Modulation and Demodulation

**Analog modulation**
- AM
- PM/FM

**Digital modulation**
- Binary Modulation
- Quadrature Modulation

**Power Efficiency**

**Noncoherent Detection**
Modulation

- Important fact: a continuous, oscillating signal will propagate farther than other signals.
- Start with a carrier signal
  - usually a sine wave that oscillates continuously.
  - Frequency of carrier fixed
Carrier Signal

• In analog transmission, the sending device produces a high-frequency signal that acts as a basis for the information signal.
  – This base signal is called the carrier signal or carrier frequency

• The receiving device is tuned to the frequency of the carrier that it expects from the sender.
Modulation

• Signal information is modulated on the carrier signal by modifying one of its characteristics (amplitude, frequency, phase).
• This modification is called modulation
• Same idea as in radio, TV transmission
• The information signal is called a modulating signal.
Modulation

• Modulation: process of changing a carrier wave to encode information.
• Modulation used with all types of media
• Why is modulation needed?
  – Allows data to be sent at a frequency which is available
  – Allows a strong carrier signal to carry a weak data signal
  – Reduces effects of noise and interference
Types of modulation

• Amplitude modulation (used in AM radio) – strength, or amplitude of carrier is modulated to encode data

• Frequency modulation (used in FM radio) – frequency of carrier is modulated to encode data

• Phase shift modulation (used for data) – changes in timing, or phase shifts encode data
Analog modulation

Analog Modulation Highlights

Carrier

Modulating Signal (Baseband)

AM (Amplitude Modulation)

FM (Frequency Modulation)
Amplitude Modulation

Modulating signal (audio)

Carrier frequency

AM signal
Amplitude modulation

DSB-SC AM

Signal: $m(t)$
Modulated signal: $A(t) = A_c m(t)$, 1-1 correspondence to $m(t)$
Carrier: $\cos \omega_c t$

General form: $x_c(t) = \frac{1}{2} A(t) \cos \omega_c t$

Carrier fixed

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Doubled-Sideband Modulation (Suppressed Carrier): DSB-SC

General form: \( x_c(t) = A(t) \times \cos \omega_c t \)

\[
X_c(f) = \frac{1}{2} A_c M_4(f + f_c) + \frac{1}{2} A_c M_4(f - f_c), \quad f_c = \frac{\omega_c}{2\pi}
\]

1 4 4 4 4 4 4 4 4 4 4 4 1 2 4 4 4 4 4 4 4 4 4 4 3
translation of \( M(f) \)

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Coherent (Synchronous) Demodulator (Detector):

*The receiver knows exactly the phase and frequency of the received signal.*

\[ x_c(t) = A_c m(t) \cos \omega_c t \]

\[ d(t) = [A_c m(t) \cos \omega_c t] 2 \cos \omega_c t \]

\[ = A_c m(t) \text{Message we want!} + 2A_c m(t) \cos \omega_c t \]

\[ y_1(t) = A_c m(t) \]
\[ X(t) \]

\[ d(t) \]

\[ Xc(f) \]

\[ D(f) \]
Power Analysis

\[ <x_c^2(t)> = <[A+m(t)]^2 (A_c)^2 \cos^2 \omega_c t> \]

Assume \( m(t) \) varies slowly w.r.t. \( \cos 2\omega_c t \)

\[ = <(1/2)(A_c)^2 [A + m(t)]^2 > + <(A_c)^2 [A+m(t)]^2 \cos 2\omega_c t> \]

Since \( <\cos X> = 0 \):

\[ = (1/2)(A_c)^2 < [A^2 + 2A<m(t) > + <m^2(t)>] \]

Assume \( <m(t)> = 0 \):

\[ = (1/2)(A_c)^2 [ A^2 + <m(t) >^2 ] \]

Carrier power \quad dc bias power \quad power of m(t) \quad signal power
Efficiency $E$: the percentage of total power that conveys information.

\[
E = \frac{\text{Signal Power}}{\text{Total Power}}
\]

\[
E_{\text{AM}} = \frac{[1/2(\text{Ac}^2)] < m^2(t)>}{[1/2(\text{Ac}^2)][A^2 + < m^2(t)>]} (100\%)
\]

\[
= \frac{a^2 < m_n^2(t)>}{1 + a^2 < m_n^2(t)>} (100\%)
\]

\[
= \frac{< m^2(t)>}{A^2 + < m^2(t)>} (100\%)
\]
Frequency Modulation

• Frequency of the carrier signal is modulated to follow the changing voltage level (amplitude) of the modulating signal.

