EE 330
Lecture 41

Digital Circuits

- Propagation Delay With Multiple Levels of Logic
- Overdrive

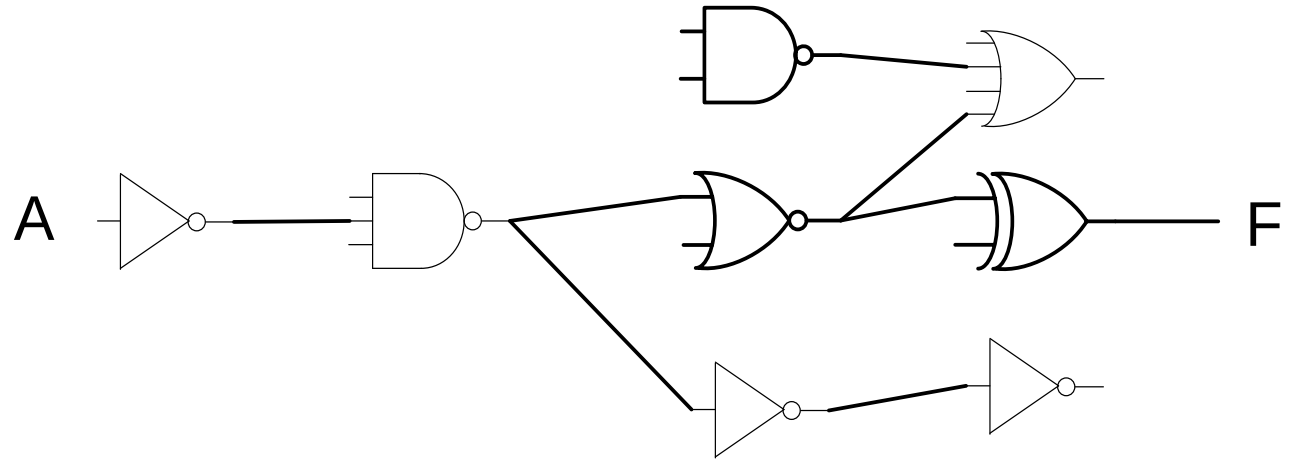
Digital Circuit Design

- Hierarchical Design
- Basic Logic Gates
- Properties of Logic Families
- Characterization of CMOS Inverter
- Static CMOS Logic Gates
  - Ratio Logic
- Propagation Delay
  - Simple analytical models
  - Elmore Delay
- Sizing of Gates

→ Propagation Delay with Multiple Levels of Logic

- Optimal driving of Large Capacitive Loads
- Power Dissipation in Logic Circuits
- Other Logic Styles
- Array Logic
- Ring Oscillators

Propagation Delay in Multiple-Levels of Logic with Stage Loading



Assume all gates sized for equal worst-case rise/fall times

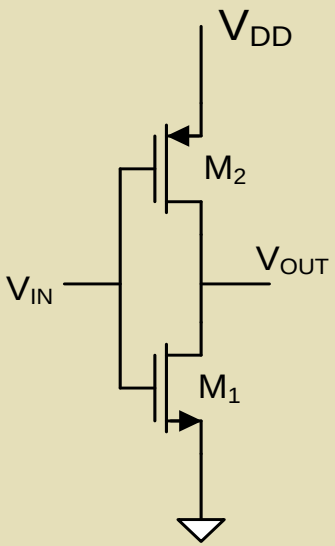
For n levels of logic between A and F

$$t_{PROP} = \sum_{k=1}^{n} t_{PROP}(k)$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Analysis strategy : Express delays in terms of those of reference inverter

Reference Inverter



Assume $\mu_n/\mu_p=3$

$W_n=W_{MIN}$, $W_p=3W_{MIN}$

In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K

$$C_{REF}=C_{IN}=4C_{OX}W_{MIN}L_{MIN}$$

FI= 1

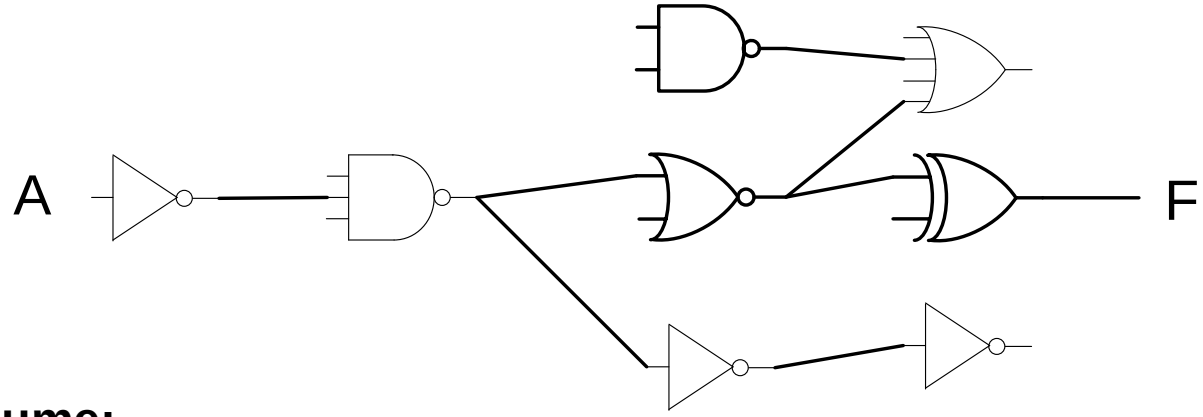
$$R_{PDREF}=\frac{L_{MIN}}{\mu_n C_{OX} W_{MIN}\left(V_{DD}-V_{Tn}\right)} \overset{V_{Tn}=.2V_{DD}}{=} \frac{L_{MIN}}{\mu_n C_{OX} W_{MIN}\left(0.8V_{DD}\right)}$$

$$t_{REF}=t_{HLREF}+t_{LHREF}=2R_{PDREF}C_{REF}$$

$$L_n=L_p=L_{MIN}$$

(Note: This C_{OX} is somewhat larger than that in the 0.5u ON process)

Propagation Delay in Multiple-Levels of Logic with Stage Loading



Assume:

- **all gates sized for equal worst-case rise/fall times**
- **all gates sized to have rise and fall times equal to that of ref inverter when driving C_{REF}**

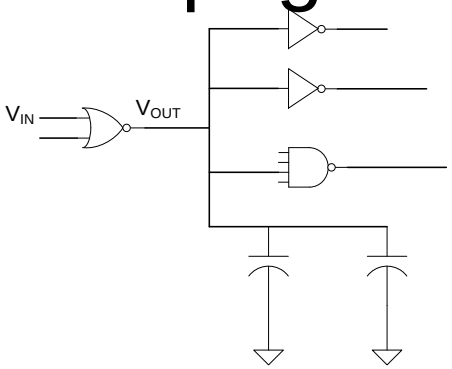
Observe:

- **Propagation delay of these gates will be scaled by the ratio of the total load capacitance on each gate to C_{REF}**

What loading will a gate see?

- **Input capacitance to other gates**
- **Any load capacitors**
- **Parasitic interconnect capacitnaces**

Propagation Delay with Stage Loading



$$t_{REF} = 2R_{PDref}C_{REF}$$

$$C_{REF} = 4C_{OX}W_{MIN}L_{MIN}$$

FI of a capacitor

$$FI_C = \frac{C}{C_{REF}}$$

FI of a gate (input k)

$$FI_G = \frac{C_{INk}}{C_{REF}}$$

FI of an interconnect

$$FI_I = \frac{C_{INI}}{C_{REF}}$$

Overall FI

$$FI = \frac{\sum_{\text{Gates}} C_{INGi} + \sum_{\text{Capacitances}} C_{INCi} + \sum_{\text{Interconnects}} C_{INIi}}{C_{REF}}$$

FI can be expressed either in units of capacitance or normalized to C_{REF}

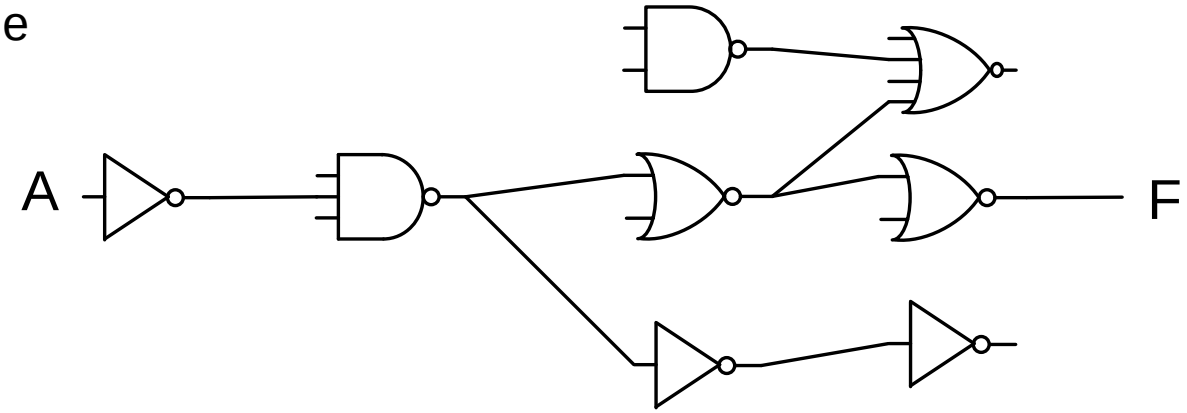
Most commonly FI is normalized but must determine from context

