Chapter 1

sensitivity, bandwidth, and accuracy. Finally, various types of control systems were categorized according to the system signals, linearity, and control objectives. Several typical control-system examples were given to illustrate the analysis and design of control systems. Most systems encountered in real life are nonlinear and time-varying to some extent. The concentration on the studies of linear systems is due primarily to the availability of unified and simple-to-understand analytical methods in the analysis and design of linear systems.

► REVIEW QUESTIONS

1. List the advantages and disadvantages of an open-loop system.
2. List the advantages and disadvantages of a closed-loop system.
3. Give the definitions of ac and dc control systems.
4. Give the advantages of a digital control system over a continuous-data control system.
5. A closed-loop control system is usually more accurate than an open-loop system. **(T) (F)**
6. Feedback is sometimes used to improve the sensitivity of a control system. **(T) (F)**
7. If an open-loop system is unstable, then applying feedback will always improve its stability. **(T) (F)**
8. Feedback can increase the gain of a system in one frequency range but decrease it in another. **(T) (F)**
9. Nonlinear elements are sometimes intentionally introduced to a control system to improve its performance. **(T) (F)**
10. Discrete-data control systems are more susceptible to noise due to the nature of their signals. **(T) (F)**

Answers to these review questions can be found on this book's companion Web site: www.wiley.com/college/golnaraghi.

Chapter 2

REVIEW QUESTIONS

1. Give the definitions of the poles and zeros of a function of the complex variable s.

2. What are the advantages of the Laplace transform method of solving linear ordinary differential equations over the classical method?

3. What are state equations?

4. What is a causal system?

5. Give the defining equation of the one-sided Laplace transform.

6. Give the defining equation of the inverse Laplace transform.

7. Give the expression of the final-value theorem of the Laplace transform. What is the condition under which the theorem is valid?

8. Give the Laplace transform of the unit-step function, $u_s(t)$.

9. What is the Laplace transform of the unit-ramp function, $tu_s(t)$?

10. Give the Laplace transform of $f(t)$ shifted to the right (delayed) by T_d in terms of the Laplace transform of $f(t)$, $F(s)$.

11. If $\mathcal{L}[f_1(t)] = F_1(s)$ and $\mathcal{L}[f_2(t)] = F_2(s)$, then find $\mathcal{L}[f_1(t)]f_2(t)]$ in terms of $F_1(s)$ and $F_2(s)$.

12. Do you know how to handle the exponential term in performing the partial-fraction expansion of

$$F(s) = \frac{10}{(s+1)(s+2)}e^{-2s}$$

13. Do you know how to handle the partial-fraction expansion of a function whose denominator order is not greater than that of the numerator, for example,

$$F(s) = \frac{10(s^2+5s+1)}{(s+1)(s+2)}$$

14. In trying to find the inverse Laplace transform of the following function, do you have to perform the partial-fraction expansion?

$$F(s) = \frac{1}{(s+5)^3}$$

15. Can the Routh-Hurwitz criterion be directly applied to the stability analysis of the following systems?

(a) Continuous-data system with the characteristic equation

$$s^4 + 5s^3 + 2s^2 + 3s + 2e^{-2s} = 0$$

(b) Continuous-data system with the characteristic equation

$$s^4 - 5s^3 + 3s^2 + Ks + K^2 = 0$$

16. The first two rows of Routh's tabulation of a third-order system are

$$\begin{array}{ccc} s^3 & 2 & 2 \\ s^2 & 4 & 4 \end{array}$$

Select the correct answer from the following choices:

(a) The equation has one root in the right-half s-plane.

(b) The equation has two roots on the $j\omega$-axis at $s = j$ and $-j$. The third root is in the left-half s-plane.

(c) The equation has two roots on the $j\omega$-axis at $s = 2j$ and $s = -2j$. The third root is in the left-half s-plane.

(d) The equation has two roots on the $j\omega$-axis at $s = 2j$ and $s = -2j$. The third root is in the right-half s-plane.

