Features of Square Law Mixers

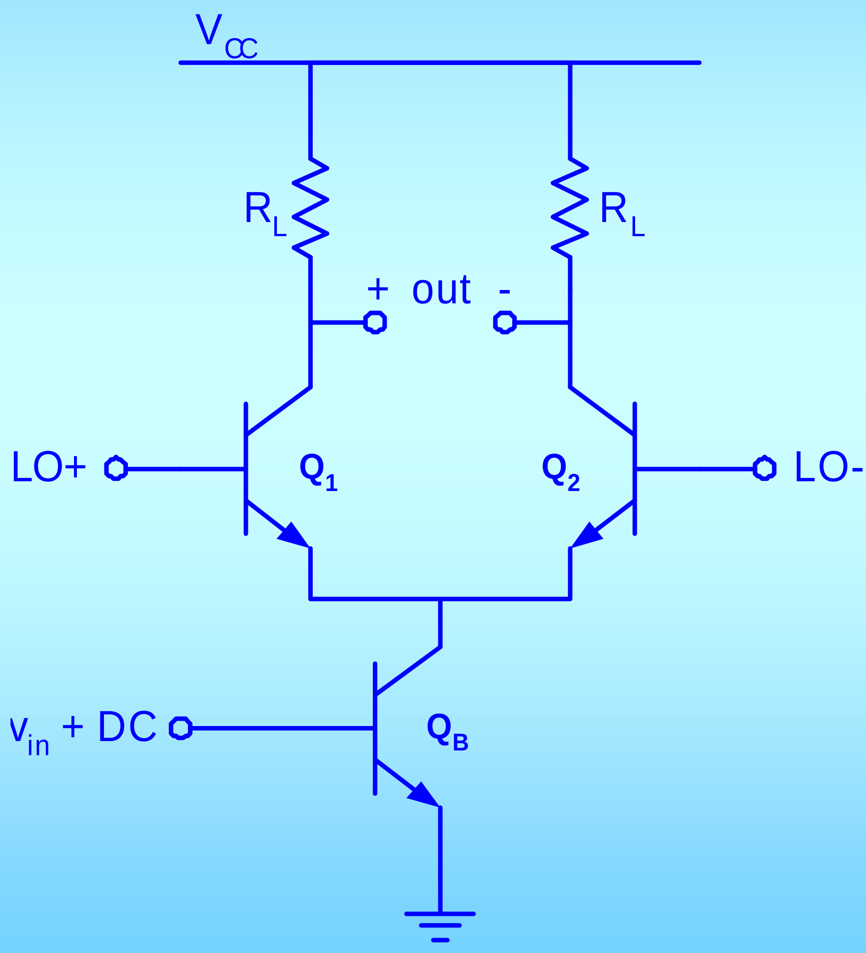
- Noise Figure: The square law MOSFET mixer can be designed to have very low noise figure.
- Linearity: true square law MOSFET mixer produces only DC, original tones, difference, and sum tones
- The corresponding BJT mixer produces a host of non-linear components due to the exponential function
- Power Dissipation: The square law mixer can be designed with very low power dissipation.
- Power Gain: Reasonable power gain can be achieved through the use of square law mixers.
- Isolation: Square law mixers offer poor isolation from LO to RF port. This is by far the biggest short coming of the square law mixers.

Mixer performance analysis

- Analyze major metrics
  - Conversion gain
  - Port isolation
  - Noise figure/factor
  - Linearity, IIP3
- Gain insights into design constraints and compromise

Common Emitter Mixer

- Single-ended input
- Differential LO
- Differential output
- Q_B provides gain for v_{in}
- Q_1 and Q_2 steer the current left and right at ω_{LO}



Common Emitter Mixer

- Conversion gain

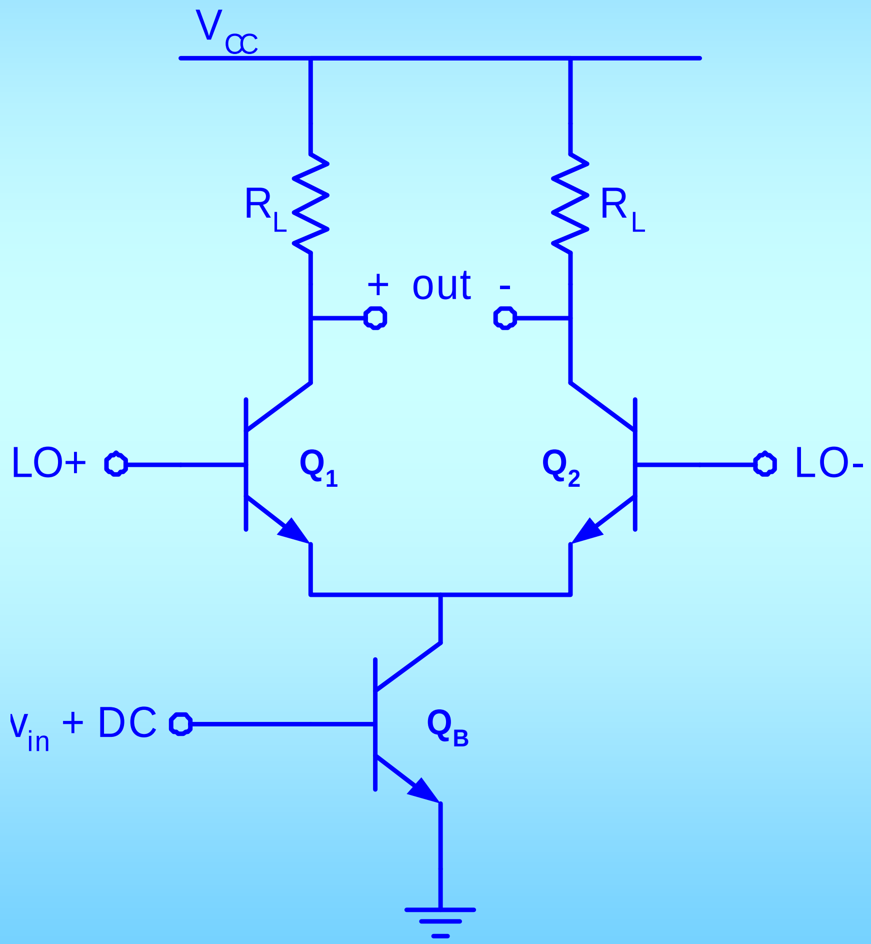
Two output component:

$$v_{out1} = \pm g_m v_{in} R_L$$

$$v_{out2} = \pm I_{Q_B DC} R_L$$

IF signal is the ω_{RF}-ω_{LO} component in v_{out1}

So gain = ?



Common Emitter Mixer

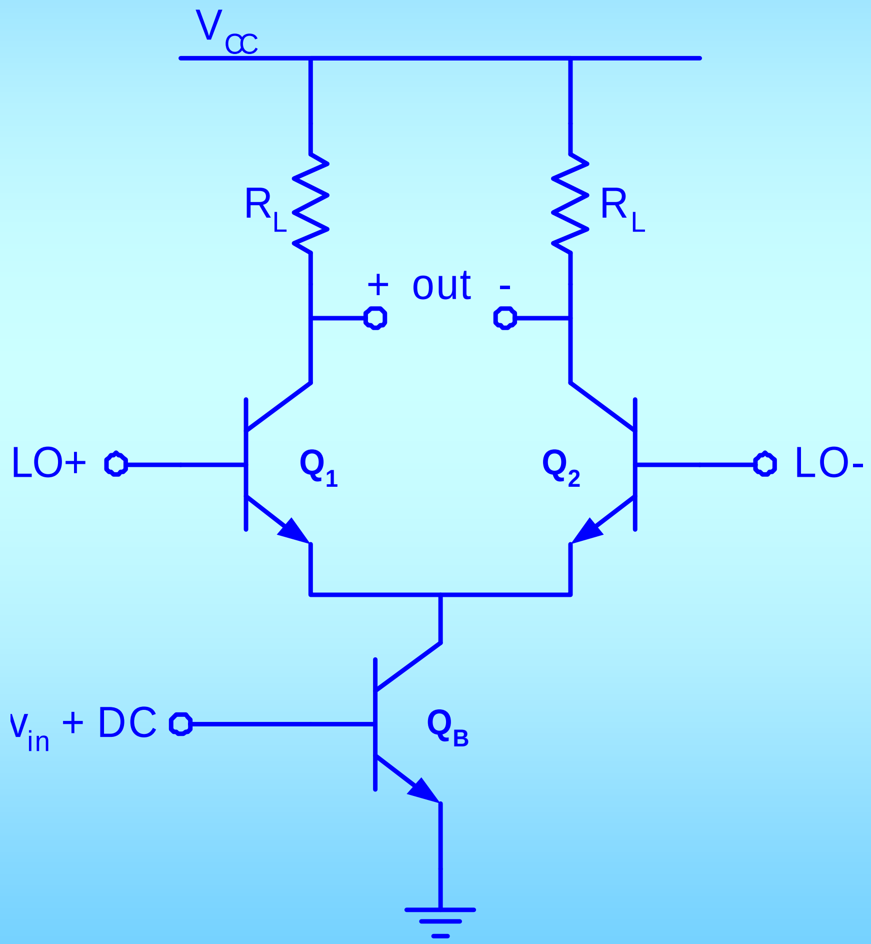
- Port isolation

At what frequency is V_{out2} switching?:

$$v_{out2} = \pm I_{Q_B DC} R_L$$

$$v_{out2} = SW(\omega_{LO}) I_{Q_B DC} R_L$$

This is feed through from LO to output

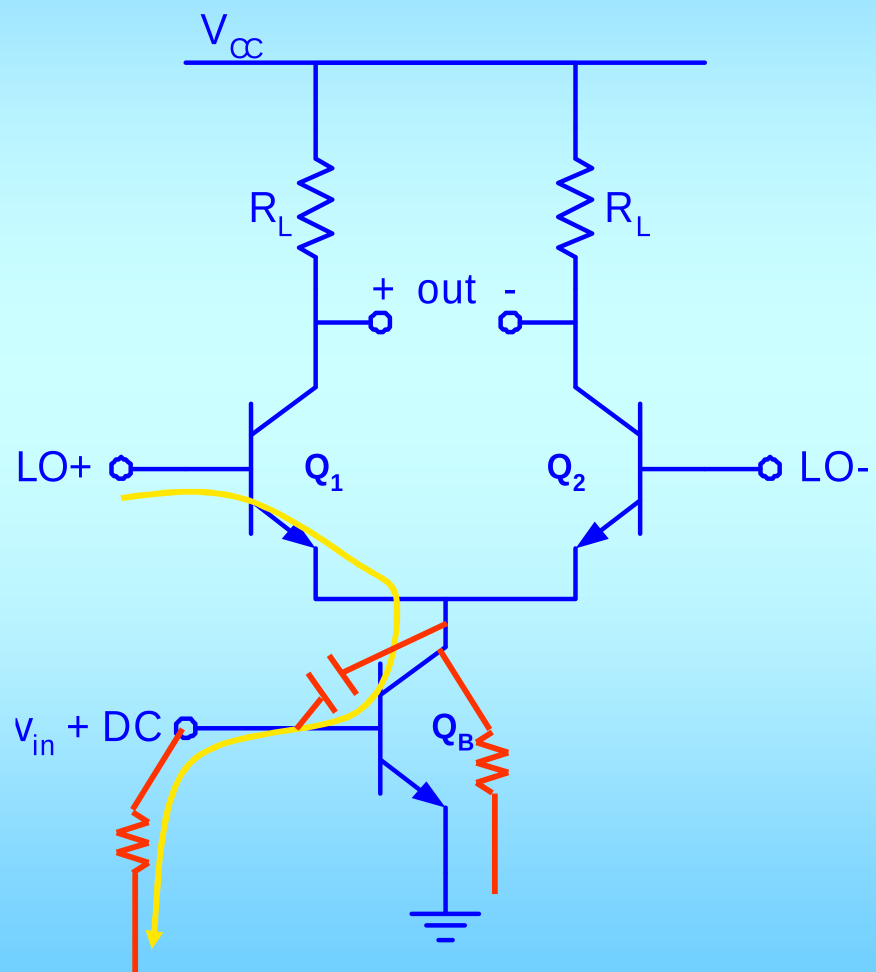


Common Emitter Mixer

- Port isolation

How about LO to RF?

This feed through is much smaller than LO to output

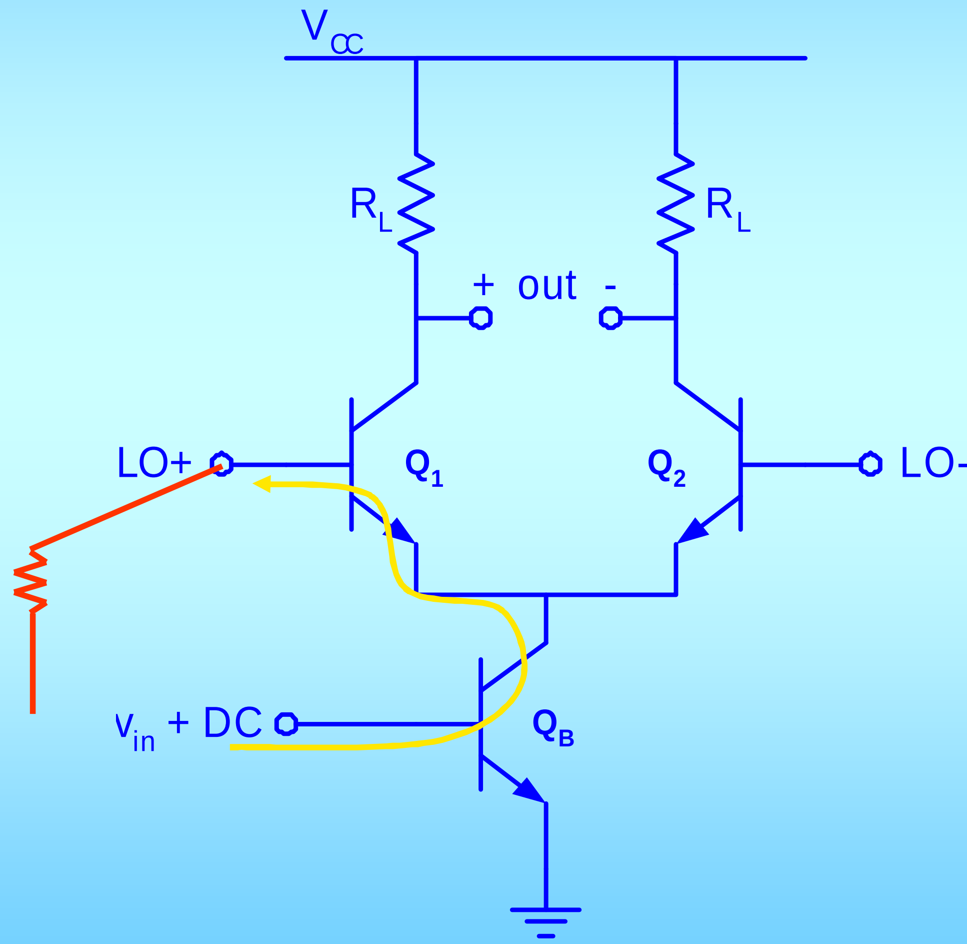


Common Emitter Mixer

- Port isolation

How about RF to LO?

If LO is generating a square wave signal, its output impedance is very small, resulting in small feed through from RF to LO to output.



Common Emitter Mixer

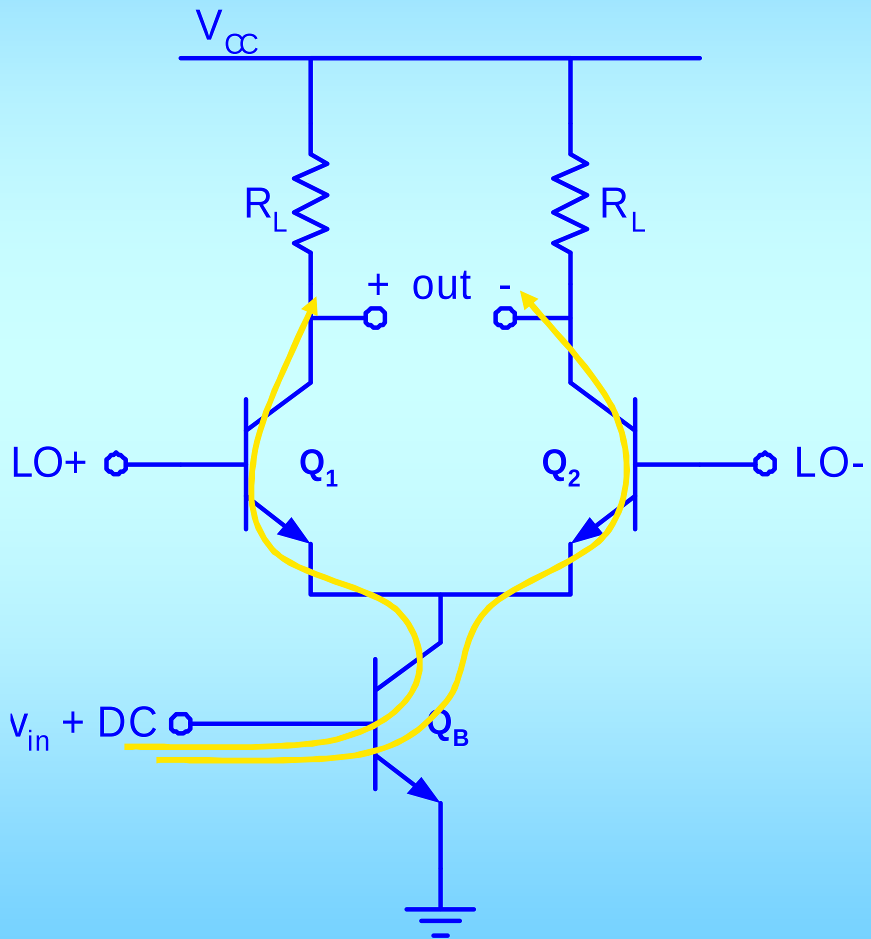
- Port isolation

What about RF to output?

Ideally, contribution to output is:

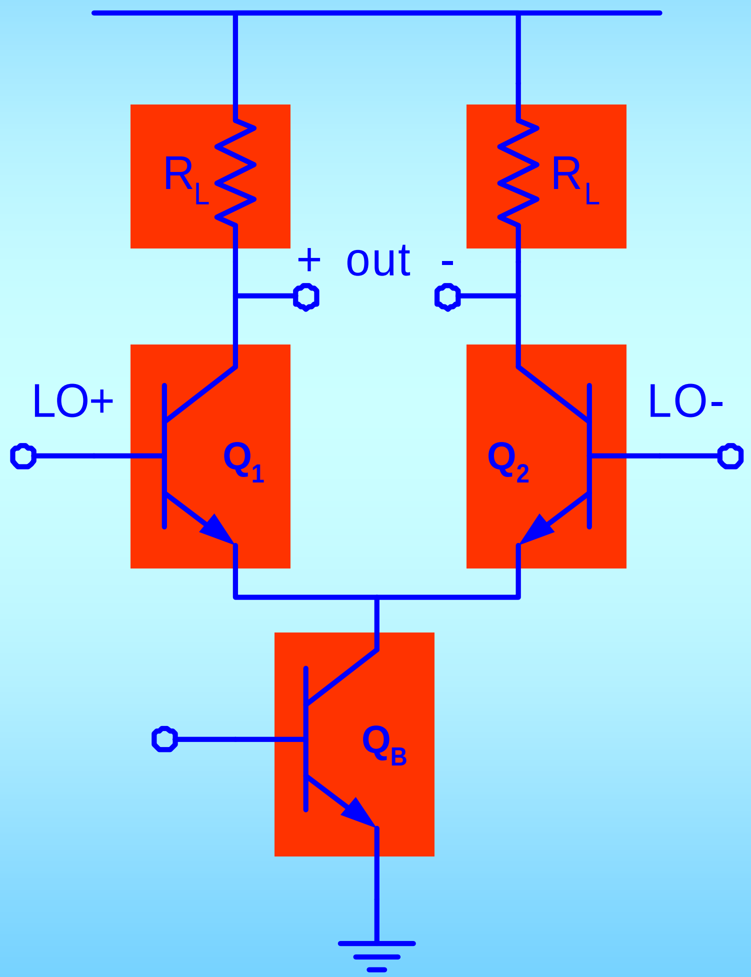
$SW(\omega_{LO})*g_m v_{in} R_L$

What can go wrong and cause an RF component at the output?