If gates sized to have same drive as ref inverter

$$t_{prop\text{-}k} = t_{REF} \bullet FI_{LOAD\text{-}k}$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Example



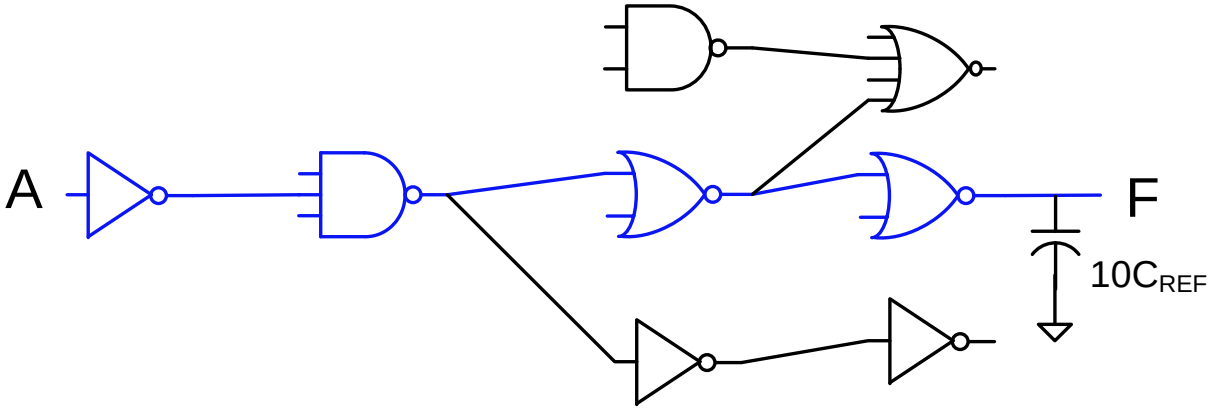
Assume all gates sized for equal worst-case rise/fall times

Assume all gate drives are the same as that of reference inverter

Neglect interconnect capacitance, assume load of $10C_{REF}$ on F output

Determine propagation delay from A to F

Propagation Delay in Multiple-Levels of Logic with Stage Loading



$$FI_{NOR} = \left(\frac{3k+1}{4}\right) C_{REF}$$

$$FI_{NAND} = \left(\frac{3+k}{4}\right) C_{REF}$$

Assume all gates sized for equal worst-case rise/fall times

Assume all gate drives are the same as that of reference inverter

Neglect interconnect capacitance, assume load of $10C_{REF}$ on F output

Determine propagation delay from A to F

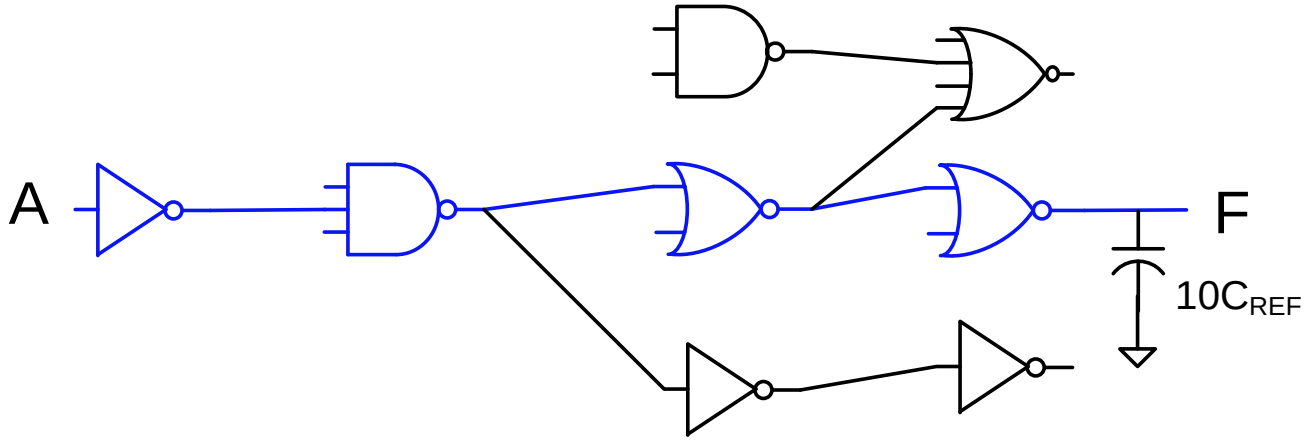
What loading will a gate see?

Derivation:

$$FI_2 = \frac{6}{4} C_{REF} \qquad FI_3 = C_{REF} + \frac{7}{4} C_{REF} \qquad FI_4 = \frac{7}{4} C_{REF} + \frac{13}{4} C_{REF} \qquad FI_{LOAD} = FI_{"5"} = 10C_{REF}$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Example



Assume all gates sized for equal worst-case rise/fall times

Assume all gate drives are the same as that of reference inverter

Neglect interconnect capacitance, assume load of $10C_{REF}$ on F output

Determine propagation delay from A to F

DERIVATIONS

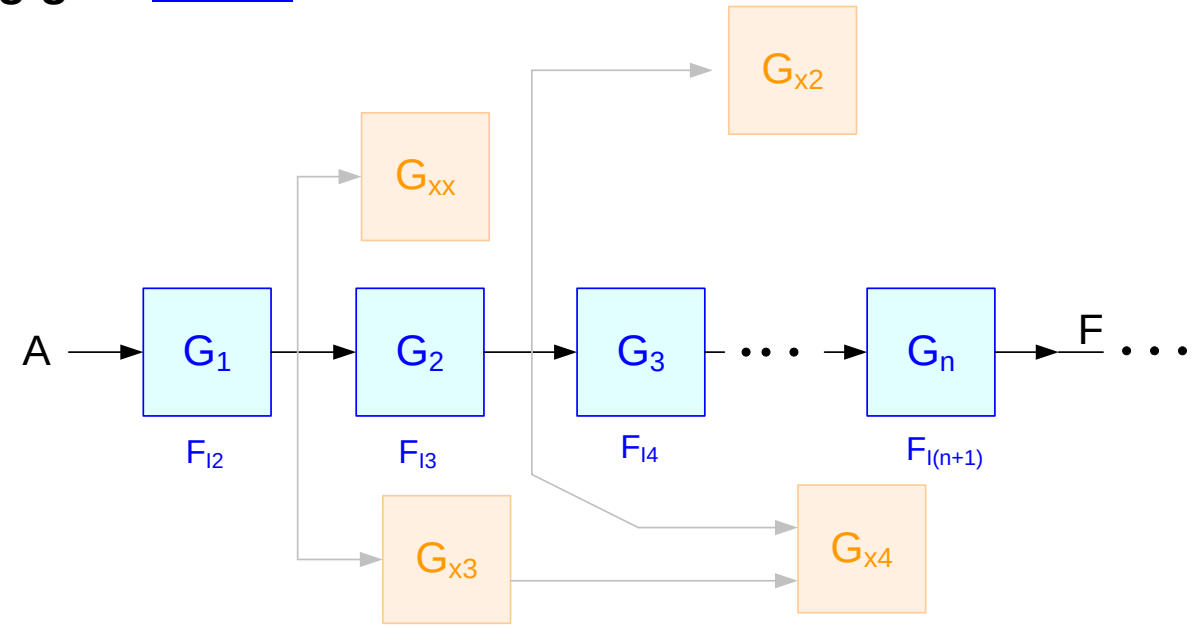
$$FI_2 = \frac{6}{4}C_{REF} \qquad FI_3 = C_{REF} + \frac{7}{4}C_{REF} \qquad FI_4 = \frac{7}{4}C_{REF} + \frac{13}{4}C_{REF} \qquad FI_5 = 10C_{REF}$$

$$t_{PROP1} = \frac{6}{4}t_{REF} \qquad t_{PROP2} = \left(1 + \frac{7}{4}\right)t_{REF} \qquad t_{PROP3} = \left(\frac{7}{4} + \frac{13}{4}\right)t_{REF} \qquad t_{PROP4} = 10t_{REF}$$

$$t_{PROP} = \sum_{k=1}^{n} t_{PROPk} = t_{REF}\sum_{k=1}^{n} FI_{(k+1)} = t_{REF}\left(\frac{6}{4} + \frac{11}{4} + \frac{20}{4} + 10\right) = t_{REF}\left(19.25\right)$$

Propagation Delay Through Multiple Stages of Logic with Stage Loading

(assuming gate drives are all same as that of reference inverter)



Identify the gate path from A to F

$$t_{PROPk} = t_{REF} FI_{(k+1)}$$

Propagation delay from A to F:

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} FI_{(k+1)}$$

This approach is analytically manageable, provides modest accuracy and is "faithful"