• Peak amplitude and phase of the carrier signal remain constant, but as the amplitude of the info signal changes, the frequency of the carrier changes accordingly.
Frequency Modulation

Modulating signal (audio)

Carrier frequency

FM signal
Phase-Shift Modulation

- vary phase of carrier
- may use more than simply 180 degree shift (binary)
- this allows higher bit rate than baud rate
  - Eight angles results in 3 bits per signal element. Or 3 bits per baud!
Angle Modulation: FM and PM

General form: \( x_c(t) = A_c \cos[\omega_c t + \phi(t)] \)

Phase modulation: \( \phi(t) = K_p m(t), \quad K_p: \text{deviation constant} \)

Frequency modulation: \( \frac{d\phi(t)}{dt} = K_f m(t) \)

\( K_f: \text{deviation constant}; \)

\( \Phi(t) = K_f \int_{t_0}^{t} m(\alpha) d\alpha + \Phi_0 \)

\( = 2\pi f_d \int_{t_0}^{t} m(\alpha) d\alpha + \Phi_0 \)

\[
\begin{cases}
\text{PM: } x_c(t) = A_c \cos[\omega_c t + K_p m(t)] \\
\text{FM: } x_c(t) = A_c \cos[\omega_c t + 2\pi f_d \int_{t_0}^{t} m(\alpha) d\alpha]
\end{cases}
\]
FM/PM waveforms

\[ f_c = \frac{1}{2\pi \sqrt{LC}} \]

\( x_{BB} \) controls the value of C
Indirect implementation (Armstrong)
using a mixer and summer

\[ x_c(t) = A_c (\cos \omega_c t - \phi(t) \sin \omega_c t) \]

If \( \phi(t) \) very small:
\[ x_c(t) = A_c \cos(\omega_c t + \phi(t)) \]
Direct method using PLL

\[ V_c(t) \]

\[ x_c(t) \]

\[ \sum \]

LPF

VCO

Frequency Divider \( \div N \)

Crystal oscillator

\[ m(t) \]

\[ \sum \]

\[ V_c(t) \]

\[ x_c(t) \]

Vc controls frequency, so is m(t)
FM/PM spectrum

Ideally

Reality
Theorem

- If $|\Phi(t)| << 1$, 

\[ X_c(f) = \frac{A_c}{2} \left\{ [\delta(f - f_c) + \delta(f + f_c)] + j[\Phi(f - f_c) - \Phi(f + f_c)] \right\} \]

- Or if 

\[ \frac{K_f \max[m(t)]}{2\pi B} > 1 \quad \text{where } B \text{ is the bandwidth of } m \]

\[ X_c(f) = \frac{\pi A_c^2}{2K_f} \left[ M\left(\frac{2\pi}{K_f} (f - f_c)\right) + M\left(\frac{2\pi}{K_f} (-f - f_c)\right) \right] \]
Digital Modulation

**FIGURE 7.13** Waveforms for ASK, PSK, and FSK modulation

Digital sequence: 1 0 1 1 0

Antipodal baseband signal:

ASK:

PSK:

FSK:

Phase difference = $2\cos^{-1}m$
Design Parameters

• **Power efficiency**: describes the ability of a modulation technique to preserve the fidelity of the digital message at low power levels.
  
  \[ \eta_P : \frac{E_b}{N_0} \]

• **Bandwidth efficiency**: describes the ability of a modulation scheme to accommodate data within a limited bandwidth.
  
  \[ \eta_B : \frac{R}{B} \text{ bps/Hz} \]
• Channel capacity formula
  
  - $R_{\text{max}} \leq C = \log_2(1+S/N)$
  
  - $\eta_{\text{Bmax}} = C/B = \log_2(1+S/N)$
    - C is the channel capacity (in bps)
    - B is the RF bandwidth
    - S/N is the signal-to-noise ratio
Bandwidth

• Absolute bandwidth: The range of frequencies over which the signal has a ‘non-zero’ power spectral density.

• Null-to-null bandwidth: equal to the width of main spectral lobe.

• Half-power (3-dB) bandwidth: the interval between frequencies at which the PSD has dropped to half power, or 3dB below the peak value.