17. If the numbers in the first column of Routh's tabulation turn out to be all negative, then the equation for which the tabulation is made has at least one root not in the left half of the s-plane. **(T) (F)**

18. The roots of the auxiliary equation, $A(s) = 0$, of Routh's tabulation of a characteristic equation must also be the roots of the latter. **(T) (F)**

19. The following characteristic equation of a continuous-data system represents an unstable system because it contains a negative coefficient.

$$s^3 - s^2 + 5s + 10 = 0$$ **(T) (F)**

20. The following characteristic equation of a continuous-data system represents an unstable system because there is a zero coefficient.

$$s^3 + 5s^2 + 4 = 0$$ **(T) (F)**

21. When a row of Routh's tabulation contains all zeros before the tabulation ends, this means that the equation has roots on the imaginary axis of the s-plane. **(T) (F)**

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▶ REFERENCES

Complex Variables, Laplace Transforms

1. F. B. Hildebrand, *Methods of Applied Mathematics*, 2nd Ed., Prentice Hall, Englewood Cliffs, NJ, 1965.
2. B. C. Kuo, *Linear Networks and Systems*, McGraw-Hill Book Company, New York, 1967.
3. C. R. Wylie, Jr., *Advanced Engineering Mathematics*, 2nd Ed., McGraw-Hill Book Company, New York, 1960.
4. K. Ogata, *Modern Control Engineering*, 4th ed., Prentice-Hall, NJ, 2002.
5. J. J. Distefano, A. R. Stubberud, and I. J. Williams, *Feedback and Control Systems*, 2nd ed., McGraw-Hill, 1990.
6. "ChE-400: Applied Chemical Engineering Calculations," http://www.ent.ohiou.edu/che/che400/Handouts%20and%20Class%20notes.htm.

Partial-Fraction Expansion

7. C. Pottle, "On the Partial Fraction Expansion of a Rational Function with Multiple Poles by Digital Computer," *IEEE Trans. Circuit Theory*, Vol. CT-11, pp. 161–162, Mar. 1964.
8. B. O. Watkins, "A Partial Fraction Algorithm," *IEEE Trans. Automatic Control*, Vol. AC-16, pp. 489–491, Oct. 1971.

Additional References

9. W. J. Palm, III, *Modeling, Analysis, and Control of Dynamic Systems*, 2nd Ed., John Wiley & Sons, New York, 1999.
10. K. Ogata, *Modern Control Engineering*, 4th Ed., Prentice Hall, NJ, 2002.
11. I. Cochin and W. Cadwallender, *Analysis and Design of Dynamic Systems*, 3rd Ed., Addison-Wesley, 1997.
12. A. Esposito, *Fluid Power with Applications*, 5th Ed. Prentice Hall, NJ, 2000.
13. H. V. Vu and R. S. Esfandiari, *Dynamic Systems*, Irwin/McGraw-Hill, 1997.
14. J. L. Shearer, B. T. Kulakowski, and J. F. Gardner, *Dynamic Modeling and Control of Engineering Systems*, 2nd Ed., Prentice Hall, NJ, 1997.
15. R. L. Woods and K. L. Lawrence, *Modeling and Simulation of Dynamic Systems*, Prentice Hall, NJ, 1997.

16. B. C. Kuo, *Automatic Control Systems*, 7th Ed., Prentice Hall, NJ, 1995.
17. B. C. Kuo and F. Golnaraghi, *Automatic Control Systems*, 8th Ed., John Wiley and Sons, NY, 2003.

Stability

18. F. R. Gantmacher, *Matrix Theory*, Vol. II, Chelsea Publishing Company, New York, 1964.
19. K. J. Khatwani, "On Routh-Hurwitz Criterion," *IEEE Trans. Automatic Control*, Vol. AC-26, p. 583, April 1981.
20. S. K. Pillai, "The ε Method of the Routh-Hurwitz Criterion," *IEEE Trans. Automatic Control*, Vol. AC-26, 584, April1981.

PROBLEMS

PROBLEMS FOR SECTION 2-1

2-1. Find the poles and zeros of the following functions (including the ones at infinity, if any). Mark the finite poles with × and the finite zeros with o in the s-plane.