Digital Circuit Design

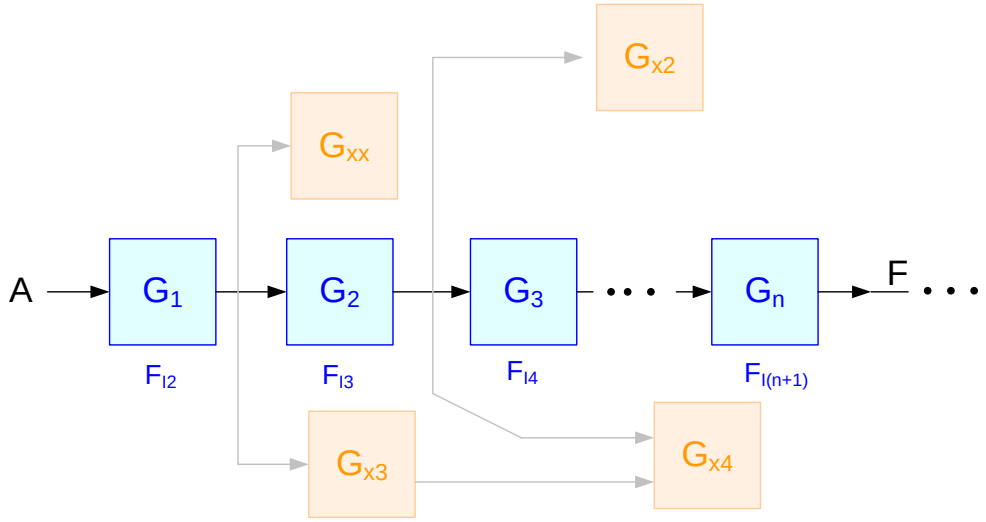
- Hierarchical Design
- Basic Logic Gates
- Properties of Logic Families
- Characterization of CMOS Inverter
- Static CMOS Logic Gates
  - Ratio Logic
- Propagation Delay
  - Simple analytical models
  - Elmore Delay
- Sizing of Gates
- Propagation Delay with Multiple Levels of Logic
- Optimal driving of Large Capacitive Loads
- Power Dissipation in Logic Circuits
- Other Logic Styles
- Array Logic
- Ring Oscillators

---

done

partial

What if the propagation delay is too long (or too short)?



Propagation delay from A to F:

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} FI_{(k+1)}$$

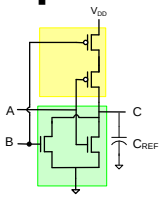
$$\mathbf{t_{PROPk} = t_{REF} FI_{(k+1)}}$$

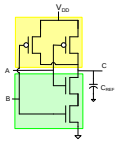
Recall:

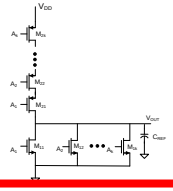
Device Sizing

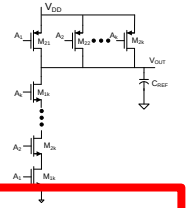
Multiple Input Gates:

2-input NOR **2-input NAND** **k-input NOR** **k-input NAND**









Equal Worst Case Rise/Fall **(and equal to that of ref inverter when driving C_{REF})**

consider the fine print !

W_n=?

W_p=?

Fastest response (t_{HL} or t_{LH}) = ?

Worst case response (t_{PROP}, usually of most interest)?

Input capacitance (FI) = ?

Minimum Sized **(assume driving a load of C_{REF})**

$W_n=W_{min}$

$W_p=W_{min}$

Fastest response (t_{HL} or t_{LH}) = ?

Slowest response (t_{HL} or t_{LH}) = ?

Worst case response (t_{PROP}, usually of most interest)?

Input capacitance (FI) = ?

Recall:

Device Sizing

Equal Worst Case Rise/Fall

(and equal to that of ref inverter when driving C_{REF})

Multiple Input Gates: 2-input NOR

(n-channel devices sized same, p-channel devices sized the same)

Assume $L_n=L_p=L_{min}$ and driving a load of C_{REF}

W_n=?

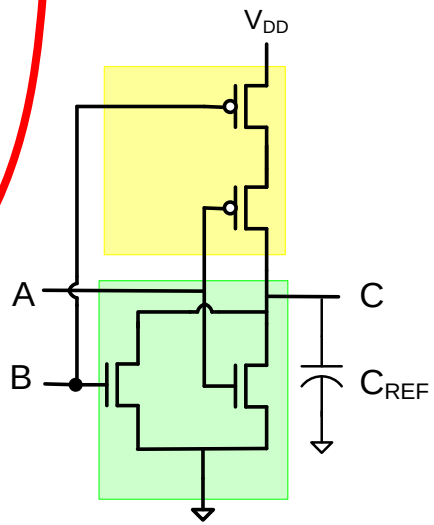
W_p=?

Input capacitance = ?

FI=?

t_{PROP}=? (worst case)

DERIVATIONS



One degree of freedom was used to satisfy the constraint indicated

$W_n=W_{MIN}$

$W_p=6W_{MIN}$

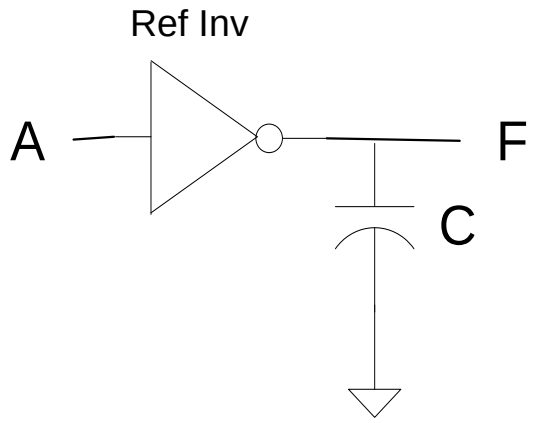
Other degree of freedom was used to achieve equal rise and fall times

$$C_{INA}=C_{INB}=C_{OX}W_{MIN}L_{MIN}+6C_{OX}W_{MIN}L_{MIN}=7C_{OX}W_{MIN}L_{MIN}=\left(\frac{7}{4}\right)4C_{OX}W_{MIN}L_{MIN}=\left(\frac{7}{4}\right)C_{REF}$$

$$FI=\left(\frac{7}{4}\right)C_{REF} \quad or \quad FI=\frac{7}{4}$$

$t_{PROP} = t_{REF}$ (worst case)

Overdrive Factors



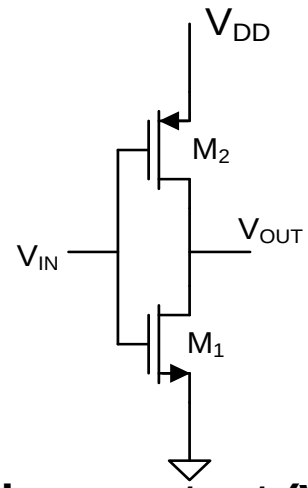
Example: Determine t_{prop} in 0.5u process if C=10pF **In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K**

$$t_{PROP} = t_{REF} \bullet FI = t_{REF} \bullet \frac{10\,pF}{4\,fF} = t_{REF} \bullet 2500$$

$$t_{PROP} = 20ps \bullet 2500 = 50nsec$$

Note this is unacceptably long !

Overdrive Factors



Scaling widths of ALL devices by constant (W_{scaled}=WxOD) will change "drive" capability relative to that of the reference inverter but not change relative value of t_{HL} and t_{LH}

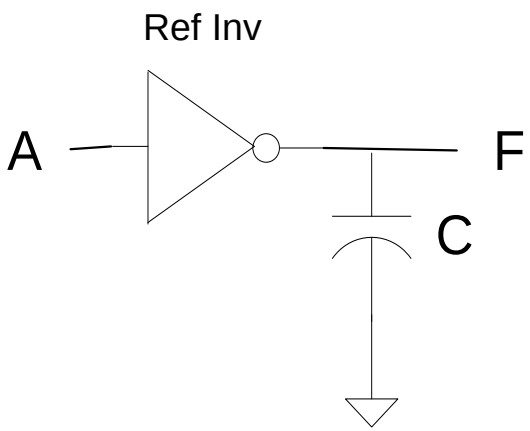
$$R_{PD}=\frac{L_1}{\mu_n C_{OX} W_1 (V_{DD}-V_{Tn})} \rightarrow R_{PDOD}=\frac{L_1}{\mu_n C_{OX} [OD \bullet W_1](V_{DD}-V_{Tn})}=\frac{R_{PD}}{OD}$$

$$R_{PU}=\frac{L_2}{\mu_p C_{OX} W_2 (V_{DD}+V_{Tp})} \rightarrow R_{PUOD}=\frac{L_2}{\mu_p C_{OX} [OD \bullet W_2](V_{DD}+V_{Tp})}=\frac{R_{PU}}{OD}$$

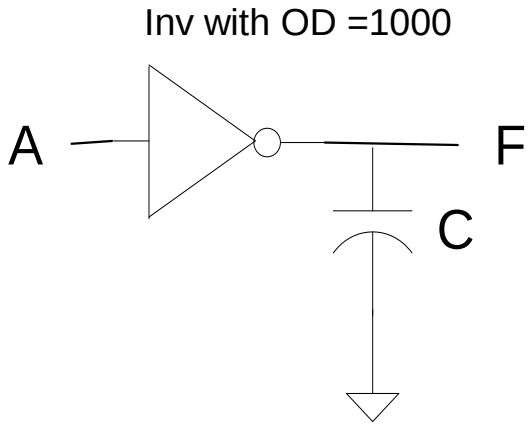
Scaling widths of ALL devices by constant will change FI by OD

$$C_{IN}=C_{OX}(W_1 L_1+W_2 L_2) \rightarrow C_{INOD}=C_{OX}([OD \bullet W_1]L_1+[OD \bullet W_2]L_2)=OD \bullet C_{IN}$$

Overdrive Factors



t_{PROP}=50nsec



t_{PROP}=?

