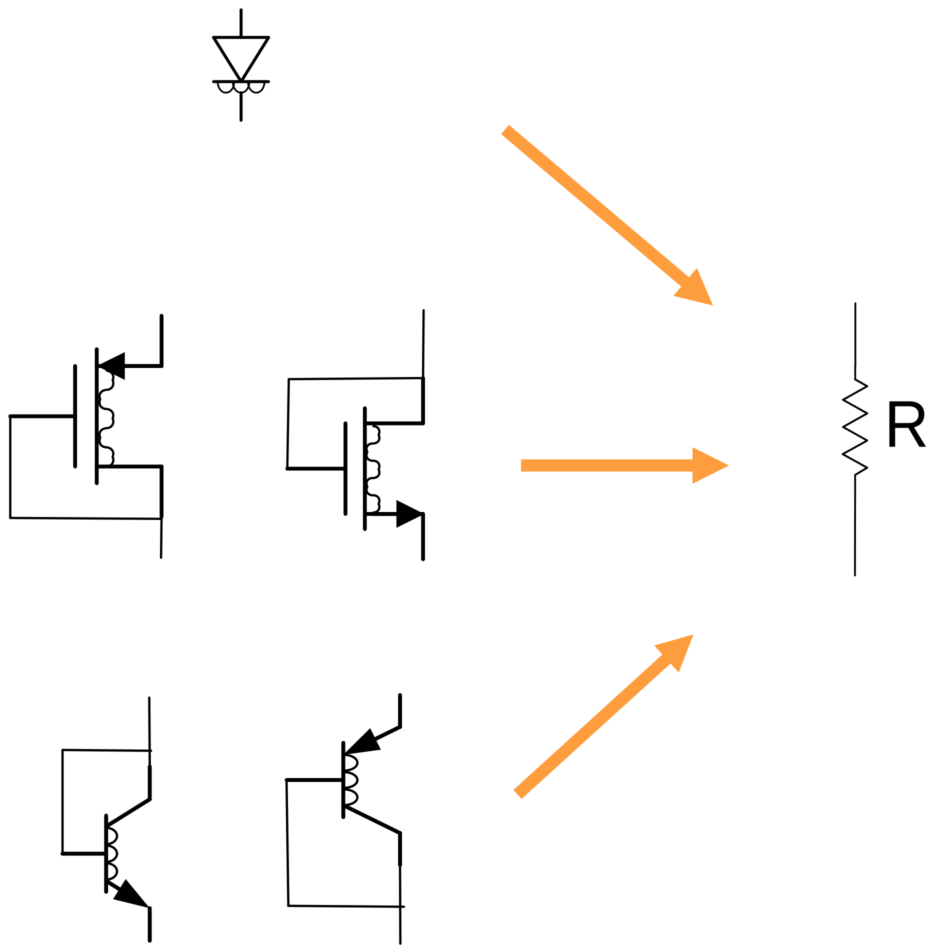




$$g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_m = \frac{I_{CQ}}{V_t} \qquad g_o = \frac{I_{CQ}}{V_{AF}}$$

Recall:

Small Signal Model for Active Devices

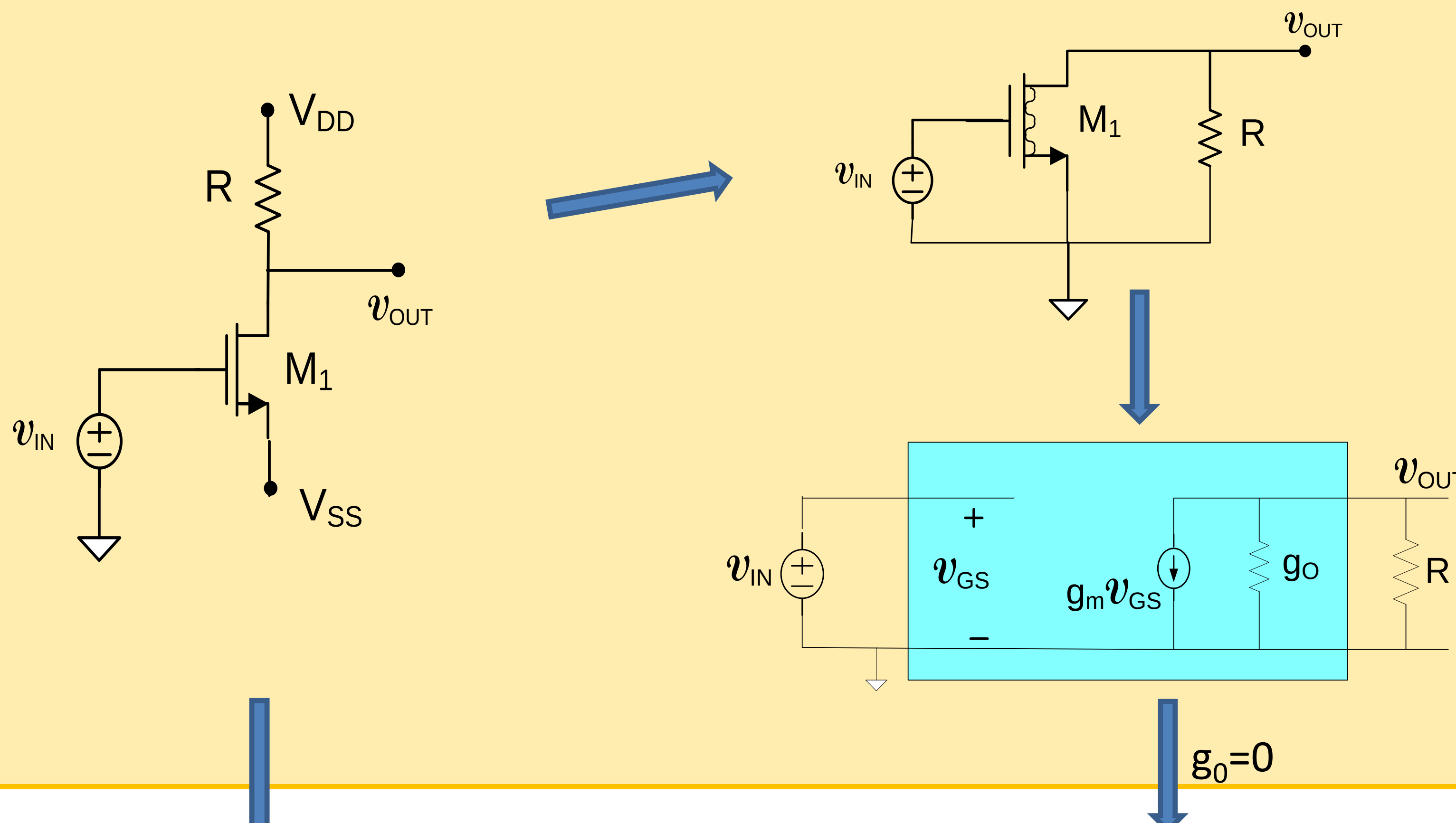


$$\frac{1}{R} = g_{diode} = \frac{I_{diodeQ}}{V_t}$$

$$\frac{1}{R} = g_m = \frac{2I_{DQ}}{V_{GSQ} - V_{TH}}$$

$$\frac{1}{R} = g_m = \frac{I_{CQ}}{V_t}$$

Recall: Small-signal analysis example



$$A_v = -g_m R$$

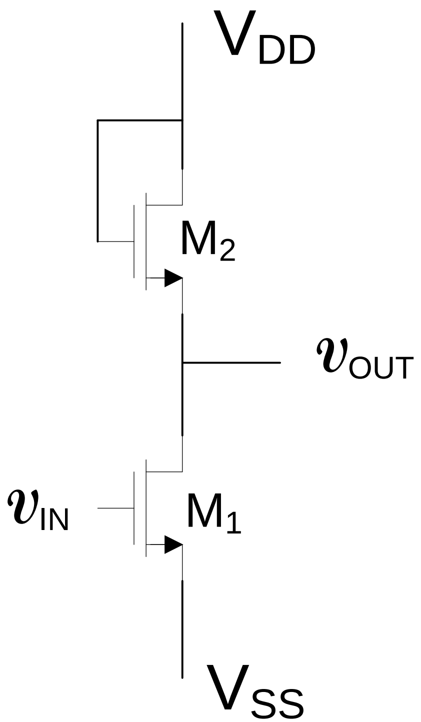
$$A_v = \frac{2I_{DQ}R}{[V_{SS}+V_T]}$$

$$A_v = \frac{2I_{DQ}R}{[V_{SS}+V_T]}$$

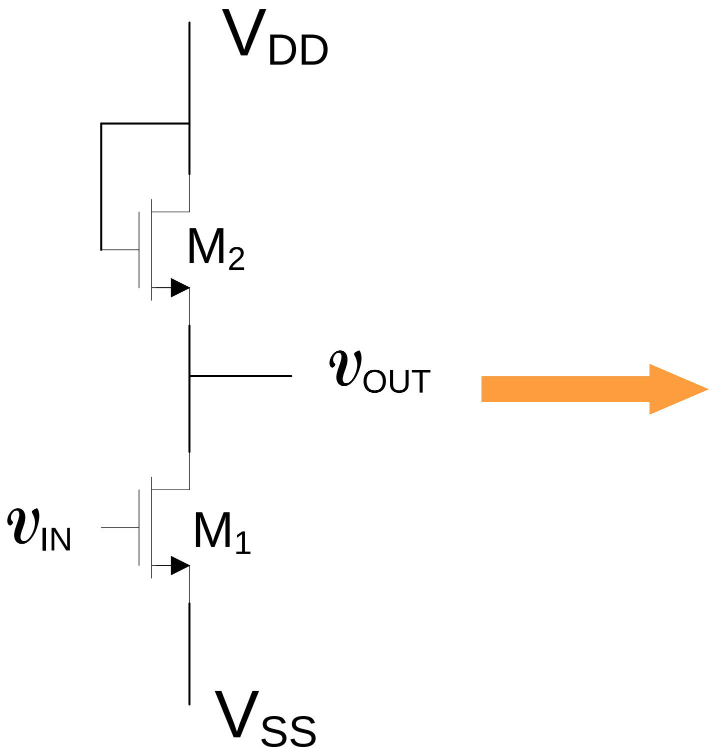
Example:

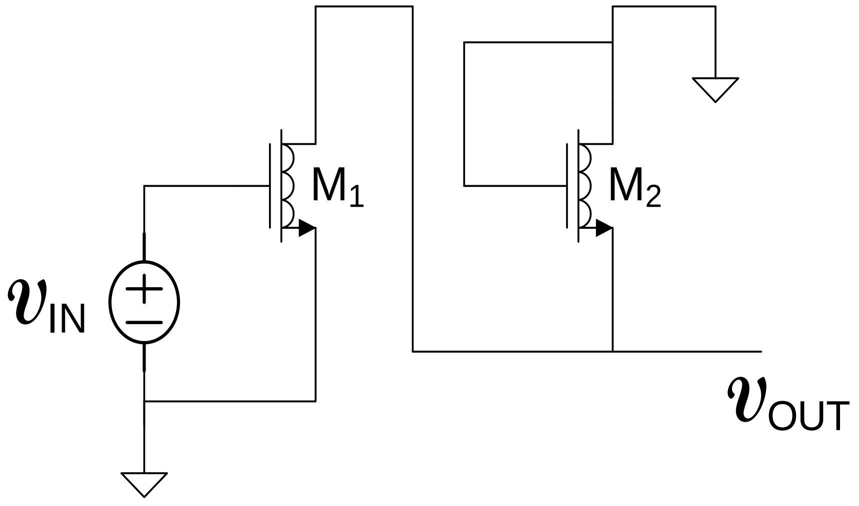
Assume M_1 and M_2 are operating in the saturation region and that $\lambda=0$

a) Determine the small signal voltage gain $A_V=v_{OUT}/v_{IN}$ in terms of the small-signal model parameters
b) Determine the small signal voltage gain $A_V=v_{OUT}/v_{IN}$ in terms of the Q-points and the nonlinear model parameters



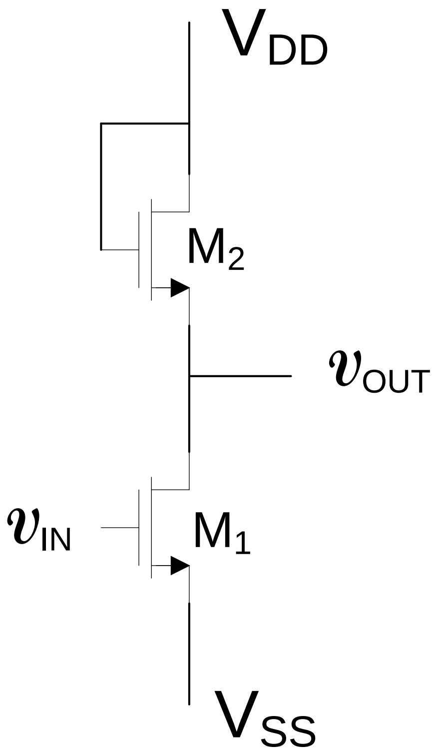
Example: *Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that $\lambda=0$*