(a) $G(s) = \dfrac{10(s+2)}{s^2(s+1)(s+10)}$

(b) $G(s) = \dfrac{10s(s+1)}{(s+2)(s^2+3s+2)}$

(c) $G(s) = \dfrac{10(s+2)}{s(s^2+2s+2)}$

(d) $G(s) = \dfrac{e^{-2s}}{10s(s+1)(s+2)}$

2-2. Poles and zeros of a function are given; find the function:

(a) Simple poles: 0, −2; poles of order 2: −3; zeros: −1, ∞

(b) Simple poles: −1, −4; zeros: 0

(c) Simple poles: −3, ∞; poles of order 2: 0,−1; zeros: $\pm j$, ∞

2-3. Use MATLAB to find the poles and zeros of the functions in Problem 2-1.

PROBLEMS FOR SECTION 2-2

2-4. Find the polar representation of $G(s)$ given in Problem 2-1 for $s = j\omega$, where ω is a constant varying from zero to infinity.

2-5. Find the polar plot of the following functions:

(a) $G(j\omega) = \dfrac{10}{(j\omega - 2)}$

(b) $G(j\omega) = \dfrac{1}{1 + 2\zeta\left(j\dfrac{\omega}{\omega_n}\right) + \left(j\dfrac{\omega}{\omega_n}\right)^2} \quad 0 < \zeta < 1$

(c) $G(j\omega) = \dfrac{1}{1 + 2\zeta\left(j\dfrac{\omega}{\omega_n}\right) + \left(j\dfrac{\omega}{\omega_n}\right)^2} \quad \zeta > 1$

(d) $G(j\omega) = \dfrac{1}{j\omega(jT\omega + 1)}$

(e) $G(j\omega) = \dfrac{e^{-j\omega L}}{(jT\omega + 1)}$

2-6. Use MATLAB to find the polar plot of the functions in Problem 2-5.

2-7. Draw the Bode plot of the following functions:

(a) $G(j\omega) = \dfrac{2000(j\omega + 0.5)}{j\omega(j\omega + 10)(j\omega + 50)}$

(b) $G(j\omega) = \dfrac{25}{j\omega(j\omega - 2.5\omega^2 + 10)}$

(c) $G(j\omega) = \dfrac{(j\omega - 100\omega^2 + 100)}{-\omega^2(j\omega - 25\omega^2 + 100)}$

(d) $G(j\omega) = \dfrac{1}{1 + 2\zeta\left(j\dfrac{\omega}{\omega_n}\right) + \left(j\dfrac{\omega}{\omega_n}\right)^2} \quad 0 \le \zeta \le 1$

(e) $G(j\omega) = \dfrac{0.03(e^{j\omega t} + 1)^2}{(e^{j\omega t} - 1)(3e^{j\omega t} + 1)(e^{j\omega t} + 0.5)}$

2-8. Use MATLAB to draw the Bode plot of the functions in Problem 2-7.

PROBLEMS FOR SECTION 2-3

2-9. Express the following set of first-order differential equations in the vector-matrix form of $\dfrac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$.

(a)
$$\frac{dx_1(t)}{dt} = -x_1(t) + 2x_2(t)$$
$$\frac{dx_2(t)}{dt} = -2x_2(t) + 3x_3(t) + u_1(t)$$
$$\frac{dx_3(t)}{dt} = -x_1(t) - 3x_2(t) - x_3(t) + u_2(t)$$

(b)
$$\frac{dx_1(t)}{dt} = -x_1(t) + 2x_2(t) + 2u_1(t)$$
$$\frac{dx_2(t)}{dt} = 2x_1(t) - x_3(t) + u_2(t)$$
$$\frac{dx_3(t)}{dt} = 3x_1(t) - 4x_2(t) - x_3(t)$$

PROBLEMS FOR SECTION 2-4

2-10. Prove theorem 3 in Section 2-4-3.
2-11. Prove the integration theorem 4 in Section 2-4-3.
2-12. Prove the shift-in-time theorem, which is

$$\mathcal{L}[g(t-T)u_s(t-T) = e^{-Ts}G(s)]$$

2-13. Prove the convolution theorem in both time and s domain, which is

$$\mathcal{L}[g_1(t) * g_2(t)] = G_1(s)G_2(s)$$
$$\mathcal{L}[g_1(t)g_2(t)] = G_1(s) * G_2(s)$$

2-14. Prove theorems 6 and 7.

