EE 475 Fall 2007 Midterm Exam #2

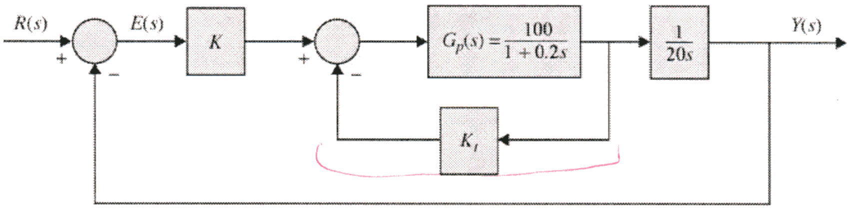
Name Sol.

Instructions:

1. This is a closed-book and closed-notes exam for individual work.
2. You may have two sheets of formulae of size no larger than US letter.
3. Calculator is discouraged.
4. Time for the exam is 50 minutes.
5. Do all problems, but easier ones first, since all are equally weighted.

Problems:

1. For the system given below, estimate the values of K and Kt so that a maximum percentage overshoot of 9.6% and a settling time of 0.05 sec for a tolerance band of 1% are achieved.



$$\frac{\overset{25}{\cancel{100}}K}{(1+0.2s+100K_t)\,\cancel{20}s}$$

$$s(s+4+400K_t)$$

$9.6\%\ M_p \Rightarrow \zeta = 0.6$

$\sigma = \frac{5}{0.05} \approx 100$

$\Rightarrow 25K = 100, \quad \cancel{K = 4}$

$\cancel{2\zeta\omega_n} \quad 2\sigma = 200 = 4 + 400K_t$

$K_t = \frac{196}{400}$

$\omega_n^2 = \left(\frac{\sigma}{\zeta}\right)^2 = \left(\frac{100}{0.6}\right)^2 = 25K$

$K = \left(\frac{100}{0.6}\right)^2 \frac{1}{25} = \left(\frac{100}{3}\right)^2$

2. For the following closed loop transfer functions, determine whether the system is marginally stable, asymptotically stable, or unstable. (no pole/zero cancelation happened)

a) $M(s) = \dfrac{5s^2 + 6s + 3}{(s+2)(s-2)}$: U

b) $M(s) = \dfrac{(s-3)(s+4)}{(s+2)(s+5)}$: a.s.

c) $M(s) = \dfrac{10s^2 + 6s + 1}{s(s+100)}$: M.S.

d) $M(s) = \dfrac{(s+1)}{s^2(s+2)}$: U.

3. Using the Routh-Hurwitz criterion, determine the number of roots of each polynomial that are in the open right half plane.

a) $d(s) = s^4 + s^3 + 6s^2 + 2s + 1$

1	6	1
1	2	
4	1	
$\frac{7}{4}$		
1		

0 root in ORHP.

b) $d(s) = 2s^4 + s^3 + 4s^2 + 5s + 12$

~~2~~ 1	~~4~~ 2	~~12~~ 6
1	5	
~~-3~~ -1	~~6~~ 2	
7		
2		

2 sign changes.
2 roots in OR.HP.

4. Determine the step, ramp, and parabolic error constants of the following unity-feedback control systems. The forward-path transfer functions are given. Assume the closed loop systems are stable.

a) $G(s) = \dfrac{50}{(1+s)(1+20s)}$

K_p= 50, K_v= 0, K_a 0

b) $G(s) = \dfrac{2}{s(s+1)(s+2)}$

K_p= ∞, K_v= 1, K_a 0

c) $G(s) = \dfrac{5s+1}{s^2(s^2+5s+6)}$

K_p= ∞, K_v= ∞, K_a 1/6

5. For the systems in the above Problem, find the steady state error values due to a unit step input, a unit ramp input, and a unit acceleration input.