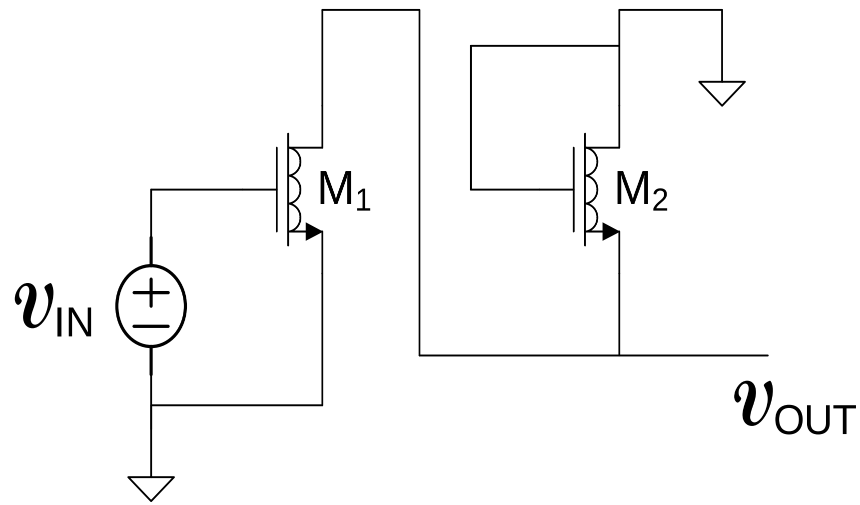




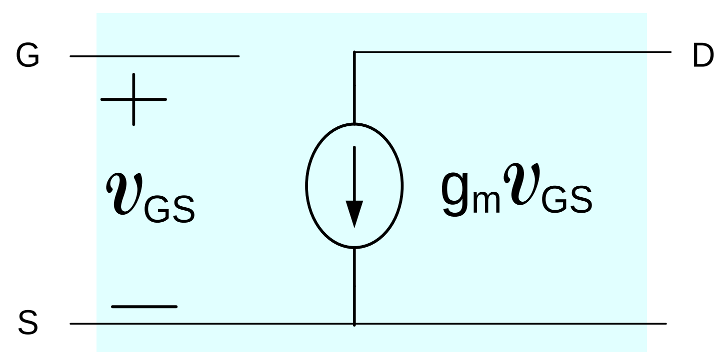
Small-signal circuit

Example: *Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that $\lambda=0$*



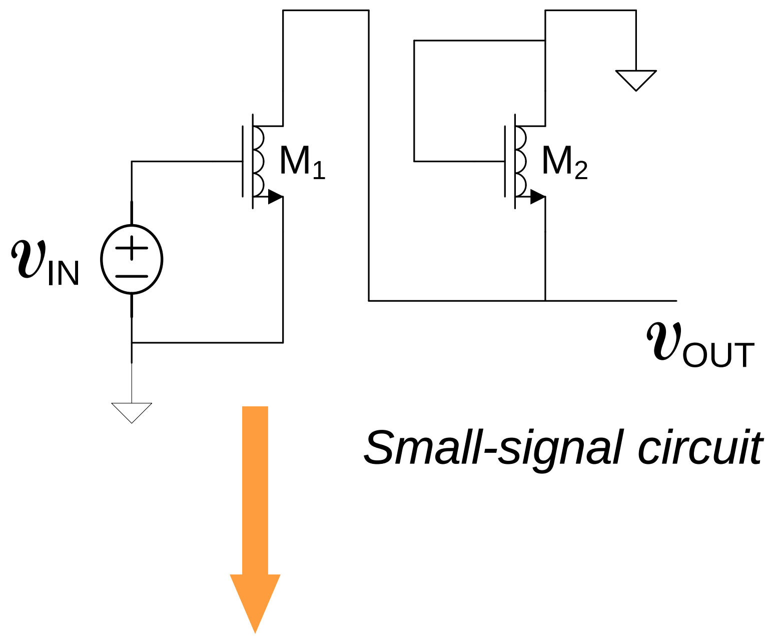


Small-signal circuit

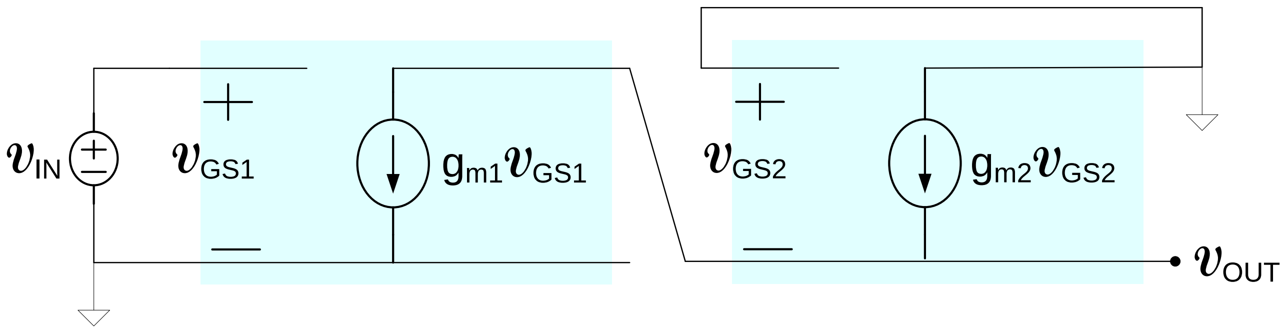


Small-signal MOSFET model for $\lambda=0$

Example: *Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that $\lambda=0$*

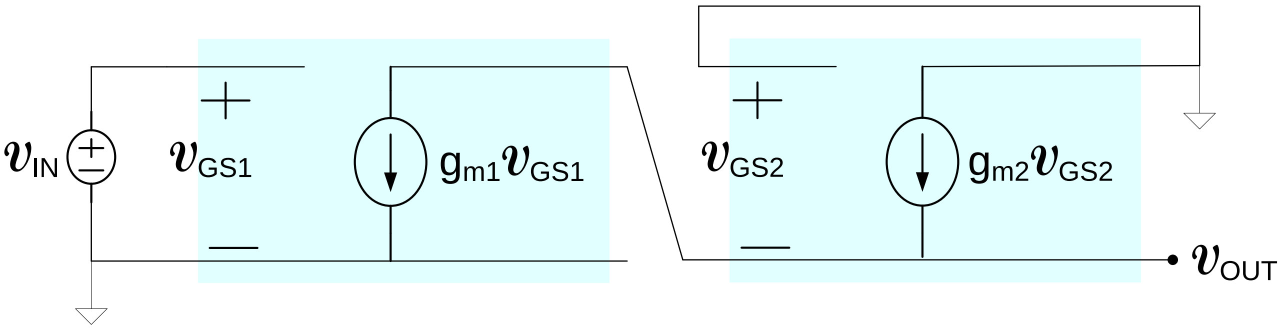


Small-signal circuit for λ=0



Small-signal circuit

Example:



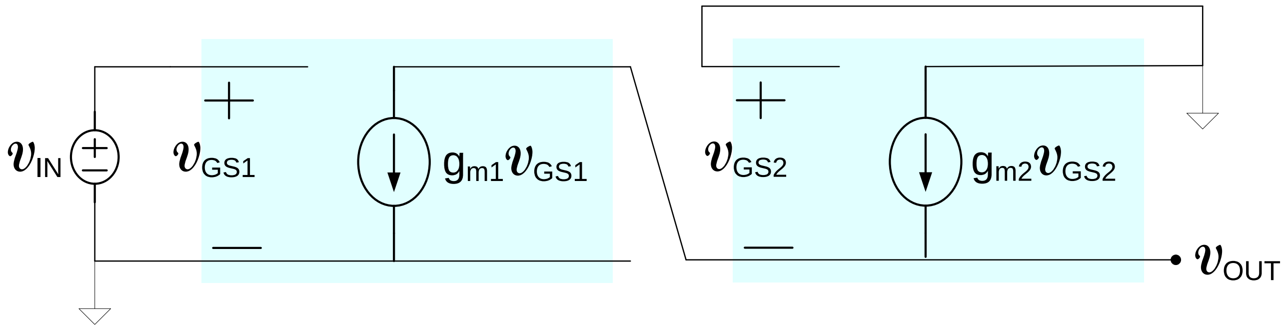
Small-signal circuit for λ=0

Analysis:

By KCL

$$\left.\begin{aligned} g_{m1}v_{GS1} &= g_{m2}v_{GS2} \\ v_{GS1} &= v_{IN} \\ -v_{GS2} &= v_{OUT} \end{aligned}\right\} \quad \text{thus:} \quad A_V = \frac{v_{OUT}}{v_{IN}} = -\frac{g_{m1}}{g_{m2}}$$

Example:



Small-signal circuit for λ=0

Analysis:

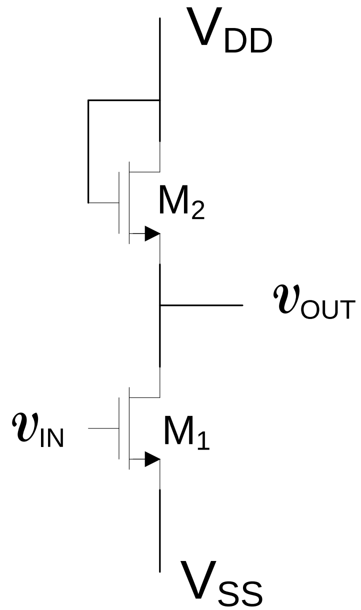
$$A_V = \frac{v_{OUT}}{v_{IN}} = -\frac{g_{m1}}{g_{m2}}$$

Recall:

$$g_m = -\sqrt{2I_D \mu C_{OX}}\sqrt{\frac{W_1}{L_1}}$$

$$A_V = -\frac{\sqrt{2I_D \mu C_{OX} \frac{W_1}{L_1}}}{\sqrt{2I_D \mu C_{OX} \frac{W_2}{L_2}}} = -\sqrt{\frac{W_1}{W_2}}\sqrt{\frac{L_2}{L_1}}$$

Example Summary:



Assumed $\lambda=0$

$$A_V = -\frac{g_{m1}}{g_{m2}}$$

In terms of small-signal parameters

$$A_V = -\sqrt{\frac{W_1}{W_2}}\sqrt{\frac{L_2}{L_1}}$$

In terms of Q-point and nonlinear model parameters

If $L_1=L_2$, obtain

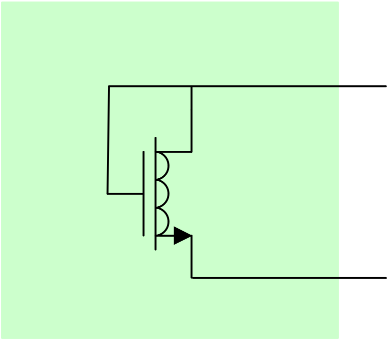
$$A_V = -\sqrt{\frac{W_1}{W_2}}\sqrt{\frac{L_2}{L_1}} = -\sqrt{\frac{W_1}{W_2}}$$

The width and length ratios can be accurately controlled with good layout when designed in a standard CMOS process !

Example:

Obtain the small signal model of the following circuit. Assume MOSFET is operating in the saturation region

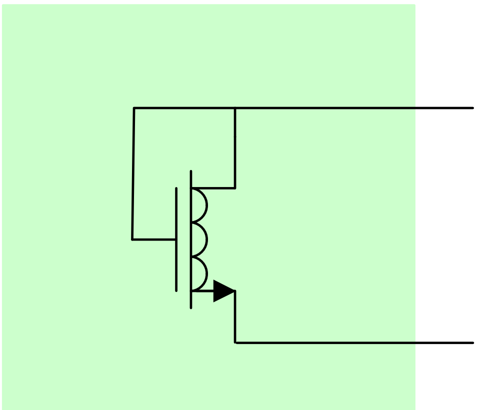
Note: $g_m >> g_o$ for MOS devices in most processes so also obtain model under this assumption

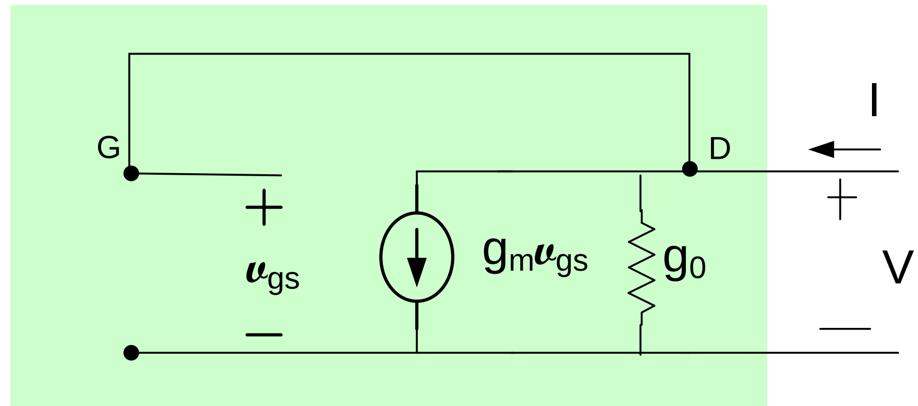


Example:

Obtain the small signal model of the following circuit. Assume MOSFET is operating in the saturation region.