2-15. Use MATLAB to obtain $\mathcal{L}\{\sin^2 2t\}$. Then, calculate $\mathcal{L}\{\cos^2 2t\}$ when you know $\mathcal{L}\{\sin^2 2t\}$. Verify your answer by calculating $\mathcal{L}\{\cos^2 2t\}$ in MATLAB.

2-16. Find the Laplace transforms of the following functions. Use the theorems on Laplace transforms, if applicable.

(a) $g(t) = 5te^{-5t}u_s(t)$

(b) $g(t) = \left(t\sin 2t + e^{-2t}\right)u_s(t)$

(c) $g(t) = 2e^{-2t}\sin 2t\, u_s(t)$

(d) $g(t) = \sin 2t \cos 2t\, u_s(t)$

(e) $g(t) = \sum_{k=0}^{\infty} e^{-5kT}\delta(t - kT)$ where $\delta(t)$ = unit-impulse function

2-17. Use MATLAB to solve Problem 2-16.

2-18. Find the Laplace transforms of the functions shown in Fig. 2P-18. First, write a complete expression for $g(t)$, and then take the Laplace transform. Let $gT(t)$ be the description of the function over the basic period and then delay $gT(t)$ appropriately to get $g(t)$. Take the Laplace transform of $g(t)$ to get the following:

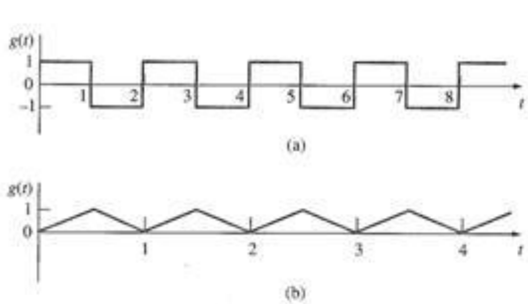


Figure 2P-18

2-19. Find the Laplace transform of the following function.

$$g(t) = \begin{cases} t+1 & 0 \le t < 1 \\ 0 & 1 \le t < 2 \\ 2-t & 2 \le t < 3 \\ 0 & t \ge 3 \end{cases}$$

2-20. Find the Laplace transform of the periodic function in Fig. 2P-20.

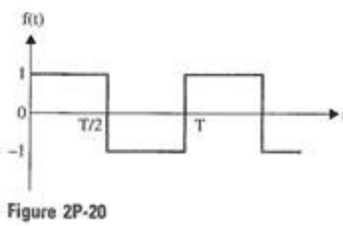


Figure 2P-20

2-21. Find the Laplace transform of the function in Fig. 2P-21.

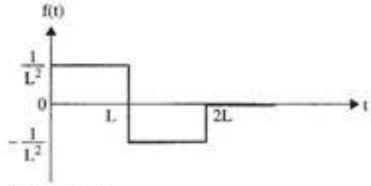


Figure 2P-21

2-22. Solve the following differential equations by means of the Laplace transform.

(a) $\frac{d^2 f(t)}{dt^2} + 5\frac{df(t)}{dt} + 4f(t) = e^{-2t}u_s(t)$ Assume zero initial conditions.

(b) $\begin{cases} \frac{dx_1(t)}{dt} = x_2(t) \\ \frac{dx_2(t)}{dt} = -2x_1(t) - 3x_2(t) + u_s(t)\, x_1(0) = 1,\ x_2(0) = 0 \end{cases}$

(c) $\frac{d^3y(t)}{dt^2} + 2\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + 2y(t) = -e^{-t}u_s(t)$

$\frac{d^2y}{dt^2}(0) = -1 \quad \frac{dy}{dt}(0) = 1 \quad y(0) = 0$

2-23. Use MATLAB to find the Laplace transform of the functions in Problem 2-22.

2-24. Use MATLAB to solve the following differential equation:

$$\frac{d^2y}{dt^2} - y = e^t \text{ (Assuming zero initial conditions)}$$

2-25. A series of a three-reactor tank is arranged as shown in Fig. 2P-25 for chemical reaction.