a) ess_step = 1/51, ess_ramp= ∞, ess_acc= ∞

b) ess_step = 0, ess_ramp= 1, ess_acc= ∞

c) ess_step = 0, ess_ramp= 0, ess_acc= 6

6. Given the following prototype 2nd order systems, answer the questions below.

a) $\dfrac{400}{s^2+20s+400}$ $\zeta = 0.5$ b) $\dfrac{1600}{s^2+40s+1600}$ $= 0.5$

c) $\dfrac{100}{s^2+10s+100}$ $\zeta = 0.5$ d) $\dfrac{900}{s^2+20s+900}$ 0.33

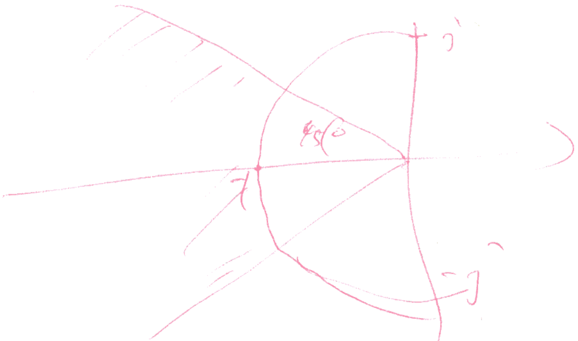
Which system's transient decays the fastest? a b c d

Which system has the highest percent overshoot? a b c d

Which system has the longest rise time? a b c d

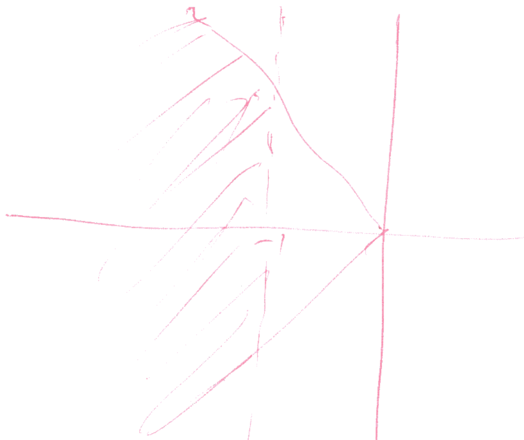
Which system has the longest settling time? a b c d

7. Sketch the s-domain diagram that corresponds to the following specifications for a prototype 2nd order system.
   a) Percent overshoot less than or equal to 5% $\Rightarrow \zeta = 0.707$
   b) Rise time less than or equal to 1.8 second $\Rightarrow \omega_n \approx \frac{1.8}{t_r} = 1$



8. Sketch the s-domain diagram that corresponds to the following specifications for a prototype 2nd order system.
   a) Percent overshoot less than or equal to 5%
   b) Settling time less than or equal to 4 second (tolerance band is ±2%)

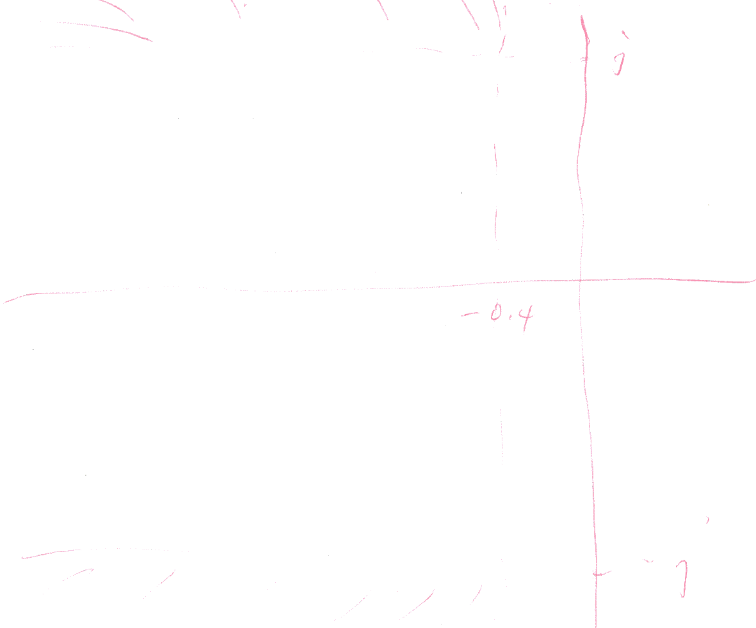
$\sigma \approx \frac{4}{t_s} = 1$



9. Sketch the s-domain diagram that corresponds to the following specifications for a prototype 2nd order system.
   a) Settling time less than or equal to 10 second (tolerance band is ±2%)
   b) Peak time less than π seconds