Solution:





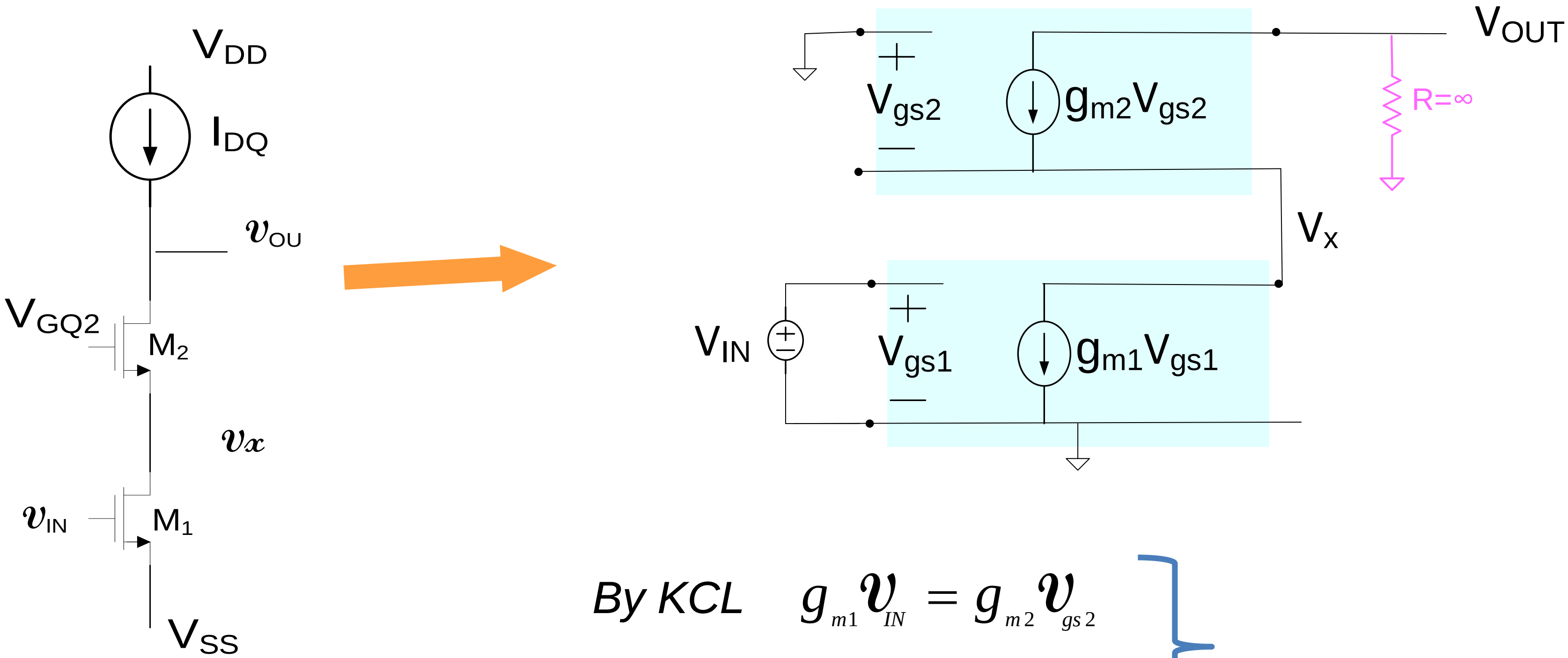
$$V\left(g_m + g_0\right) = I$$

$$R_{EQ} = \frac{V}{I} = \frac{1}{g_m + g_0}$$

for $g_m >> g_o$

$$R_{EQ} \cong \frac{1}{g_m}$$

Example: Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that $\lambda=0$



By KCL

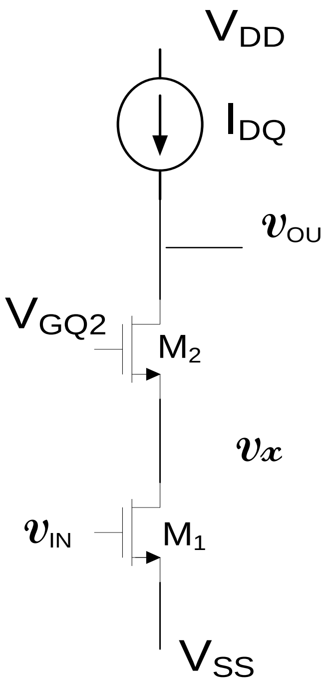
$$\left.\begin{aligned} g_{m1} v_{IN} &= g_{m2} v_{gs2} \\ g_{m2} v_{gs2} R &= -v_{OUT} \end{aligned}\right\}$$

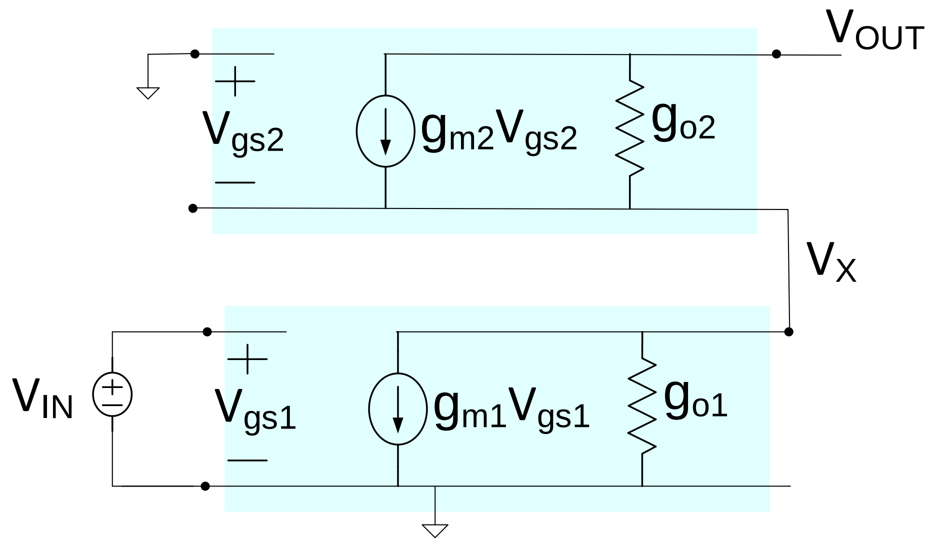
Solving obtain:

$$A_V = \frac{v_{OUT}}{v_{IN}} = -g_{m1} R \underset{R=\infty}{\rightarrow} \infty$$

Unexpectedly large, need better device models!

Example: Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that $\lambda \neq 0$





By KCL

$$\left.\begin{aligned} & g_{m1} v_{IN} + g_{o1} v_X = g_{m2} v_{GS2} + g_{o2}(v_{OUT} - v_X) \\ & v_{GS2} = -v_X \\ & (v_{OUT} - v_X) g_{o2} + g_{m2} v_{GS2} = 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} & g_{m1} v_{IN} + (g_{m2} + g_{o1} + g_{o2}) v_X = g_{o2} v_{OUT} \\ & v_{OUT} g_{o2} = (g_{m2} + g_{o2}) v_X \end{aligned}\right\}$$

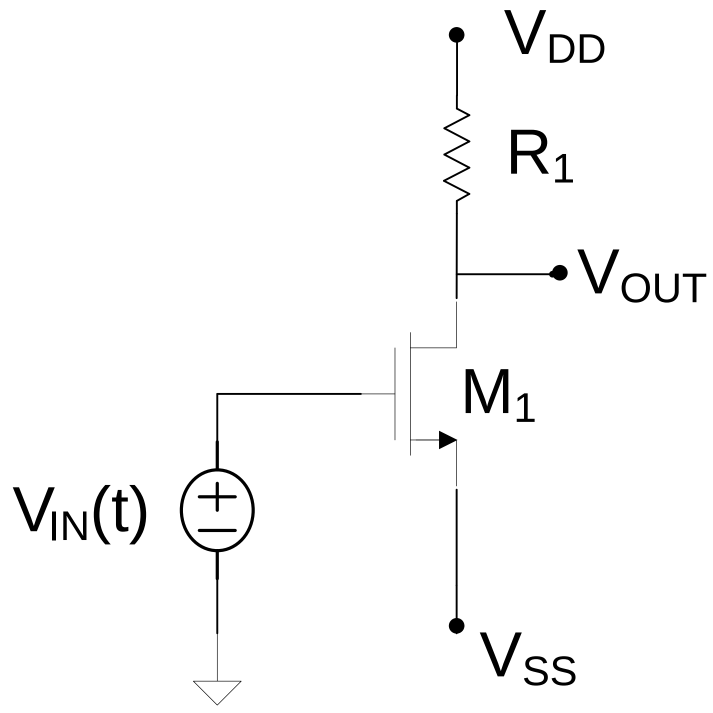
thus:

$$A_V = \frac{v_{OUT}}{v_{IN}} = -\frac{g_{m1} g_{m2} + g_{m1} g_{o2}}{g_{o1} g_{o2}} \cong -\frac{g_{m1}}{g_{o1}} \frac{g_{m2}}{g_{o2}}$$

- Analysis is straightforward but a bit tedious
- A_V is very large and would go to ∞ if g_{01} and g_{02} were both 0

Graphical Analysis and Interpretation

Consider Again



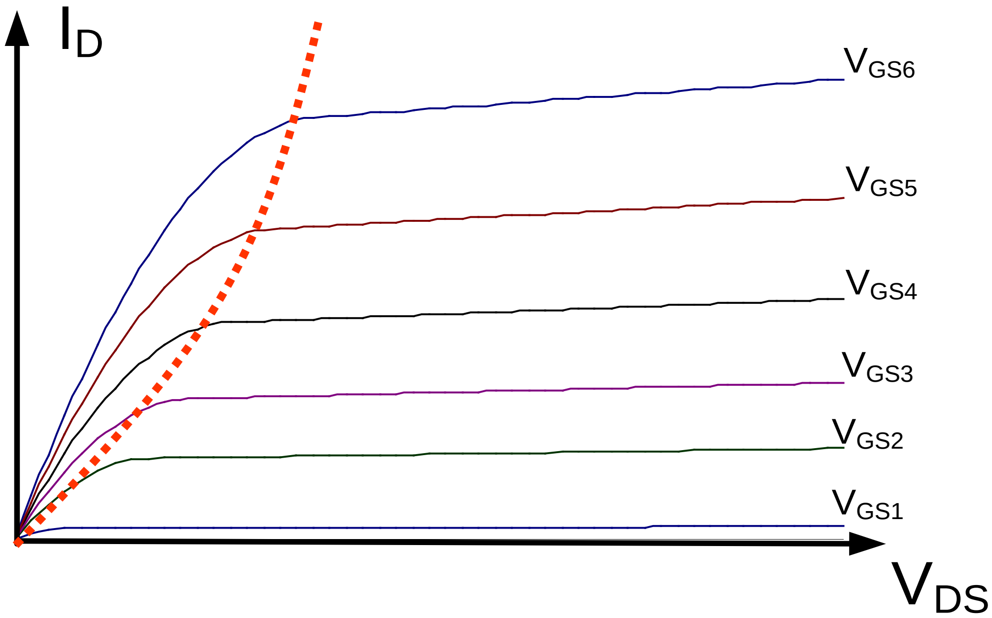
$$V_{OUT} = V_{DD} - I_D R$$

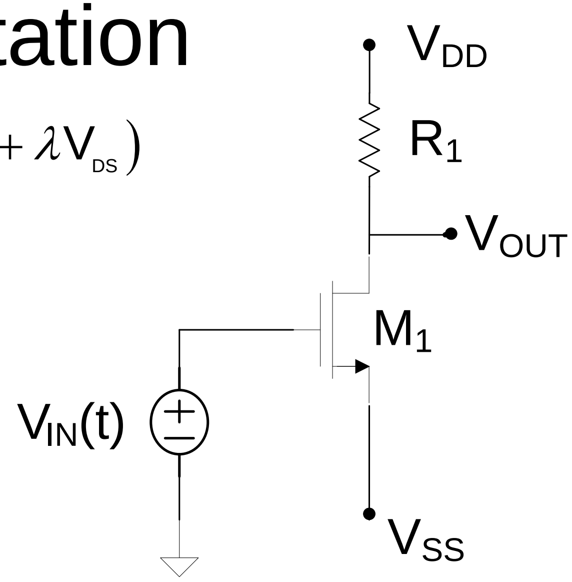
$$I_D = \frac{\mu C_{OX} W}{2L}\left(V_{IN} - V_{SS} - V_T\right)^2$$

$$I_{DQ} = \frac{\mu C_{OX} W}{2L}\left(V_{SS} + V_T\right)^2$$

Graphical Analysis and Interpretation

Device Model (family of curves) $$I_D = \frac{\mu C_{OX} W}{2L}(V_{GS}-V_T)^2(1+\lambda V_{DS})$$