• 99 percent bandwidth (by Federal Communication Commission): occupied 99 percent of signal power.
General Digital Modulation

- **Geometric** Representation of Modulation
  - set of modulation signal:
    \[ S = \{s_1(t), s_2(t), \ldots, s_M(t)\} \]
  - vector representation with orthogonal basis functions:
    \[ s_i(t) = \sum_{j=1}^{N} s_{ij} \Phi_j(t) \]
    \[ \int_{-\infty}^{\infty} \Phi_i(t)\Phi_j(t) dt = 0 \quad i \neq j \]
    \[ E = \int_{-\infty}^{\infty} \Phi_i^2(t) dt = 1 \]
Geometric Representation of Modulation

- **BPSK:**

\[ s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad 0 \leq t \leq T_b \]

\[ s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad 0 \leq t \leq T_b \]

\[ \Phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \quad 0 \leq t \leq T_b \]

\[ S_{BPSK} = \left\{ \sqrt{E_b} \Phi_1(t), -\sqrt{E_b} \Phi_1(t) \right\} \]
Geometric Representation of Modulation

- Bit error analysis: in AWGN channel with a noise spectral density No.

\[ P_s(\varepsilon \mid s_i) \leq \sum_{j=1, j \neq i} Q \left( \frac{d_{ij}}{\sqrt{2N_0}} \right) \]

\( d_{ij} \): distance between the ith and jth signal point in the constellation

\[ Q(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx \]

\[ P_s(\varepsilon) = P_s(\varepsilon \mid s_i) P(s_i) = \frac{1}{M} \sum_{i=1}^{M} P_s(\varepsilon \mid s_i) \]
Linear Modulation Techniques

• Transmitted signal amp. is proportional to modulated signal
• Good spectral efficiency
• Bad power efficiency since linear AMP is needed.
• E.g. QPSK, OQPSK, π/4QPSK...

\[
s(t) = \text{Re}[Am(t) \exp(j2\pi f_c t)] \\
= A[m_R(t) \cos(2\pi f_c t) - m_I(t) \sin(2\pi f_c t)]
\]

Figure 3.13 Four-level digital representation of a binary data stream.
Linear Modulation Techniques

- BPSK Transmitter

\[ S_{BPSK}(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta_c) \quad (\text{bit 1}) \]

\[ S_{BPSK}(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta_c) \quad (\text{bit 0}) \]

\[ P_{BPSK} = P_p(f + f_c) + P_p(f - f_c), \quad P_p(f) = A_c^2 T_b \left( \frac{\sin \pi f T_b}{\pi f T_b} \right)^2 \]
Demodulation Techniques

\[ X_{\text{BPSK}}(t) \xrightarrow{\times} 2\cos(2\pi f_c t + \theta_c) \xrightarrow{} \text{Notch filter} \xrightarrow{2f_c} X_{\text{in}} \]
Linear Modulation Techniques

- Waveform of BPSK after adding noise
Detection with Matched Filter:

Filter impulse response is a square pulse, matching the pulse shapes in $x_{in}$.
\[
y(t) = p(t) \ast h(t) = \int_{-\infty}^{+\infty} p(t - \tau) h(\tau) d\tau,
\]
\[
E_p = \int_{-\infty}^{+\infty} |p(t)|^2 dt
\]
\[
n_o(t) = p(t) \ast n(t)\]
\[
\overline{n_o^2 * t} = \int_{-\infty}^{+\infty} |H(f)|^2 N(f) df\]
\[
\overline{n_o^2} = N_o / 2\]
\[
SNR_{\text{max}} = \frac{2E_p}{N_0}
\]
Optimum Detection:

\[ y(T_b) = \int_{-\infty}^{+\infty} x(\tau) h(T_b - \tau) \, d\tau \]

\[ = \int_{-\infty}^{+\infty} x(\tau) p(\tau) \, d\tau , \]

\[ y(T_b) = \int_{\tau=0}^{\tau=T_b} x(\tau) p(\tau) \, d\tau \]
Demodulation

• BPSK Coherent Receiver:

\[ s_{\text{BPSK}}(t) = m(t) \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta_c + \theta_{ch}) \]

\[ = m(t) \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta) \]

Using Carrier Recovery to get carrier frequency:

\[ 2m(t) \sqrt{\frac{2E_b}{T_b}} \cos^2(2\pi f_c t + \theta) = m(t) \sqrt{\frac{2E_b}{T_b}} [1 + 1 \cos 2(2\pi f_c t + \theta_c)] \]

Bit Error Rate: \( P_{e,\text{BPSK}} = Q \left( \sqrt{\frac{2E_b}{N_0}} \right) \)
• BPSK Coherent Receiver Architecture

Received signal → AGC → Symbol/clock recovery → Carrier recovery

\[ \cos(2\pi f_c t + \phi) \]