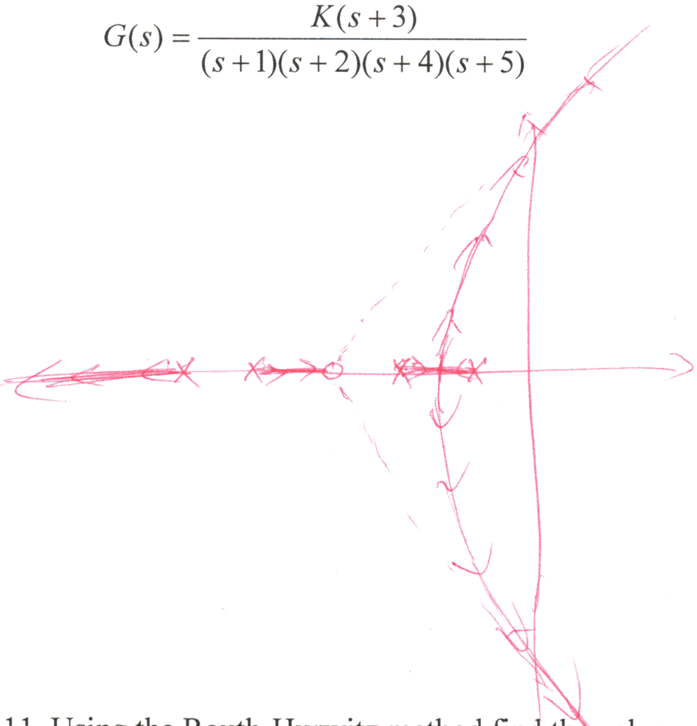
$\sigma = \frac{4}{10} = 0.4$

$\omega_d = \frac{\pi}{t_p} = 1$



10. Given the forward-path transfer function of a unity-feedback control system, quickly sketch the root locus for $K \geq 0$. Do not calculate the jw-axis crossings or departure/arrival angles. Approximately locate any breakaway points.

$$G(s) = \frac{K(s+3)}{(s+1)(s+2)(s+4)(s+5)}$$



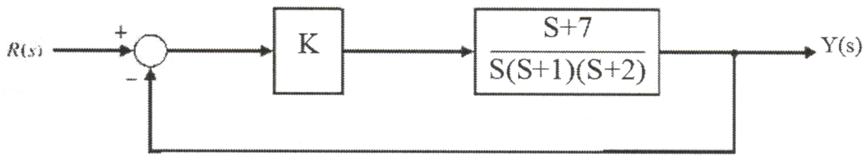
3 asymptotes

intersection: $\frac{-1-2-4-5+3}{3} = -3$

angle: $\pm 60^\circ$ & -180°

breakaway ≈ -1.5

11. Using the Routh-Hurwitz method find the values of K that make the system stable, the value of K that yields sustained oscillation, and the oscillation frequency.



$d(s) = s(s+1)(s+2) + K(s+7) = s^3 + 3s^2 + (2+K)s + 7K$

Routh table:

s^3	1	$2+K$
s^2	3	$7K$
s^1	$\frac{3(2+K)-7K}{3}$	
s^0	$7K$	

for stability:

$$\begin{cases} 3(2+K) - 7K > 0 \\ 7K > 0 \end{cases}$$

$\Rightarrow 0 < K < \frac{6}{4}$

when $K = \frac{6}{4} = \frac{3}{2}$ system will have sustained osc.