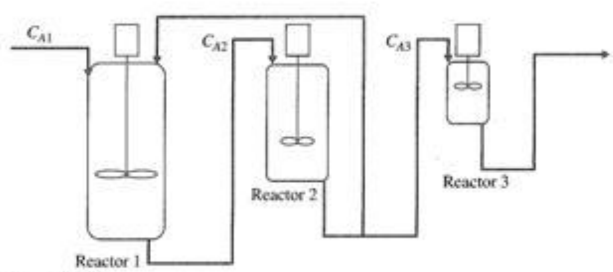


Figure 2P-25

The state equation for each reactor is defined as follows:

$$R1: \frac{dC_{A1}}{dt} = \frac{1}{V_1}[1000 + 100C_{A2} - 1100C_{A1} - k_1V_1C_{A1}]$$

$$R2: \frac{dC_{A2}}{dt} = \frac{1}{V_2}[1100C_{A1} - 1100C_{A2} - k_2V_2C_{A2}]$$

$$R3: \frac{dC_{A3}}{dt} = \frac{1}{V_3}[1000C_{A2} - 1000C_{A3} - k_3V_3C_{A3}]$$

when V_i and k_i represent the volume and the temperature constant of each tank as shown in the following table:

Reactor	V_i	k_i
1	1000	0.1
2	1500	0.2
3	100	0.4

Use MATLAB to solve the differential equations assuming $C_{A1} = C_{A2} = C_{A3} = 0$ at $t = 0$.

PROBLEMS FOR SECTION 2-5

2-26. Find the inverse Laplace transforms of the following functions. First, perform partial-fraction expansion on $G(s)$; then, use the Laplace transform table.

(a) $G(s) = \dfrac{1}{s(s+2)(s+3)}$

(b) $G(s) = \dfrac{10}{(s+1)^2(s+3)}$

(c) $G(s) = \dfrac{100(s+2)}{s(s^2+4)(s+1)}e^{-s}$

(d) $G(s) = \dfrac{2(s+1)}{s(s^2+s+2)}$

(e) $G(s) = \dfrac{1}{(s+1)^3}$

(f) $G(s) = \dfrac{2(s^2+s+1)}{s(s+1.5)(s^2+5s+5)}$

(g) $G(s) = \dfrac{2+2se^{-s}+4e^{-2s}}{s^2+3s+2}$

(h) $G(s) = \dfrac{2s+1}{s^3+6s^2+11s+6}$

(i) $G(s) = \dfrac{3s^3+10s^2+8s+5}{s^4+5s^3+7s^2+5s+6}$

2-27. Use MATLAB to find the inverse Laplace transforms of the functions in Problem 2-26. First, perform partial-fraction expansion on $G(s)$; then, use the inverse Laplace transform.

2-28. Given the state equation of the system, convert it to the set of first-order differential equation.

(a) $A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 1 \\ -1 & -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

2-29. The following differential equations represent linear time-invariant systems, where $r(t)$ denotes the input and $y(t)$ the output. Find the transfer function $Y(s)/R(s)$ for each of the systems. (Assume zero initial conditions.)

(a) $\frac{d^3y(t)}{dt^3} + 2\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 3\frac{dr(t)}{dt} + r(t)$

(b) $\frac{d^4y(t)}{dt^4} + 10\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + 5y(t) = 5r(t)$

(c) $\frac{d^3y(t)}{dt^3} + 10\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) + 2\int_0^t y(\tau)d\tau = \frac{dr(t)}{dt} + 2r(t)$

(d) $2\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + 5y(t) = r(t) + 2r(t-1)$

(e) $\frac{d^2y(t+1)}{dt^2} + 4\frac{dy(t+1)}{dt} + 5y(t+1) = \frac{dr(t)}{dt} + 2r(t) + 2\int_{-\infty}^{t} r(\tau)d\tau$

(f) $\frac{d^3y(t)}{dt^2} + 2\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + 2y(t) + 2\int_{-\infty}^{t} y(\tau)d\tau = \frac{dr(t-2)}{dt} + 2r(t-2)$

2-30. Use MATLAB to find $Y(s)/R(s)$ for the differential equations in Problem 2-29.

2-31. Use MATLAB to find the partial-fraction expansion to the following functions.