Matched filter → sampler → Detect decision

Signal Pulse generator
Linear Modulation Techniques

- From BPSK to QPSK

Constellation diagram
Linear Modulation Techniques

- Constellation diagram after adding noise
QPSK transmitter

\[ s_{QPSK}(t) = \sqrt{\frac{2E_b}{T_s}} \cos[2\pi f_c t + (i - 1) \frac{\pi}{2}] \]

\[ 0 \leq t \leq T_s \quad i = 1,2,3,4 \]

\[ s_{QPSK}(t) = \sqrt{\frac{2E_b}{T_s}} \cos[(i - 1) \frac{\pi}{2}] \cos(2\pi f_c t) \]

\[ - \sqrt{\frac{2E_b}{T_s}} \sin[(i - 1) \frac{\pi}{2}] \sin(2\pi f_c t) \]

\[ s_{QPSK}(t) = \left\{ \sqrt{E_b} \cos[(i - 1) \frac{\pi}{2}]\phi_1(t) - \sqrt{E_b} \sin[(i - 1) \frac{\pi}{2}]\phi_2(t) \right\} \]

\[ P_{BPSK} = P_p(f + f_c) + P_p(f - f_c), \quad P_p(f) = 2CT_b \left( \frac{\sin \frac{2\pi f T_b}{2\pi f T_b}}{2\pi f T_b} \right)^2 \]

C is average magnitude squared
QPSK transmitter

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QPSK Receiver

Bit Error Rate: \[ P_{e,QPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \]
BER & BW efficiency comparison

• BPSK vs QPSK
  – BER is the same !!!

\[
P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)
\]

But QPSK only need \(\frac{1}{2}\) the bandwidth of BPSK !!!

BPSK: symbol rate = bit rate
QPSK: symbol rate = \(\frac{1}{2}\) bit rate
Reducing the maximum phase jump

• Offset QPSK
  – To overcome 180° phase shift in QPSK thus prevent the overhead to overcome signal envelop distortion caused in QPSK
Reducing the maximum phase jump

- Offset QPSK Tx
Reducing the maximum phase jump

- Offset QPSK Rx
Further reduction of phase jump

- \( \pi/4\)-QPSK
  - Alternate between the 2 QPSK constellations
  - If the current two bits corresponds to a point in the left constellation
  - Next two bits will be represented by a point on the right constellation
  - Vice versa

- Also called \( \pi/4\)-DQPSK
Concerns over linear modulation

• On each of the I/Q axis, we are performing AM modulation

• Transmitted signal amplitude changes with time
  – Cause receiver challenges
  – Transmitter power utilization
  – Sensitive to additive noise

Ę use nonlinear modulation
Constant Envelope Modulation

- Class C Amp. can be used
  - saving power
- Limiter-discriminator detection can be used
  - easy and simple architecture
- Good performance against random noise and signal fluctuation due to Rayleigh fading
  - good performance
- But! BW is larger than linear modulation
Binary frequency shift keying

- BFSK
  - General Form
  
  \[ s_{FSK}(t) = v_H(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c + 2\pi \Delta f)t \quad 0 \leq t \leq T_b \text{ (bit 1)} \]

  \[ s_{FSK}(t) = v_L(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c - 2\pi \Delta f)t \quad 0 \leq t \leq T_b \text{ (bit 0)} \]

  - Discontinuous phase FSK

  \[ s_{FSK}(t) = v_H(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_{H,1}t + \theta_1) \quad 0 \leq t \leq T_b \text{ (bit 1)} \]

  \[ s_{FSK}(t) = v_L(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_{L,1}t - \theta_1) \quad 0 \leq t \leq T_b \text{ (bit 0)} \]
Discontinuous phase FSK

- Discontinuous phase FSK
  - can be combined by two OOK
  - cause spectral spreading and spurious
Continuous phase FSK

- similar to FM except that m(t) is binary

\[ s_{FSK}(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta(t)) \]

\[ = \sqrt{\frac{2E_b}{T_b}} \cos[2\pi f_c t + 2\pi f_d \int_{-\infty}^{t} m(\eta) d\eta] \]

\[ = \sqrt{\frac{2E_b}{T_b}} \left[ \cos 2\pi f_c t \cdot \cos \theta(t) - \sin 2\pi f_c t \cdot \sin \theta(t) \right] \]
Binary frequency shift keying

- Coherent detection of BFSK

\[ P_{e, \text{FSK}} = Q \left( \sqrt{\frac{E_b}{N_0}} \right) \]
Binary frequency shift keying