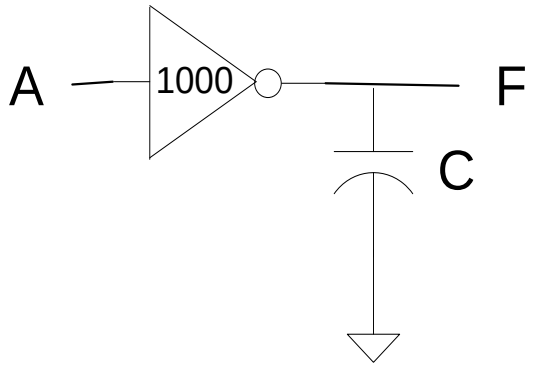
Example: Determine t_{prop} in 0.5u process if C=10pF and OD=1000

$$t_{PROP} = t_{REF} \bullet FI \bullet \frac{1}{OD} = t_{REF} \bullet \frac{10\,pF}{4\,fF} = t_{REF} \bullet 2500$$

$$t_{PROP} = t_{REF} \bullet FI \bullet \frac{1}{OD} = t_{REF} \bullet \frac{10\,pF}{4\,fF} \bullet \frac{1}{1000} = t_{REF} \bullet 2.5$$

Note sizing the inverter with the OD improved delay by a factor of 1000 !

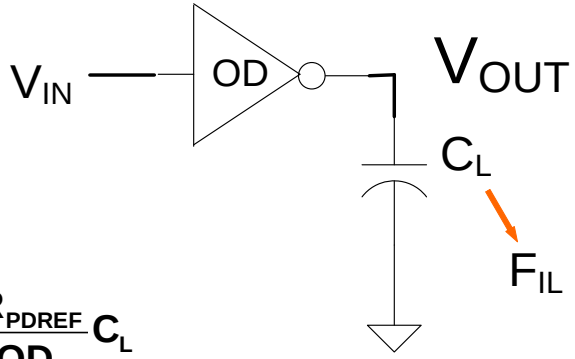
Overdrive Factors



- **By definition, the factor by which the W/L of all devices are scaled above those of the reference inverter is termed the overdrive factor, OD**
- **Scaling widths by overdrive factor DECREASES resistance by same factor**
- **Scaling all widths by a constant does not compromise the symmetry between the rise and fall times (i.e. $t_{HL}=t_{LH}$)**
- **Judicious use of overdrive can dramatically improve the speed of digital circuits**
- **Large overdrive factors are often used**
- **Scaling widths by overdrive factor INCREASES input capacitance by same factor - So is there any net gain in speed?**

Propagation Delay with Over-drive Capability

Overdrive



$$t_{HL} = t_{LH} = \frac{R_{PDREF}}{OD} C_L$$

$$t_{PROP} = t_{HL} + t_{LH} = 2\frac{R_{PDREF} C_L}{OD} = \frac{t_{REF}}{OD}$$

Asymmetric Overdrive

Define the Asymmetric Overdrive Factors of the stage to be the factor by which PU and PD resistors are scaled relative to those of the reference inverter.

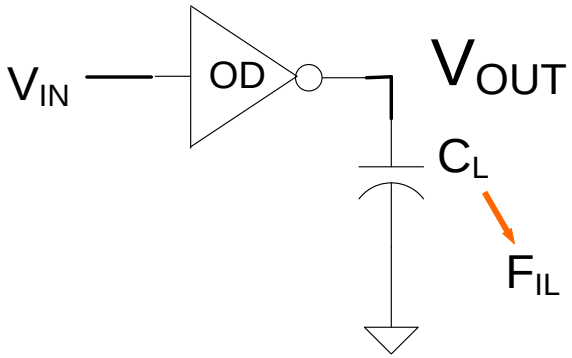
$$R_{PDEFF} = \frac{R_{PDREF}}{OD_{HL}} \qquad R_{PUEFF} = \frac{R_{PUREF}}{OD_{LH}}$$

$$t_{HL} = \frac{R_{PDREF}}{OD_{HL}} C_L \qquad t_{LH} = \frac{R_{PDREF}}{OD_{LH}} C_L$$

$$t_{PROP} = t_{HL} + t_{LH} = \frac{R_{PDREF}}{OD_{HL}} C_L + \frac{R_{PDREF}}{OD_{LH}} C_L = R_{PDREF} C_L \left[\frac{1}{OD_{HL}} + \frac{1}{OD_{LH}} \right] = \frac{t_{REF}}{2} \left[\frac{1}{OD_{HL}} + \frac{1}{OD_{LH}} \right] F_{IL}$$

Propagation Delay with Over-drive Capability

Overdrive



If inverter with OD is sized for equal rise/fall, $OD_{HL}=OD_{LH}=OD$

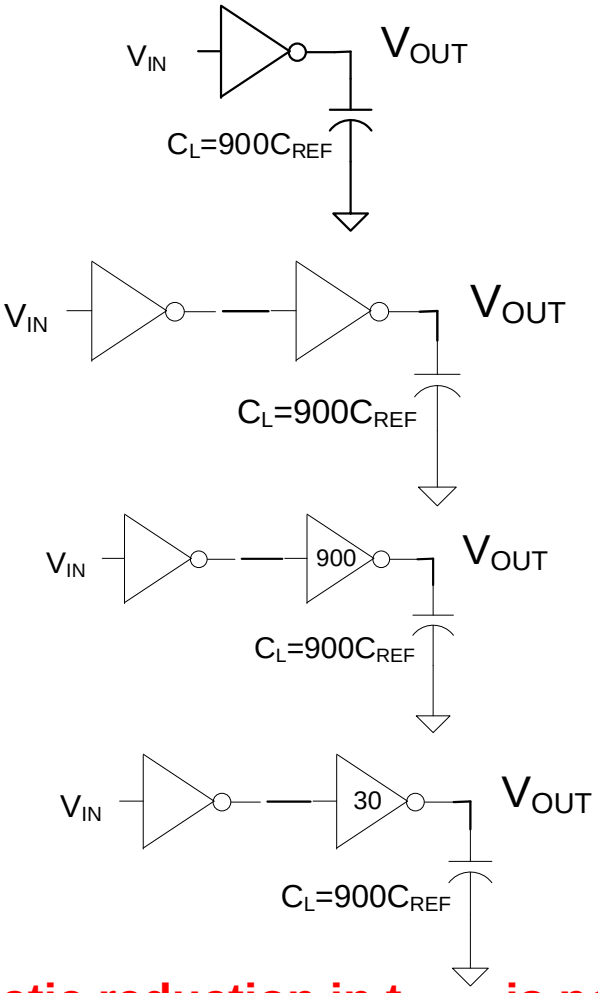
$$t_{PROP}=R_{PDREF}C_L\left[\frac{1}{OD_{HL}}+\frac{1}{OD_{LH}}\right]=R_{PDREF}C_L\frac{2}{OD}=t_{REF}\frac{F_{IL}}{OD}$$

OD may be larger or smaller than 1

Propagation Delay with Over-drive Capability

Example

Compare the propagation delays. Assume the OD is 900 in the third case and 30 in the fourth case. Don't worry about the extra inversion at this time.



$$t_{PROP}=900t_{REF}$$

$$t_{PROP}=t_{REF}+900t_{REF}=901t_{REF}$$

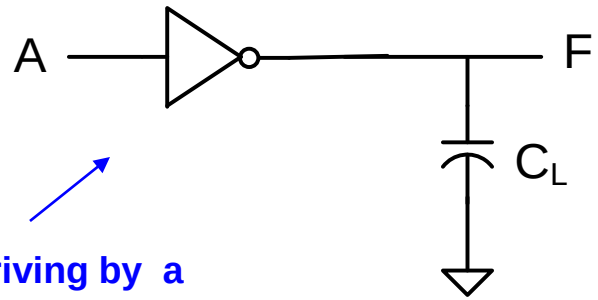
$$t_{PROP}=900t_{REF}+t_{REF}=901t_{REF}$$

$$t_{PROP}=30t_{REF}+30t_{REF}=60t_{REF}$$

- **Dramatic reduction in t_{PROP} is possible** (input is driving same in last 3 cases)
- **Will later determine what optimal number of stages and sizing is**

Driving Large Capacitive Loads

Example



Assume driving by a reference inverter

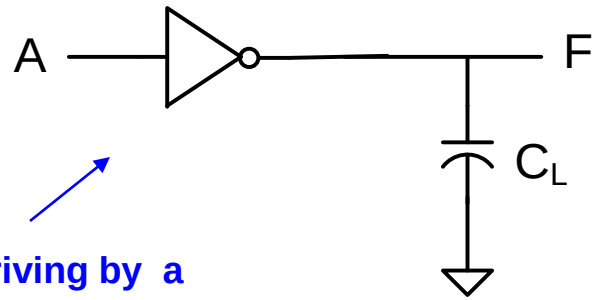
Assume $C_L=1000C_{REF}$

t_{PROP}=?

In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K

Driving Large Capacitive Loads

Example



Assume driving by a reference inverter

Assume $C_L=1000C_{REF}$

$t_{PROP}=1000t_{REF}$

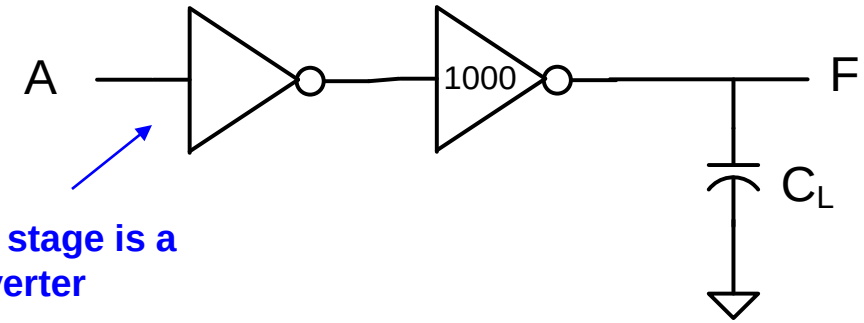
t_{PROP} is too long !