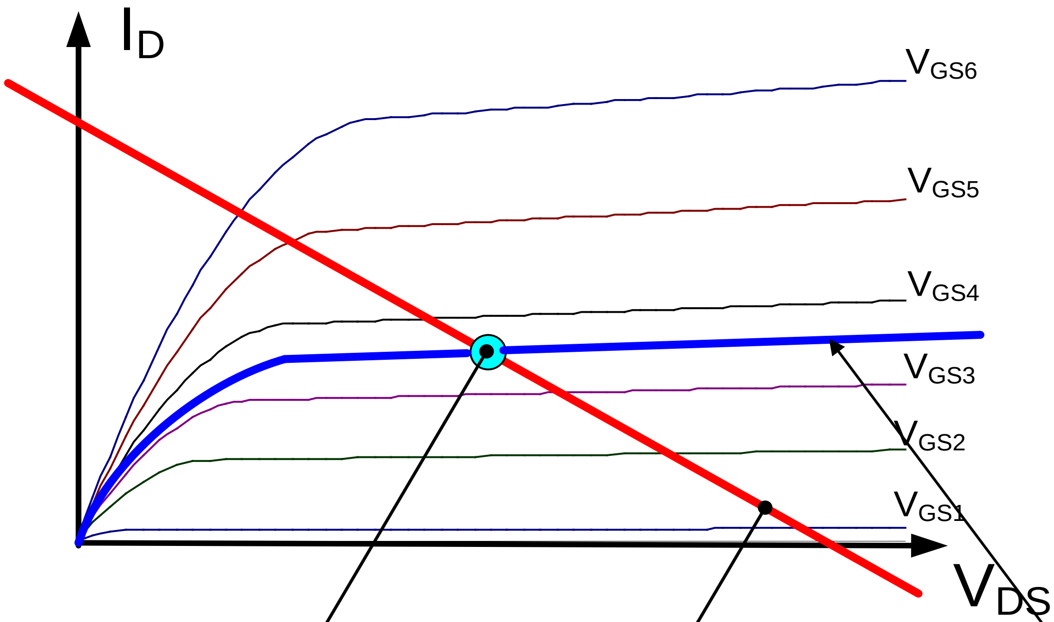
Load Line $$V_{OUT}=V_{DD}-I_D R$$

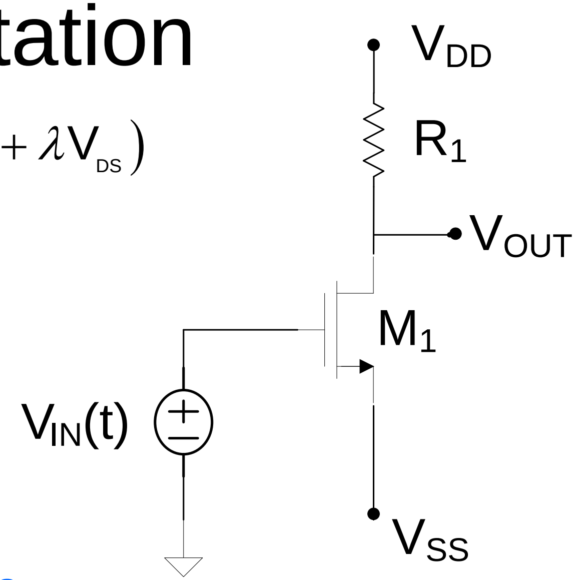
Device Model $$I_D = \frac{\mu C_{OX} W}{2L}(V_{IN}-V_{SS}-V_T)^2$$

Device Model at Operating Point $$I_{DQ} = \frac{\mu C_{OX} W}{2L}(V_{SS}+V_T)^2$$

Graphical Analysis and Interpretation

Device Model (family of curves) $I_D = \frac{\mu C_{OX} W}{2L}(V_{GS}-V_T)^2(1+\lambda V_{DS})$





$V_{GSQ}=-V_{SS}$

$$I_{DQ} \cong \frac{\mu C_{OX} W}{2L}(V_{SS}+V_T)^2$$

Q-Point

Load Line

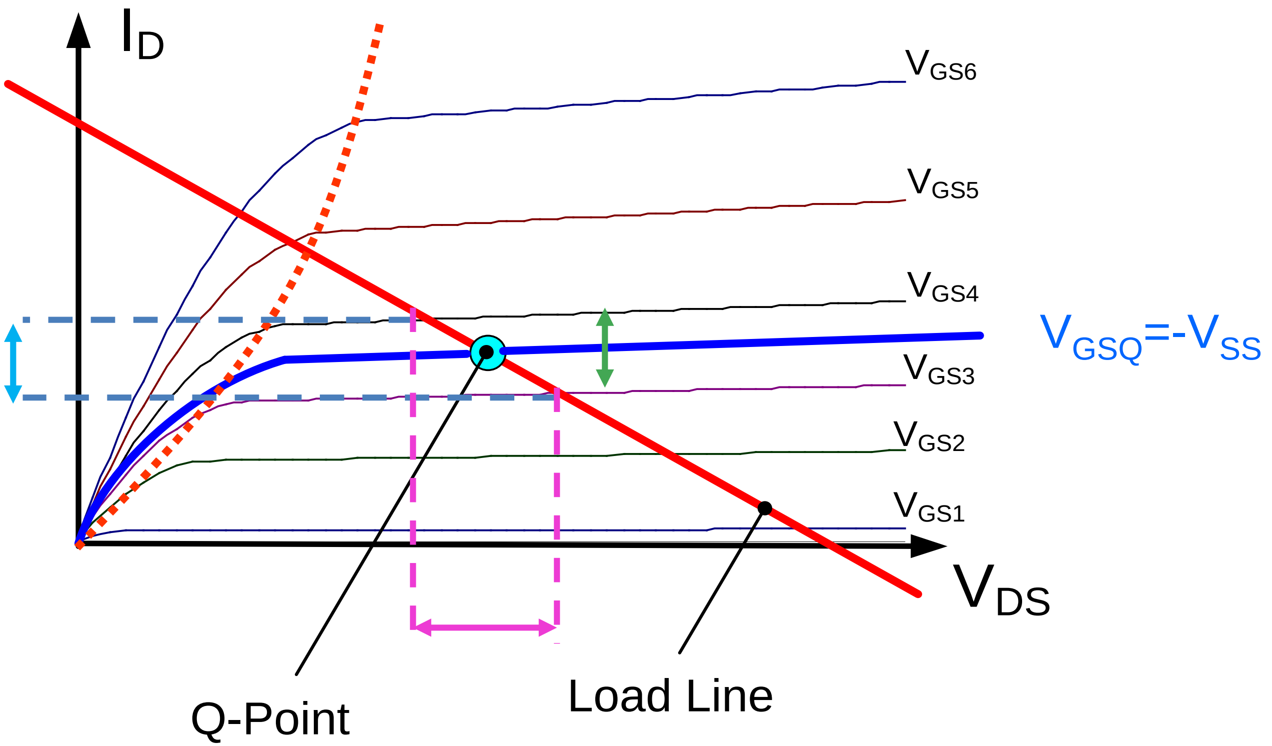
$$V_{OUT}=V_{DD}-I_D R$$

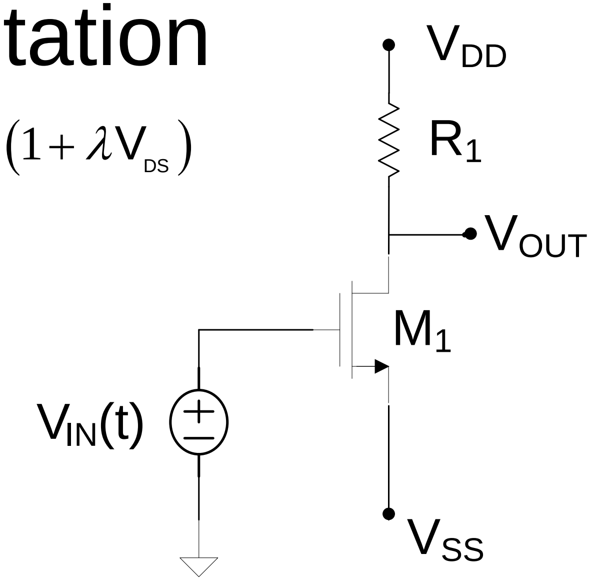
$$I_D = \frac{\mu C_{OX} W}{2L}(V_{IN}-V_{SS}-V_T)^2 \quad ?$$

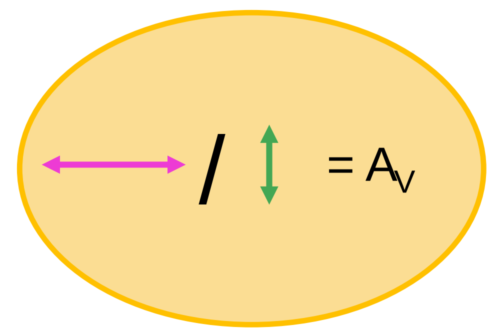
Must satisfy both equations all of the time !

Graphical Analysis and Interpretation

Device Model (family of curves) $I_D = \frac{\mu C_{ox} W}{2L}(V_{IN}-V_{SS}-V_T)^2(1+\lambda V_{DS})$



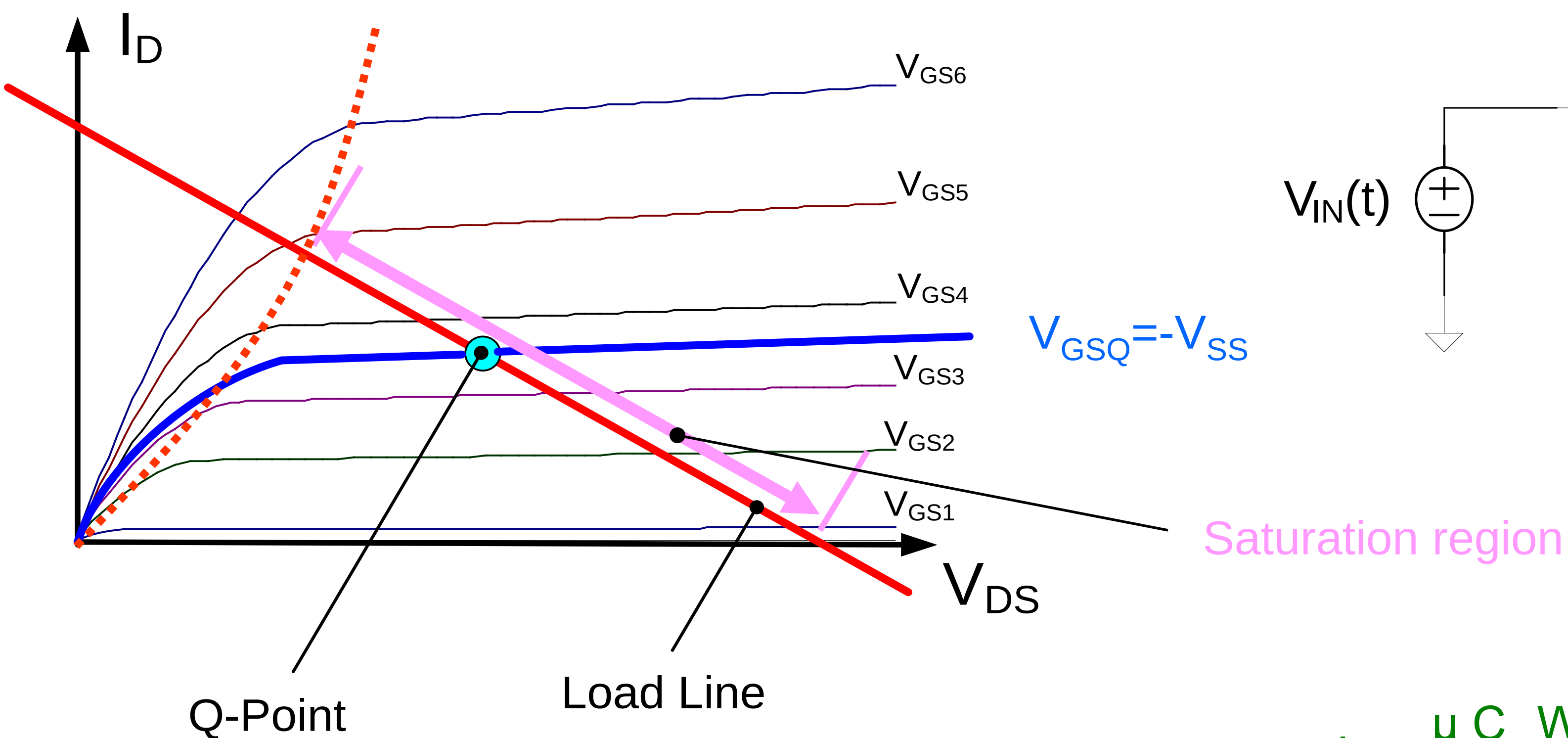


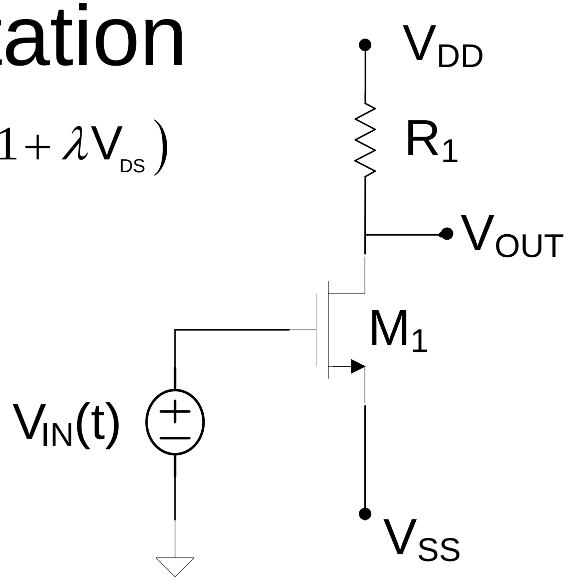


- As V_{IN} changes around Q-point, V_{IN} induces changes in V_{GS} . The operating point must remain on the load line!
- Small sinusoidal changes of V_{IN} will be nearly symmetric around the V_{GSQ} line
- This will cause nearly symmetric changes in both I_D and V_{DS} !
- Since V_{SS} is constant, change in V_{DS} is equal to change in V_{OUT}