(a) $G(s) = \frac{10(s+1)}{s^2(s+4)(s+6)}$

(b) $G(s) = \frac{(s+1)}{s(s+2)(s^2+2s+2)}$

(c) $G(s) = \frac{5(s+2)}{s^2(s+1)(s+5)}$

(d) $G(s) = \frac{5e^{-2s}}{(s+1)(s^2+s+1)}$

(e) $G(s) = \frac{100(s^2+s+3)}{s(s^2+5s+3)}$

(f) $G(s) = \frac{1}{s(s^2+1)(s+0.5)^2}$

(g) $G(s) = \frac{2s^3+s^2+8s+6}{(s^2+4)(s^2+2s+2)}$

(h) $G(s) = \frac{2s^4+9s^3+15s^2+s+2}{s^2(s+2)(s+1)^2}$

2-32. Use MATLAB to find the inverse Laplace transforms of the functions in Problem 2-31.

PROBLEMS FOR SECTIONS 2-7 THROUGH 2-13

2-33. Without using the Routh-Hurwitz criterion, determine if the following systems are asymptotically stable, marginally stable, or unstable. In each case, the closed-loop system transfer function is given.

(a) $M(s) = \dfrac{10(s+2)}{s^3 + 3s^2 + 5s}$

(b) $M(s) = \dfrac{s-1}{(s+5)(s^2+2)}$

(c) $M(s) = \dfrac{K}{s^3 + 5s + 5}$

(d) $M(s) = \dfrac{100(s-1)}{(s+5)(s^2+2s+2)}$

(e) $M(s) = \dfrac{100}{s^3 - 2s^2 + 3s + 10}$

(f) $M(s) = \dfrac{10(s+12.5)}{s^4 + 3s^3 + 50s^2 + s + 10^6}$

2-34. Use the ROOTS command in MATLAB to solve Problem 2-33.

2-35. Using the Routh-Hurwitz criterion, determine the stability of the closed-loop system that has the following characteristic equations. Determine the number of roots of each equation that are in the right-half s-plane and on the $j\omega$-axis.

(a) $s^3 + 25s^2 + 10s + 450 = 0$

(b) $s^3 + 25s^2 + 10s + 50 = 0$

(c) $s^3 + 25s^2 + 250s + 10 = 0$

(d) $2s^4 + 10s^3 + 5.5s^2 + 5.5s + 10 = 0$

(e) $s^6 + 2s^5 + 8s^4 + 15s^3 + 20s^2 + 16s + 16 = 0$

(f) $s^4 + 2s^3 + 10s^2 + 20s + 5 = 0$

(g) $s^8 + 2s^7 + 8s^6 + 12s^5 + 20s^4 + 16s^3 + 16s^2 = 0$

2-36. Use MATLAB to solve Problem 2-35.

2-37. Use MATLAB Toolbox 2-13-1 to find the roots of the following characteristic equations of linear continuous-data systems and determine the stability condition of the systems.

(a) $s^3 + 10s^2 + 10s + 130 = 0$

(b) $s^4 + 12s^3 + s^2 + 2s + 10 = 0$

(c) $s^4 + 12s^3 + 10s^2 + 10s + 10 = 0$

(d) $s^4 + 12s^3 + s^2 + 10s + 1 = 0$

(e) $s^6 + 6s^5 + 125s^4 + 100s^3 + 100s^2 + 20s + 10 = 0$

(f) $s^5 + 125s^4 + 100s^3 + 100s^2 + 20s + 10 = 0$

2-38. For each of the characteristic equations of feedback control systems given, use MATLAB to determine the range of K so that the system is asymptotically stable. Determine the value of K so that the system is marginally stable and determine the frequency of sustained oscillation, if applicable.

(a) $s^4 + 25s^3 + 15s^2 + 20s + K = 0$

(b) $s^4 + Ks^3 + 2s^2 + (K+1)s + 10 = 0$

(c) $s^3 + (K+2)s^2 + 2Ks + 10 = 0$

(d) $s^3 + 20s^2 + 5s + 10K = 0$

(e) $s^4 + Ks^3 + 5s^2 + 10s + 10K = 0$

(f) $s^4 + 12.5s^3 + s^2 + 5s + K = 0$

2-39. The loop transfer function of a single-loop feedback control system is given as

$$G(s)H(s) = \frac{K(s+5)}{s(s+2)(1+Ts)}$$

The parameters K and T may be represented in a plane with K as the horizontal axis and T as the vertical axis. Determine the regions in the T-versus-K parameter plane where the closed-loop system is asymptotically stable and where it is unstable. Indicate the boundary on which the system is marginally stable.