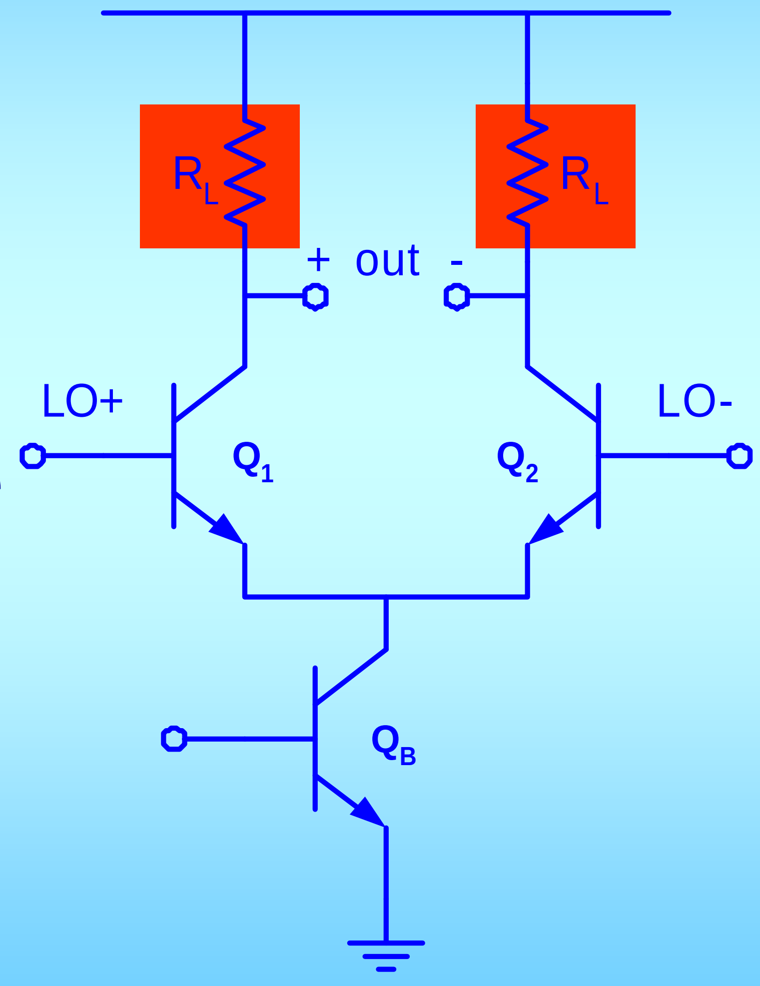
Common Emitter Mixer

- Noise Components:
  1. Noise due to loads
  2. Noise due to the input transistor (Q_B)
  3. Noise due to switches (Q_1 and Q_2)



Common Emitter Mixer

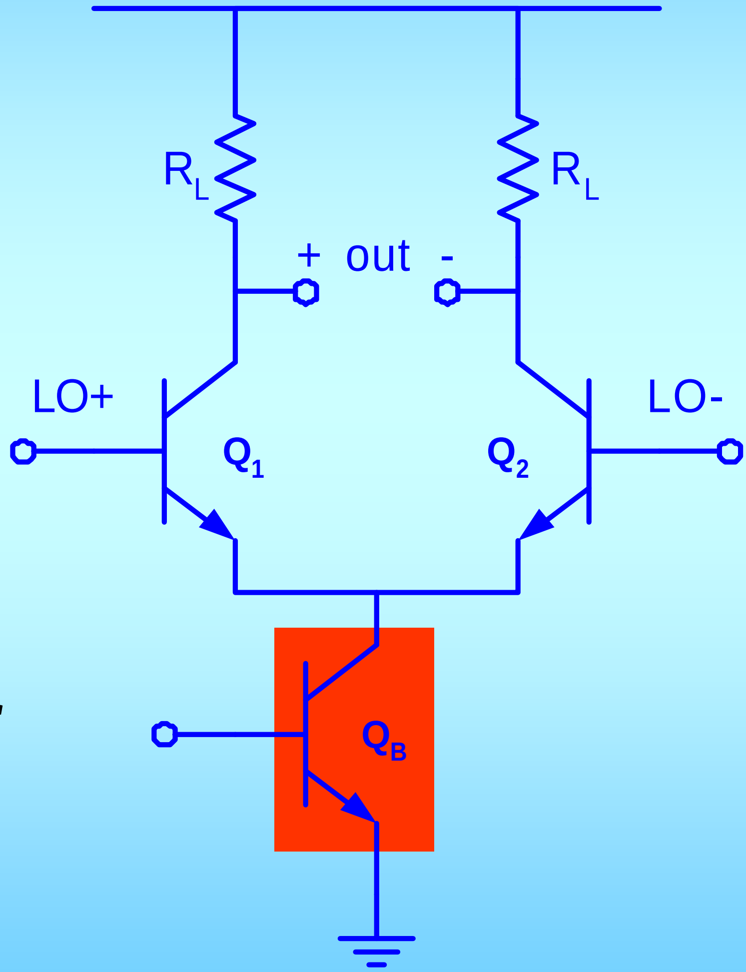
1. Noise due to loads:
   - Each R_L contributes $v_{RL}^2 = 4kTR_L\Delta f$
   - Since they are uncorrelated with each other, their noise power's add
   - Total contribution of R_L's: $v_{oRL}^2 = 8kTR_L\Delta f$



Common Emitter Mixer

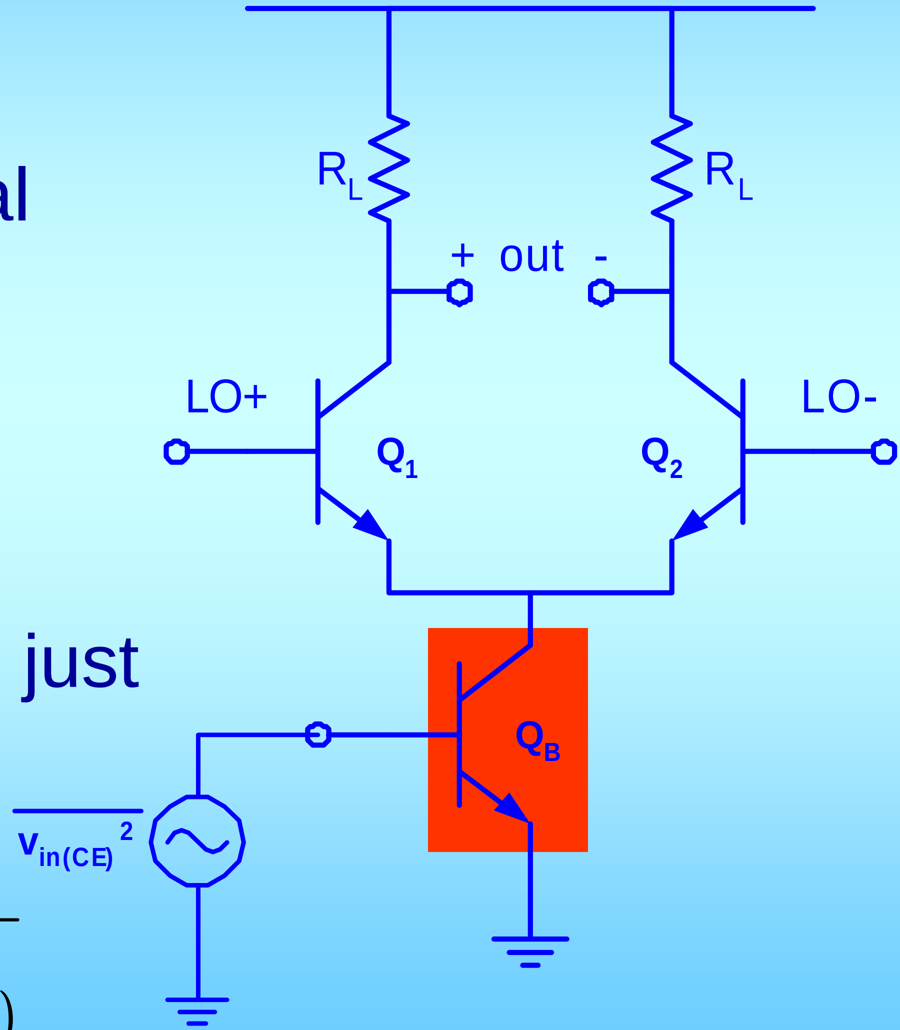
2. Noise due input transistor (the transducer):
   – From BJT device model, equivalent input noise voltage of a CE amplifier is:

$$\overline{v_{in(CE)}^2} = 4kT\left(r_b + \frac{1}{2g_m}\right)\Delta f$$



Common Emitter Mixer

2. Noise due to input transistor:
   - If this is a differential amplifier, QB noise would be common mode
   - But Q1 and Q2 just switching, the noise just appears at either terminal of out:



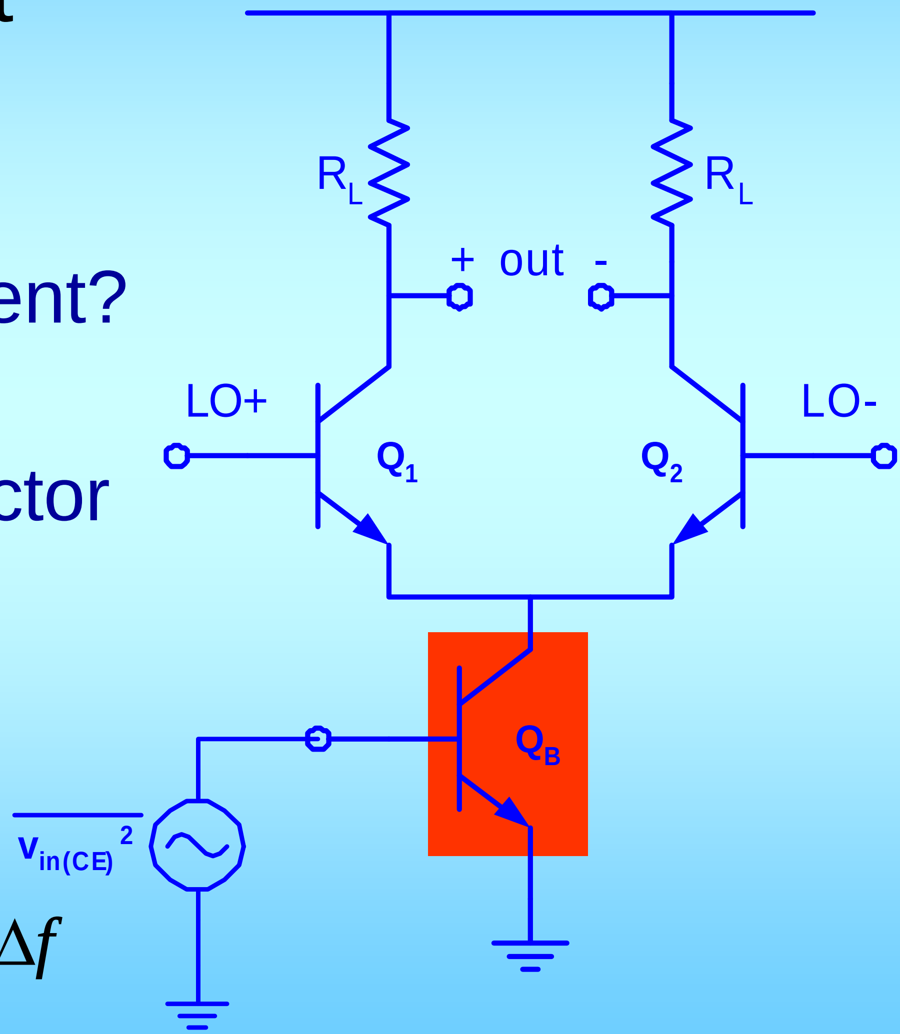
$$\overline{v_{out,Q_B}^2} = (gain)^2 \cdot \overline{v_{in(CE)}^2}$$

Common Emitter Mixer

2. Noise due to input transistor:
   - Noise at the two terminals dependent?
   - Accounted for by incorporating a factor "n".

$$\overline{v_{out,Q_B}^2} = n \cdot (gain)^2 \cdot \overline{v_{in(CE)}^2}$$

$$\overline{v_{out,Q_B}^2} = (g_m R_L)^2 \cdot 4kT\left(r_b + \frac{1}{2g_m}\right)\Delta f$$



Common Emitter Mixer

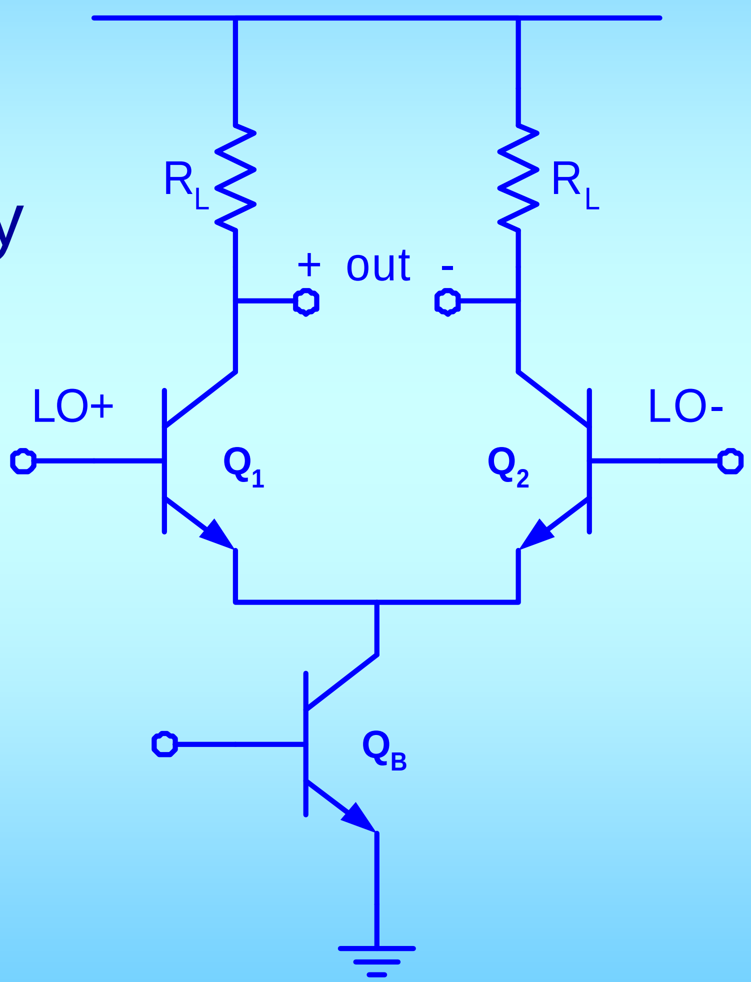
- Total Noise due to RL and QB:
  - If we assume rb is very small:

$$\frac{\overline{v_T^2}}{\Delta f} \cong 8kTR_L\left(1+\frac{g_m R_L}{4}\right)$$

When:

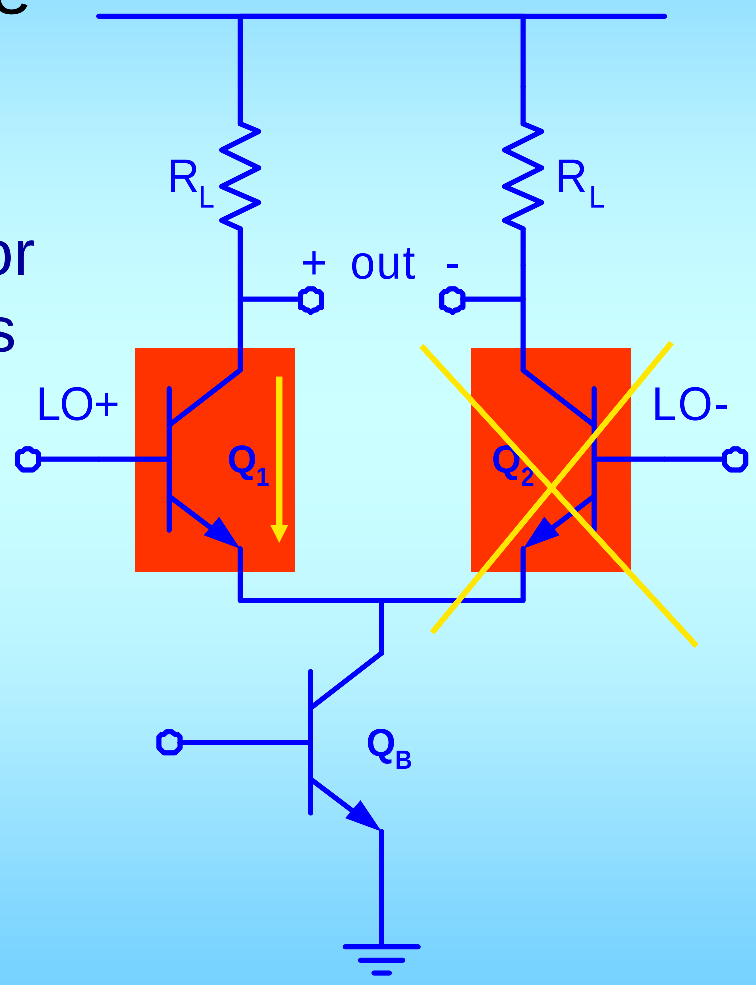
$r_b << 1/(2g_m)$ and

$n=1$



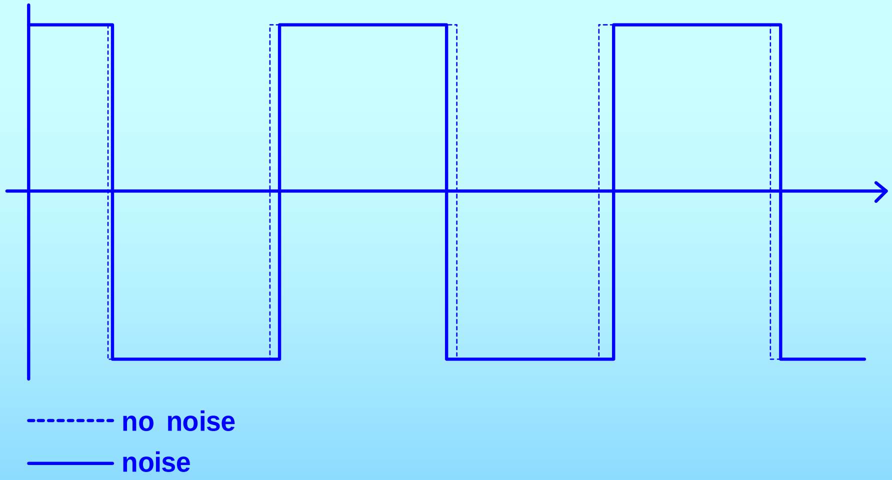
Common Emitter Mixer

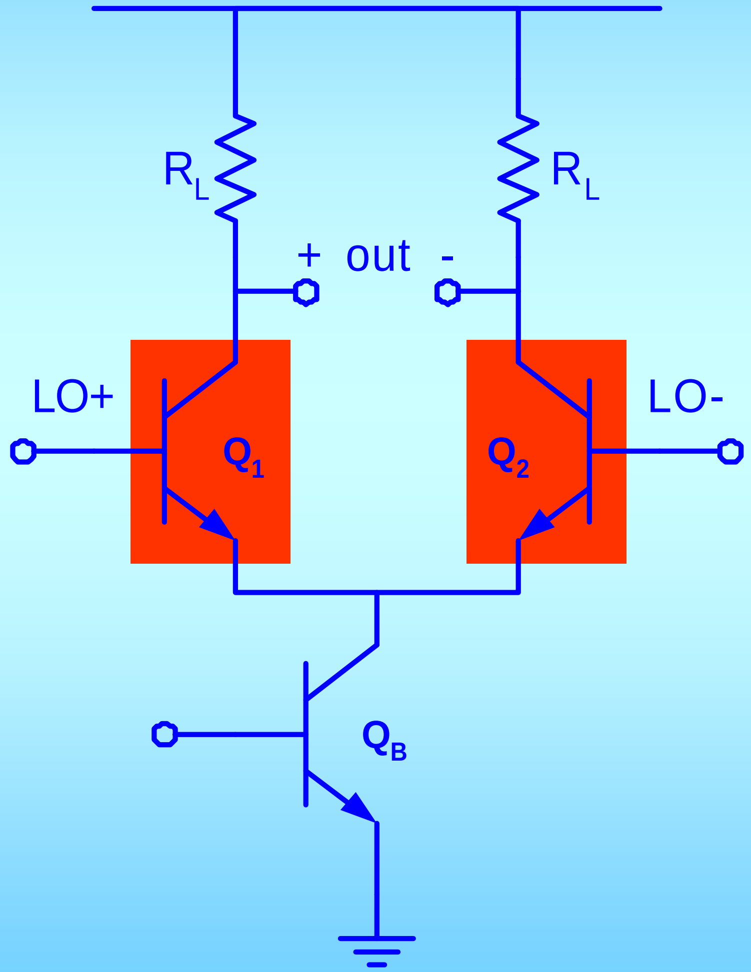
3. What about the noise due to switches?
   – When Q2 is off and Q1 is on, acting like a cascode or more like a resister if LO is strong
   – Can show that Q1's noise has little effect on V_{out}
   – V_{E1}~V_{C1}, V_{BE1} has similar noise as V_{C1}, which cause jitter in the time for Q1 to turn off if the edges of LO are not infinitely steep



Common Emitter Mixer

3. What about the noise due to switches:
   – Transition time "jitter" in the switching signal:

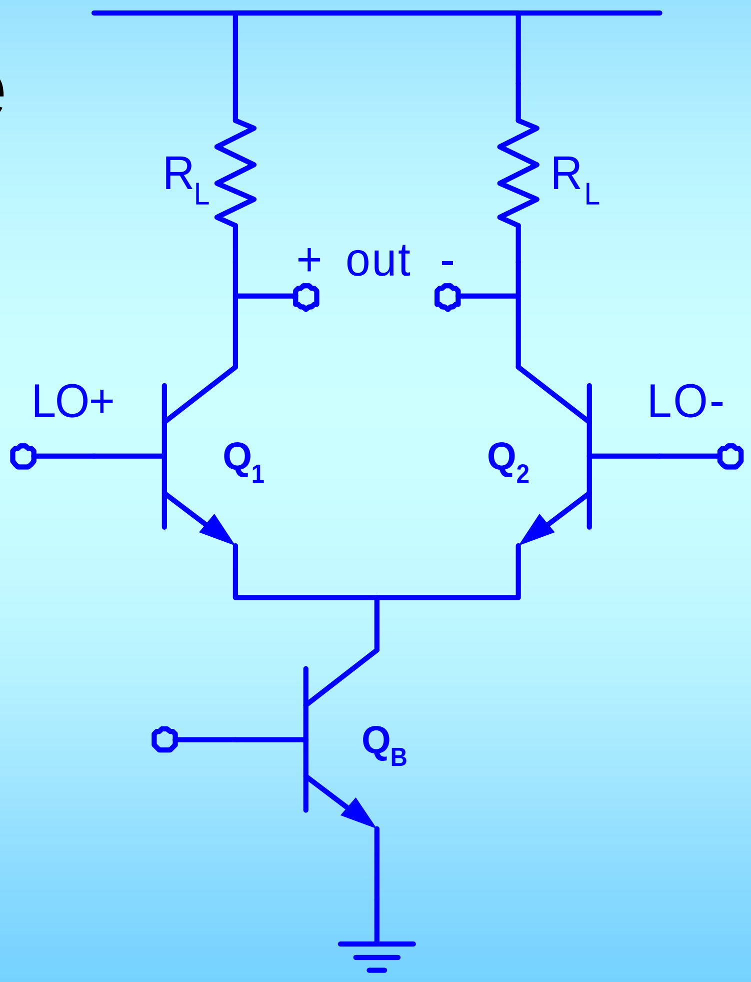




Effect is quite complex, quantitative analysis later

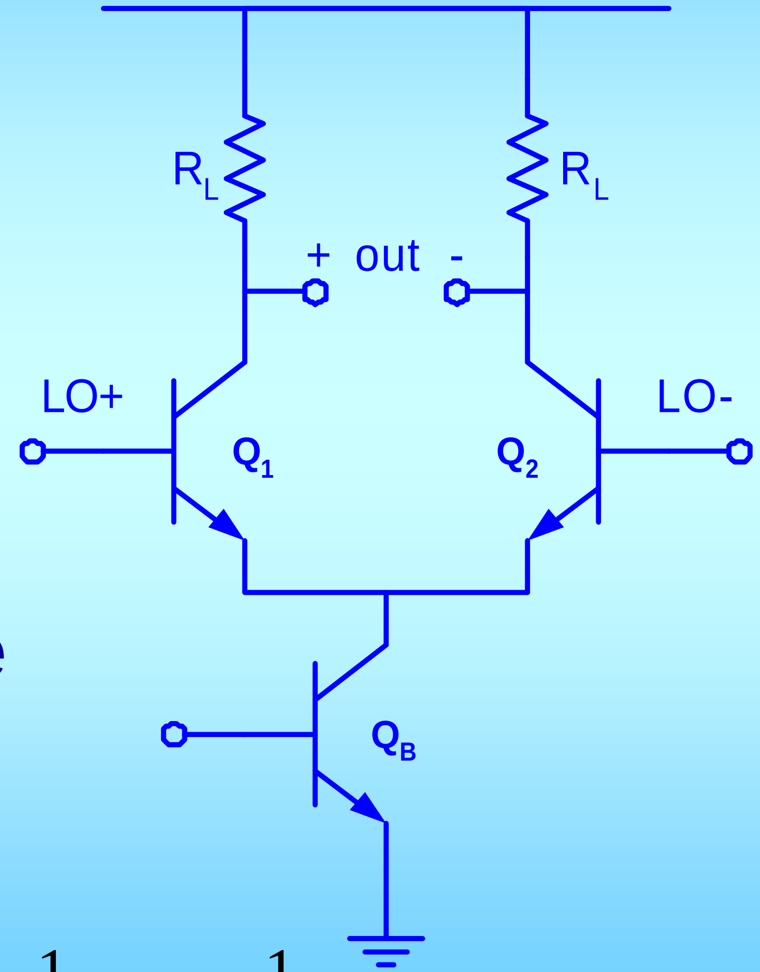
Common Emitter Mixer

- How to improve Noise Figure of mixer:
  - Reduce RL
  - Increase gm and reduce r_b of Q_B
  - Faster switches
  - Steeper rise or fall edge in LO
  - Less jitter in LO