Graphical Analysis and Interpretation

Device Model (family of curves) $I_D = \dfrac{\mu C_{OX} W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2\left(1+\lambda V_{DS}\right)$



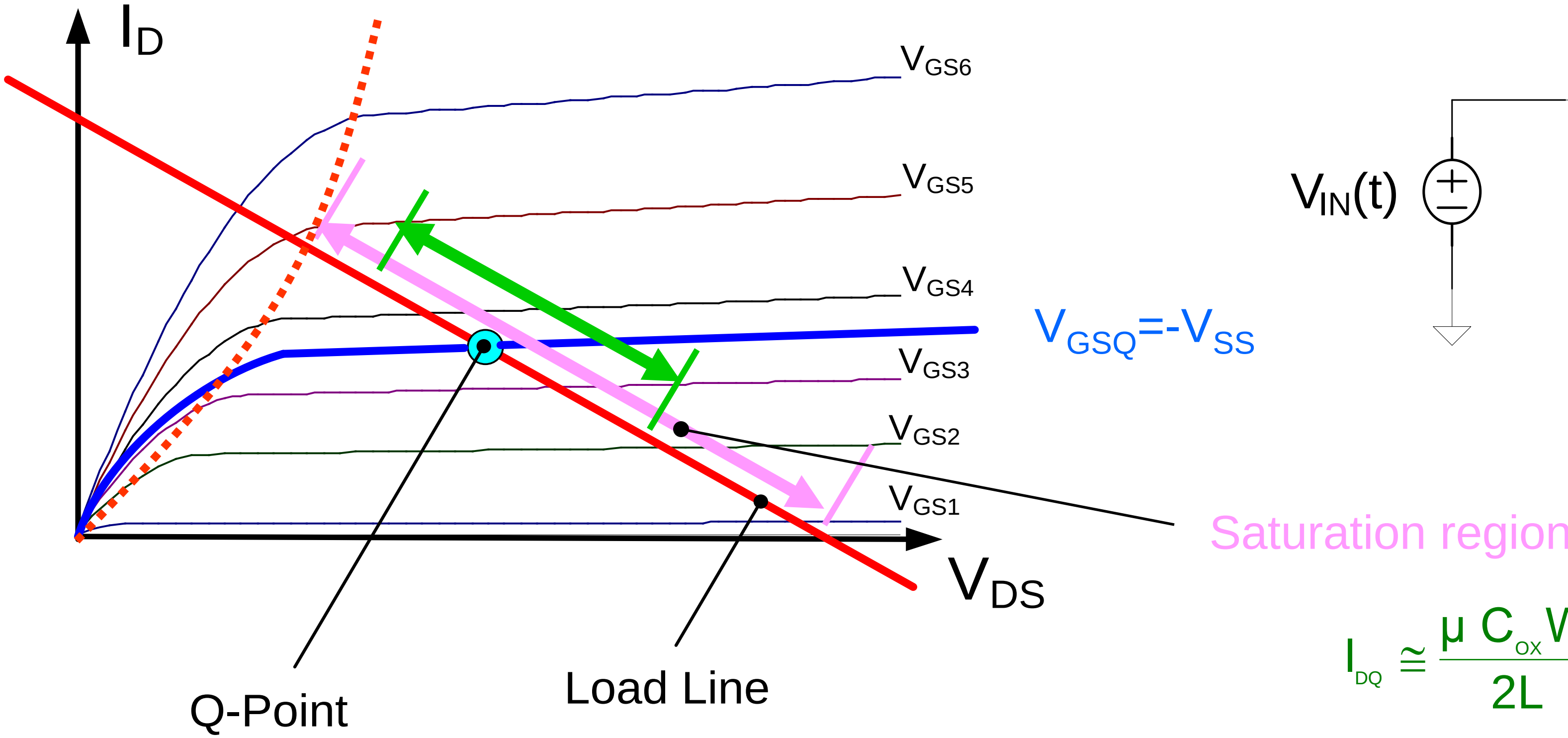


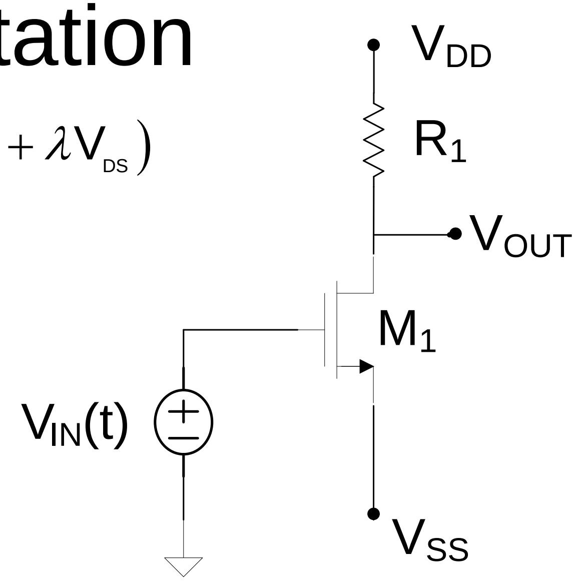
$$I_{DQ} \cong \frac{\mu C_{OX} W}{2L}\left(V_{SS}+V_T\right)^2$$

As V_{IN} changes around Q-point, due to changes V_{IN} induces in V_{GS}, the operating point must remain on the load line!

Graphical Analysis and Interpretation

Device Model (family of curves) $$I_{DQ} = \frac{\mu C_{OX} W}{2L}(V_{GS}-V_T)^2(1+\lambda V_{DS})$$



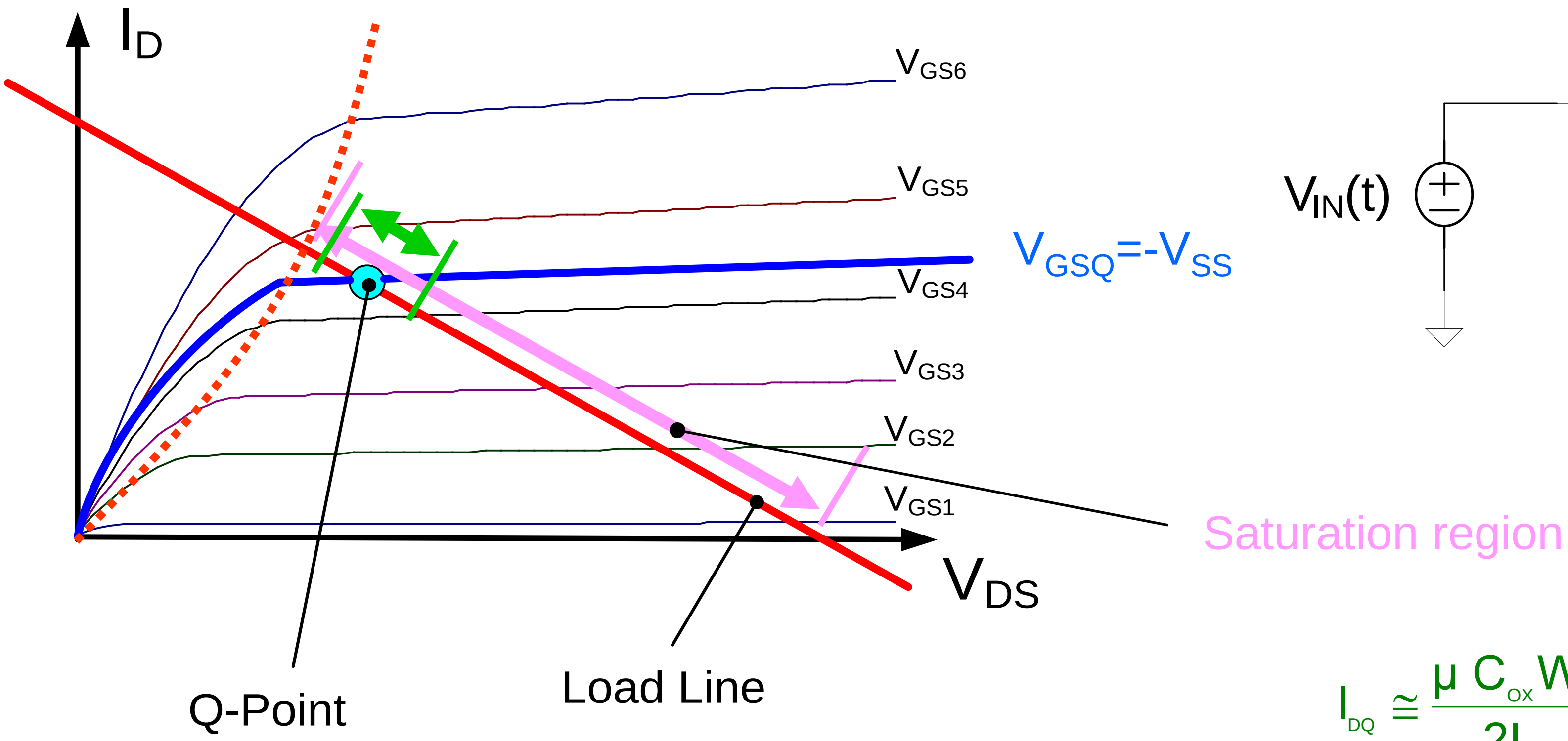


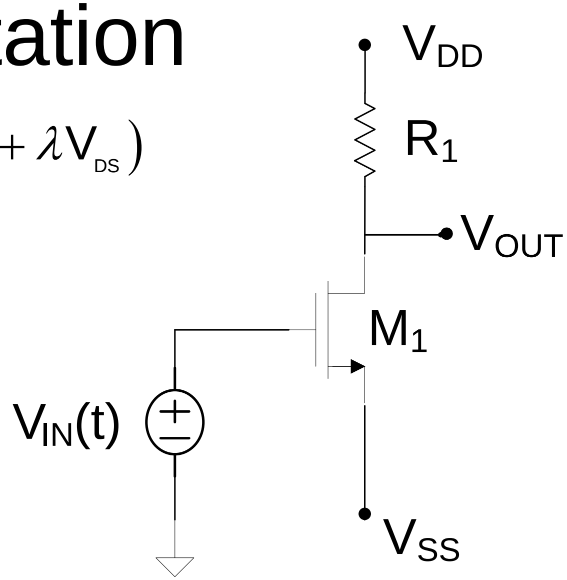
$$I_{DQ} \cong \frac{\mu C_{OX} W}{2L}(V_{SS}+V_T)^2$$

- Linear signal swing region smaller than saturation region
- Modest nonlinear distortion provided saturation region operation maintained
- Symmetric swing about Q-point
- Signal swing can be maximized by judicious location of Q-point

Graphical Analysis and Interpretation

Device Model (family of curves) $I_{DQ} = \frac{\mu C_{OX} W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$