- Non coherent detection of BFSK

\[ P_{e, \text{FSK, NC}} = \frac{1}{2} \exp \left( - \frac{E_b}{2 N_0} \right) \]
Modulation Index

- Modulation Index of FSK:

\[ h = \frac{(2\Delta F)}{R_b}, \]

where \( \Delta F \) is the peak RF frequency deviation and \( R_b \) is the bits rate

Example:

\[ \Delta F = f_H - f_c = \frac{1}{4} R_b \]

\[ 2 \times \frac{1}{4} R_b \]

\[ \Rightarrow h = \frac{2 \times \frac{1}{4} R_b}{R_b} = 0.5 \]
CPFSK and modulation index

\[ h = \frac{1}{4} \]

\[ h = \frac{1}{3} \]

\[ h = \frac{1}{2} \]

\[ h = \frac{2}{3} \]
Minimum shift keying

- a special type of Continuous phase FSK
- modulation index \( h = 0.5 \)
- peak RF frequency deviation=\( R_b/4 \)
- coherently orthogonal. i.e. \( \int_{0}^{T_b} H(t) L(t) dt = 0 \)
- MSK=fast FSK
- MSK=OQPSK with baseband rectangular being replaced with half-sinusoidal
- MSK = FSK with binary signaling freq. of \( f_c \pm 1/4T_b \)
Minimum shift keying

• Advantage of MSK: particularly attractive for use in mobile radio communication systems:
  – constant envelope
  – spectral efficiency ?
  – good BER
  – self-synchronizing capability
Minimum shift keying

- **MSK as OQPSK:**
  
  \[
  s_{\text{MSK}}(t) = m_I(t) \cos\left(\frac{\pi t}{2T_b}\right) \cos(2\pi f_c t) + m_Q(t) \sin\left(\frac{\pi t}{2T_b}\right) \sin(2\pi f_c t)
  \]

- **MSK as CPFSK:**
  
  \[
  s_{\text{MSK}}(t) = \cos\left[2\pi f_c t - m_I(t)m_Q(t) \frac{\pi t}{2T_b} + \phi_k\right]
  \]

- **MSK power spectrum**

\[
P_{\text{MSK}} = P_p(f + f_c) + P_p(f - f_c), \quad P_p(f) = \frac{16 A_c^2}{\pi^2} \left(\frac{\cos 2\pi f T_b}{1 - 16 f^2 T_b^2}\right)
\]
Minimum shift keying

- MSK Transceiver

\[
x_{\text{MSK}}(t) = \sqrt{2}A_c \cos \left[ \omega_c t + \int_{-\infty}^{t} \sum_{m} b_m p(t - mT_b) dt \right]
\]
Minimum shift keying

I

Q

MSK

1/31/2005
Comparison of MSK, OQPSK, QPSK

90° phase shift in each symbol
MSK

• Receiver of MSK

Figure 5.40
Block diagram of an MSK receiver.
Why is MSK more spectrally efficient?

- Power spectral density of MSK
  - 99% BW of MSK = $1.2/T_b$
  - 99% BW of QPSK or OQPSK = $8/T_b$

Figure 5.38
Power spectral density of MSK signals as compared to QPSK and OQPSK signals.
Even better spectral efficiency? Use Gaussian MSK

\[ H(f) = \exp\left(-\frac{f^2 \ln 2}{B^2 \cdot 2}\right) \]

**Figure 3.44** GMSK modulation.
• GMSK & GFSK
  – GMSK reduces the sidelobe levels of MSK
  – GMSK = Gaussian filter + MSK
  – MSK = GMSK with $B = \infty$
  – Important parameter: $3\text{dB-BW} – \text{bit duration product}$
    $(B_{3\text{dB}}T_b)$
Transmitter of GMSK:
QUAD architecture

\[ s(t) = \cos(2\pi f_c t + \phi(t)) \]

\[ = \cos 2\pi f_c t \cdot \cos \phi(t) - \sin 2\pi f_c t \cdot \sin \phi(t) \]
Deciding frequency modulation index \( K_f \)

\[
s_{\text{FSK}}(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \phi(t))
\]

\[
= \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + K_f \int_{-\infty}^{t} m(\eta) \, d\eta)
\]

\[
\phi(t) = K_f \int_{-\infty}^{t} m(\eta) \, d\eta = K_f \sum_{n=-\infty}^{\infty} a_n r(\eta - nT) \, d\eta
\]