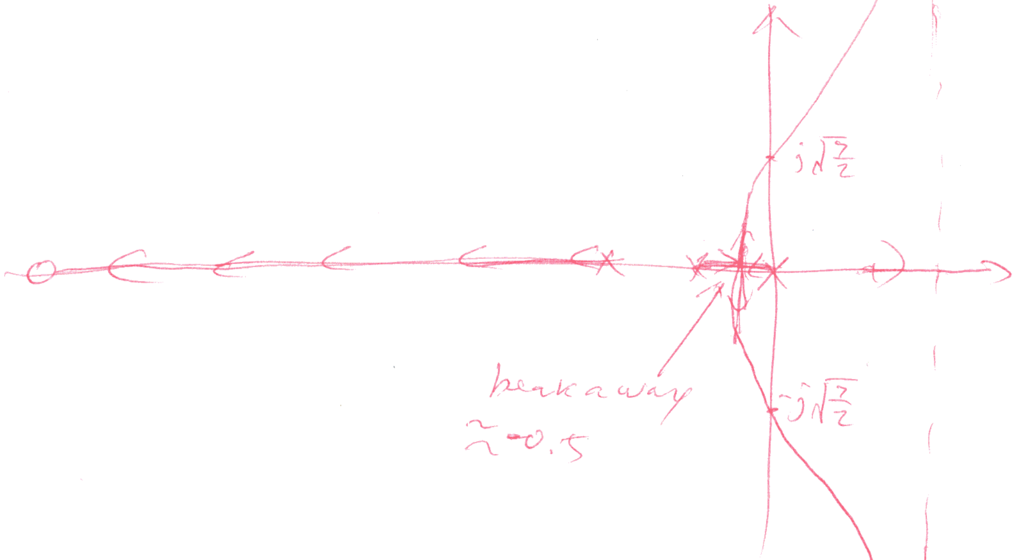
Osc. freq:

$3s^2 + 7K = 3s^2 + \frac{21}{2} = 0$

$s^2 = -\frac{7}{2}$

$s_{1,2} = \pm j\sqrt{\frac{7}{2}}$, $\omega_{osc.} = \sqrt{7/2}$

12. Sketch the root locus of the system in the previous problem for K>=0. Make sure your real axis and asymptotes are correct. Estimate breakaway points.

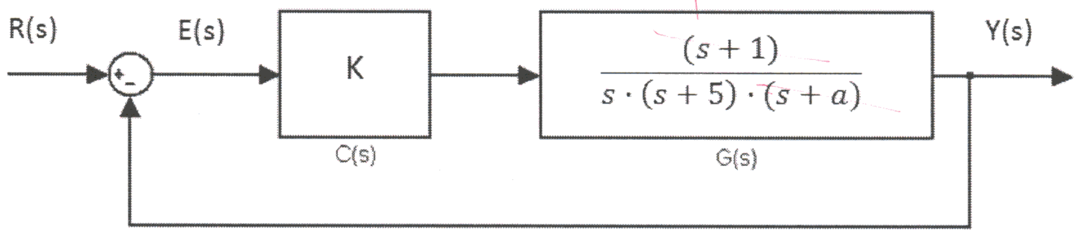


2 asymptotes meet at:

$$\frac{0-1-2+7}{2} = +2$$

$$\text{angle} = \pm\frac{\pi}{2}$$

13. When K = 10, the system below is stable and the steady state error due to ramp input is 0.5. Find the range of K which results in the system being asymptotically stable.



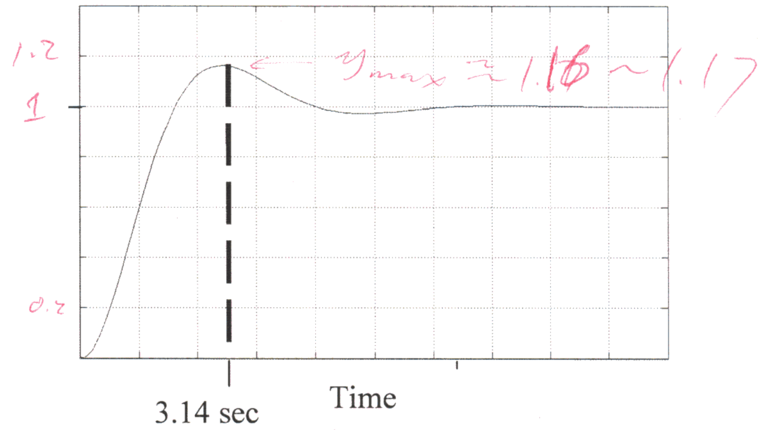
e_{ss} to ramp $= 0.5$, $K_v = 2$.