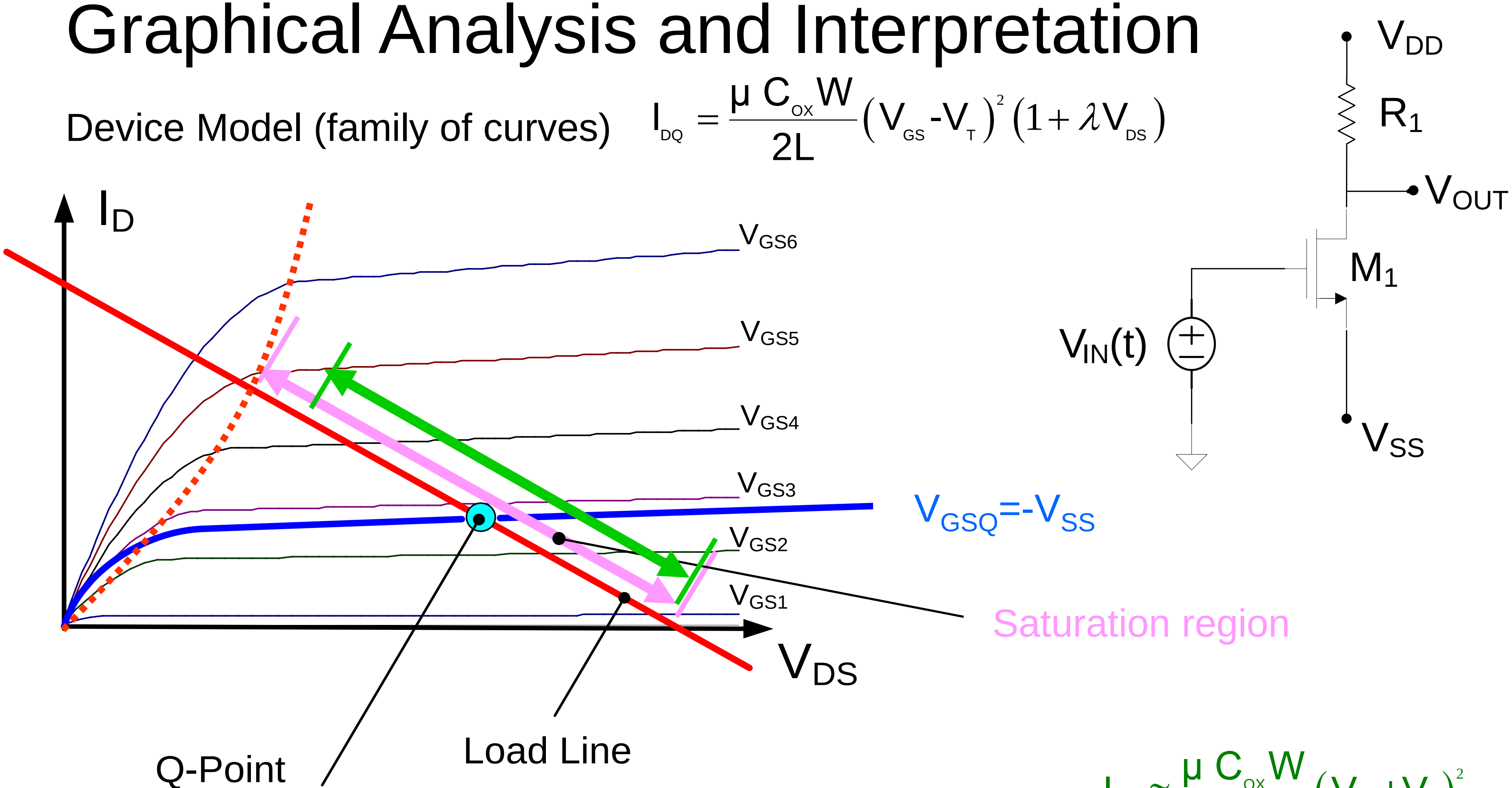


$$I_{DQ} \cong \frac{\mu C_{OX} W}{2L} (V_{SS} + V_T)^2$$

Very limited signal swing with non-optimal Q-point location

Graphical Analysis and Interpretation

Device Model (family of curves) $I_{DQ} = \frac{\mu C_{OX} W}{2L}(V_{GS}-V_T)^2(1+\lambda V_{DS})$



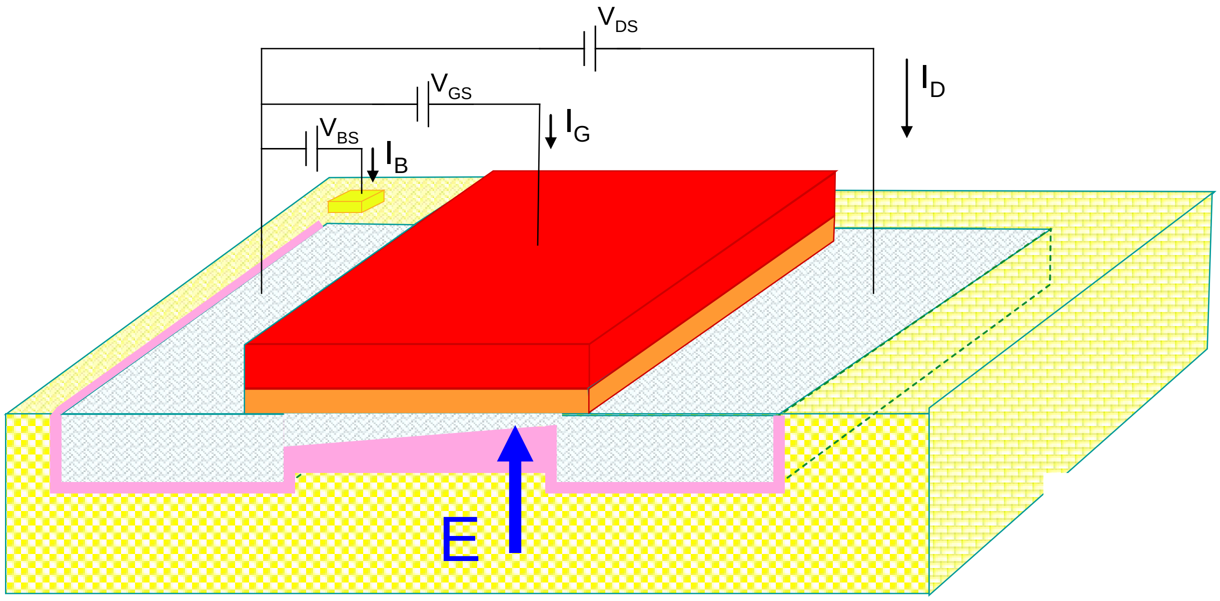
$I_{DQ} \cong \frac{\mu C_{OX} W}{2L}(V_{SS}+V_T)^2$

- Signal swing can be maximized by judicious location of Q-point
- Often selected to be at middle of load line in saturation region

Small-Signal MOSFET Model Extension

Existing 3-terminal small-signal model does not depend upon the bulk voltage !

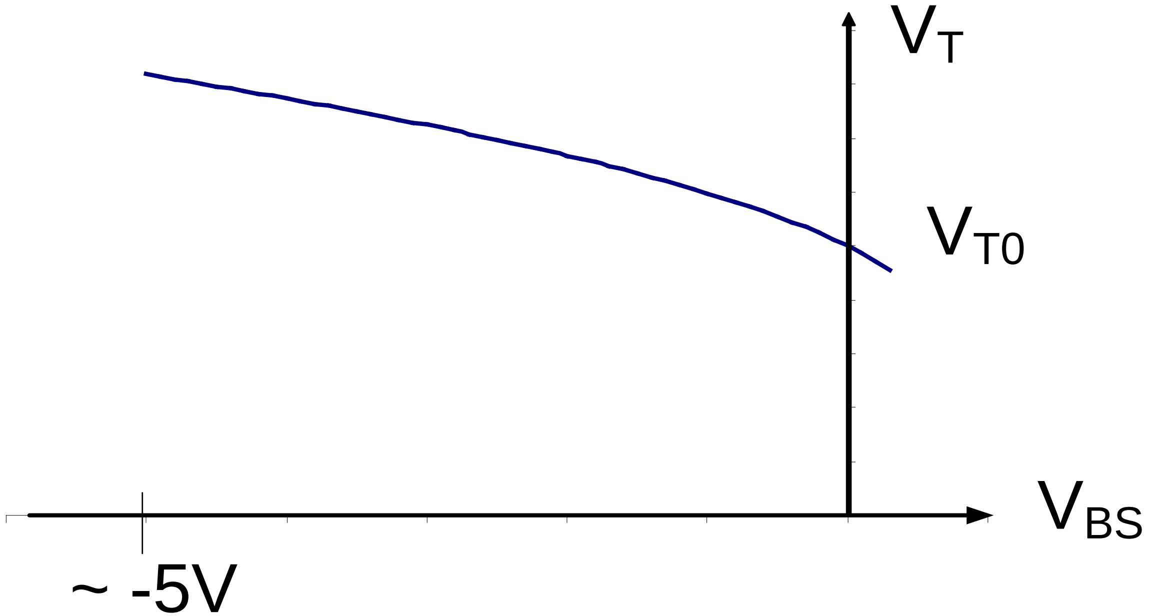
Recall that changing the bulk voltage changes the electric field in the channel region and thus the threshold voltage!



Recall: Typical Effects of Bulk on Threshold Voltage for n-channel Device

$$V_T = V_{T0} + \gamma\left[\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right]$$

$$\gamma \cong 0.4V^{-\frac{1}{2}} \qquad \phi \cong 0.6V$$



Bulk-Diffusion Generally Reverse Biased (V_{BS}< 0 or at least less than 0.3V) for n-channel

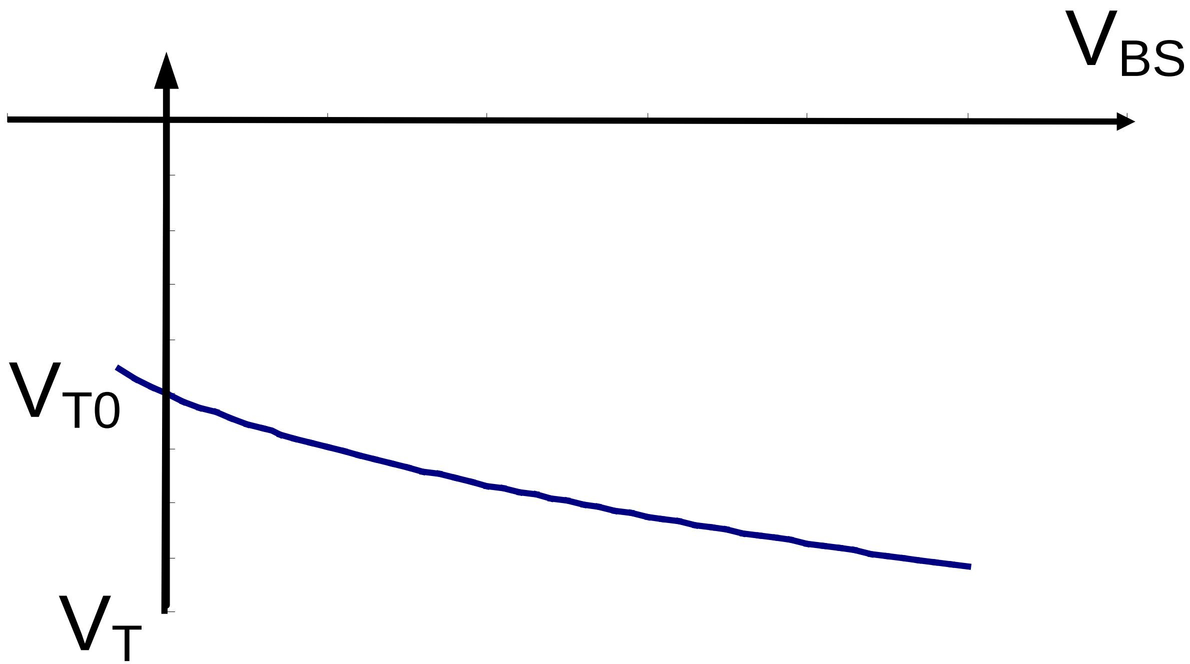
Shift in threshold voltage with bulk voltage can be substantial

Often V_{BS}=0

Recall: Typical Effects of Bulk on Threshold Voltage for p-channel Device

$$V_T = V_{T0} - \gamma\left[\sqrt{\phi + V_{BS}} - \sqrt{\phi}\right]$$

$\gamma \cong 0.4V^{-\frac{1}{2}}$ $\qquad$ $\phi \cong 0.6V$

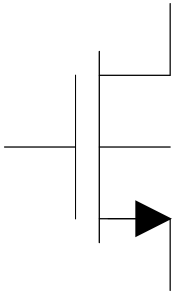


Bulk-Diffusion Generally Reverse Biased (V_{BS} > 0 or at least greater than -0.3V) for n-channel

Same functional form as for n-channel devices but V_{T0} is now negative and the magnitude of V_T still increases with the magnitude of the reverse bias

Recall:

4-terminal model extension



$$I_G = 0$$

$$I_B = 0$$

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Model Parameters : $\{\mu, C_{OX}, V_{T0}, \varphi, \gamma, \lambda\}$

Design Parameters : {W,L} but only one degree of freedom W/L

biasing or quiescent point

Small-Signal 4-terminal Model Extension

$$I_G = 0$$

$$I_B = 0$$

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

$$\mathbf{y_{11}} = \left.\frac{\partial \mathbf{I_G}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad \mathbf{y_{12}} = \left.\frac{\partial \mathbf{I_G}}{\partial \mathbf{V_{DS}}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad \mathbf{y_{13}} = \left.\frac{\partial \mathbf{I_G}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

$$y_{21} = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = g_m \qquad y_{22} = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{\vec{V}=\vec{V}_Q} = g_o \qquad y_{23} = \left.\frac{\partial I_D}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} = g_{mb}$$

$$\mathbf{y_{31}} = \left.\frac{\partial \mathbf{I_B}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad \mathbf{y_{32}} = \left.\frac{\partial \mathbf{I_B}}{\partial \mathbf{V_{DS}}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad \mathbf{y_{33}} = \left.\frac{\partial \mathbf{I_B}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

Small-Signal 4-terminal Model Extension

$$\mathbf{I_D = \mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2 \bullet (1 + \lambda V_{DS})}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Definition:

$$V_{EB} = V_{GS} - V_T$$

$$V_{EBQ} = V_{GSQ} - V_{TQ}$$

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = \mu C_{OX} \frac{W}{\not{2}L} \not{2}(V_{GS} - V_T)^1 \bullet (1 + \lambda V_{DS})\Big|_{\vec{V}=\vec{V}_Q} \cong \mu C_{OX} \frac{W}{L} V_{EBQ}$$