Common Emitter Mixer

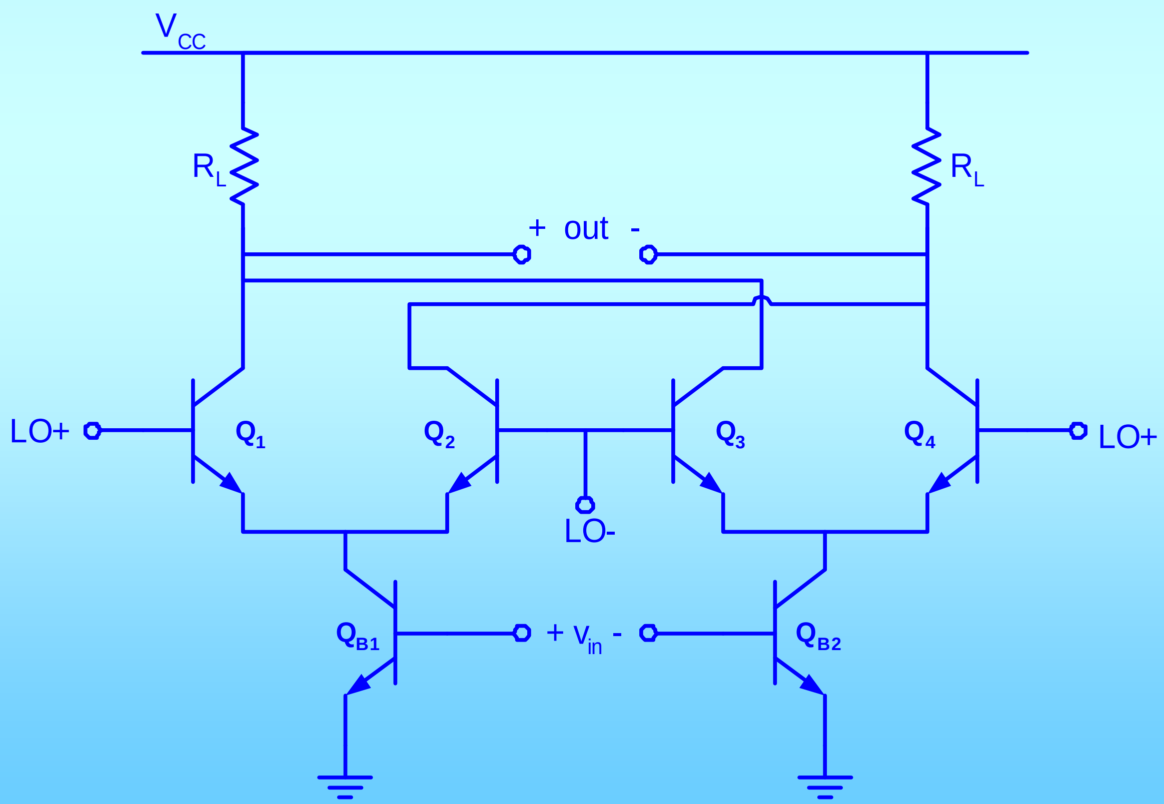
- IP3:
  - The CE input transistor (Q_B) converts v_{in} to I_{in}
    - BJTs cause 3rd-order harmonics
  - Multiplying by RL is linear operation
  - Q1 & Q2 only modulate the frequency
  - $\therefore IP3_{mixer} = IP3_{CE's\ Vbe\text{->}I}$



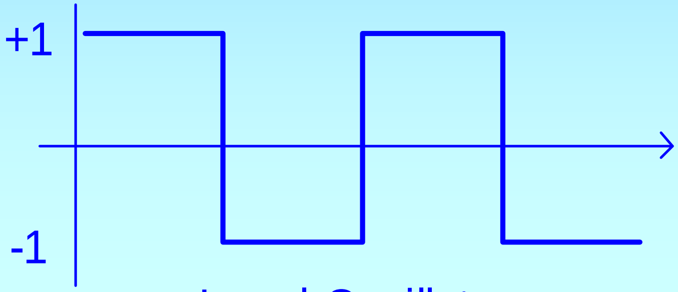
$$I_{Q_B} = aI_s e^{(V_{BB}+v_{in})/v_t} = I_{DC}(1+\frac{1}{v_t}v_{in}+\frac{1}{2v_t^2}v_{in}^2+\frac{1}{6v_t^3}v_{in}^3+...)$$

Double Balanced Mixer

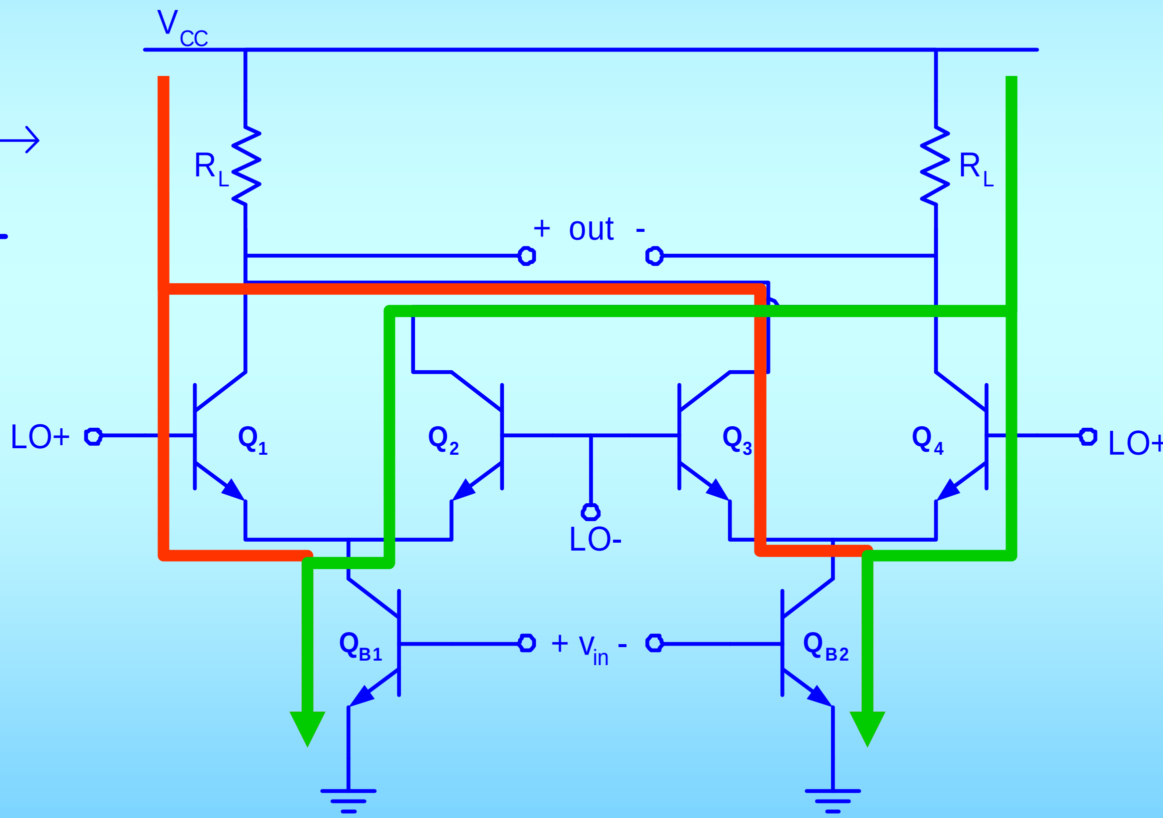
- Basically two CE mixers
  - One gets $+v_{in}/2$, the other gets $–v_{in}/2$



Double Balanced Mixer



Local Oscillator



$$v_{out} = g_m v_{in} R_L$$

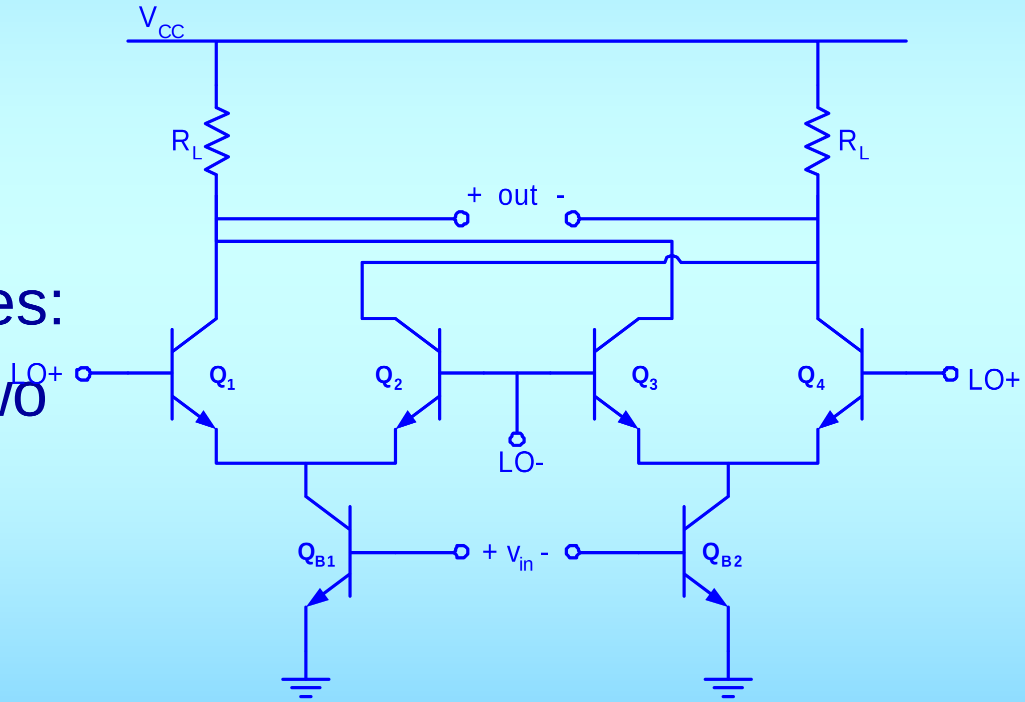
$$v_{out} = -g_m v_{in} R_L$$

Double Balanced Mixer

- Benefits:
  - Fully Differential
  - No output signal at ω_{LO}
- Three stages:
  - CE input stages
  - Switches
  - Output load

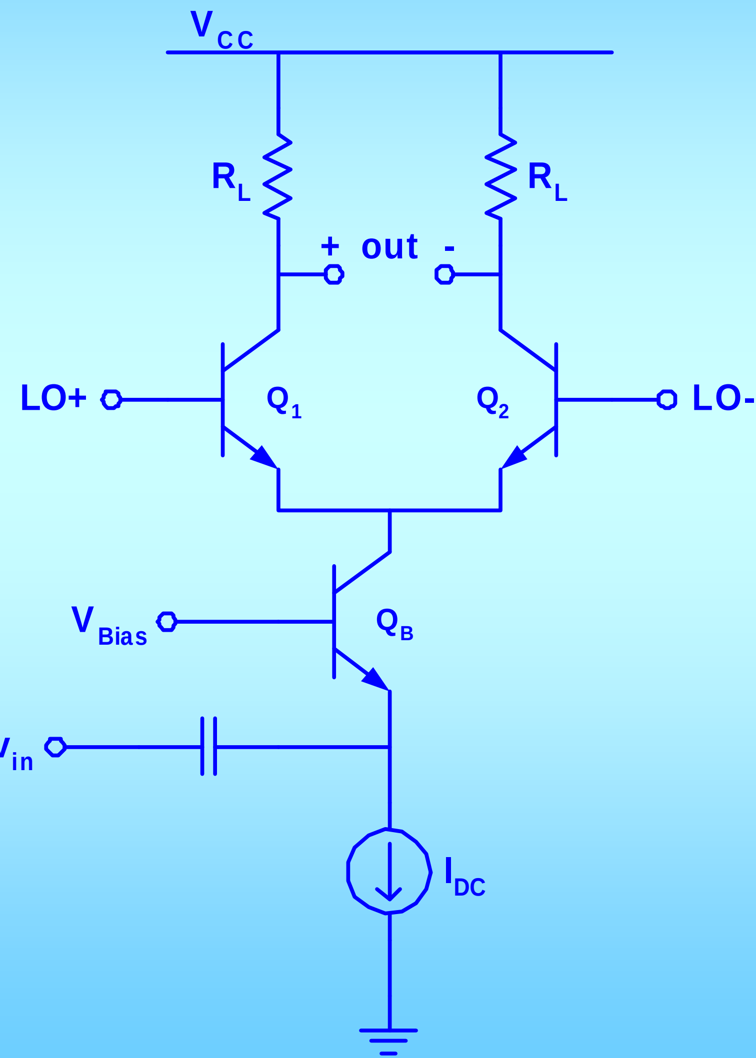
Double Balanced Mixer

- Noise:
  - Similar to CE Mixer
- IP3:
  - Expansion of differential gain gives:
  - Vin split between two Q's, it can double before reaching the same level of nonlinearity
  - IIP3 improved by 3 dB



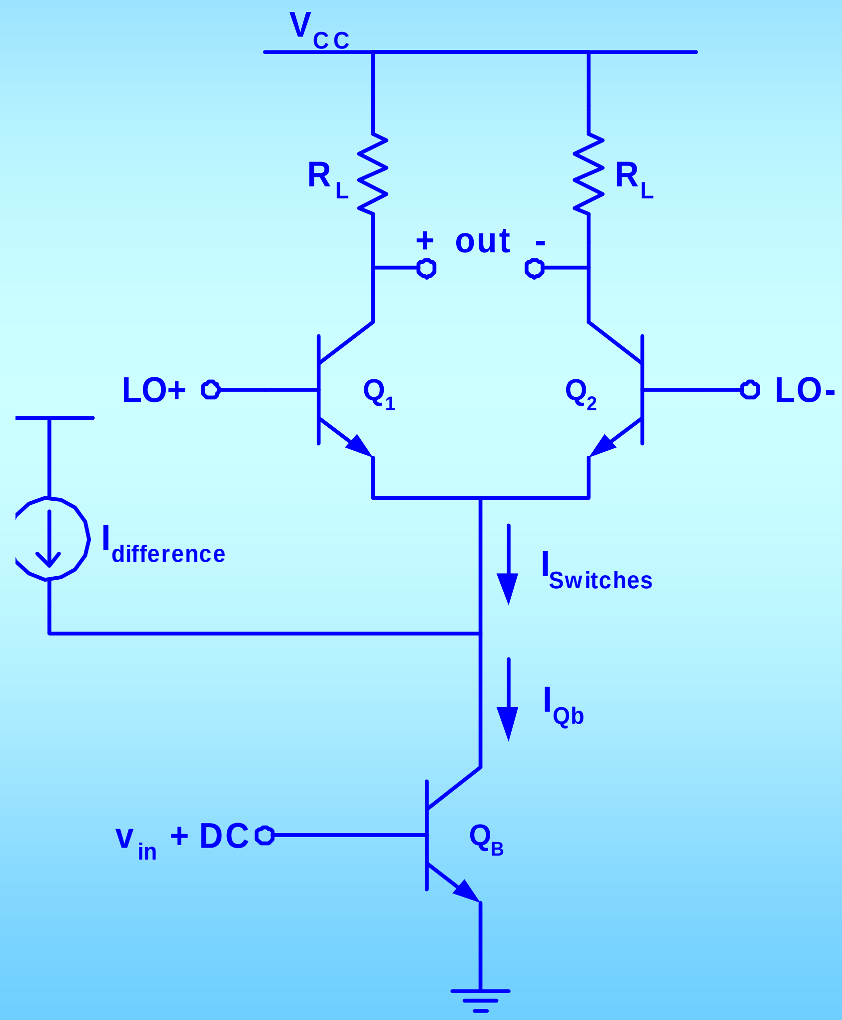
Common Base Mixers

- Similar operation to CE mixers
- Different input stage
  - Q_B is CB
- Slightly different output noise
  - Different CB input noise
- Better linearity



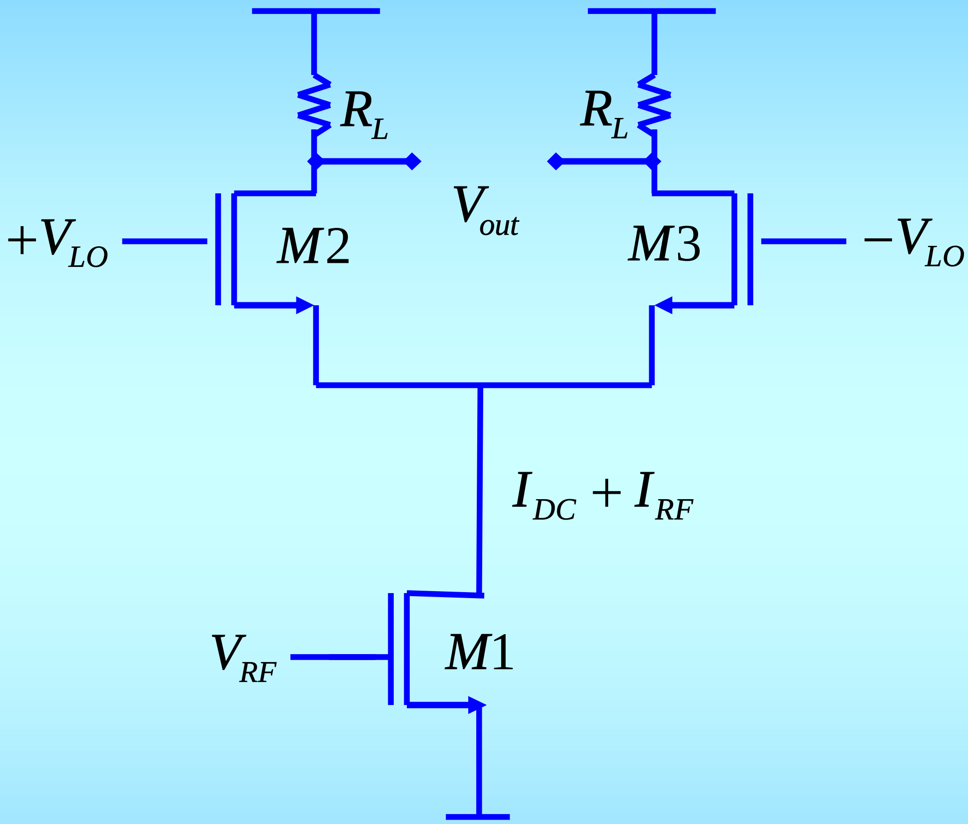
Mixer Improvements

- Debiasing switches from input transistors:
  - To lower NF we want high g_m, but low Q_1 and Q_2 current
    - Conflicting!
  - We can set low $I_{Switches}$ and high I_{Qb} using a current source

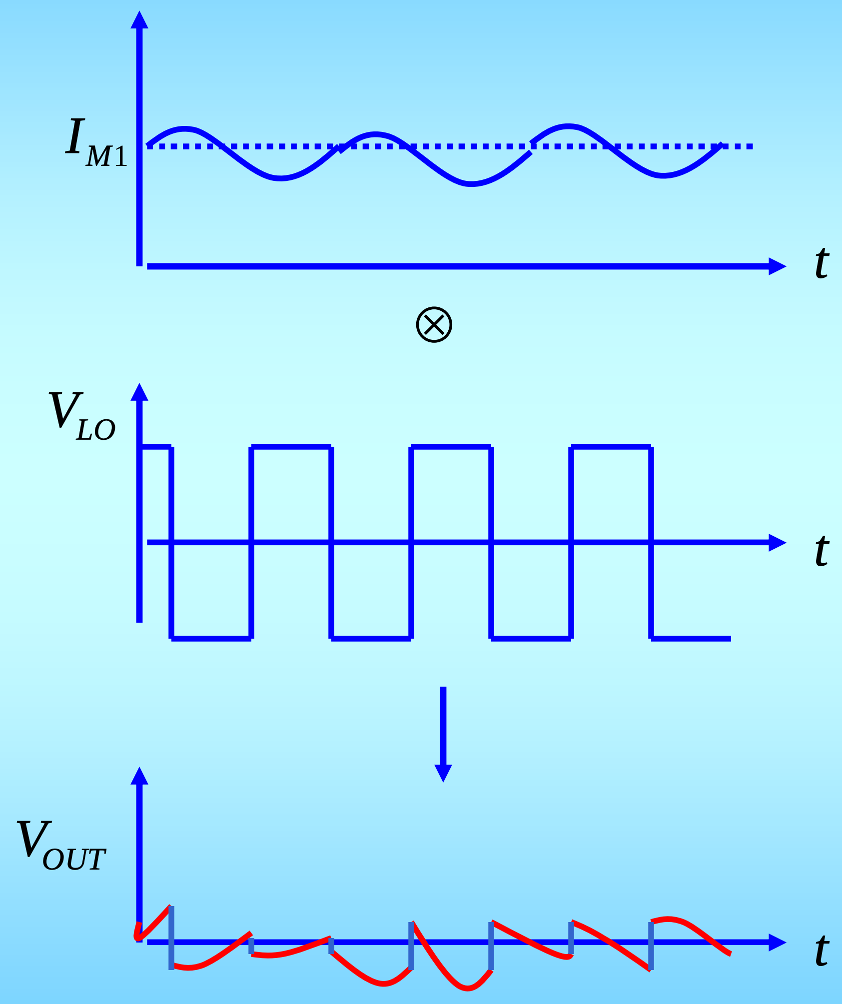


MOS Single Balanced Mixer

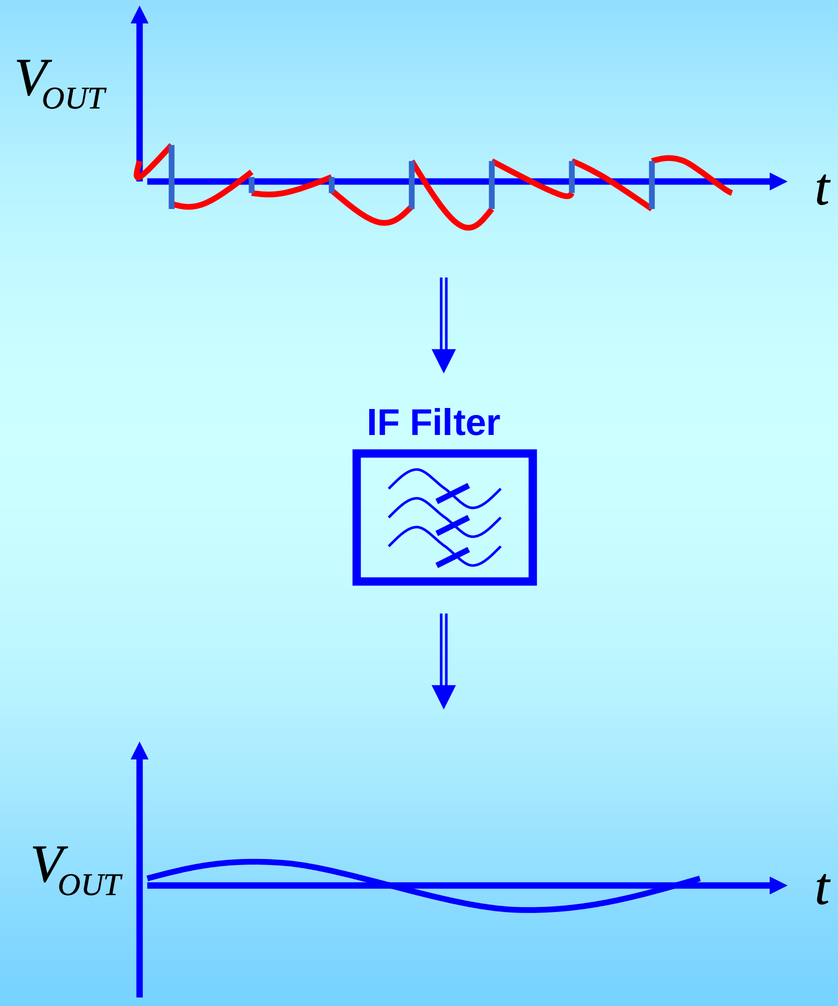
- The transistor M1 converts the RF voltage signal to the current signal.
- Transistors M2 and M3 commute the current between the two branches.