\[
\Rightarrow K_f \int_{-\infty}^{\infty} r(t) \, dt = \pi / 2
\]

\[
\Rightarrow K_f \int_{-\infty}^{\infty} \Pi(t) * h_G(t) \, dt = \pi / 2
\]

\[
\therefore K_f = \frac{\pi / 2}{\int_{-\infty}^{\infty} \Pi(t) * h_G(t) \, dt}
\]
• Power spectral density of a GMSK signal
• Receiver of GMSK

\[ P_e = Q \left( \sqrt{\frac{2\alpha E_b}{N_0}} \right) \]

\[ \alpha \equiv \begin{cases} 0.68 & \text{for GMSK with } BT = 0.25 \\ 0.85 & \text{for simple MSK (} BT = \infty \text{)} \end{cases} \]
- Occupied RF Bandwidth

Table 5.3 Occupied RF Bandwidth (for GMSK and MSK as a fraction of $R_b$) Containing a Given Percentage of Power [Mur81]. Notice that GMSK is spectrally tighter than MSK.

<table>
<thead>
<tr>
<th>BT</th>
<th>90%</th>
<th>99%</th>
<th>99.9%</th>
<th>99.99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 GMSK</td>
<td>0.52</td>
<td>0.79</td>
<td>0.99</td>
<td>1.22</td>
</tr>
<tr>
<td>0.25 GMSK</td>
<td>0.57</td>
<td>0.86</td>
<td>1.09</td>
<td>1.37</td>
</tr>
<tr>
<td>0.5 GMSK</td>
<td>0.69</td>
<td>1.04</td>
<td>1.33</td>
<td>2.08</td>
</tr>
<tr>
<td>MSK</td>
<td>0.78</td>
<td>1.20</td>
<td>2.76</td>
<td>6.00</td>
</tr>
</tbody>
</table>
Simulated power spectral density

• GSM GMSK v.s MSK
Simulated power spectral density

- Bluetooth v.s GSM

![Graph showing simulated power spectral density for Bluetooth and GSM with m values and BT values.](image-url)
M-ary PSK

• Two or more bits are grouped to form symbols and one of the possible M symbols may be sent

\[ S_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos \left( 2\pi f_c t + \frac{2\pi}{M} (i - 1) \right), \quad 0 \leq t \leq T_s \quad i = 1, 2, K, M \]

\[ = \sqrt{\frac{2E_s}{T_s}} \cos \left( (i - 1) \frac{2\pi}{M} \right) \cos(2\pi f_c t) - \sqrt{\frac{2E_s}{T_s}} \sin \left( (i - 1) \frac{2\pi}{M} \right) \sin(2\pi f_c t) \]

\[ i = 1, 2, K, M \]

\[ S_{M-PSK}(t) = \left\{ \sqrt{E_s} \cos \left( (i - 1) \frac{2\pi}{M} \right) \phi_1(t) - \sqrt{E_s} \sin \left( (i - 1) \frac{2\pi}{M} \right) \phi_2(t) \right\} \]

\[ i = 1, 2, K, M \]
M-ary PSK

- Constellation diagram of 8PSK
Quadrature amplitude modulation

- $E_{\text{min}}$: energy of the signal with the lowest amplitude
- $a_i, b_i$, index of the signal point

\[
S_i(t) = \sqrt{\frac{2E_{\text{min}}}{T_s}} a_i \cos(2\pi f_c t) - \sqrt{\frac{2E_{\text{min}}}{T_s}} b_i \sin(2\pi f_c t)
\]

$0 \leq t \leq T \quad i = 1, 2, K, M$

\[
\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t) \quad 0 \leq t \leq T_s
\]

\[
\phi_2(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t) \quad 0 \leq t \leq T_s
\]
Constellation diagram of 16QAM
Digital QAM Modulator

\( d[n] \) bit stream

\( s(t) \)

Matched delay matches delay through 90° phase shifter
Phase Shift by 90 Degrees

- 90° phase shift performed by Hilbert transformer

\[
\cos(2\pi f_0 t) \Rightarrow \frac{1}{2} \delta(f + f_0) + \frac{1}{2} \delta(f - f_0)
\]

\[
\text{cosine} \Rightarrow \text{sine}
\]

\[
\sin(2\pi f_0 t) \Rightarrow \frac{j}{2} \delta(f + f_0) - \frac{j}{2} \delta(f - f_0)
\]

\[
\text{sine} \Rightarrow -\cosine
\]