2-40. Given the forward-path transfer function of unity-feedback control systems, apply the Routh-Hurwitz criterion to determine the stability of the closed-loop system as a function of K. Determine the value of K that will cause sustained constant-amplitude oscillations in the system. Determine the frequency of oscillation.

(a) $G(s) = \dfrac{K(s+4)(s+20)}{s^3(s+100)(s+500)}$

(b) $G(s) = \dfrac{K(s+10)(s+20)}{s^2(s+2)}$

(c) $G(s) = \dfrac{K}{s(s+10)(s+20)}$

(d) $G(s) = \dfrac{K(s+1)}{s^3 + 2s^2 + 3s + 1}$

2-41. Use MATLAB to solve Problem 2-40.

2-42. A controlled process is modeled by the following state equations.

$$\frac{dx_1(t)}{dt} = x_1(t) - 2x_2(t) \quad \frac{dx_2(t)}{dt} = 10x_1(t) + u(t)$$

The control $u(t)$ is obtained from state feedback such that

$$u(t) = -k_1x_1(t) - k_2x_2(t)$$

where k_1 and k_2 are real constants. Determine the region in the k_1-versus-k_2 parameter plane in which the closed-loop system is asymptotically stable.

2-43. A linear time-invariant system is described by the following state equations.

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(\mathbf{t})$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -3 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The closed-loop system is implemented by state feedback, so that $u(t) = -\mathbf{K}\mathbf{x}(t)$, where $\mathbf{K} = [k_1\, k_2\, k_3]$ and k_1, k_2, and k_3 are real constants. Determine the constraints on the elements of $\mathbf{K}$ so that the closed-loop system is asymptotically stable.

2-44. Given the system in state equation form,

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(\mathbf{t})$$

(a) $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

Can the system be stabilized by state feedback $u(t) = -\mathbf{K}\mathbf{x}(t)$, where $\mathbf{K} = [k_1\ k_2\ k_3]$?

2-45. Consider the open-loop system in Fig. 2P-45(a).

Figure 2P-45a

where $\frac{d^2y}{dt^2} - \frac{g}{l}y = z$ and $f(t) = \tau\frac{dz}{dt} + z$.

Our goal is to stabilize this system so the closed-loop feedback control will be defined as shown in the block diagram in Fig. 2P-45(b).

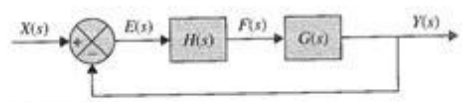


Figure 2P-45b

Assuming $f(t) = k_p e + k_d \frac{de}{dt}$.

(a) Find the open-loop transfer function.

(b) Find the closed-loop transfer function.

(c) Find the range of k_p and k_d in which the system is stable.

(d) Suppose $\frac{g}{l} = 10$ and $\tau = 0.1$. If $y(0) = 10$ and $\frac{dy}{dt} = 0$, then plot the step response of the system with three different values for k_p and k_d. Then show that some values are better than others; however, all values must satisfy the Routh-Hurwitz criterion.

2-46. The block diagram of a motor-control system with tachometer feedback is shown in Fig. 2P-46. Find the range of the tachometer constant K_t so that the system is asymptotically stable.

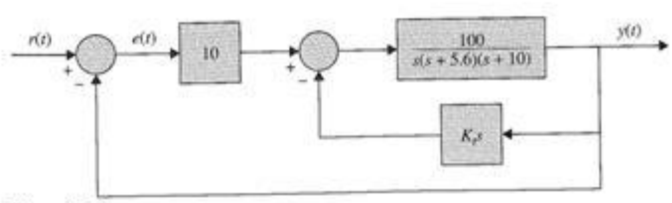


Figure 2P-46

2-47. The block diagram of a control system is shown in Fig. 2P-47. Find the region in the K-versus-α plane for the system to be asymptotically stable. (Use K as the vertical and α as the horizontal axis.)