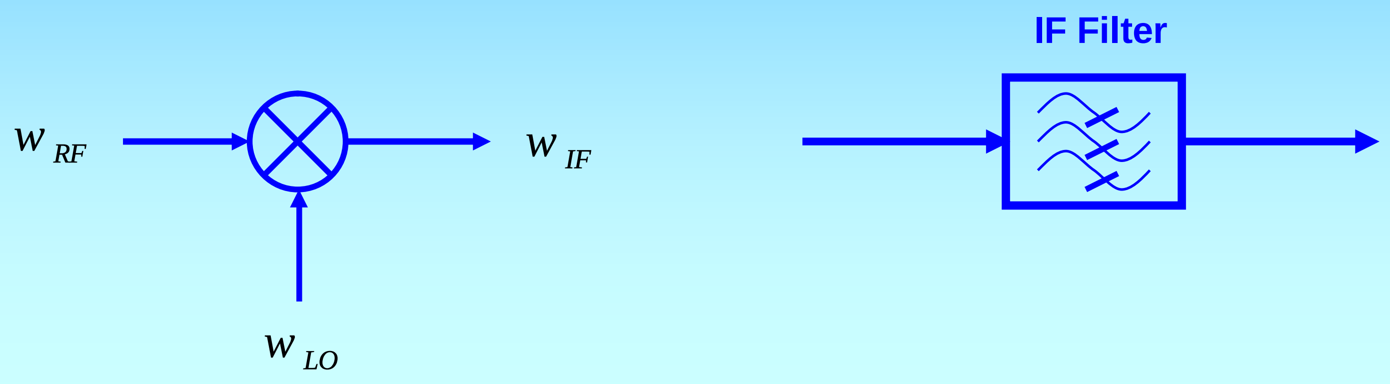
MOS Single Balanced Mixer

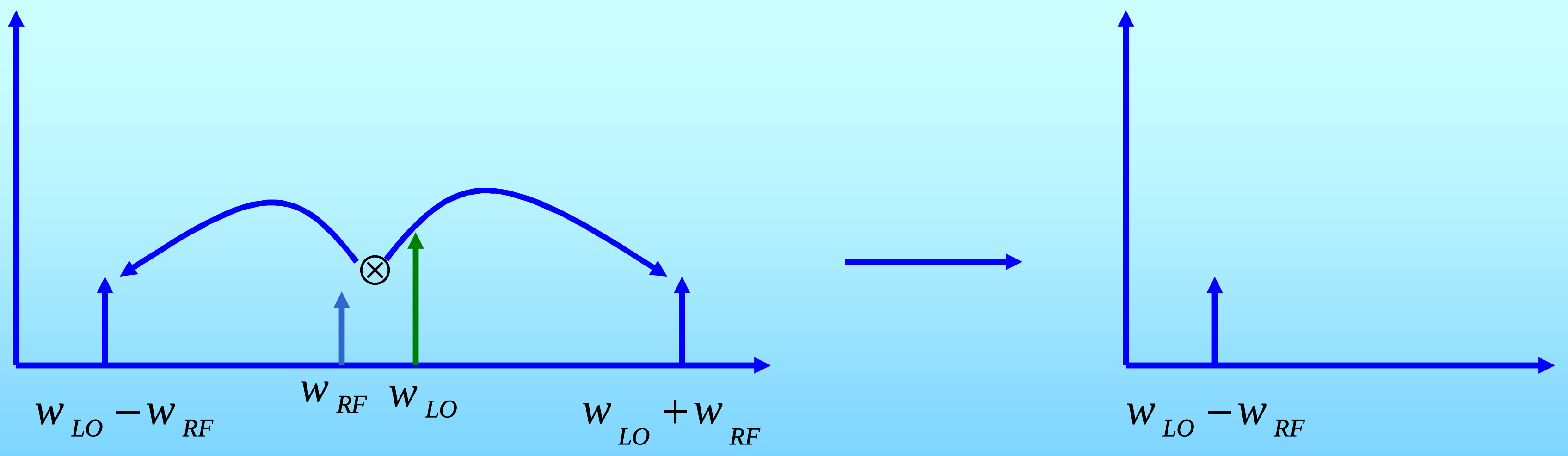


MOS Single Balanced Mixer

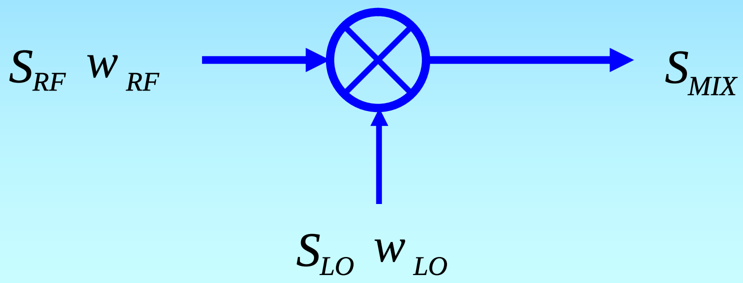


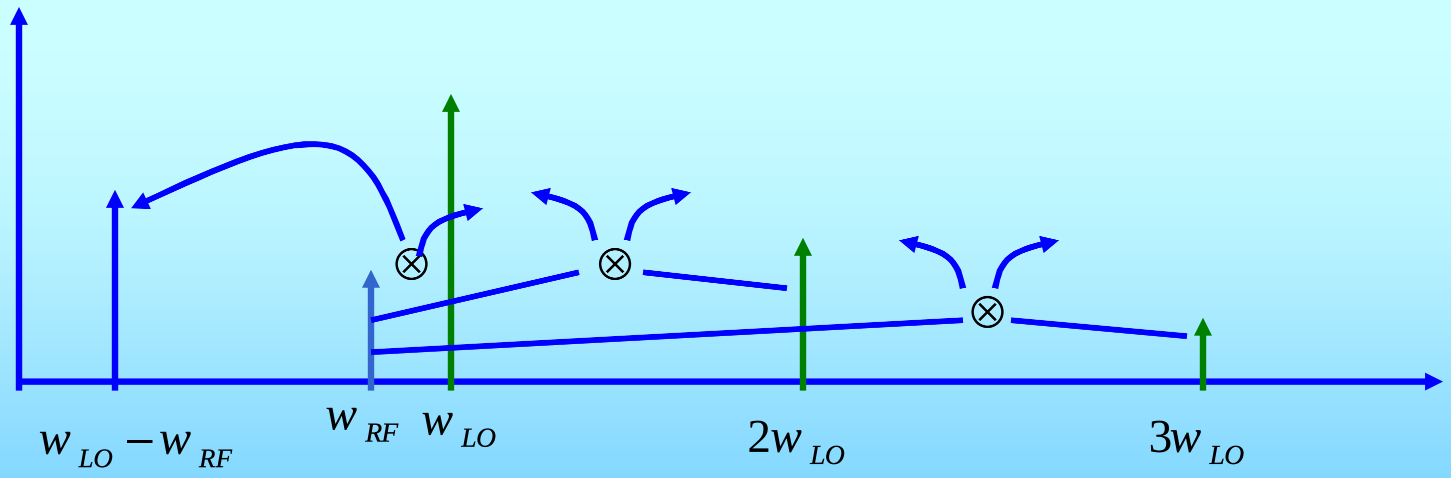
MOS Single Balanced Mixer



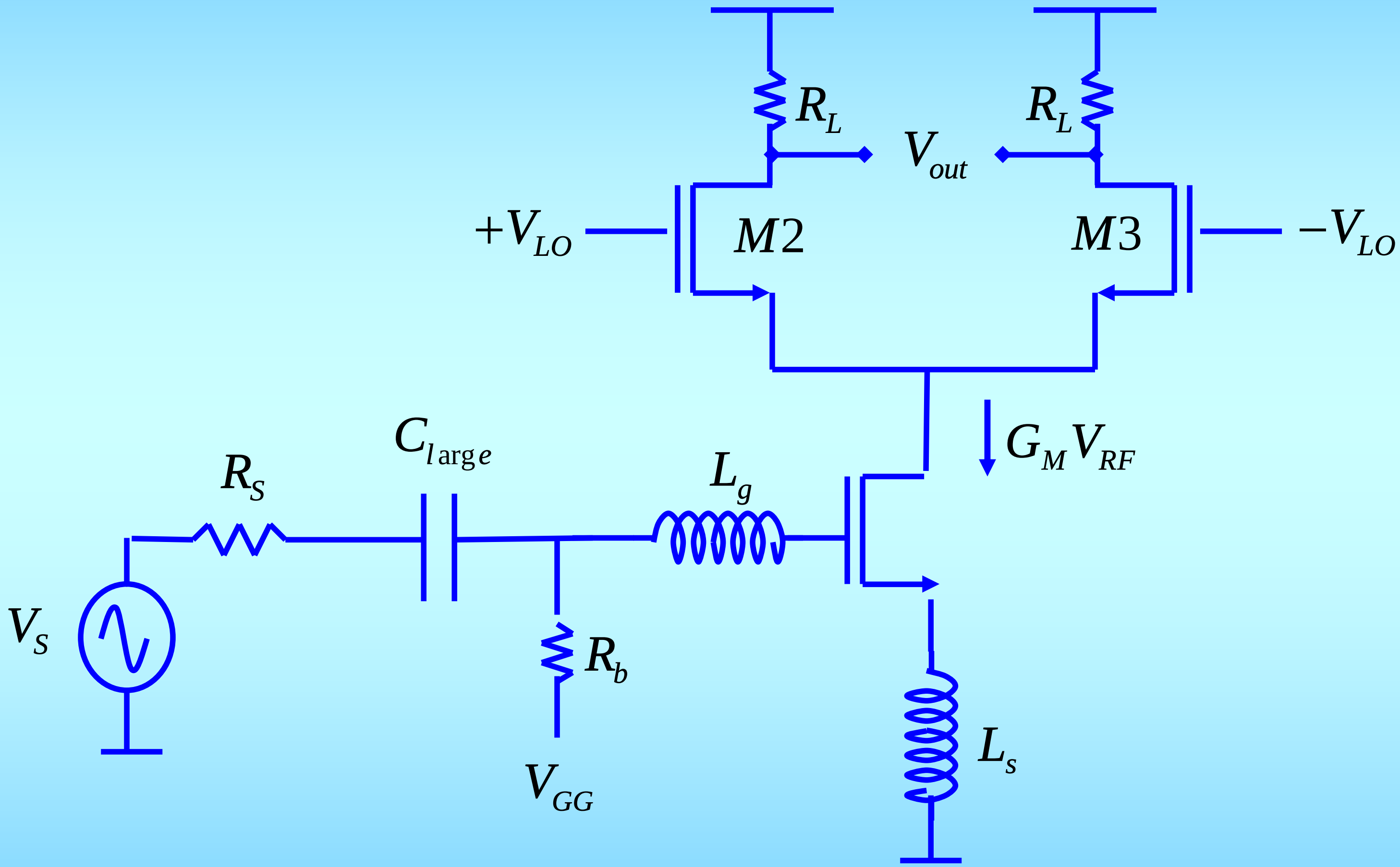


MOS Single Balanced Mixer



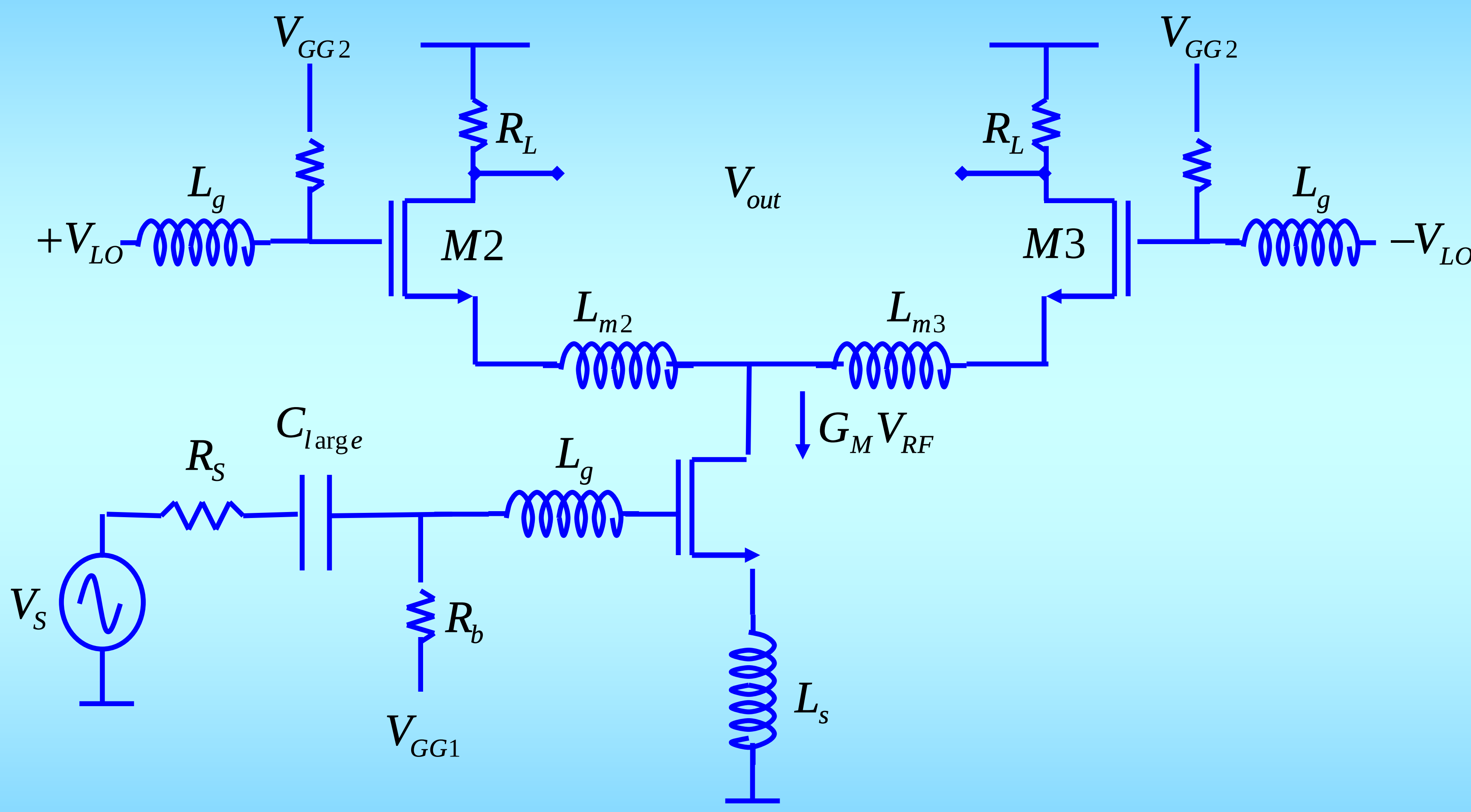


Single Balanced Mixer Analysis (Incl. Impd. Match)



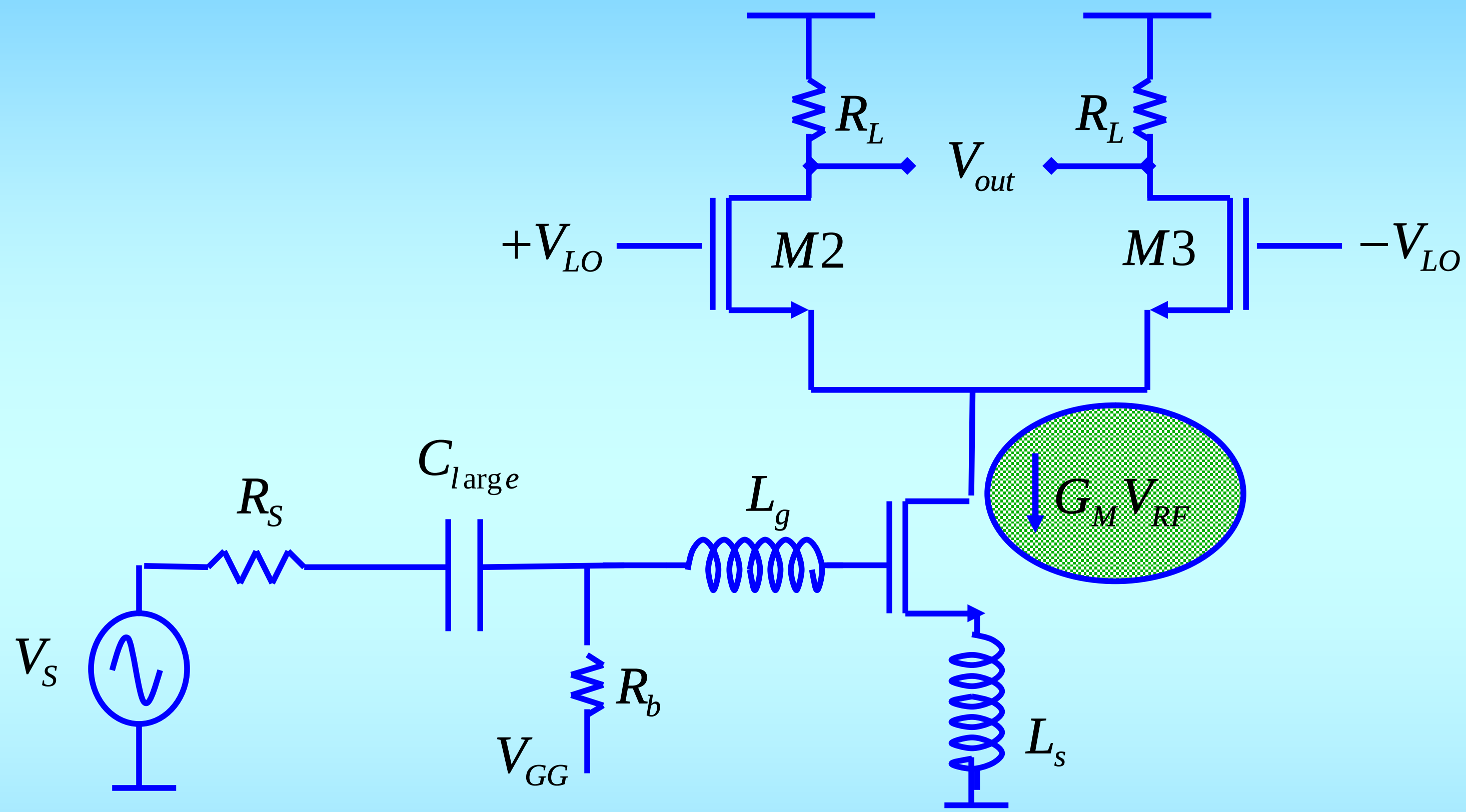
This architecture, without impedance matching for the LO port, is very commonly used in many designs.

Single Balanced Mixer Analysis (Incl. Impd. Match)



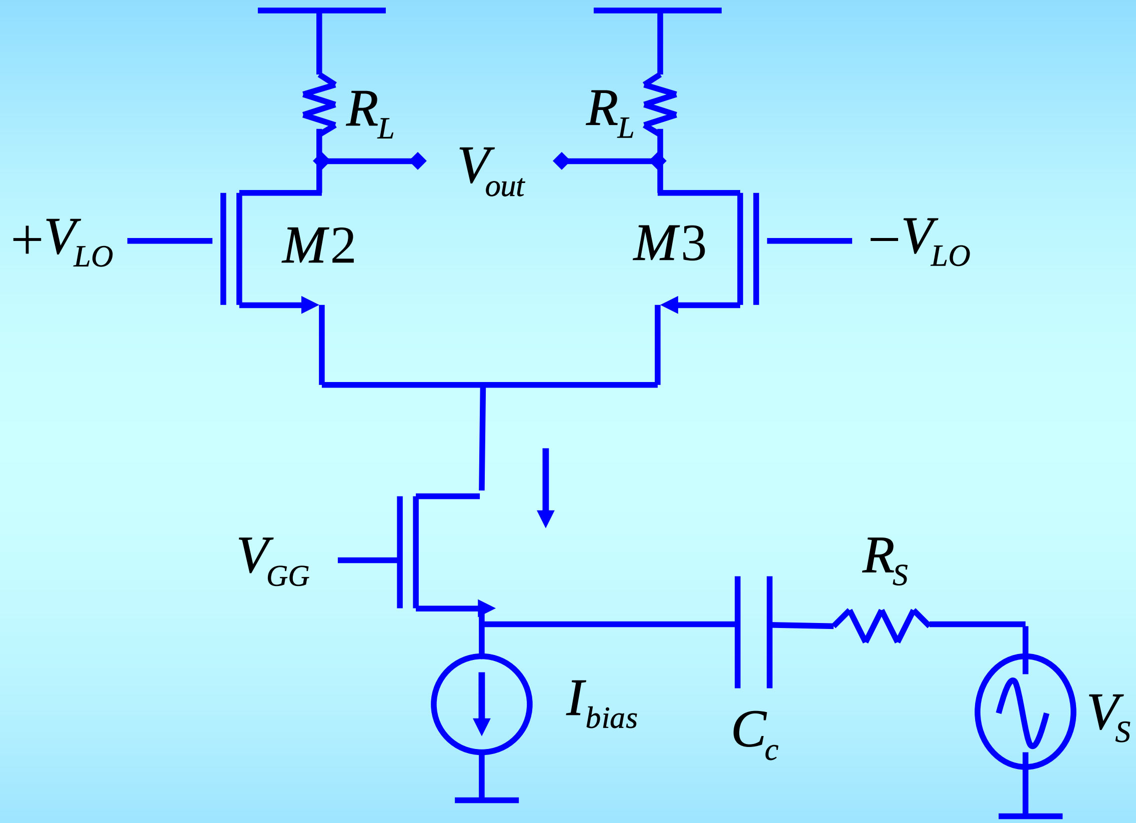
- This architecture, with impedance matching for the LO port, maximizes LO power utilization without wasting it.