In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K

Driving Large Capacitive Loads

Example

Assume $C_L = 1000 C_{REF}$



Assume first stage is a reference inverter

$$t_{PROP} = ?$$

$$t_{PROP} = t_{REF} \sum_{k=1}^{2} \frac{F_{I(k+1)}}{OD_k}$$

$$t_{PROP} = t_{REF} \left(\frac{1}{1} 1000 + \frac{1}{1000} 1000 \right) = t_{REF} \left(1000 + 1 \right)$$

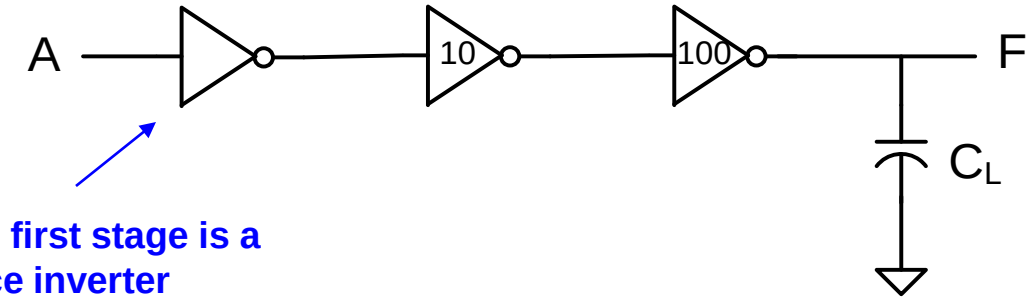
$$t_{PROP} = t_{REF} \left(1001 \right)$$

Delay of second inverter is really small but overall delay is even longer than before!

Driving Large Capacitive Loads

Example

Assume $C_L = 1000 C_{REF}$



Assume first stage is a reference inverter

$$t_{PROP} = t_{REF} \sum_{k=1}^{3} \frac{F_{I(k+1)}}{OD_k}$$

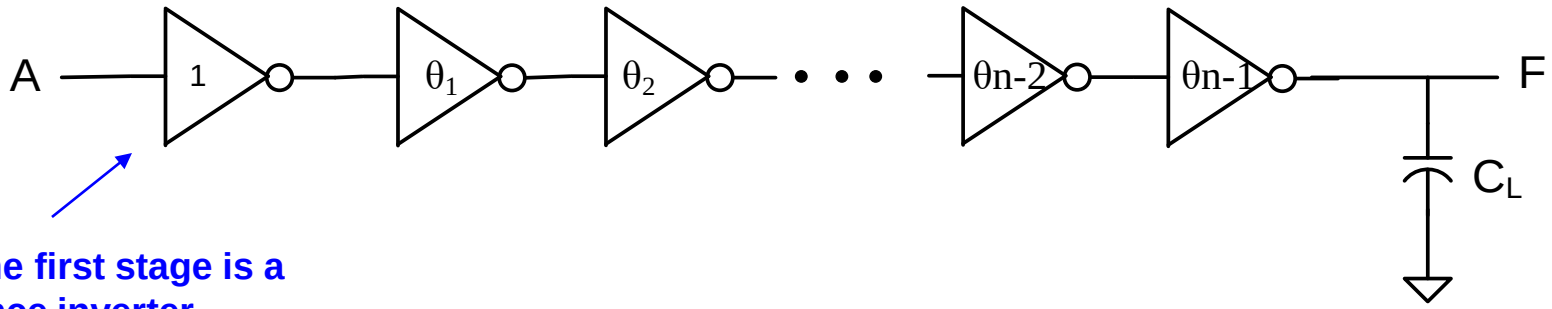
$$t_{PROP} = t_{REF} \left(\frac{1}{1} 10 + \frac{1}{10} 100 + \frac{1}{100} 1000 \right) = t_{REF} \left(10 + 10 + 10 \right)$$

$$t_{PROP} = 30 t_{REF}$$

Dramatic reduction is propagation delay (over a factor of 30!)

What is the fastest way to drive a large capacitive load?

Optimal Driving of Capacitive Loads



Assume first stage is a reference inverter

Need to determine the number of stages, n, and the OD factors for each stage to minimize t_{PROP}.

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} \frac{F_{I(k+1)}}{OD_k} \quad \rightarrow \quad t_{PROP} = t_{REF} \sum_{k=1}^{n} \frac{\theta_k}{\theta_{k-1}}$$

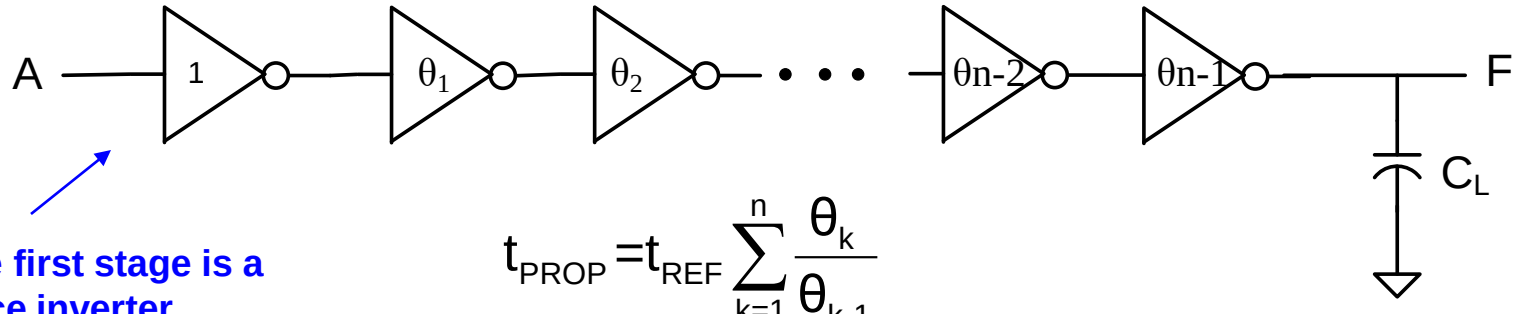
where $\theta_0 = 1$, $\theta_n = C_L / C_{REF}$

This becomes an n-parameter optimization (minimization) problem !

Unknown parameters: $\{\theta_1, \theta_2, \ldots \theta_{n-1}, n\}$

An n-parameter nonlinear optimization problem is generally difficult !!!!

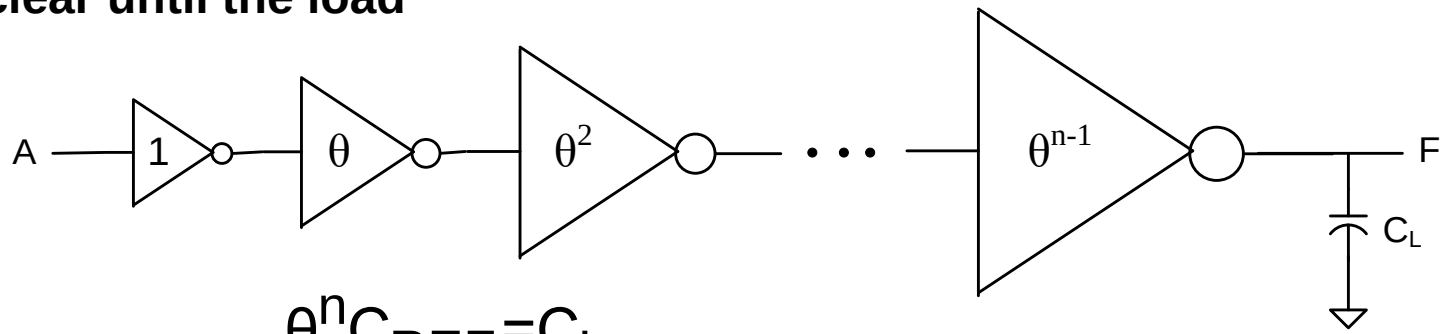
Optimal Driving of Capacitive Loads



Assume first stage is a reference inverter

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} \frac{\theta_k}{\theta_{k-1}}$$

Order reduction strategy : Assume overdrive of stages increases by the same factor clear until the load



$$\theta^n C_{REF} = C_L$$

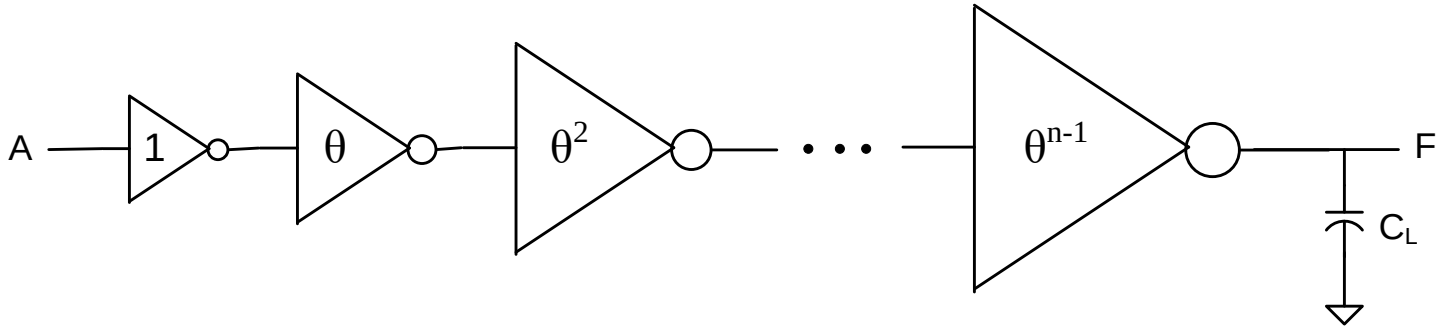
This becomes a 2-parameter optimization (minimization) problem !