Same as 3-term

$$g_o = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{\vec{V}=\vec{V}_Q} = \mu C_{OX} \frac{W}{2L} 2(V_{GS} - V_T)^2 \bullet \lambda\Big|_{\vec{V}=\vec{V}_Q} \cong \lambda I_{DQ}$$

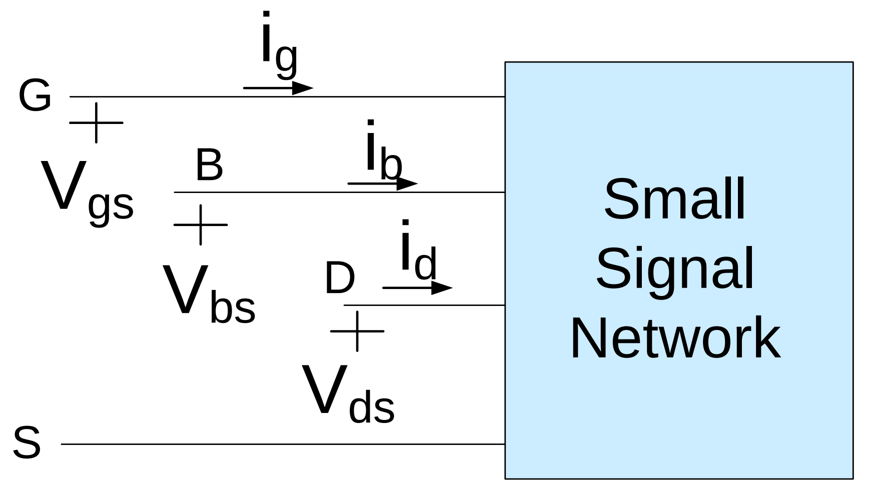
Same as 3-term

$$g_{mb} = \left.\frac{\partial I_D}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} = \mu C_{OX} \frac{W}{\not{2}L} \not{2}(V_{GS} - V_T)^1 \bullet \left(-\frac{\partial V_T}{\partial V_{BS}}\right) \bullet (1 + \lambda V_{DS})\Big|_{\vec{V}=\vec{V}_Q}$$

$$g_{mb} = \left.\frac{\partial I_D}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} \cong \mu C_{OX} \frac{W}{L} V_{EBQ} \bullet \left.\frac{\partial V_T}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} = \left(\mu C_{OX} \frac{W}{L} V_{EBQ}\right)(-\not{1})\gamma \frac{1}{2}(\phi - V_{BS})^{-\frac{1}{2}}\Big|_{\vec{V}=\vec{V}_Q} (-\not{1})$$

$$g_{mb} \cong g_m \frac{\gamma}{2\sqrt{\phi - V_{BSQ}}}$$

Small Signal Model Summary



$$i_g = 0$$

$$i_b = 0$$

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

$$g_m = \frac{\mu C_{OX} W}{L} V_{EBQ}$$

$$g_o = \lambda I_{DQ}$$

$$g_{mb} = g_m \left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right)$$

Relative Magnitude of Small Signal MOS Parameters

Consider:

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

3 alternate equivalent expressions for g_m

$$g_m = \frac{\mu C_{OX} W}{L} V_{EBQ} \qquad g_m = \sqrt{\frac{2\mu C_{OX} W}{L}}\sqrt{I_{DQ}} \qquad g_m = \frac{2I_{DQ}}{V_{EBQ}}$$

If μC_{OX}=100μA/V^2 , λ=.01V^{-1}, γ = 0.4V$^{0.5}$, V_{EBQ}=1V, W/L=1, V_{BSQ}=0V

$$I_{DQ} \cong \frac{\mu C_{ox} W}{2L} V_{EBQ}^2 = \frac{10^{-4} \cancel{W}}{2\cancel{L}}(1V)^2 = 5E\text{-}5$$

$$g_m = \frac{\mu C_{OX} W}{L} V_{EBQ} = 1E\text{-}4$$

$$g_o = \lambda I_{DQ} = 5E\text{-}7$$

$$g_{mb} = g_m\left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}}\right) = .26g_m$$

In this example

$$g_o << g_m, g_{mb}$$

$$g_{mb} < g_m$$

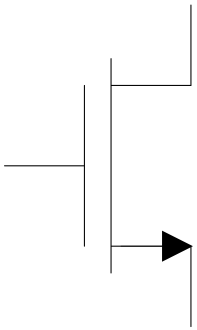
This relationship is common

In many circuits, v_{BS}=0 as well

- **Often the g_o term can be neglected in the small signal model because it is so small**
- **Be careful about neglecting g_o prior to obtaining a final expression**

Large and Small Signal Model Summary

Large Signal Model

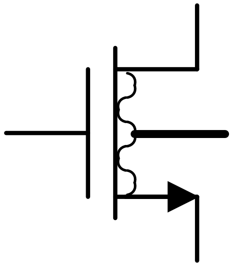


$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \frac{W}{L}\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \frac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

saturation

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Small Signal Model



saturation

$$i_g = 0$$

$$i_b = 0$$

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

where

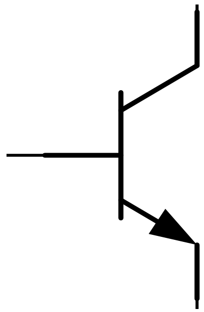
$$g_m = \frac{\mu C_{OX} W}{L} V_{EBQ}$$

$$g_{mb} = g_m \left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right)$$

$$g_o = \lambda I_{DQ}$$

Large and Small Signal Model Summary

Large Signal Model



$$I_C = \beta I_B\left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$V_{BE} > 0.4V$

$V_{BC} < 0$

Forward Active

$V_{BE} = 0.7V$
$V_{CE} = 0.2V$

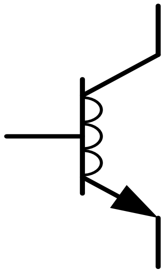
$I_C < \beta I_B$

$I_C = I_B = 0$

$V_{BE} < 0$

$V_{BC} < 0$

Small Signal Model



Forward Active

$$i_b = g_\pi v_{be}$$

$$i_c = g_m v_{be} + g_0 v_{ce}$$

where

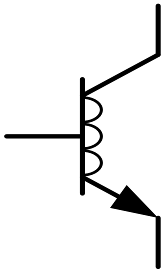
$$g_m = \frac{I_{CQ}}{V_t}$$

$$g_\pi = \frac{I_{CQ}}{\beta V_t}$$

$$g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Relative Magnitude of Small Signal BJT Parameters

$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$



$$\frac{g_m}{g_\pi} = \frac{\left[\frac{I_Q}{V_t}\right]}{\left[\frac{I_Q}{\beta V_t}\right]}$$

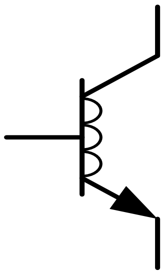
$$\frac{g_\pi}{g_o} = \frac{\left[\frac{I_Q}{\beta V_t}\right]}{\left[\frac{I_Q}{V_{AF}}\right]}$$

$$g_m >> g_\pi >> g_o$$

Often the g_o term can be neglected in the small signal model because it is so small

Relative Magnitude of Small Signal Parameters

$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$



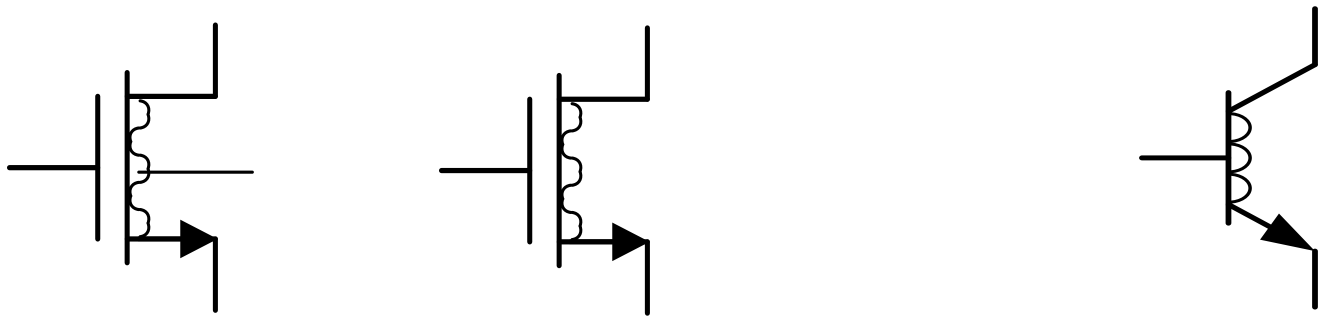
$$\frac{g_m}{g_\pi} = \frac{\left[\frac{I_Q}{V_t}\right]}{\left[\frac{I_Q}{\beta V_t}\right]} = \beta$$

$$\frac{g_\pi}{g_o} = \frac{\left[\frac{I_Q}{\beta V_t}\right]}{\left[\frac{I_Q}{V_{AF}}\right]} = \frac{V_{AF}}{\beta V_t} \approx \frac{200V}{100 \bullet 26mV} = 77$$

$$g_m >> g_\pi >> g_o$$

- **Often the g_o term can be neglected in the small signal model because it is so small**
- **Be careful about neglecting g_o prior to obtaining a final expression**

Small Signal Model Simplifications for the MOSFET and BJT

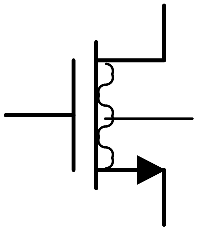


MOSFET BJT

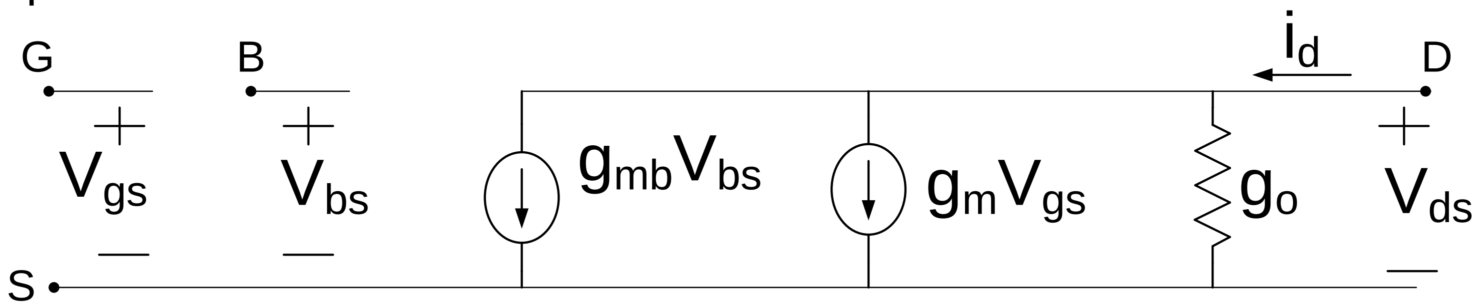
Often simplifications of the small signal model are adequate for a given application

These simplifications will be discussed next

Small Signal MOSFET Model Summary



An equivalent Circuit:



$$g_m = \frac{\mu C_{OX} W}{L}\left(V_{GSQ} - V_T\right)$$

$$g_o = \lambda I_{DQ}$$

$$g_{mb} = g_m\left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}}\right)$$

Alternate equivalent representations for g_m from $I_D \cong \mu C_{OX}\frac{W}{2L}\left(V_{GS} - V_T\right)^2$

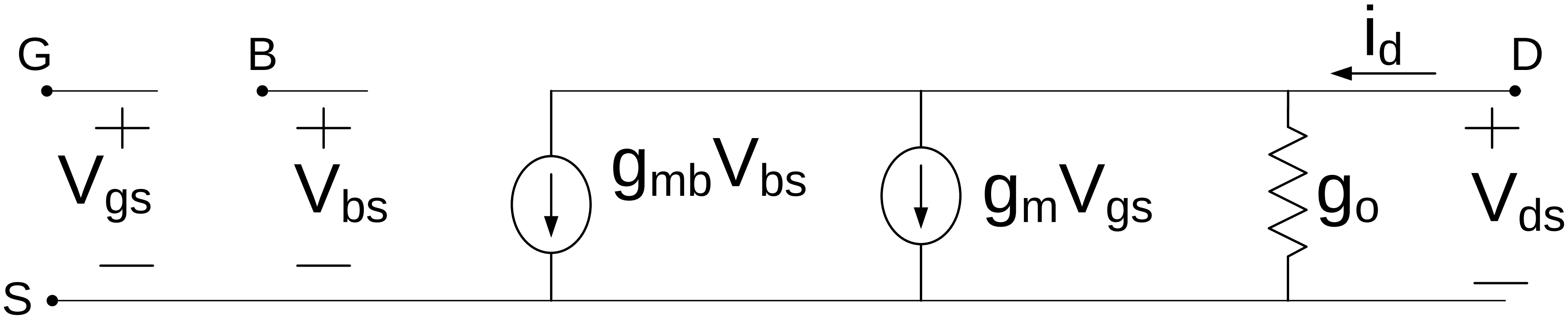
$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}}\sqrt{I_{DQ}}$$