Single Balanced Mixer Analysis: Linearity



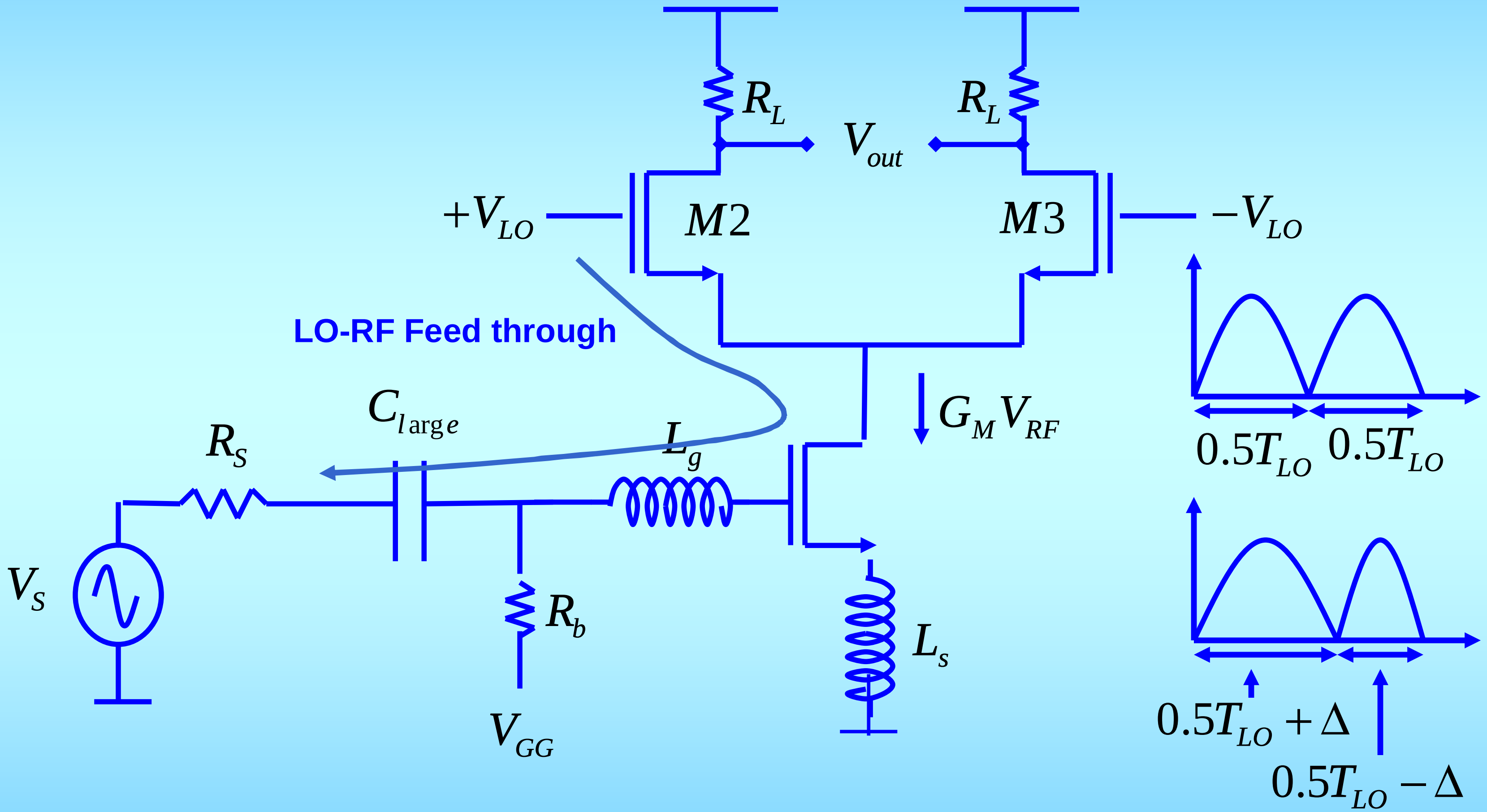
- Linearity of the Mixer primarily depends on the linearity of the transducer (I_tail=Gm*V_rf). Inductor Ls helps improve linearity of the transducer.
- The transducer transistor M1 can be biased in the linear law region to improve the linearity of the Mixer. Unfortunately this results in increasing the noise figure of the mixer (as discussed in LNA design).

Single Balanced Mixer Analysis: Linearity



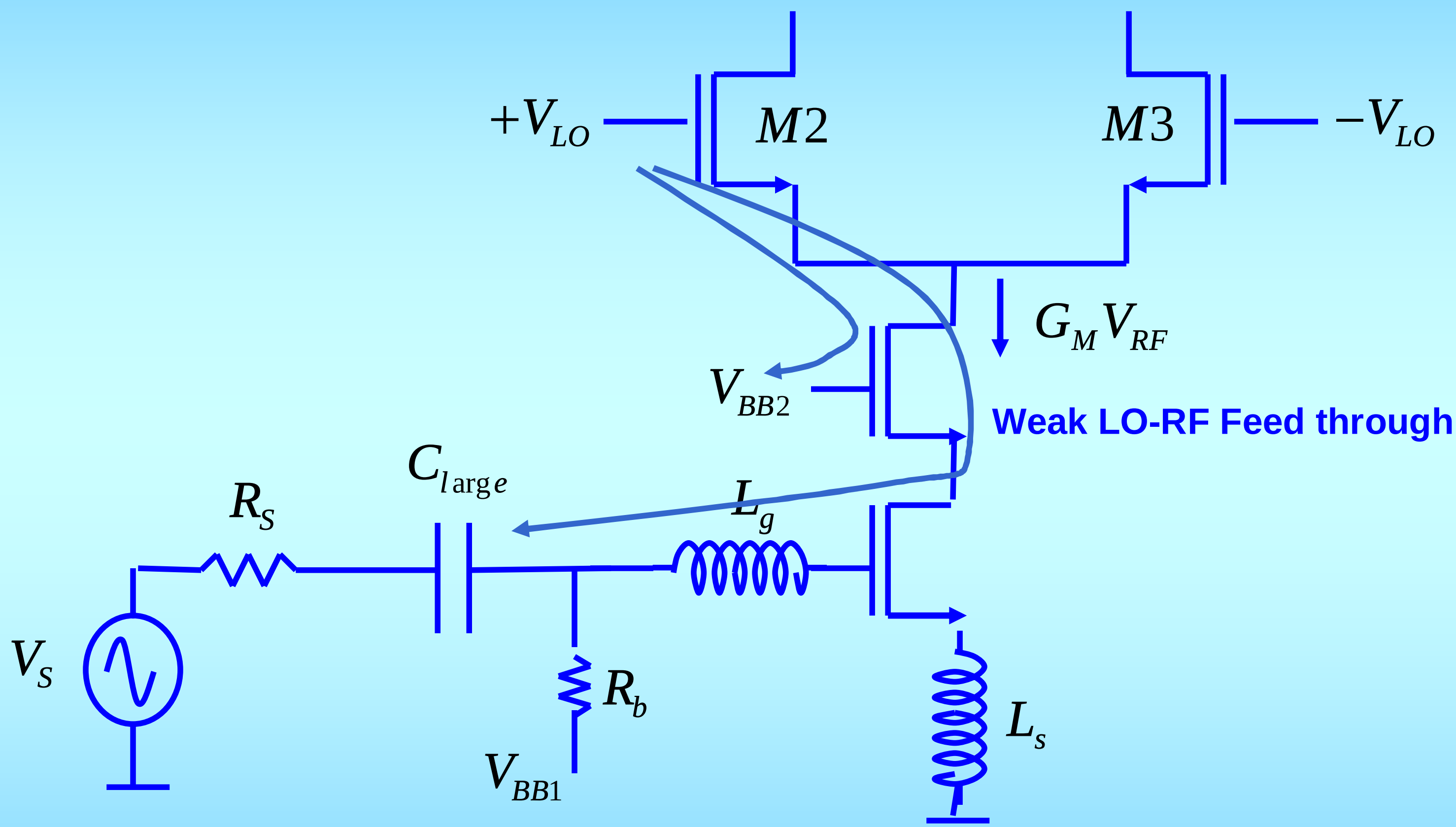
- Using the common gate or common base stage as the transducer improves the linearity of the mixer. Unfortunately the approach reduces the gain and increases the noise figure of the mixer.

Single Balanced Mixer Analysis: Isolation



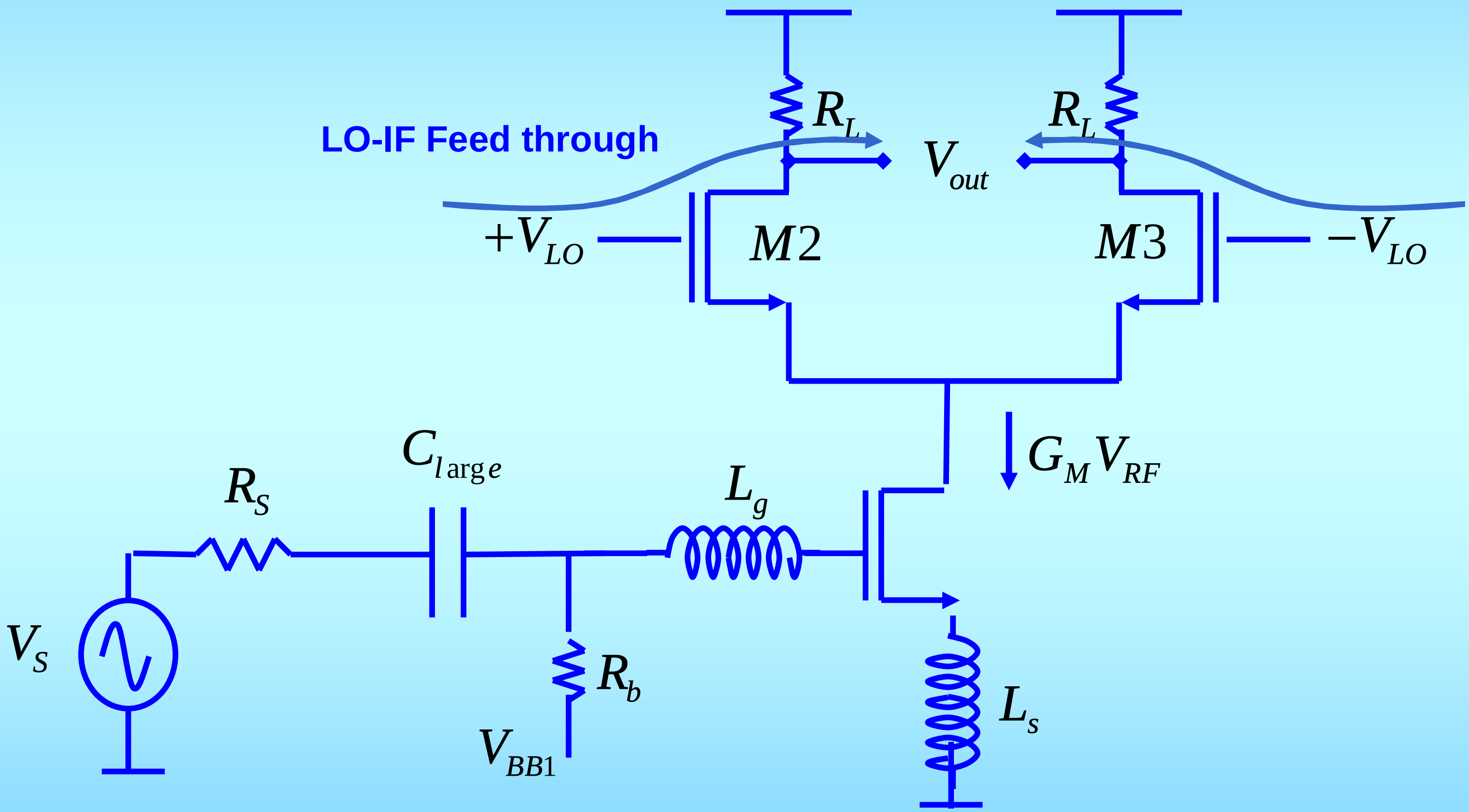
- The strong LO easily feeds through and ends up at the RF port in the above architecture especially if the LO does not have a 50% duty cycle. Why?

Single Balanced Mixer Analysis: Isolation



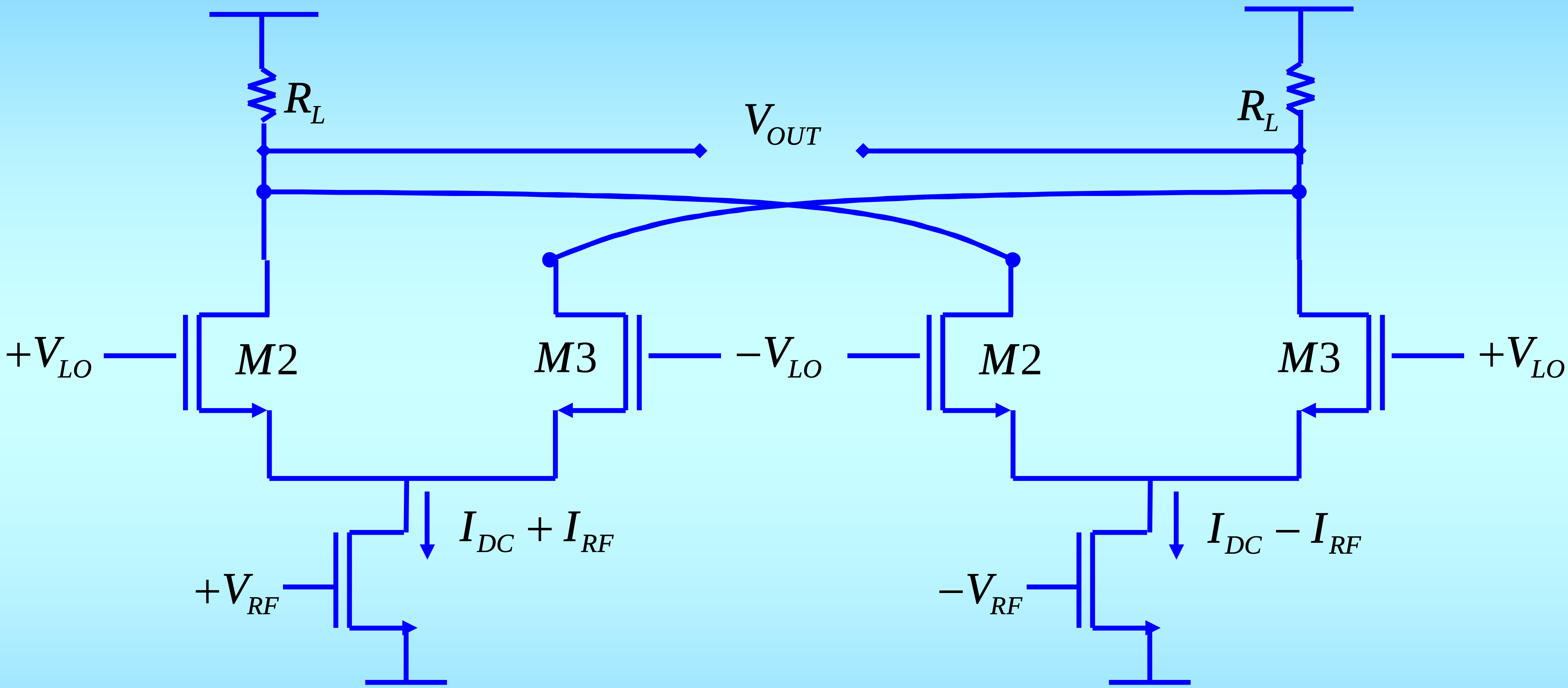
- The amplified RF signal from the transducer is passed to the commuting switches through use of a common gate stage ensuring that the mixer operation is unaffected. Adding the common gate stage suppresses the LO-RF feed through.

Single Balanced Mixer Analysis: Isolation



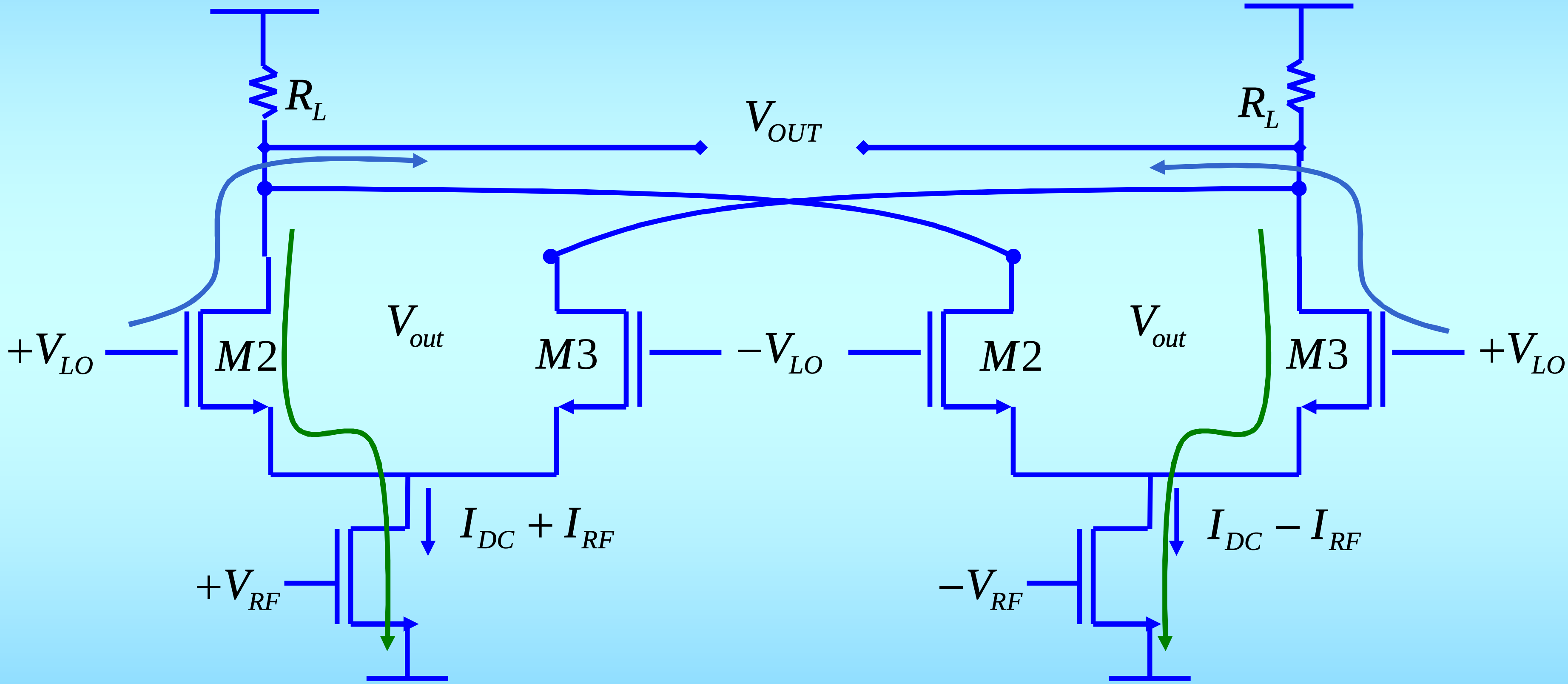
- The strong LO-IF feed-through may cause the mixer or the amplifier following the mixer to saturate. It is therefore important to minimize the LO-IF feed-through.

Double Balanced Mixer



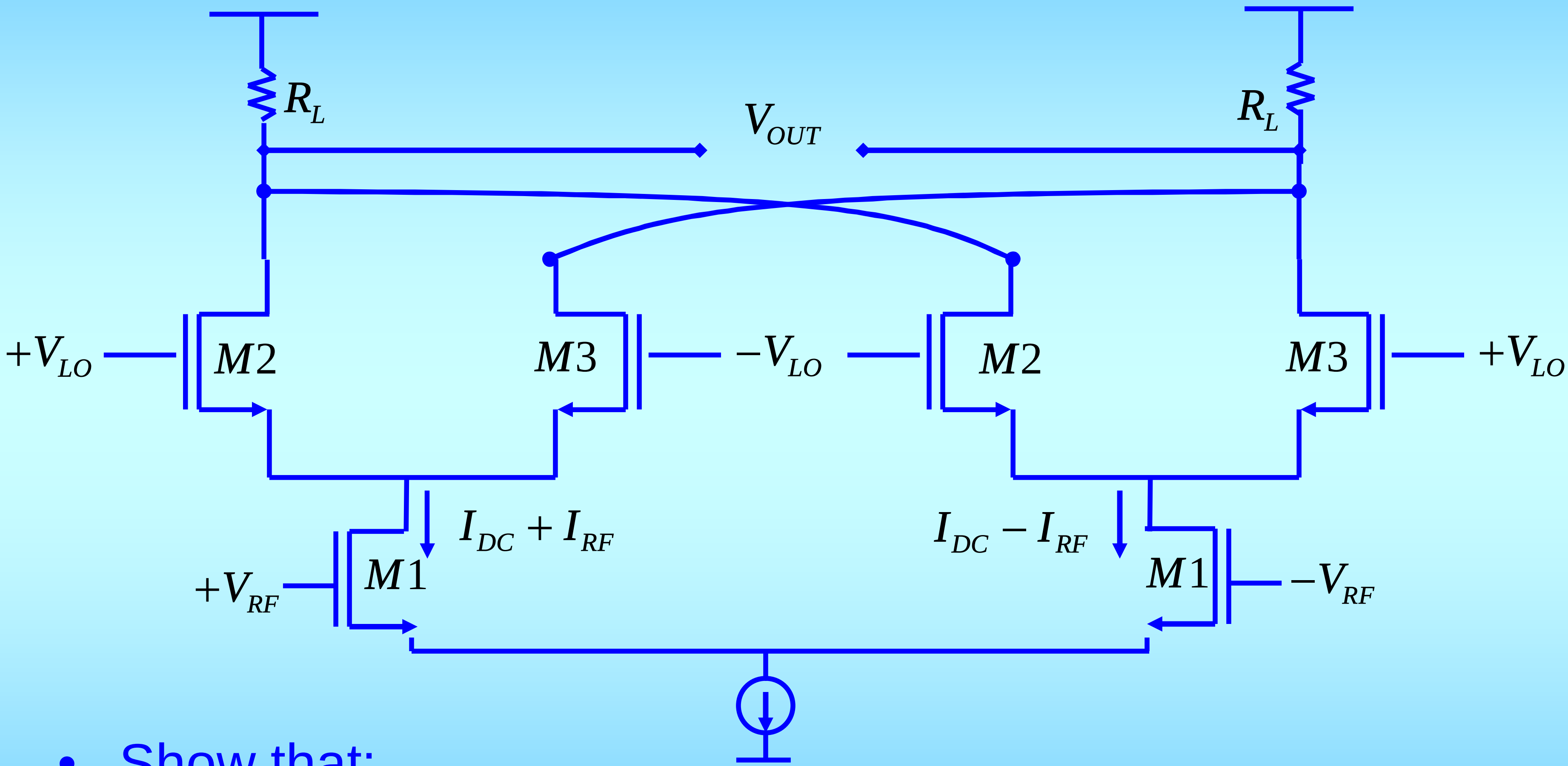
- Strong LO-IF feed suppressed by double balanced mixer.
- All the even harmonics cancelled.
- All the odd harmonics doubled (including the signal).

Double Balanced Mixer



- The LO feed through cancels.
- The output voltage due to RF signal doubles.