Unknown parameters: $\{\theta, n\}$

One degree of freedom

One constraint : $\theta^n C_{REF} = C_L$

Optimal Driving of Capacitive Loads



$$t_{PROP} = t_{REF} \sum_{k=1}^{n} \frac{\theta_k}{\theta_{k-1}} \quad \rightarrow \quad t_{PROP} = t_{REF} \sum_{k=1}^{n} \frac{\theta^k}{\theta^{k-1}} \qquad \theta^n C_{REF} = C_L$$

$$t_{PROP} = t_{REF} n\theta$$

$$\left.\begin{aligned} t_{PROP} &= t_{REF} n\theta \\ \theta^n C_{REF} &= C_L \end{aligned}\right\}$$

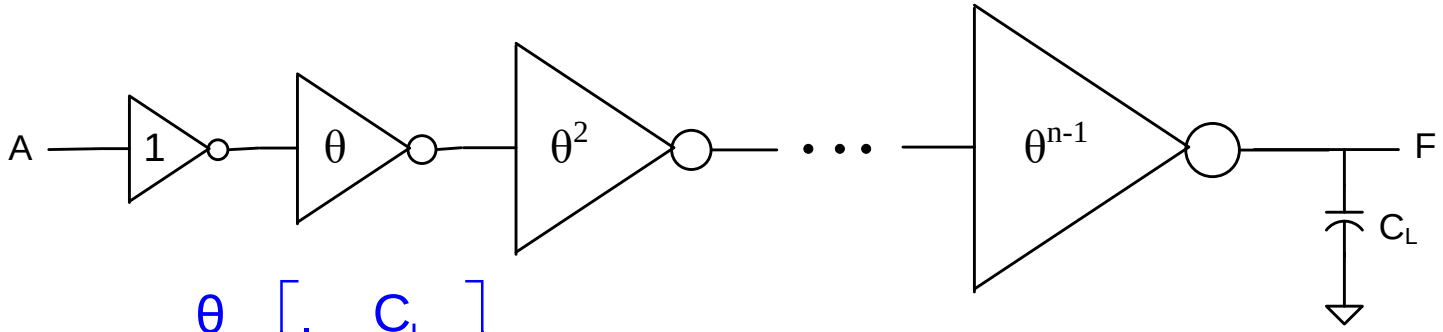
Unknown parameters: $\{\theta, n\}$

$$\theta^n C_{REF} = C_L \quad \rightarrow \quad n = \frac{1}{\ln(\theta)} \ln\left(\frac{C_L}{C_{REF}}\right)$$

Thus obtain an expression for t_{PROP} in terms of only θ

$$t_{PROP} = t_{REF} \frac{\theta}{\ln(\theta)} \left[\ln \frac{C_L}{C_{REF}}\right]$$

Optimal Driving of Capacitive Loads



$$t_{PROP} = t_{REF} \frac{\theta}{\ln(\theta)} \left[\ln \frac{C_L}{C_{REF}} \right]$$

$$\theta^n C_{REF} = C_L$$

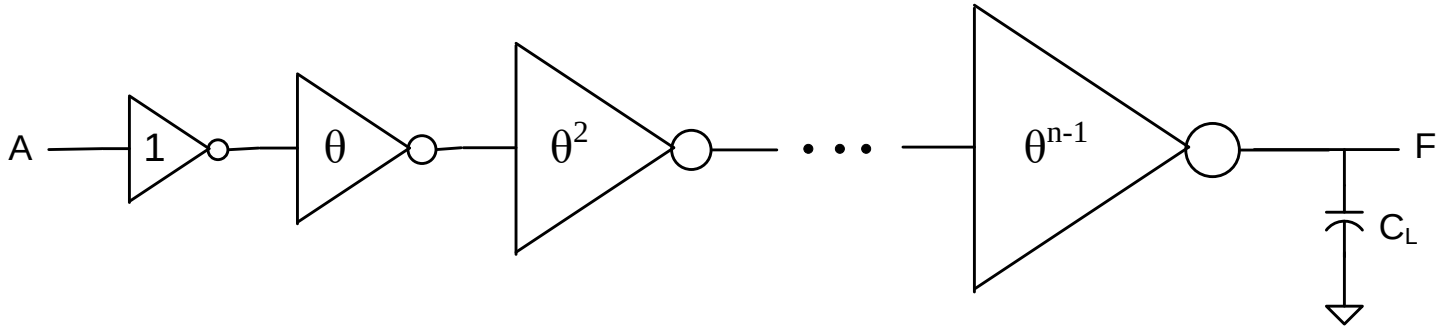
Is suffices to minimize the function $f(\theta) = \frac{\theta}{\ln(\theta)}$

$$\frac{df}{d\theta} = \frac{\ln(\theta) - \theta \bullet \left(\frac{1}{\theta}\right)}{(\ln(\theta))^2} = 0$$

$$\ln(\theta) - 1 = 0 \quad \rightarrow \quad \theta = e$$

$$n = \frac{1}{\ln(\theta)} \ln\left(\frac{C_L}{C_{REF}}\right) \quad \rightarrow \quad n = \ln\left(\frac{C_L}{C_{REF}}\right)$$

Optimal Driving of Capacitive Loads



$$\theta_{OPT} = e$$

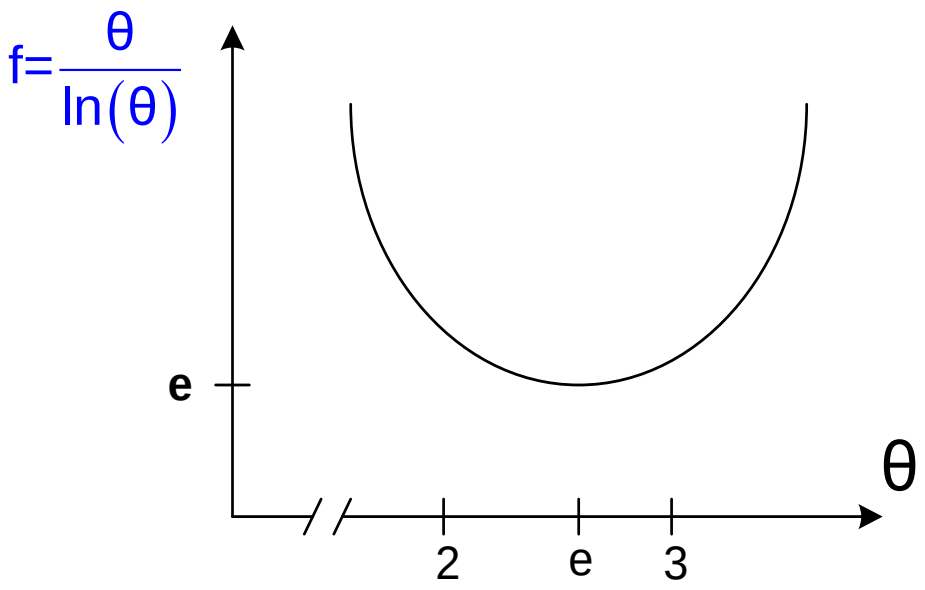
$$\theta^n C_{REF} = C_L$$

$$n_{OPT} = \ln\left(\frac{C_L}{C_{REF}}\right)$$

$$t_{PROP} = t_{REF}\frac{\theta}{\ln(\theta)}\left[\ln\frac{C_L}{C_{REF}}\right]$$

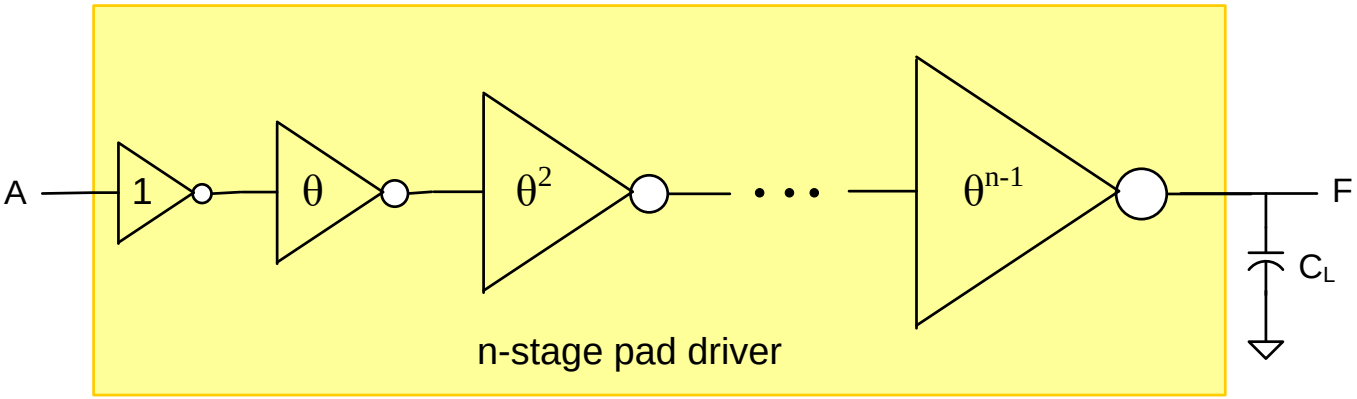
$$t_{PROP} = t_{REF} e\left[\ln\frac{C_L}{C_{REF}}\right] = n\theta t_{REF}$$