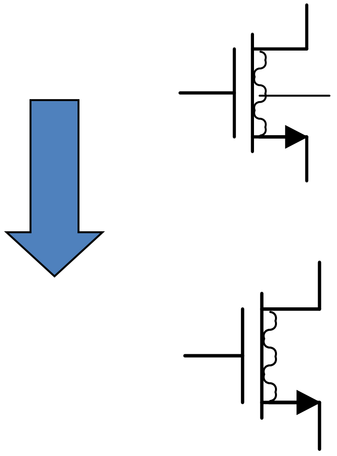
$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$$

$$g_{mb} < g_m$$

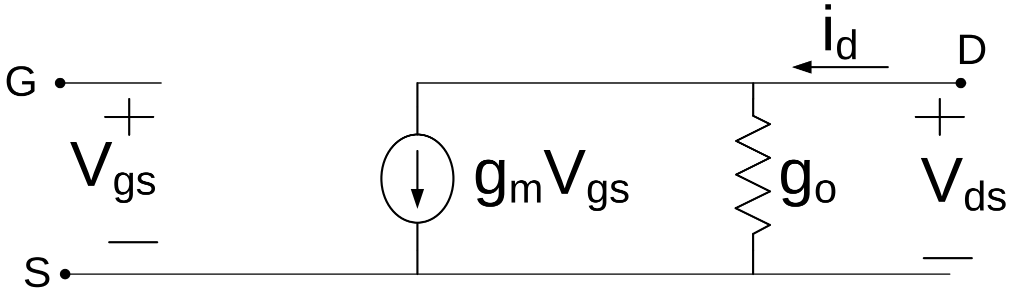
$$g_0 << g_m, g_{mb}$$

Small Signal Model Simplifications

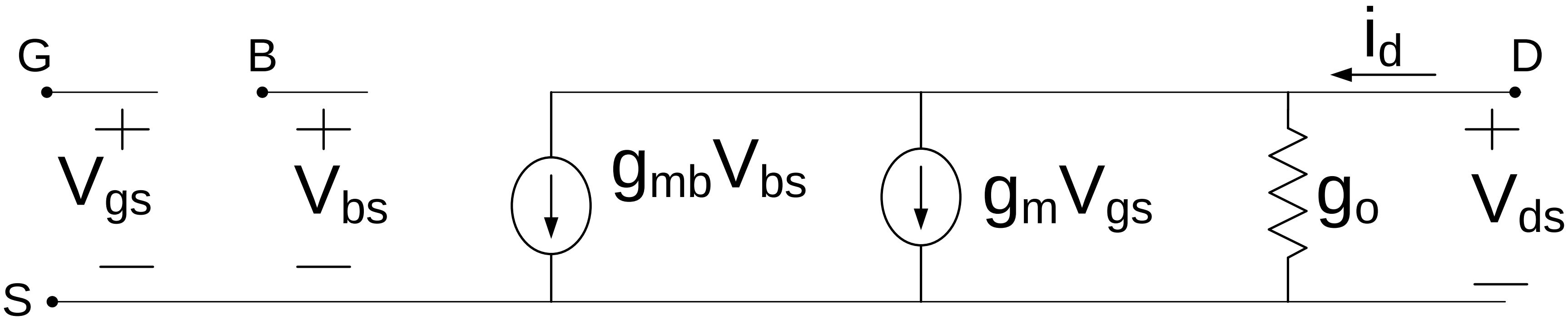


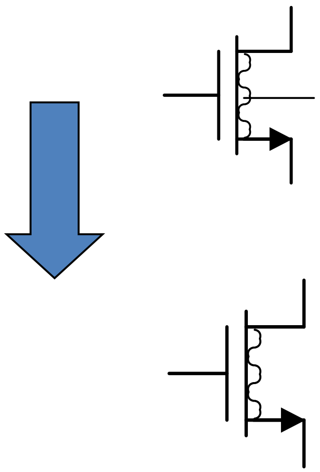


Simplification that is often adequate

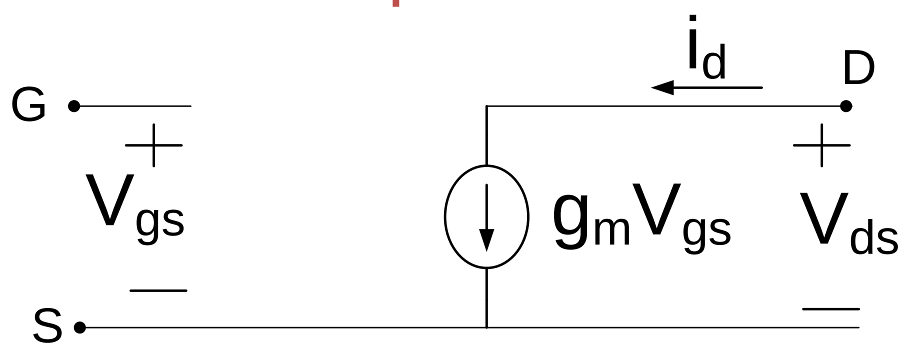


Small Signal Model Simplifications

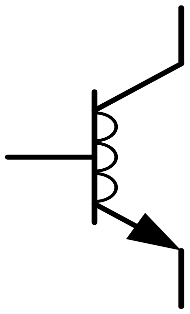




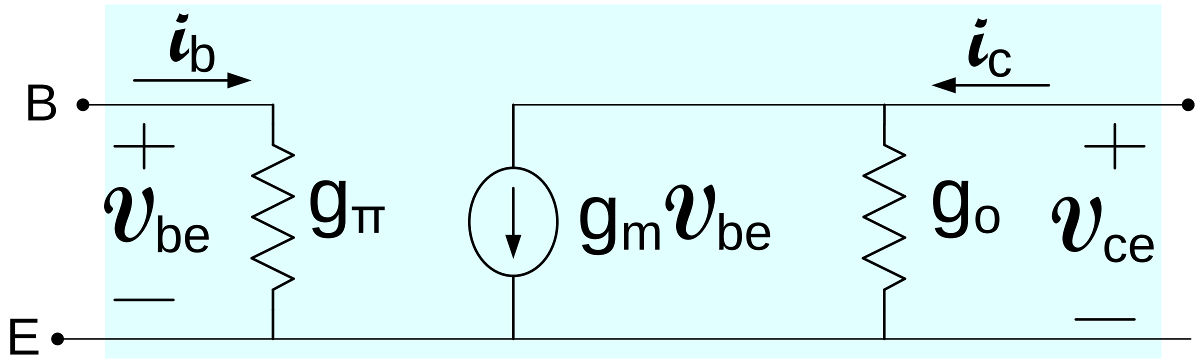
Even further simplification that is often adequate



Small Signal BJT Model Summary



An equivalent circuit

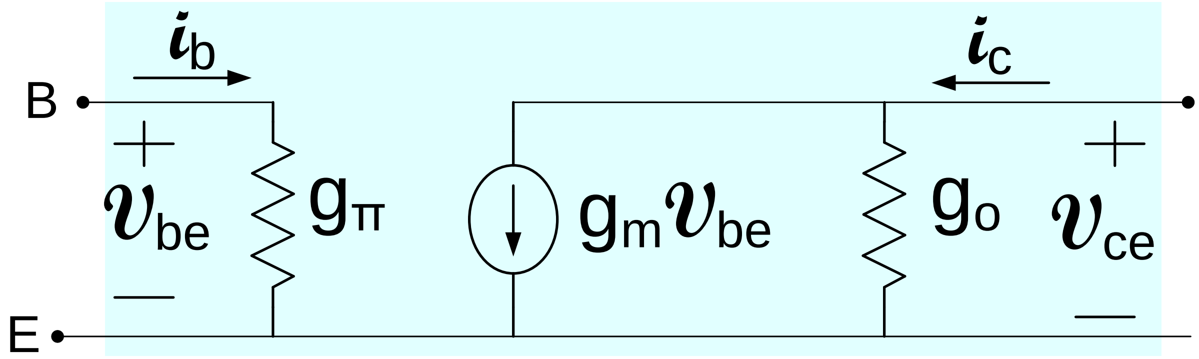


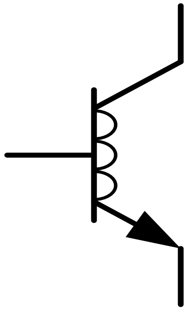
$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

$$g_m >> g_\pi >> g_o$$

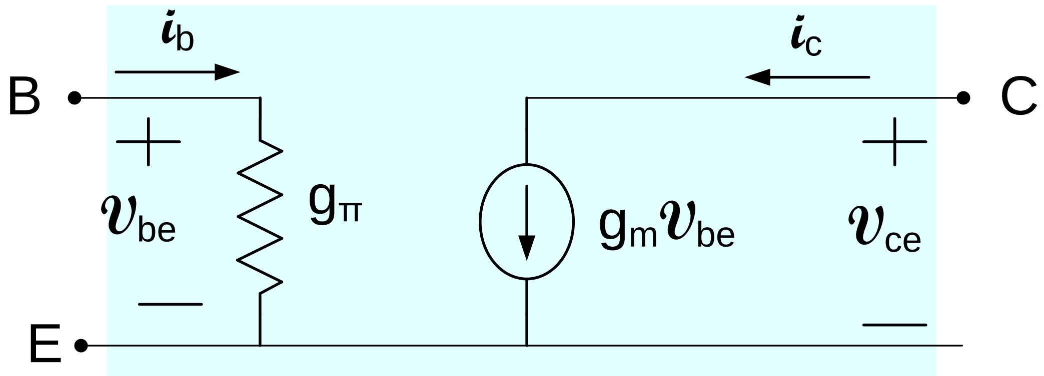
This contains absolutely no more information than the set of small-signal model equations

Small Signal BJT Model Simplifications



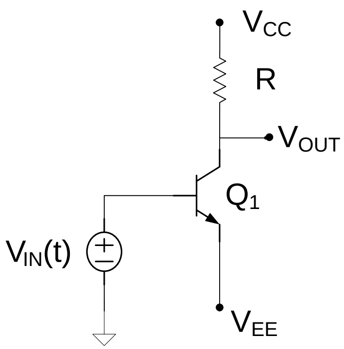


Simplification that is often adequate

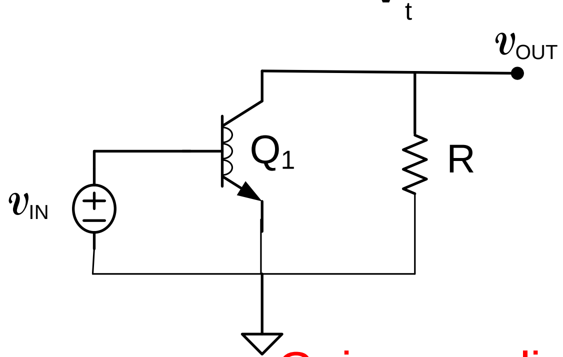


Gains for MOSFET and BJT Circuits

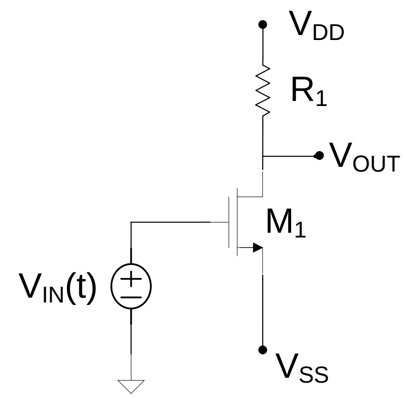
BJT



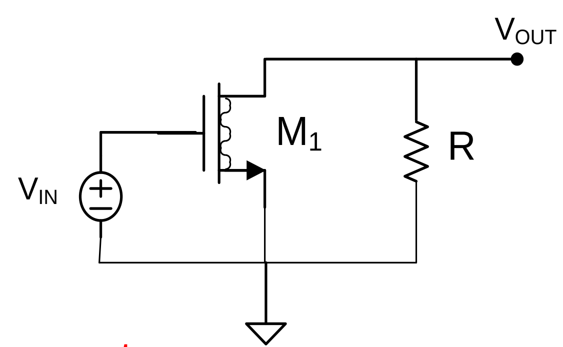
$$A_{VB} = -\frac{I_{CQ} R_1}{V_t}$$



MOSFET



$$A_{VM} = \frac{2I_{DQ} R}{[V_{SS} + V_T]}$$



For both circuits

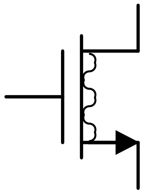
$$A_V = -g_m R$$

Gains vary linearly with small signal parameter g_m

Power is often a key resource in the design of an integrated circuit

In both circuits, power is proportional to I_{CQ}, I_{DQ}

How does g_m vary with I_{DQ}?



$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}}$$

Varies with the square root of I_{DQ}

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$$

Varies linearly with I_{DQ}

$$g_m = \frac{\mu C_{OX} W}{L}\left(V_{GSQ} - V_T\right)$$

Doesn't vary with I_{DQ}

How does g_m vary with I_{DQ}?

All of the above are true – but with qualification

g_m is a function of more than one variable (I_{DQ}) and how it varies depends upon how the remaining variables are constrained

End of Lecture 25