Double Balanced Mixer: Linearity



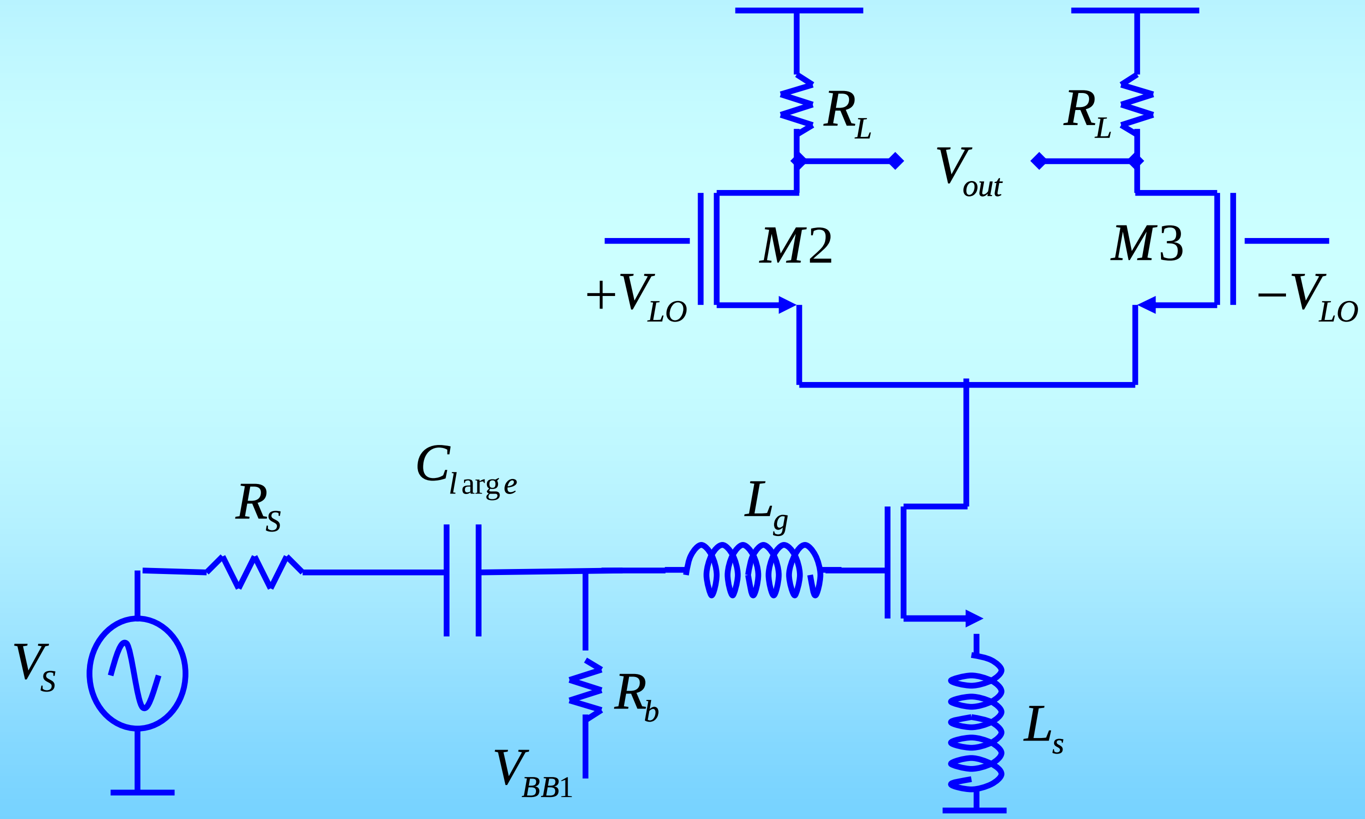
- Show that:

$$V_{IF} = 2I_{DC}R_L\left\{\left(\frac{K_{SQ}}{2I_{DC}}\right)^{1/2} * V_{RF} + \frac{1}{2}\cdot\left(\frac{K_{SQ}}{2I_{DC}}\right)^{3/2} V_{RF}^3 + \ldots\right\}$$

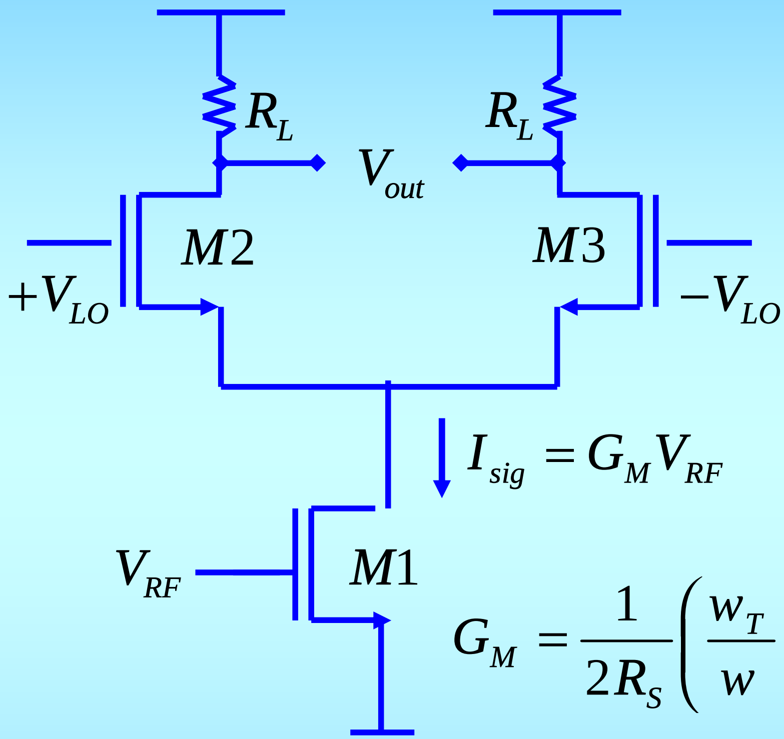
$$IIP_3 \ in-volts \ = \sqrt{\frac{8I_{DC}}{3K_{SQ}}}$$

Mixer Input Match

$$R_S = R_g + w_T L_S \qquad w\left(L_g + L_s\right) = \frac{1}{wC_{gs}}$$



Mixer Gain



$$G_M = \frac{1}{2R_S}\left(\frac{w_T}{w}\right)$$

$$0 \to \frac{T_{LO}}{2} : V_{out} = \left[V_{cc} - \left(I_{DC} + I_{sig}\right).R_L\right] - \left[V_{cc}\right] = -\left(I_{DC} + I_{sig}\right).R_L$$

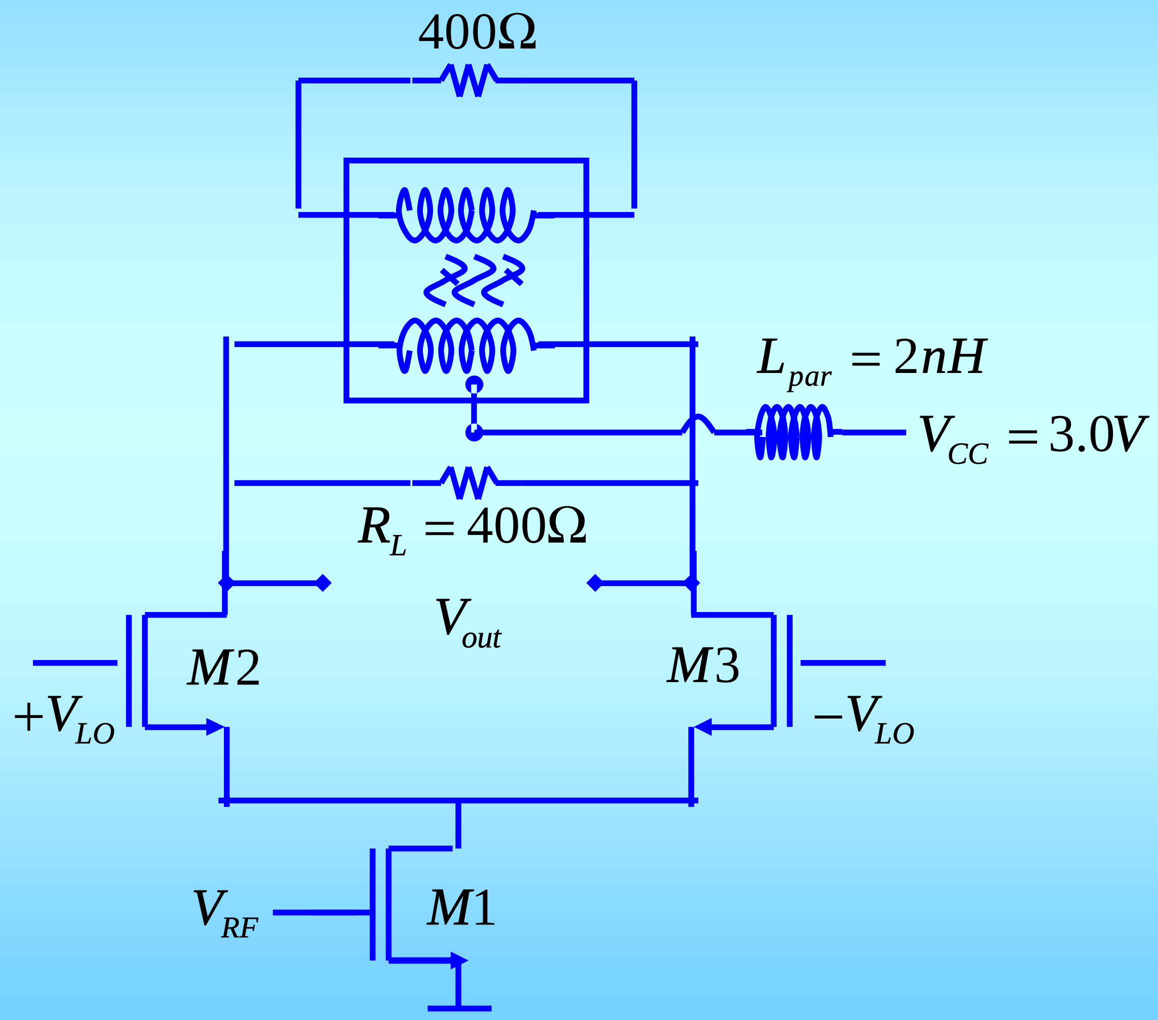
$$\frac{T_{LO}}{2} \to T_{LO} : V_{out} = \left[V_{cc}\right] - \left[V_{cc} - \left(I_{DC} + I_{sig}\right).R_L\right] = \left(I_{DC} + I_{sig}\right).R_L$$

$$V_{out-sig} = I_{sig} R_L * SW = I_{sig} R_L \frac{4}{p}\left(\cos w_{LO} t - \frac{1}{3}\cos 3w_{LO} t + \frac{1}{5}\cos 5w_{LO} t - \frac{1}{7}\cos 7w_{LO} t + \mathbf{L}\right)$$

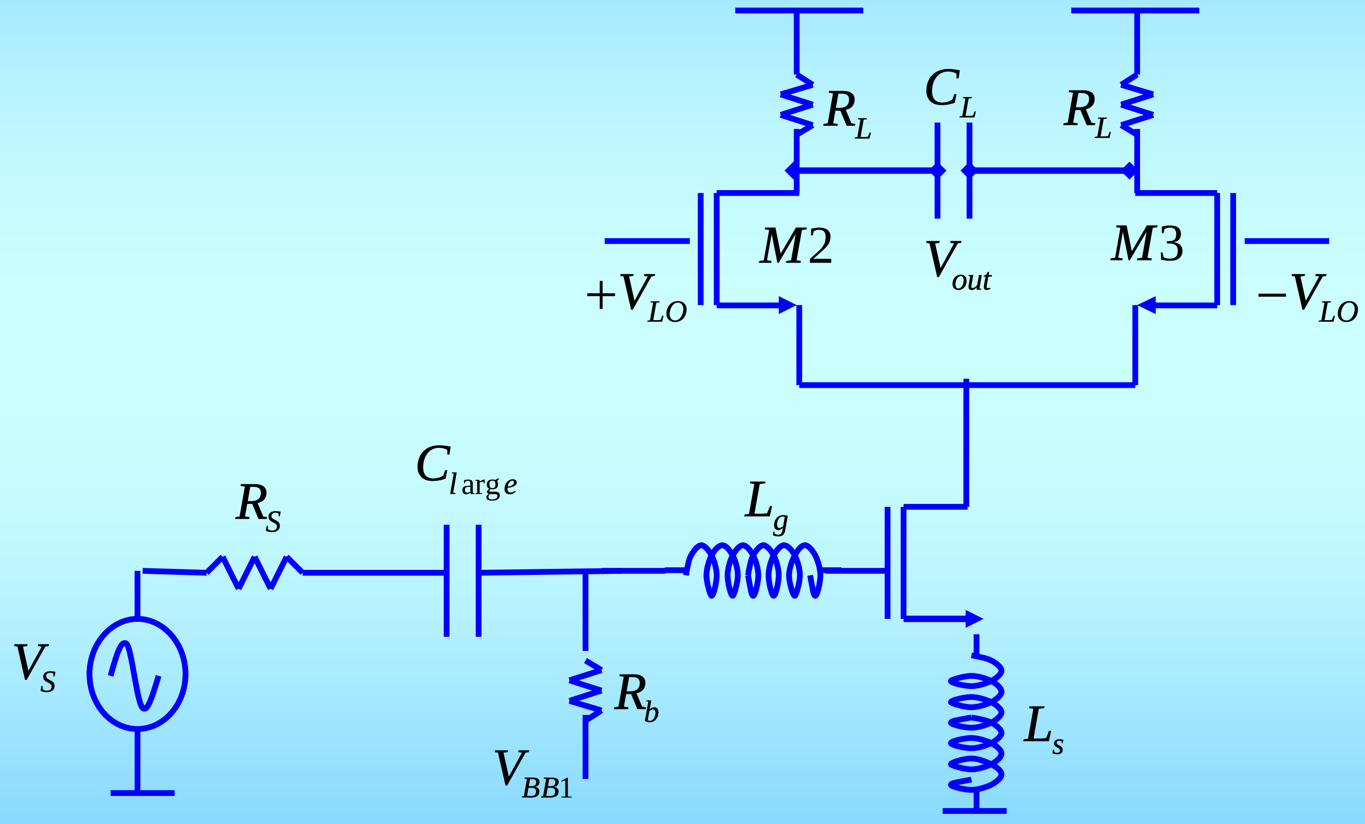
Mixer Output Match

- Heterodyne Mixer: For IF frequencies of 100-200MHz (signal bandwidth of 4MHz), no impedance matching due to:
  - The signal bandwidth is comparable to the IF frequency therefore the impedance matching would create gain and phase distortions
  - Need large inductors and capacitors to impedance match at 200MHz

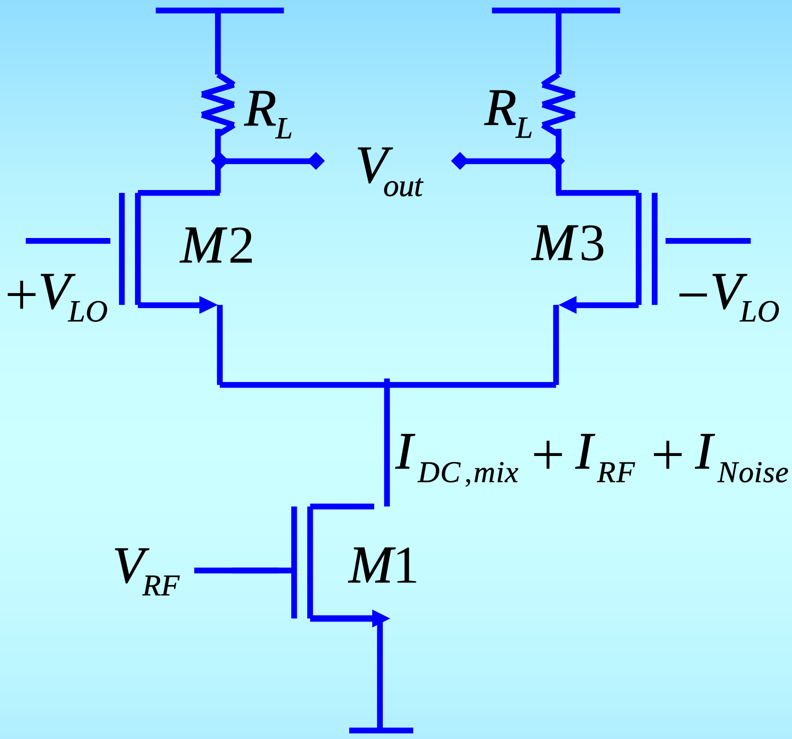
Mixer Output Match (IF)

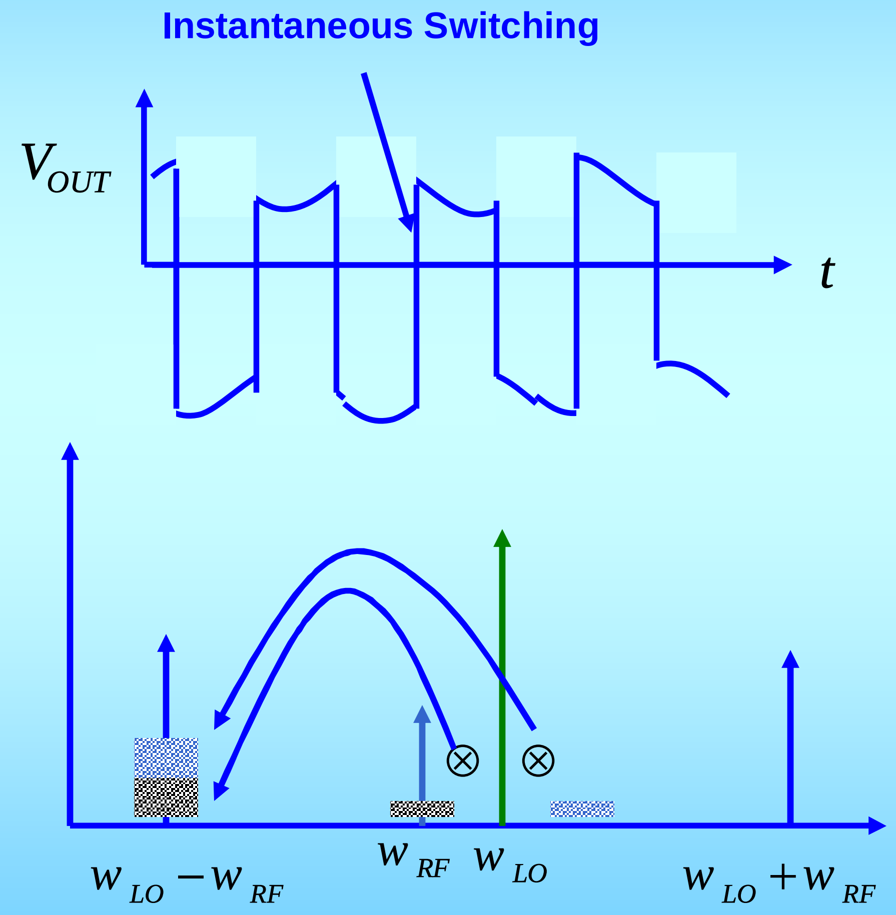


Mixer Output Match (direct conversion)

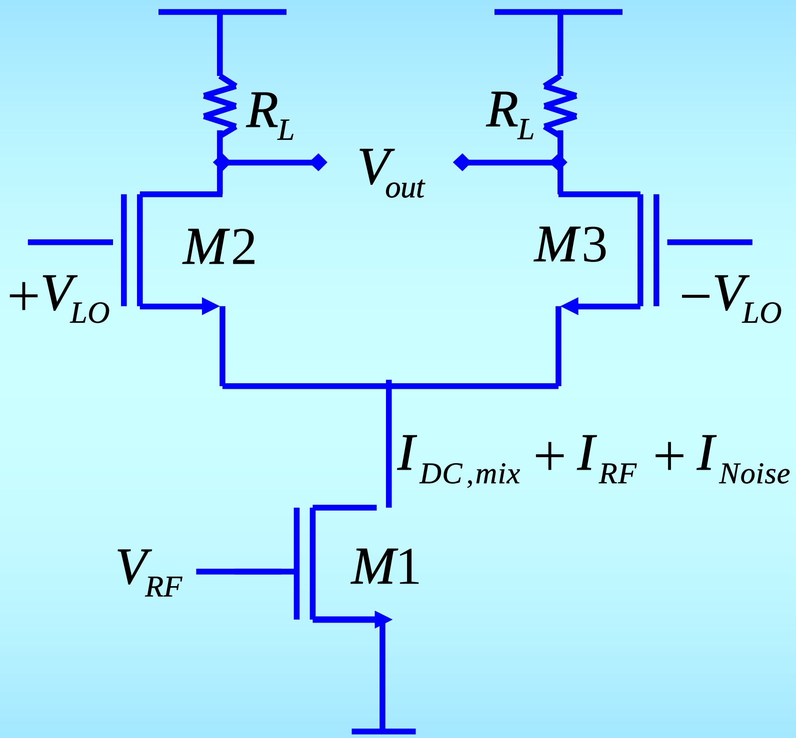


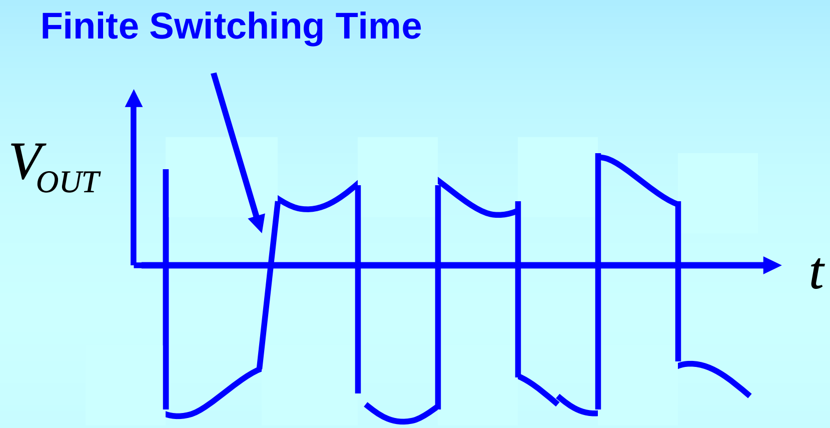
Mixer Noise Analysis





Mixer Noise Analysis

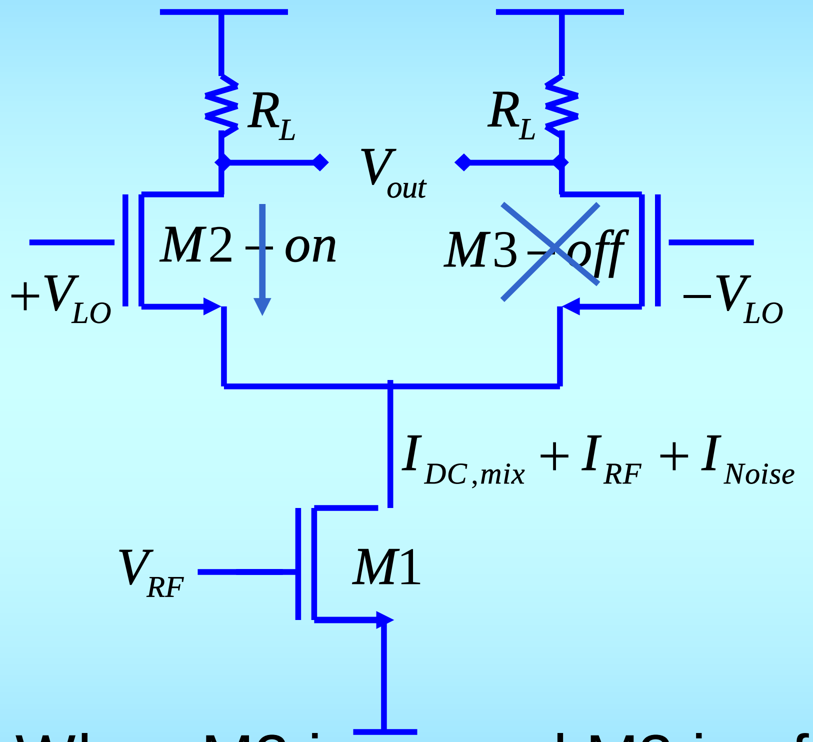


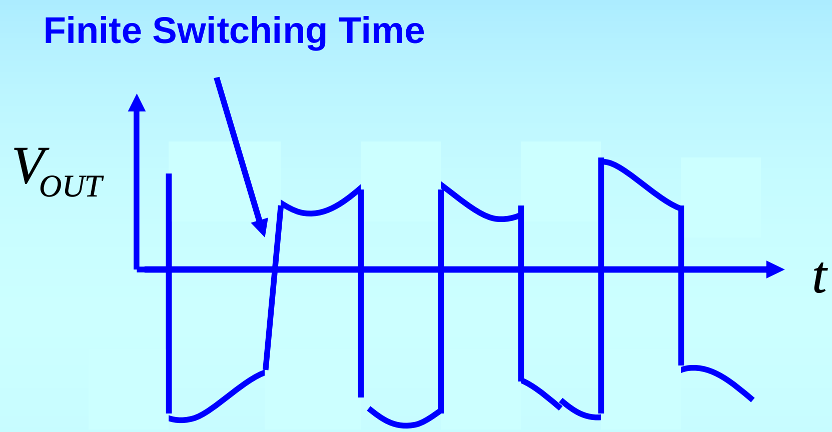


- If the switching is not instantaneous, additional noise from the switching pair will be added to the mixer output.
- Let us examine this in more detail.