Optimal Driving of Capacitive Loads



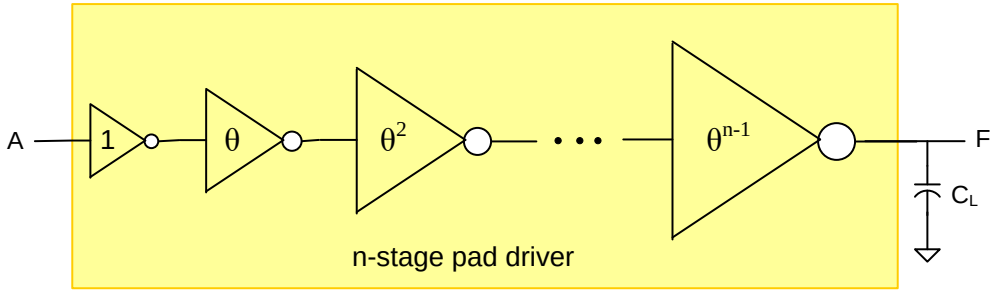
- **minimum at θ=e but shallow inflection point for 2<θ<3**
- **practically pick θ=2, θ=2.5, or θ=3**
- **since optimization may provide non-integer for n, must pick close integer**

Optimal Driving of Capacitive Loads



- **Often termed a pad driver**
- **Often used to drive large internal busses as well**
- **Generally included in standard cells or in cell library**
- **Device sizes can become very large**
- **Odd number of stages will cause signal inversion but usually not a problem**

Optimal Driving of Capacitive Loads



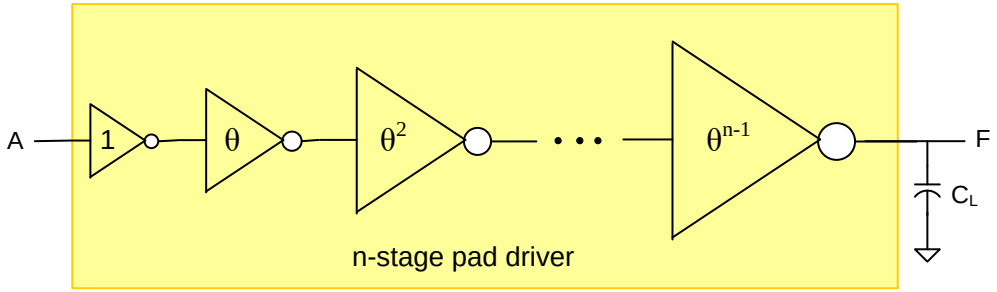
Example: Design a pad driver for driving a load capacitance of 10pF, determine t_{PROP} for the pad driver, and compare this with the propagation delay for driving the pad with a minimum-sized reference inverter. In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K

$$n_{OPT} = \ln\left(\frac{C_L}{C_{REF}}\right) = \ln\left(\frac{10pF}{4fF}\right) = 7.8$$

Select n=8, θ=2.5

$$W_{nk} = 2.5^{k-1} \bullet W_{REF}, \qquad W_{pk} = 3 \bullet 2.5^{k-1} \bullet W_{REF}$$

Optimal Driving of Capacitive Loads



Example: Design a pad driver for driving a load capacitance of 10pF, determine t_{PROP} for the pad driver, and compare this with the propagation delay for driving the pad with a minimum-sized reference inverter.

In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K

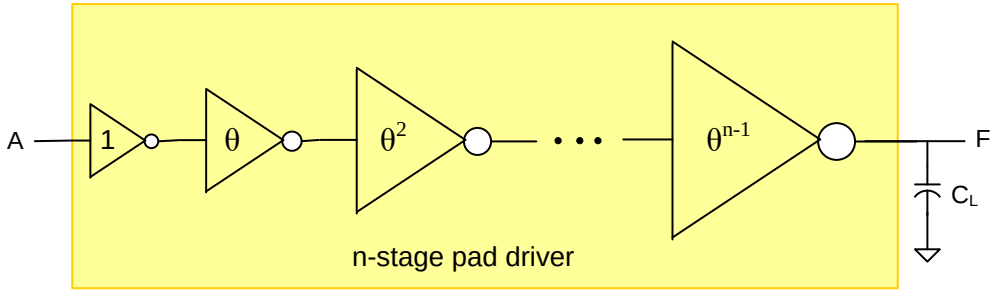
$W_{nk}=2.5^{k-1} \bullet W_{REF}$, $W_{pk}=3 \bullet 2.5^{k-1} \bullet W_{REF}$

$W_{REF}=W_{MIN}$ $L_n=L_p=L_{MIN}$

k	n-channel		p-channel	
1	1	WMIN	3	WMIN
2	2.5	WMIN	7.5	WMIN
3	6.25	WMIN	18.75	WMIN
4	15.6	WMIN	46.9	WMIN
5	39.1	WMIN	117.2	WMIN
6	97.7	WMIN	293.0	WMIN
7	244.1	WMIN	732.4	WMIN
8	610.4	WMIN	1831.1	WMIN

Note devices in last stage are very large !

Optimal Driving of Capacitive Loads



Example: Design a pad driver for driving a load capacitance of 10pF, determine t_{PROP} for the pad driver, and compare this with the propagation delay for driving the pad with a minimum-sized reference inverter.

In 0.5u proc t_{REF}=20ps, C_{REF}=4fF, R_{PDREF}=2.5K

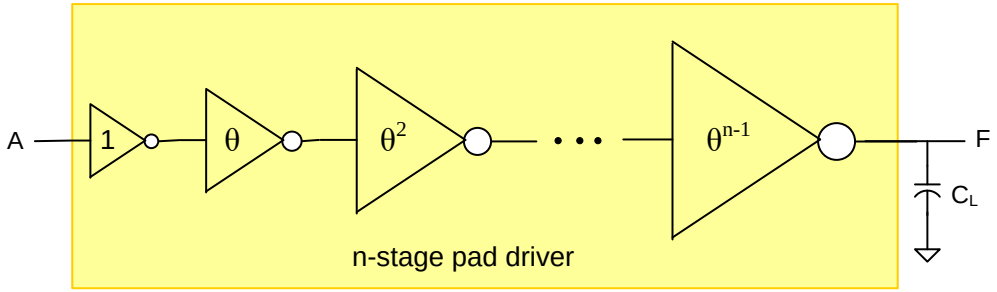
$W_{nk}=2.5^{k-1} \bullet W_{REF}$, $W_{pk}=3 \bullet 2.5^{k-1} \bullet W_{REF}$

$$t_{PROP} \cong n\theta t_{REF} = 8 \bullet 2.5 \bullet t_{REF} = 20 t_{REF}$$

More accurately:

$$t_{PROP} = t_{REF}\left(\sum_{k=1}^{7}\theta + \frac{1}{\theta^7}\frac{C_L}{C_{REF}}\right) = t_{REF}\left(17.5 + \frac{1}{610}2500\right) = 21.6 t_{REF}$$

Optimal Driving of Capacitive Loads



Example: Design a pad driver for driving a load capacitance of 10pF, determine t_{PROP} for the pad driver, and compare this with the propagation delay for driving the pad with a minimum-sized reference inverter.

In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K

$W_{nk}=2.5^{k-1} \bullet W_{REF}$, $W_{pk}=3 \bullet 2.5^{k-1} \bullet W_{REF}$

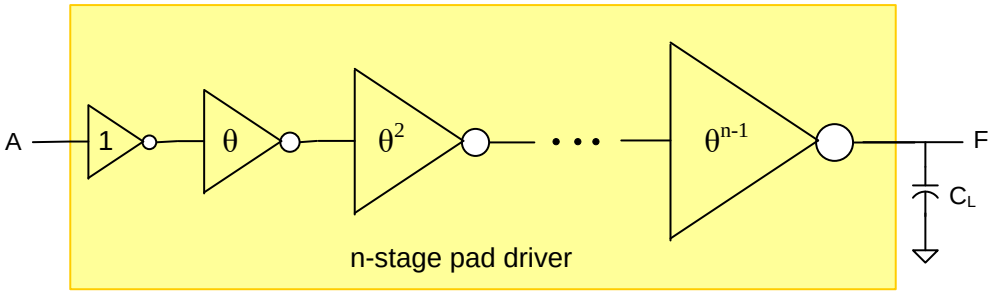
If driven directly with the minimum-sized reference inverter