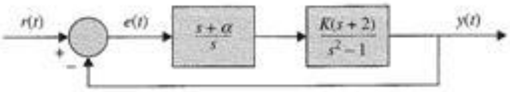


Figure 2P-47

2-48. The conventional Routh-Hurwitz criterion gives information only on the location of the zeros of a polynomial $F(s)$ with respect to the left half and right half of the s-plane. Devise a linear transformation $s = f(p, \alpha)$, where p is a complex variable, so that the Routh-Hurwitz criterion can be applied to determine whether $F(s)$ has zeros to the right of the line $s = -\alpha$, where α is a positive real number. Apply the transformation to the following characteristic equations to determine how many roots are to the right of the line $s = -1$ in the s-plane.

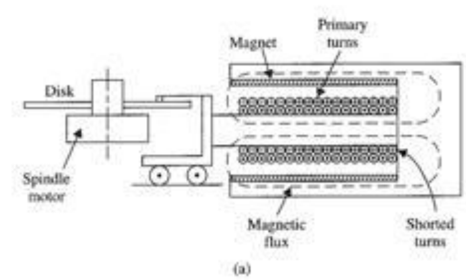
(a) $F(s) = s^2 + 5s + 3 = 0$

(b) $s^3 + 3s^2 + 3s + 1 = 0$

(c) $F(s) = s^3 + 4s^2 + 3s + 10 = 0$

(d) $s^3 + 4s^2 + 4s + 4 = 0$

2-49. The payload of a space-shuttle-pointing control system is modeled as a pure mass M. The payload is suspended by magnetic bearings so that no friction is encountered in the control. The attitude of the payload in the y direction is controlled by magnetic actuators located at the base. The total force produced by the magnetic actuators is $f(t)$. The controls of the other degrees of motion are independent and are not considered here. Because there are experiments located on the payload, electric power must be brought to the payload through cables. The linear spring with spring constant K_s is used to model the cable attachment. The dynamic system model for the



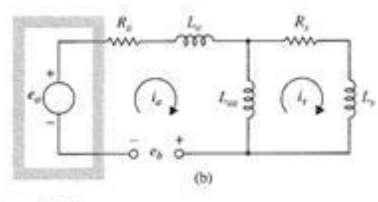


Figure 2P-49

control of the y-axis motion is shown in Figure 2P-49. The force equation of motion in the y-direction is

$$f(t) = K_s y(t) + M\frac{d^2y(t)}{dt^2}$$

where $K_s = 0.5\,\text{N-m/m}$ and $M = 500\,\text{kg}$. The magnetic actuators are controlled through state feedback, so that

$$f(t) = -K_P y(t) - K_D\frac{dy(t)}{dt}$$

(a) Draw a functional block diagram for the system.

(b) Find the characteristic equation of the closed-loop system.

(c) Find the region in the K_D-versus-K_P plane in which the system is asymptotically stable.

2-50. An inventory-control system is modeled by the following differential equations:

$$\frac{dx_1(t)}{dt} = -x_2(t) + u(t)$$
$$\frac{dx_2(t)}{dt} = -Ku(t)$$

where $x_1(t)$ is the level of inventory; $x_2(t)$, the rate of sales of product; $u(t)$, the production rate; and K, a real constant. Let the output of the system by $y(t) = x_1(t)$ and $r(t)$ be the reference set point for the desired inventory level. Let $u(t) = r(t) - y(t)$. Determine the constraint on K so that the closed-loop system is asymptotically stable.

2-51. Use MATLAB to solve Problem 2-50.

2-52. Use MATLAB to

(a) Generate symbolically the time function of $f(t)$

$$f(t) = 5 + 2e^{-2t}\sin\left(2t + \frac{\pi}{4}\right) - 4e^{-2t}\cos\left(2t - \frac{\pi}{2}\right) + 3e^{-4t}$$

(b) Generate symbolically $G(s) = \dfrac{(s+1)}{s(s+2)(s^2+2s+2)}$

(c) Find the Laplace transform of $f(t)$ and name it $F(s)$.

(d) Find the inverse Laplace transform of $G(s)$ and name it $g(t)$.

(e) If $G(s)$ is the forward-path transfer function of unity-feedback control systems, find the transfer function of the closed-loop system and apply the Routh-Hurwitz criterion to determine its stability.

(f) If $F(s)$ is the forward-path transfer function of unity-feedback control systems, find the transfer function of the closed-loop system and apply the Routh-Hurwitz criterion to determine its stability.