- Frequency response of ideal Hilbert transformer:

\[
H(f) = -j \text{sgn}(f)
\]

\[
\text{sgn}(x) = \begin{cases} 
1 & \text{if } x > 0 \\
0 & \text{if } x = 0 \\
-1 & \text{if } x < 0 
\end{cases}
\]
Hilbert Transformer

- Magnitude response
  All pass except at origin

\[ |H(f)| \]

- Phase response
  Piecewise constant

\[ \angle H(f) \]

- For \( f_c > 0 \)
  \[ \cos(2\pi f_c t + \frac{\pi}{2}) = \sin(2\pi f_c t) \]

- For \( f_c < 0 \)
  \[ \cos(2\pi f_c t - \frac{\pi}{2}) = \cos(-(2\pi f_c t + \frac{\pi}{2})) \]
  \[ = \cos(2\pi (-f_c) t + \frac{\pi}{2}) = \sin(2\pi (-f_c) t) \]

1/31/2005
Hilbert Transformer

- Continuous-time ideal Hilbert transformer
  
  \[ H(f) = -j \text{sgn}(f) \]
  
  \[ h(t) = \begin{cases} 
  \frac{1}{\pi t} & \text{if } t \neq 0 \\
  0 & \text{if } t = 0 \end{cases} \]

- Discrete-time ideal Hilbert transformer
  
  \[ H(\omega) = -j \text{sgn}(\omega) \]
  
  \[ h[n] = \begin{cases} 
  \frac{2 \sin^2(\pi n/2)}{\pi n} & \text{if } n \neq 0 \\
  0 & \text{if } n = 0 \end{cases} \]

Even-indexed samples are zero.
QAM Receiver

• Channel has linear distortion, additive noise, and nonlinear distortion

• Adaptive digital FIR filter used to equalize linear distortion (magnitude/phase distortion in channel)

  Channel equalizer coefficients adapted during startup

  At startup, transmitter sends known PN training sequence
Received signal $\rightarrow$ AGC $\rightarrow$ Carrier recovery $\rightarrow$ Symbol/clock recovery $\rightarrow$ Matched filter $\rightarrow$ sampler

$\cos(2\pi f_c t + \phi)$

$\rightarrow$ Signal Pulse generator $\rightarrow$ Detect decision

$\rightarrow$ -sin$(2\pi f_c t + \phi)$

$\rightarrow$ Matched filter $\rightarrow$ sampler

$90^\circ$ Phase shift
In-Phase/Quadrature Demodulation

- **QAM transmit signal**: \( x(t) = a(t) \cos(\omega_c t) + b(t) \sin(\omega_c t) \)
- **QAM demodulation by modulation then filtering**
  - Construct in-phase \( i(t) \) and quadrature \( q(t) \) signals
  - Lowpass filter them to obtain baseband signals \( a(t) \) and \( b(t) \)

\[
i(t) = 2x(t)\cos(\omega_c t) = 2a(t)\cos^2(\omega_c t) + 2b(t)\sin(\omega_c t)\cos(\omega_c t) \\
= a(t) + a(t)\cos(2\omega_c t) + b(t)\sin(2\omega_c t)
\]

**baseband** \quad **high frequency component centered at** \( 2\omega_c \)

\[
q(t) = 2x(t)\sin(\omega_c t) = 2a(t)\cos(\omega_c t)\sin(\omega_c t) + 2b(t)\sin^2(\omega_c t) \\
= b(t) + a(t)\sin(2\omega_c t) - b(t)\cos(2\omega_c t)
\]

**baseband** \quad **high frequency component centered at** \( 2\omega_c \)
Performance Analysis of QAM

• Received QAM signal

\[ x(nT) = s(nT) + v(nT) \]

• Information signal \( s(nT) \)

\[ s(nT) = a_n + j b_n = (2i - 1)d + j (2k - 1)d \]

where \( i, k \in \{ -1, 0, 1, 2 \} \) for 16-QAM

• Noise, \( v_I(nT) \) and \( v_Q(nT) \) are independent Gaussian random variables \( \sim N(0; \sigma^2/T) \)

\[ v(nT) = v_I(nT) + j v_Q(nT) \]
Performance Analysis of QAM

- Type 1 correct detection

\[ P_1(c) = P(|v_I(nT)| < d \& |v_Q(nT)| < d) \]

\[ = P(|v_I(nT)| < d)P(|v_Q(nT)| < d) \]

\[ = (1 - P(|v_I(nT)| > d))(1 - P(|v_Q(nT)| > d)) \]

\[ = (1 - 2Q\left(\frac{d}{\sigma}\sqrt{T}\right))^2 \]
Performance Analysis of QAM