$$t_{PROP}=t_{REF}\frac{C_L}{C_{REF}}=2500t_{REF}$$

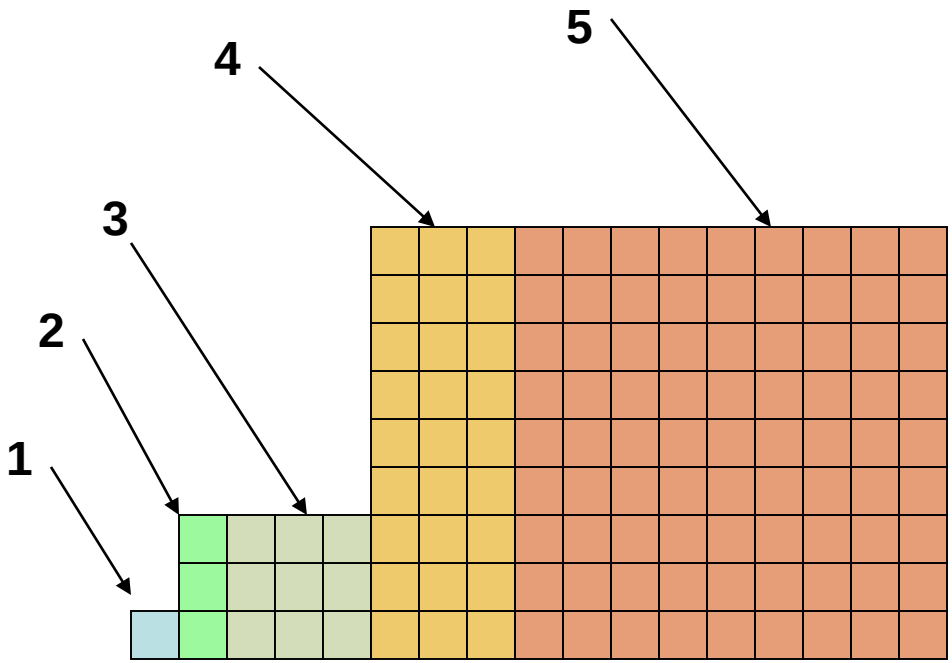
Note an improvement in speed by a factor of

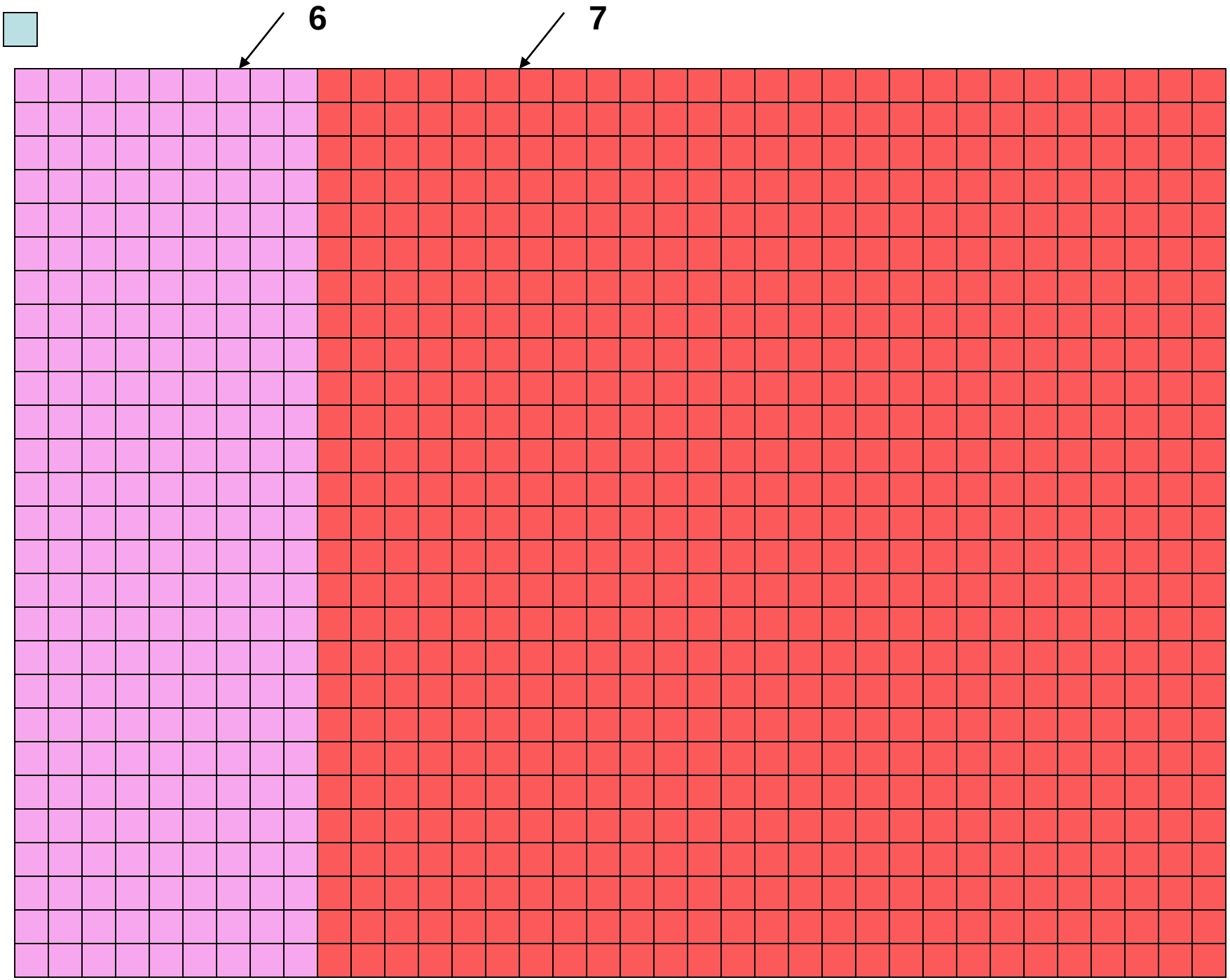
$$r=\frac{2500}{20}=125$$

Pad Driver Size Implications

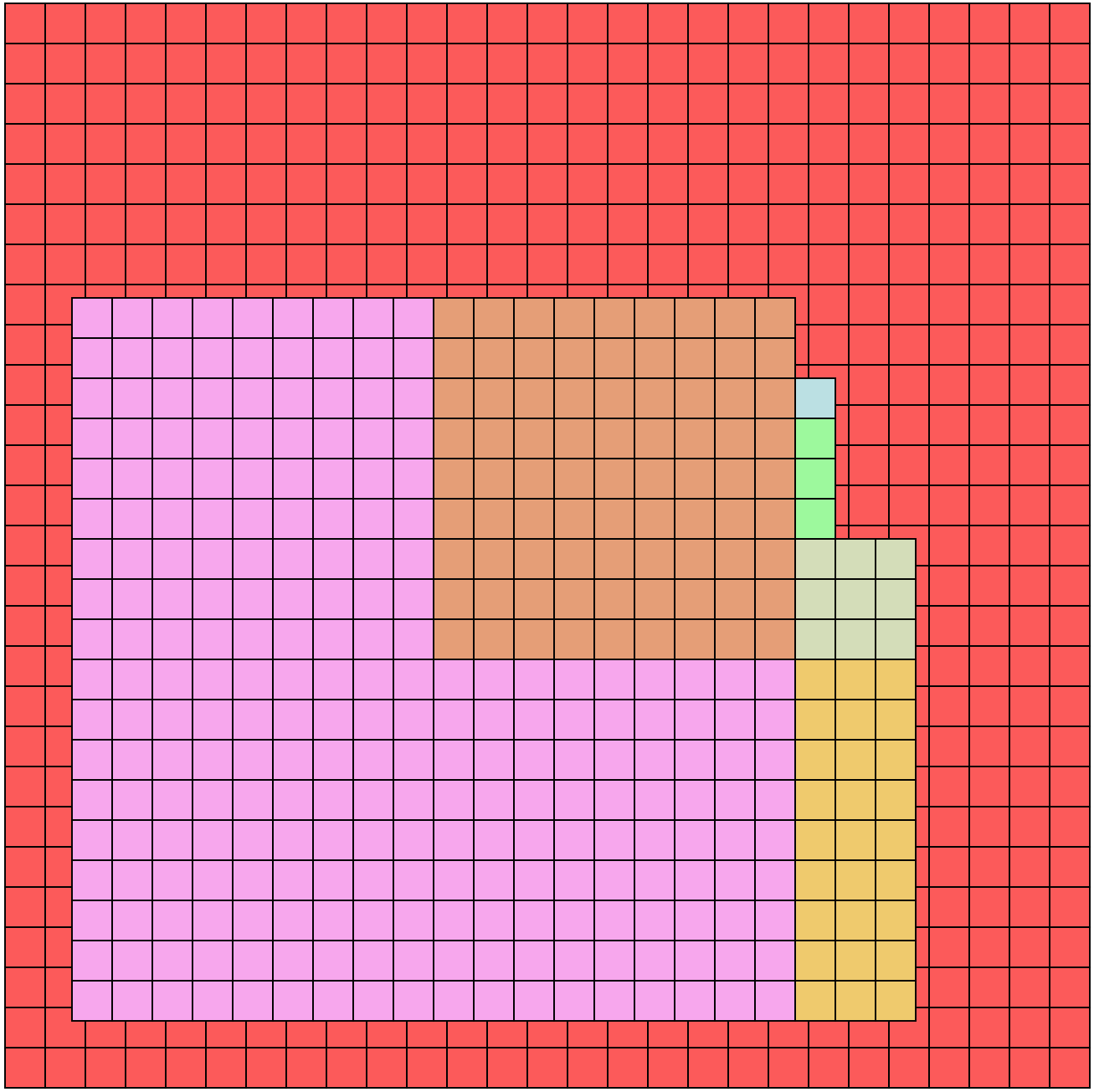


Consider a 7-stage pad driver and assume θ = 3




6
7

Area of Last Stage Larger than that of all previous stages combined!



End of Lecture 41