Mixer Noise Analysis

- Noise analysis of a single balanced mixer cont...:

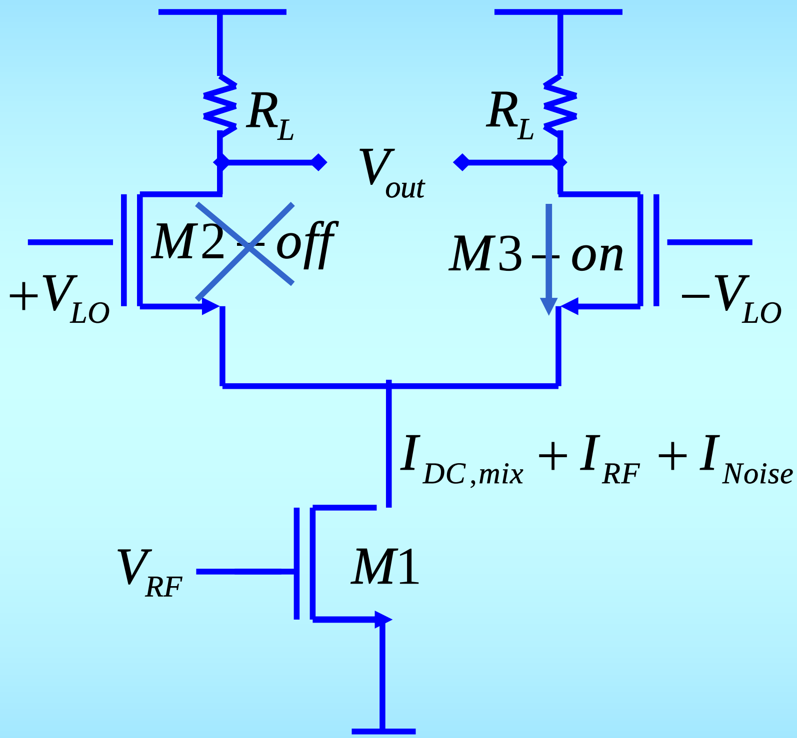


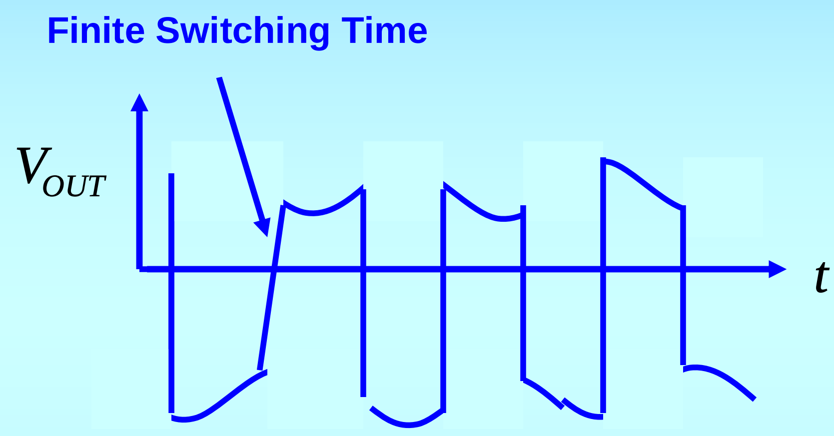


- When M2 is on and M3 is off:
  – M2 does not contribute any additional noise (M2 acts as cascode)
  – M3 does not contribute any additional noise (M3 is off)

Mixer Noise Analysis

- Noise analysis of a single balanced mixer cont...:

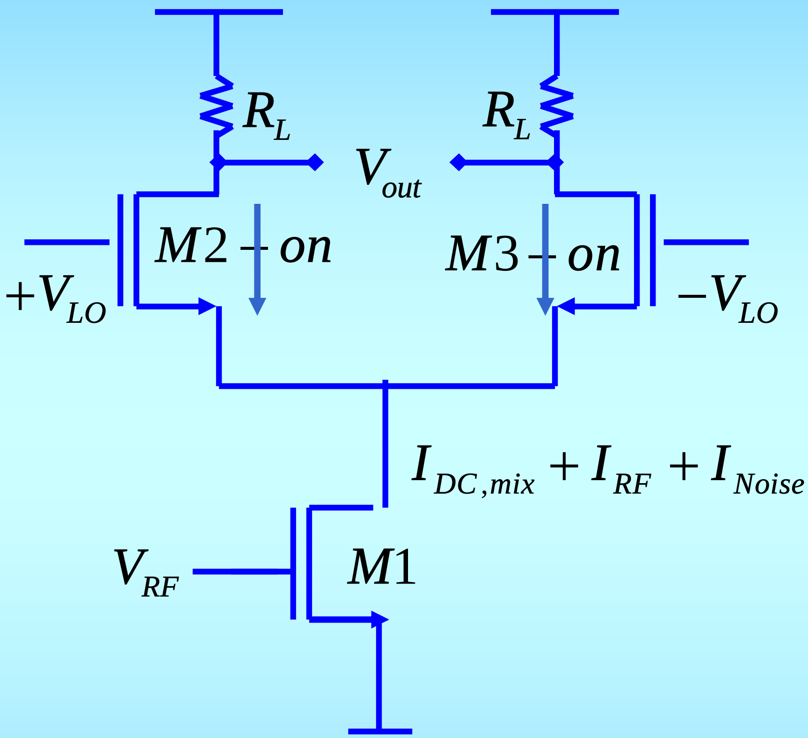


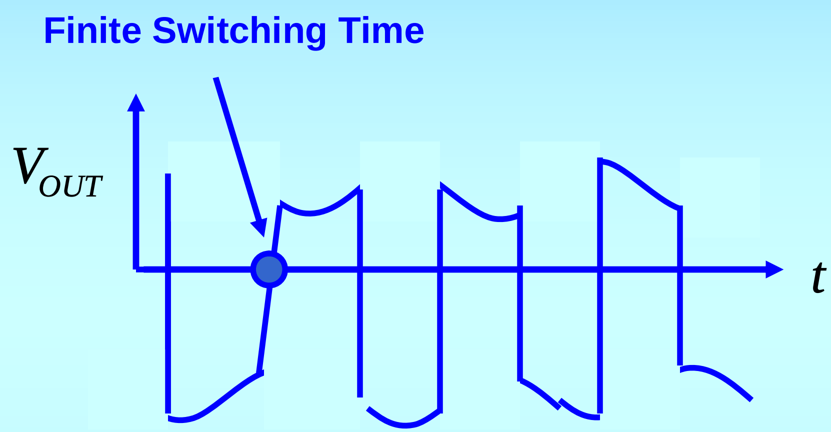


- When M2 is off and M3 is on:
  - M2 does not contribute any additional noise (M2 is off)
  - M3 does not contribute any additional noise (M3 acts as cascode)

Mixer Noise Analysis

- Noise analysis of a single balanced mixer cont...:

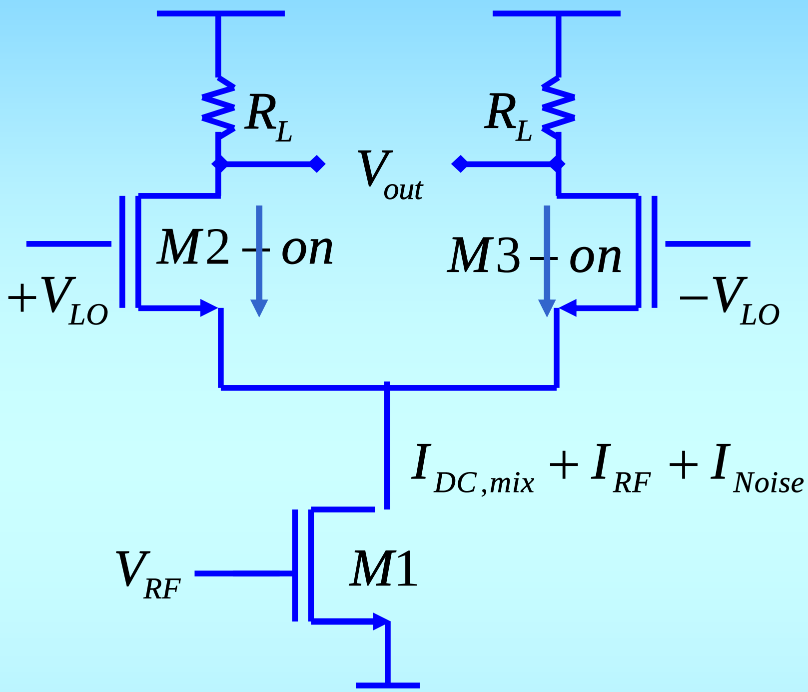


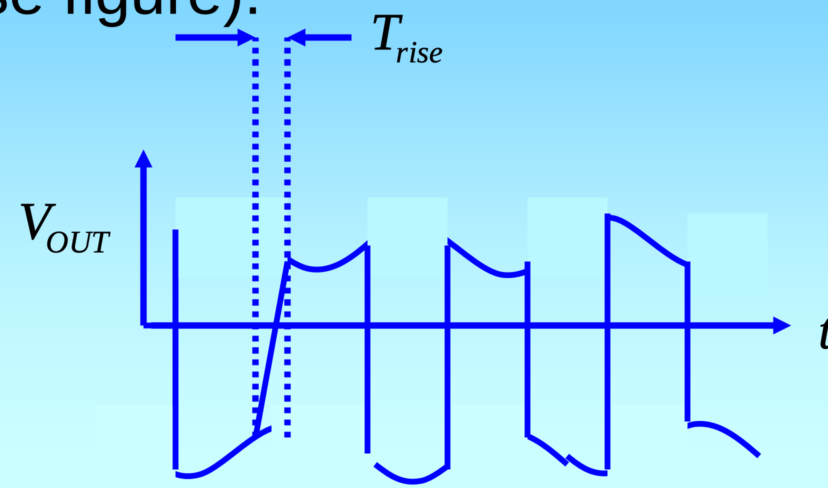


- When $V_{LO+} = V_{LO-}$ (i.e. the LO is passing through zero), the noise contribution from the transducer (M1) is zero. Why?
- However, the noise contributed from M2 and M3 is not zero because both transistors are conducting and the noise in M2 and M3 are uncorrelated.

Mixer Noise Analysis

- Optimizing the mixer (for noise figure):





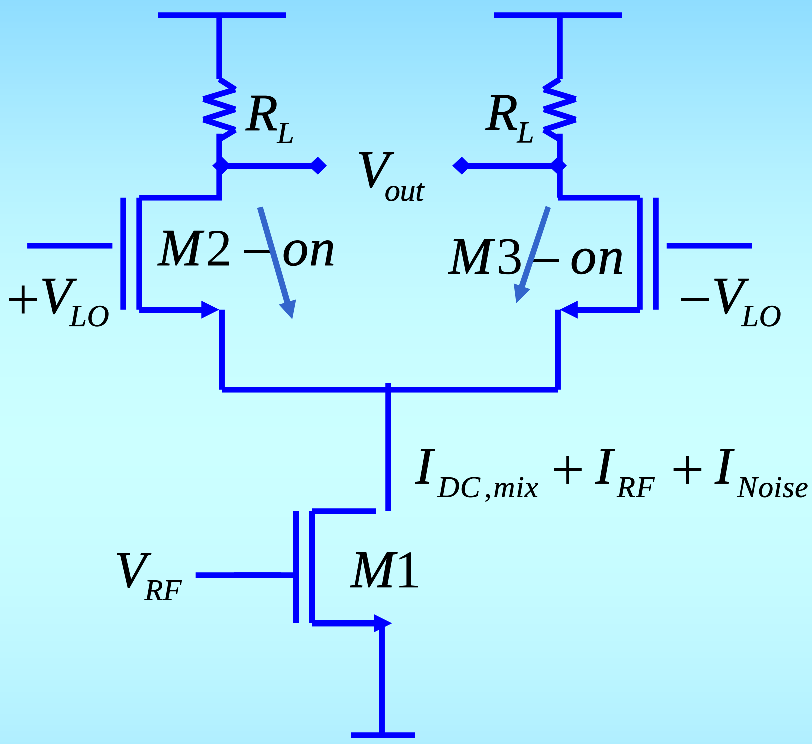
$$g_m \propto \sqrt{W} \ldots fixed - I_{DC}$$

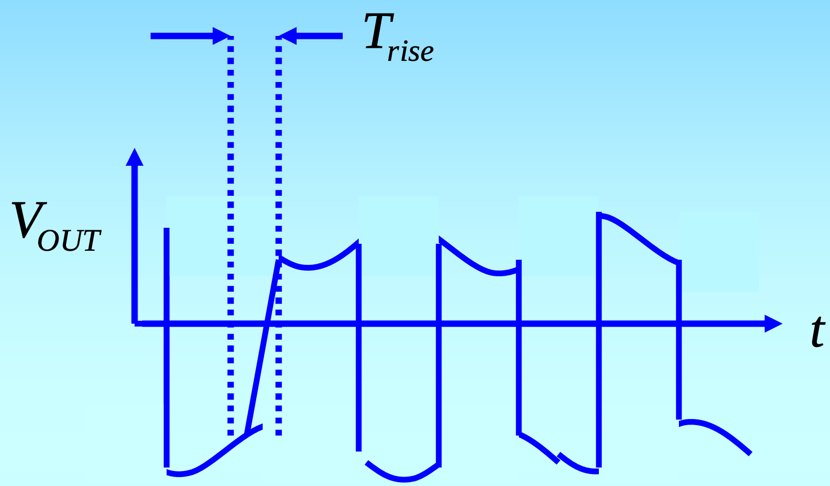
$$w_T \propto \frac{1}{\sqrt{W}} \ldots fixed - I_{DC}$$

- Design the transducer for minimum noise figure.
- Noise from M2, M3 minimized by fast switching :
  - making LO amplitude large
  - making M2 and M3 short (i.e. increasing fT of M2 and M3)
- Noise from M2, M3 can be minimized by using wide M2/M3 switches.

Mixer Noise Analysis

- Noise Figure Calculation:

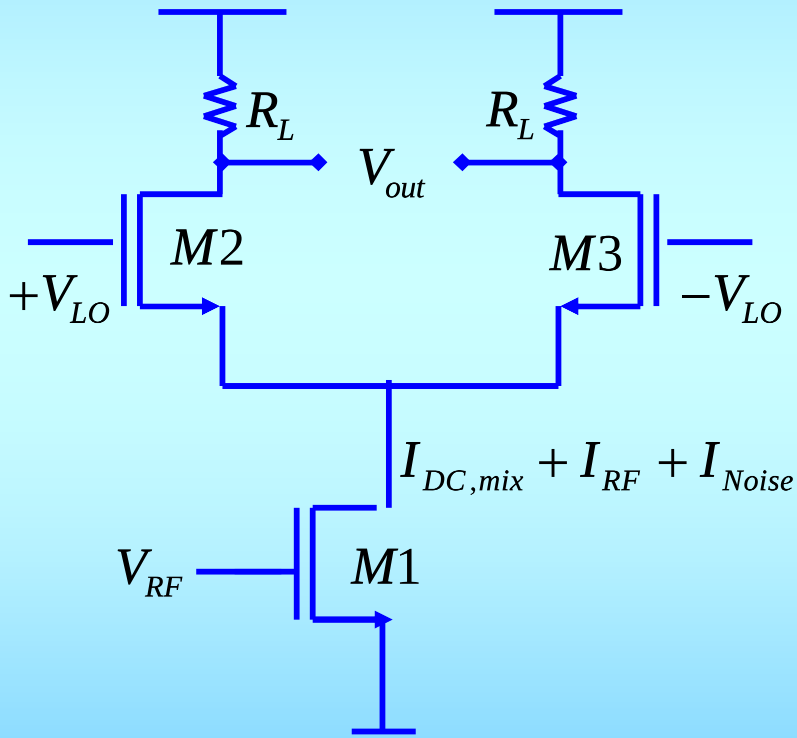


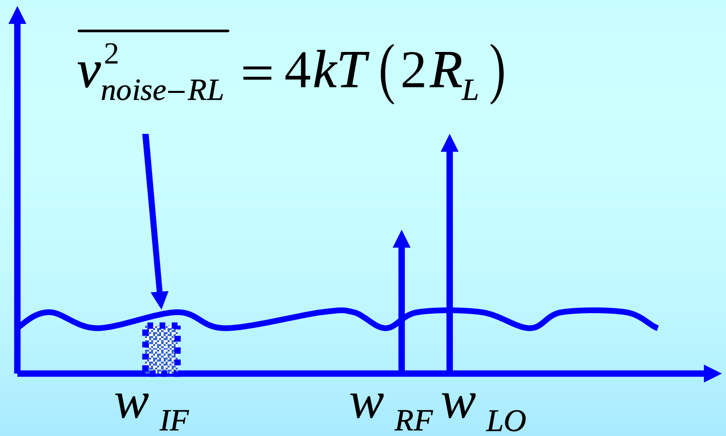


- Let us calculate the noise figure including the contribution of M2/M3 during the switching process.

Mixer Noise Analysis: RL Noise

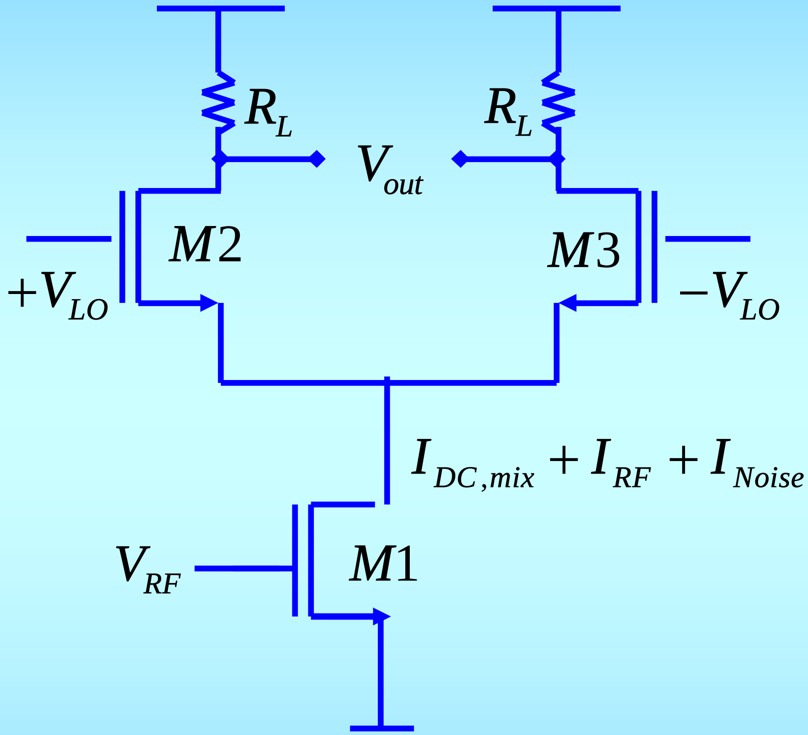
- Noise Analysis of Heterodyne Mixer (RL noise):

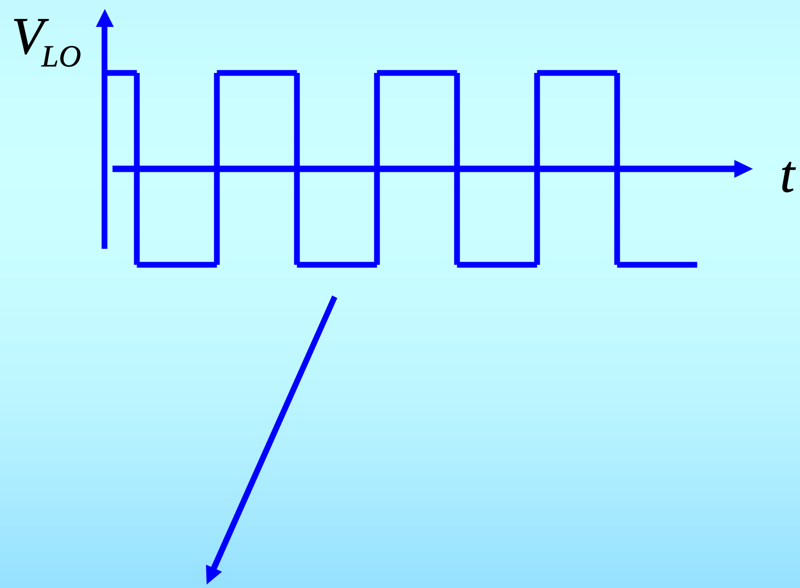




Mixer Noise Analysis: Transducer Noise

- Noise Analysis of Heterodyne Mixer (Transducer noise):





$$i_{noise-M1-switch} = i_{noise-M1}(t).SW(t)$$

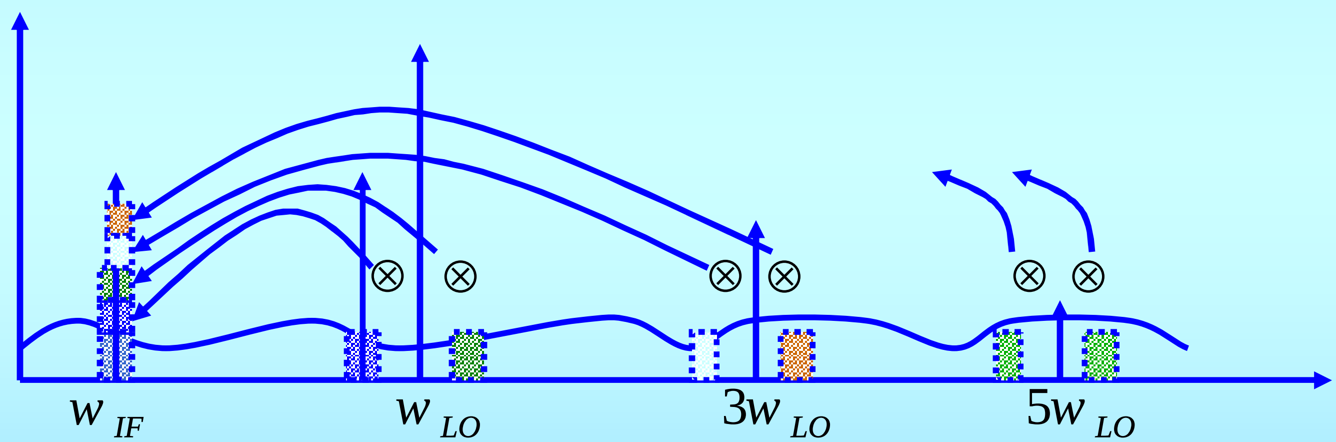
$$= i_{noise-M1}(t).\left(\frac{4}{p}Cos\{w_{LO}t\} - \frac{4}{3p}Cos\{3w_{LO}t\} + \frac{4}{5p}Cos\{5w_{LO}t\} - ...\right)$$