- Type 2 correct detection

\[ P_2(c) = P(v_I(nT) < d \& |v_Q(nT)| < d) \]
\[ = P(v_I(nT) < d)P(|v_Q(nT)| < d) \]
\[ = (1 - 2Q\left(\frac{d}{\sigma}\sqrt{T}\right))(1 - Q\left(\frac{d}{\sigma}\sqrt{T}\right)) \]

- Type 3 correct detection

\[ P_3(c) = P(v_I(nT) < d \& v_Q(nT) > -d) \]
\[ = P(v_I(nT) < d)P(v_Q(nT) > -d) \]
\[ = (1 - Q\left(\frac{d}{\sigma}\sqrt{T}\right))^2 \]
Performance Analysis of QAM

• Probability of correct detection

\[
P(c) = \frac{4}{16} (1 - 2Q\left(\frac{d}{\sigma} \sqrt{T}\right))^2 + \frac{4}{16} \left(1 - Q\left(\frac{d}{\sigma} \sqrt{T}\right)\right)^2
\]

\[
+ \frac{8}{16} (1 - 2Q\left(\frac{d}{\sigma} \sqrt{T}\right))(1 - Q\left(\frac{d}{\sigma} \sqrt{T}\right))
\]

\[
= 1 - 3Q\left(\frac{d}{\sigma} \sqrt{T}\right) + \frac{9}{4} Q^2 \left(\frac{d}{\sigma} \sqrt{T}\right)
\]

• Symbol error probability

\[
P(e) = 1 - P(c) = 3Q\left(\frac{d}{\sigma} \sqrt{T}\right) - \frac{9}{4} Q^2 \left(\frac{d}{\sigma} \sqrt{T}\right)
\]
Average Power Analysis

- PAM and QAM signals are deterministic
- For a deterministic signal $p(t)$, instantaneous power is $|p(t)|^2$
- 4-PAM constellation points: \{-3\,d, -d, \,d, \,3\,d\}
  - Total power $9\,d^2 + \,d^2 + \,d^2 + 9\,d^2 = 20\,d^2$
  - Average power per symbol $5\,d^2$
- 4-QAM constellation points: \{d + j\,d, -d + j\,d, d − j\,d, -d − j\,d\}
  - Total power $2\,d^2 + 2\,d^2 + 2\,d^2 + 2\,d^2 = 8\,d^2$
  - Average power per symbol $2\,d^2$
Summary of QAM.

\[
\begin{align*}
\{a_i, b_i\} &= \begin{bmatrix}
(-L + 1, L - 1) & (-L + 3, L - 1) & L & (L - 1, L - 1) \\
(-L + 1, L - 3) & (-L + 3, L - 3) & L & (L - 1, L - 3) \\
. & . & . & . \\
(-L + 1, -L + 1) & (-L + 3, -L + 1) & L & (L - 1, -L + 1)
\end{bmatrix} \\
\{a_i, b_i\} &= \begin{bmatrix}
(-3, 3) & (-1, 3) & (1, 3) & (3, 3) \\
(-3, 1) & (-1, 1) & (1, 1) & (3, 1) \\
(-3, -1) & (-1, -1) & (1, -1) & (3, -1) \\
(-3, -3) & (-1, -3) & (1, -3) & (3, -3)
\end{bmatrix}
\end{align*}
\]

\[
P_e \approx 4 \left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{2E_{\min}}{N_0}}\right)
\]

\[
P_e \approx 4 \left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_{av}}{(M - 1)N_0}}\right)
\]
M-ary FSK

\[ S_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos \left( \frac{\pi}{T_s} (n_c + i)t \right) \quad 0 \leq t \leq T_s \quad i = 1, 2, K, M \]

– error probability under coherent detection

\[ P_e \leq (M - 1)Q \left( \sqrt{\frac{2E_b \log_2 M}{N_0}} \right) \]

– error probability under non-coherent detection

\[ P_e = \sum_{k=1}^{M-1} \left( \frac{(-1)^{k+1}}{k + 1} \right) \left( \begin{array}{c} M - 1 \\ k \end{array} \right) \exp \left( \frac{-kE_s}{(k + 1)N_0} \right) \]
M-ary FSK

- **M\_ary FSK**
  - **BW of coherent MFSK**:
    \[ B = \frac{R_b (M + 3)}{2 \log_2 M} \]
  - **BW of noncoherent MFSK**:
    \[ B = \frac{R_b M}{2 \log_2 M} \]