$$K \frac{(s+1)}{\cancel{s}(s+5)(s+a)}\Big|_{s=0} = \frac{K}{5a} = 2 = \frac{10}{5a} = \frac{2}{a}$$

$$\Rightarrow a = 1$$

for any $K > 0$, we have A.S

14. The step response of a second-order prototype system is plotted below.



Find yss, ess to step, ymax, Mp, and the transfer function of the system.

$y_{ss} = 1$, $e_{ss} = 0$, $y_{max} \simeq 1.16$ to 1.17

$t_p = 3.14$, $= \dfrac{\pi}{\omega_d} \Rightarrow \omega_d = 1$

$M_p = 16 \sim 17\% \Rightarrow \zeta \simeq 0.5$

$\omega_n^2 = \dfrac{\omega_d^2}{1-\zeta^2} = \dfrac{1}{1-0.25} = \dfrac{1}{0.75} = \dfrac{4}{3}$

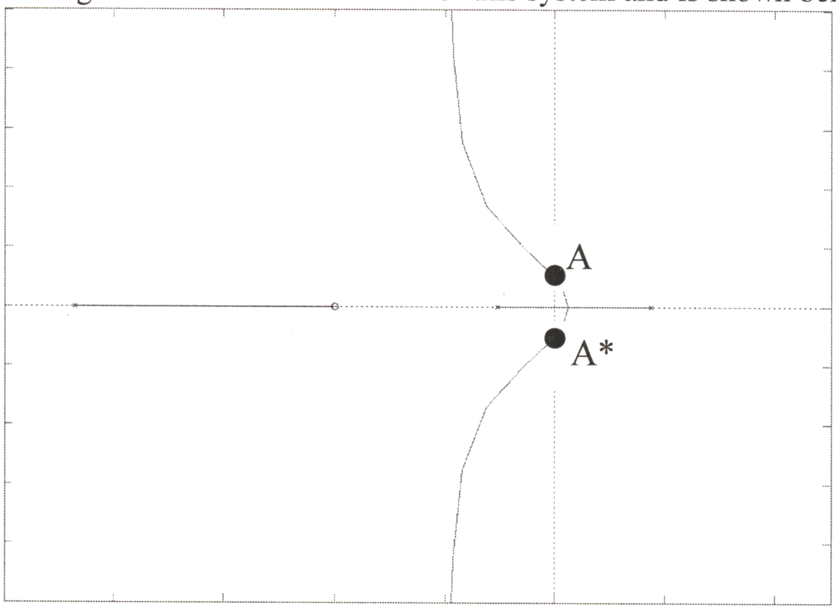
$2\zeta\omega_n = 2 * 0.5 * \sqrt{\dfrac{4}{3}} = \sqrt{\dfrac{4}{3}}$

$\therefore TF = \dfrac{4/3}{s^2 + \sqrt{\frac{4}{3}} \cdot s + \sqrt{\frac{4}{3}}}$

15. The forward-path transfer function of a unity-feedback control system is given as:

$$G(s) = K \cdot \frac{(s+2)}{s^3 + 4 \cdot s^2 - 2 \cdot s - 2}$$

The root-locus diagram has been constructed for this system and is shown below.



Find the value of K at A/A*, and the value of A.

$d(s) = s^3 + 4s^2 - 2s - 2 + k(s+2)$

Routh:

1	$k-2$
4	$2k-2$
$\frac{4(k-2)-(2k-2)}{4}$	
$2k-2$	

for $j\omega$-axis crossing:

$4(k-2) - (2k-2) = 0$

$2k = 8 - 2 = 6$

$k = 3$

$\Rightarrow A(s) = 4s^2 + (2 \cdot 3 - 2)$

$= 4s^2 + 4 = 0$

$s = \pm j \Rightarrow A = j$