Mixer Noise Analysis: Transducer Noise

- Noise Analysis of Heterodyne Mixer (Trans-conductor noise):

$$i_{noise-M1-switch} = i_{noise-M1}(t).SW(t)$$

$$= i_{noise-M1}(t).\left(\frac{4}{p}Cos\{w_{LO}t\} - \frac{4}{3p}Cos\{3w_{LO}t\} + \frac{4}{5p}Cos\{5w_{LO}t\} - \ldots\right)$$



$$\overline{i^2_{noise-M1}}(f) = g.\frac{4kT}{R_{ch}} = g.4kTg_{m1}$$

$$\overline{i^2_{noise-M1}}(w_{IF}) = 2.\left(\frac{4}{p}\right)^2.\left[1 + \frac{1}{3^2} + \frac{1}{5^2} + ..\right].g\,4kTg_{m1}$$

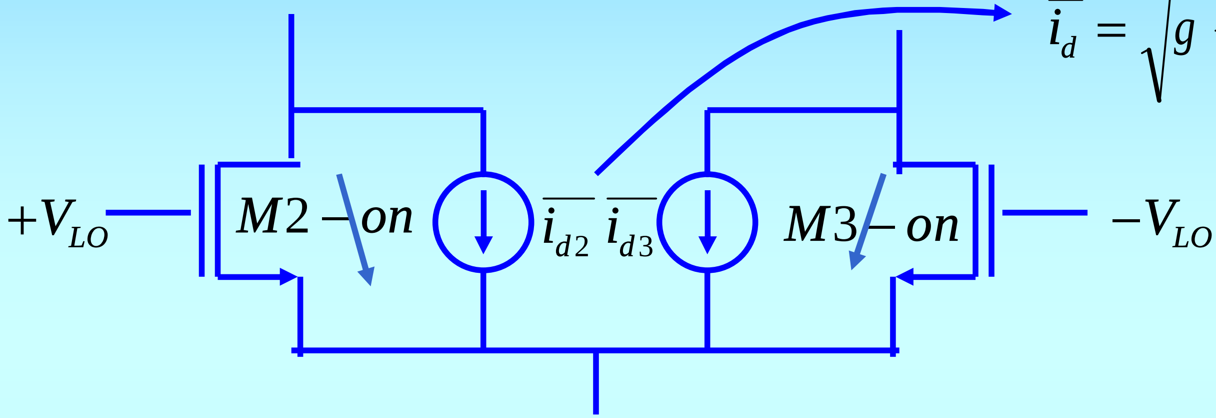
$$SW(f) = \frac{4}{p}d(w_{LO}) + \frac{4}{3p}d(3w_{LO}) + \ldots$$

$$\overline{i^2_{noise-M1}}(w_{IF}) = 4.g\,4kTg_{m1}$$

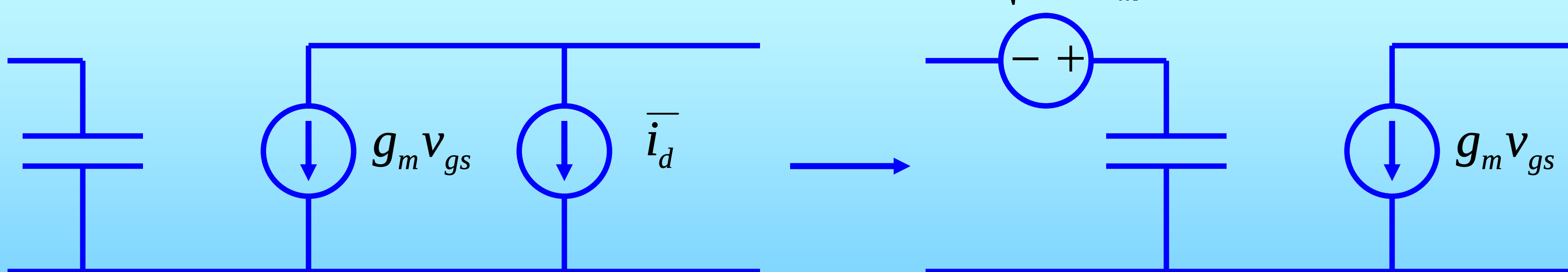
Mixer Noise Analysis: Switch Noise

- Noise Analysis of Heterodyne Mixer (switch noise):

$$\overline{i_d} = \sqrt{g \frac{4kT}{R_{ch}}} \approx \sqrt{g\, 4kTg_m}$$

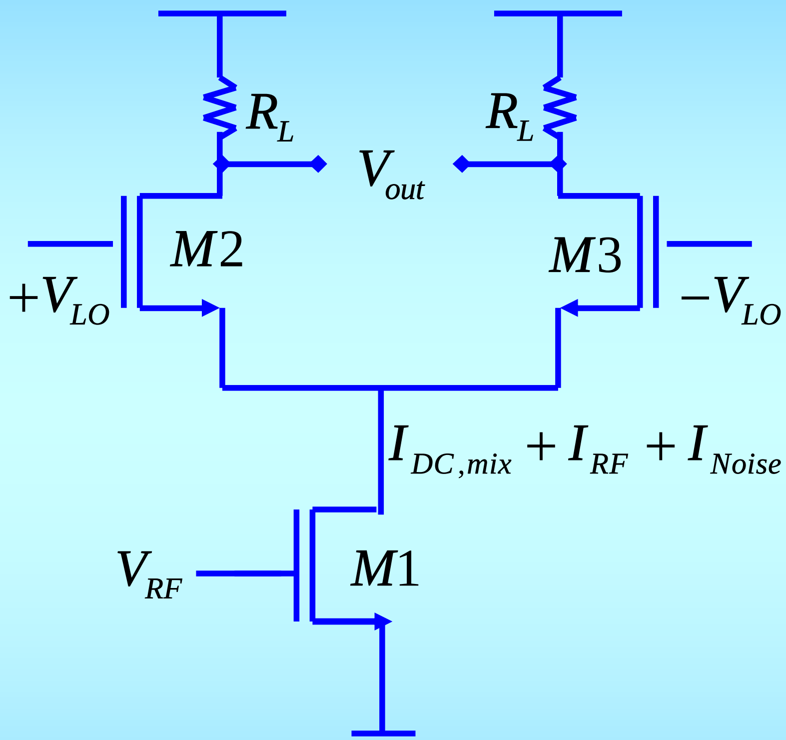


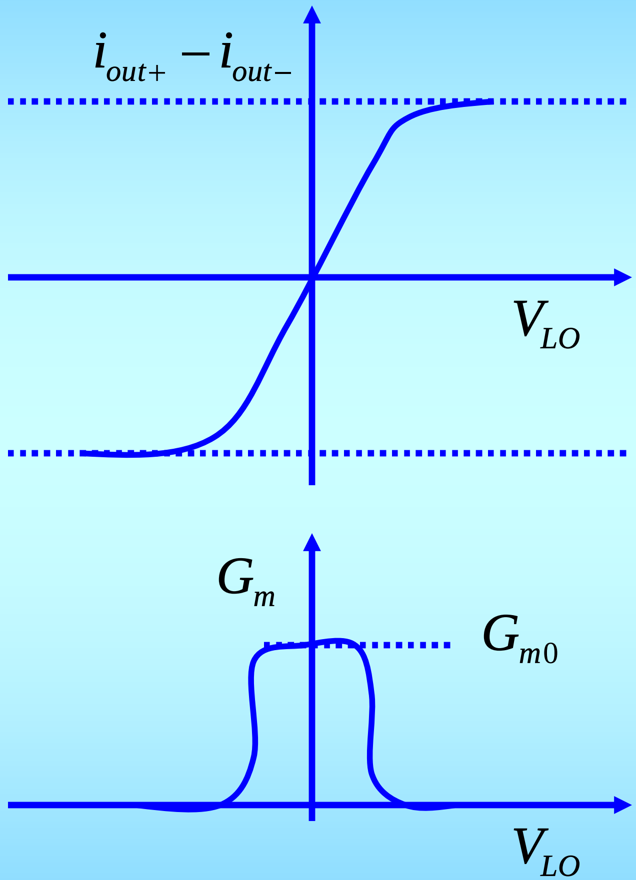
$$\overline{v_{gn}} = \sqrt{g \cdot \frac{4kT}{g_m}}$$



Mixer Noise Analysis: Switch Noise

- Noise Analysis of Heterodyne Mixer (switch noise):



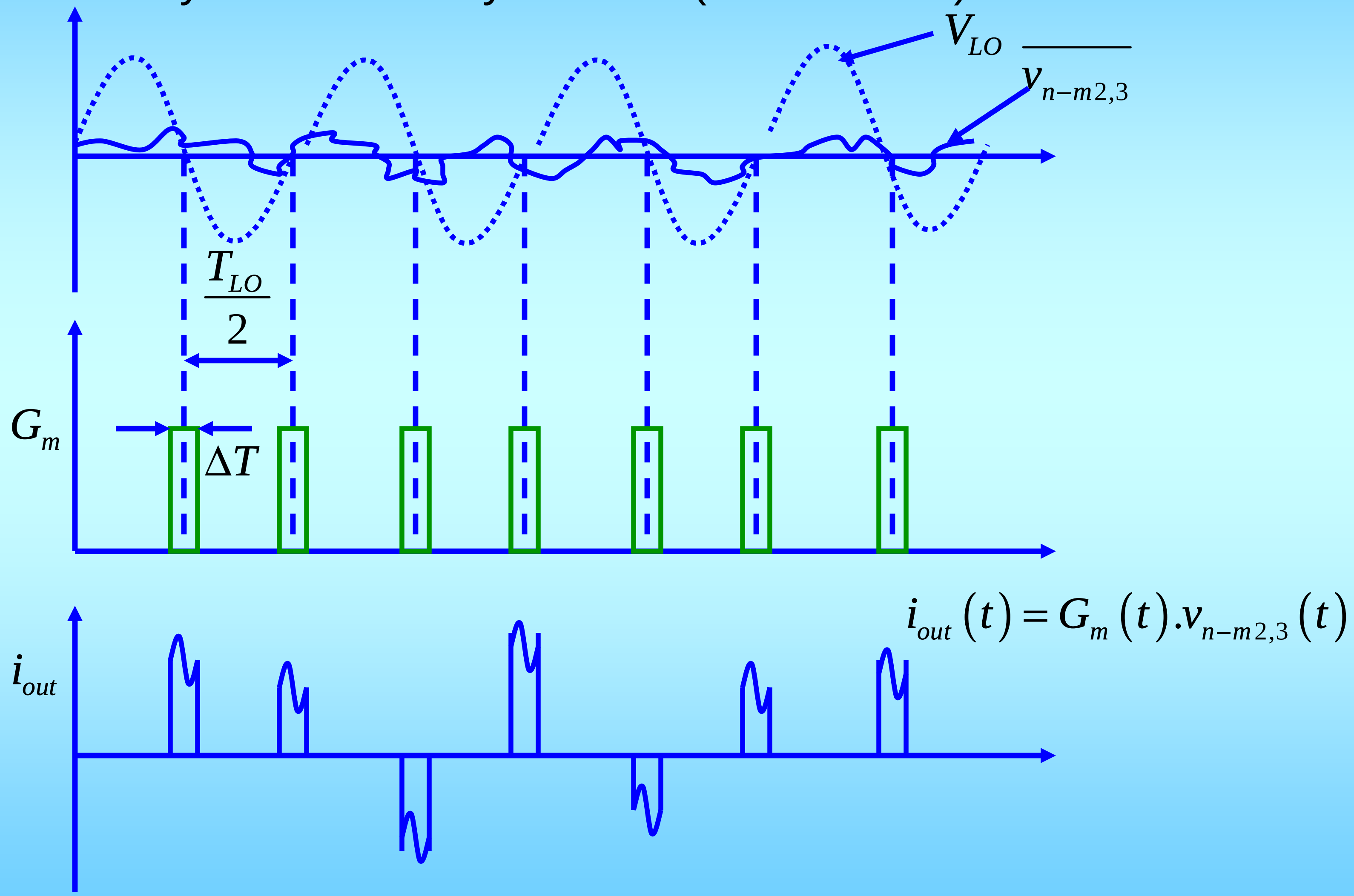


- Show that:

$$G_m = g_{m2} = g_{m3} = g_{m2,3} \approx \frac{2.I_{DC,mix}}{\Delta V}$$

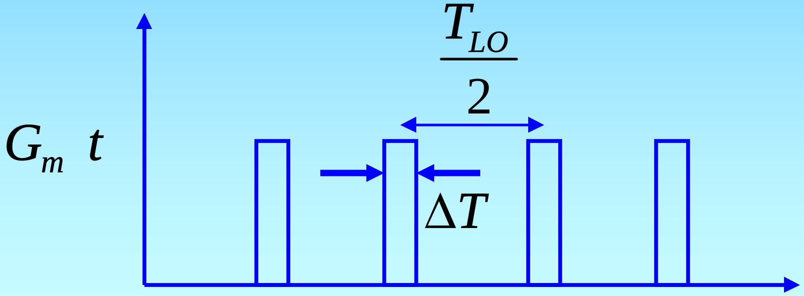
Mixer Noise Analysis: Switch Noise

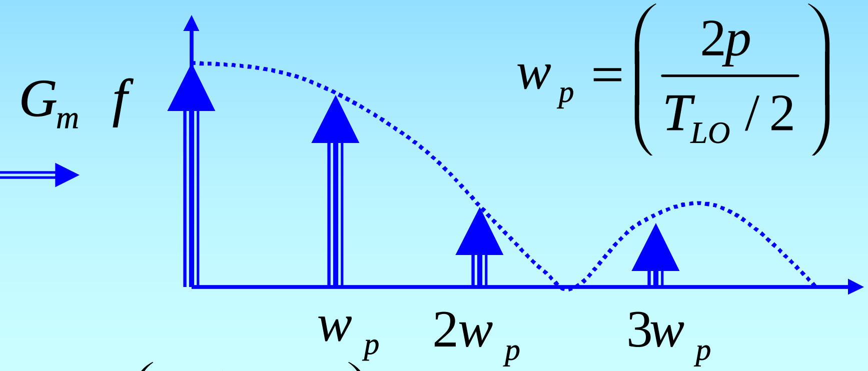
- Noise Analysis of Heterodyne Mixer (switch noise) cont...:



Mixer Noise Analysis: Switch Noise

- Noise Analysis of Heterodyne Mixer (switch noise) cont...:



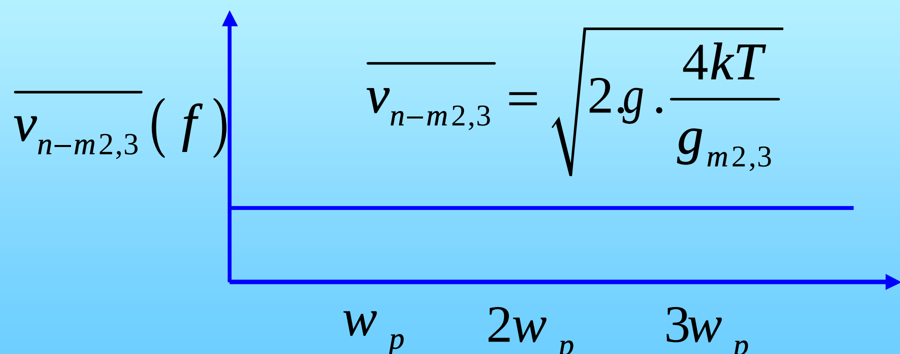


$$w_p = \left(\frac{2p}{T_{LO}/2}\right)$$

$$G_m(t) = G_{m0}.\left(\frac{\Delta T}{T_{LO}/2}\right) + \frac{1}{\left(\frac{T_{LO}}{2}\right)}\sum_{k=1}^{\infty} \Delta T.G_{m0}.\frac{Sin\left(k.\frac{\Delta T w_p}{2}\right)}{\left(k.\frac{\Delta T w_p}{2}\right)}.2Cos\left(k w_p t\right)$$

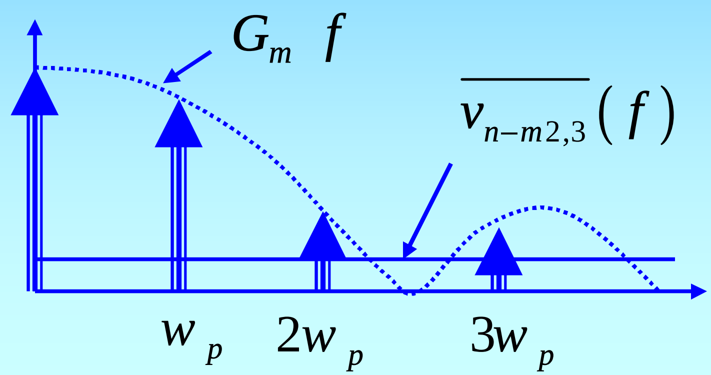
$$\overline{v_{n-m2,3}} = \sqrt{\overline{v^2_{n-m2}} + \overline{v^2_{n-m3}}}$$

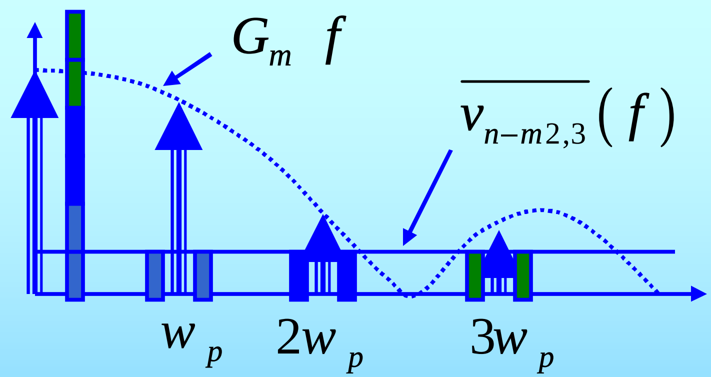
$$\overline{v_{n-m2,3}} = \sqrt{2.g\,.\frac{4kT}{g_{m2,3}}}$$



Mixer Noise Analysis: Switch Noise

- Noise Analysis of Heterodyne Mixer (switch noise) cont...:





$$\overline{i^2_{noise-M\,2,3}}(w_{IF}) = \frac{1}{\left(\frac{T_{LO}}{2}\right)} . G^2_{m0} . \Delta T . \overline{v^2_{n-m\,2,3}}$$

Mixer Noise Analysis: Switch Noise

- Noise Analysis of Heterodyne Mixer (switch noise) cont...:

$$\overline{i^2_{noise-M2,3}}(w_{IF}) = \frac{1}{\left(\frac{T_{LO}}{2}\right)}.G^2_{m0}.\Delta T.\overline{v^2_{n-m2,3}} \qquad G_m = g_{m2} = g_{m3} = g_{m2,3} \approx \frac{2.I_{DC,mix}}{\Delta V}$$

$$G_{m0} = \frac{2I_{DC,mix}}{\Delta V} \qquad \Delta V = Slope.\Delta T \qquad V_{LO}(t) = A_{LO}Cos(w_{LO}t)$$

$$Slope\ \Big]_{w_{LO}t=90} = \left[\frac{dV_{LO}(t)}{dt}\right]_{w_{LO}t=90} = A_{LO}w_{LO} \qquad \overline{v_{n-m2,3}} = \sqrt{2.g.\frac{4kT}{g_{m2,3}}}$$

$$\overline{i^2_{noise-M2,3}}(w_{IF}) = \frac{1}{T_{LO}/2}.G^2_{m0}.\Delta T.\overline{v^2_{n-m2,3}} = \frac{1}{T_{LO}/2}.G^2_{m0}.\Delta T.\left(2.g.\frac{4kT}{g_{m2,3}}\right)$$

$$= \frac{1}{T_{LO}/2}.G_{m0}.\Delta T.(2.g.4kT) = \frac{1}{T_{LO}/2}.\frac{2.I_{DC,mix}}{\Delta V}.\Delta T.(2.g.4kT)$$

$$= \frac{2I_{DC,mix}}{T_{LO}/2}.(2.g.4kT).\frac{\Delta T}{\Delta V} = \frac{2I_{DC,mix}}{T_{LO}/2}.(2.g.4kT).\frac{1}{A_{LO}w_{LO}}$$

$$= 4.g\,4kT\left(\frac{I_{DC,mix}}{p\,A_{LO}}\right)$$

⟸ **Total Noise Contribution due to switches M2 and M3**

Mixer Noise Analysis: Total Noise

- Noise Analysis of Heterodyne Mixer (total noise):

$$\overline{v_{noise-RL}^{2}} = 4kT\left(2R_L\right) \qquad g_{m-short} = \frac{dI_{DS-short}}{dV_{GS}} = \frac{1}{2}WC_{ox}v_{sat} = \frac{I_{DS-short}}{\left(V_{GSQ} - V_{T0}\right)}$$

$$\overline{i_{noise-M1}^{2}}\left(w_{IF}\right) = 4.g\ 4kTg_{m1} = 4.g\ 4kT.\frac{I_{DC,mix}}{\left(V_{GSQ} - V_{T0}\right)}$$

$$\overline{i_{noise-M2,3}^{2}}\left(w_{IF}\right) = 4.g\ 4kT\left(\frac{I_{DC,mix}}{p\ A_{LO}}\right)$$

$$\overline{v_{noise-MIX}^{2}}\left(w_{IF}\right) = \overline{v_{noise-RL}^{2}} + R_L^2\,\overline{i_{noise-M1}^{2}} + R_L^2\,\overline{i_{noise-M2,3}^{2}}$$

$$\overline{v_{noise-MIX}^{2}}\left(w_{IF}\right) = 4kTR_L\left\{1 + 4.g\,.\frac{I_{DC,mix}}{\left(V_{GSQ} - V_{T0}\right)}.R_L + 4.g\,.\frac{I_{DC,mix}}{p\ A_{LO}}.R_L\right\}$$

Mixer Noise Analysis: Total Noise

- Noise Analysis of Heterodyne Mixer (total noise):

$$\overline{v^2_{noise-MIX}}\left(w_{IF}\right) = 4kTR_L \left\{ 1 + 4.g.\frac{I_{DC,mix}}{\left(V_{GSQ} - V_{T0}\right)}.R_L + 4.g.\frac{I_{DC,mix}}{p\,A_{LO}}.R_L \right\}$$

$(V_{GSQ}$-$V_{T0})$ ↑ è M1 linearity ↑ and noise ↓

A_{LO} ↑ è noise contribution from M2/M3 ↓