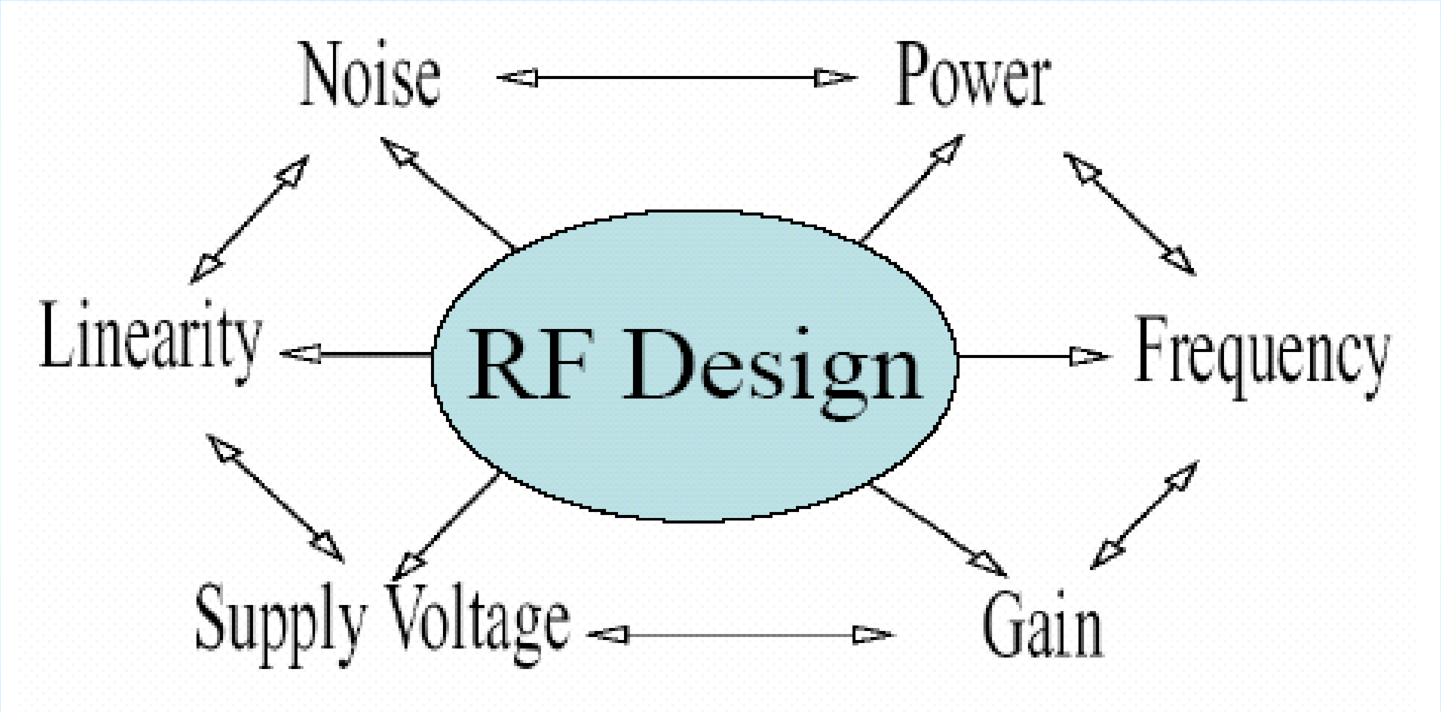
RF Circuit Design Hexagon



Linear Time-Invariant systems

- Linear

$$If \quad x_1(t) \to y_1(t), x_2(t) \to y_2(t),$$
$$then \qquad ax_1(t) + bx_2(t) \to ay_1(t) + by_2(t)$$

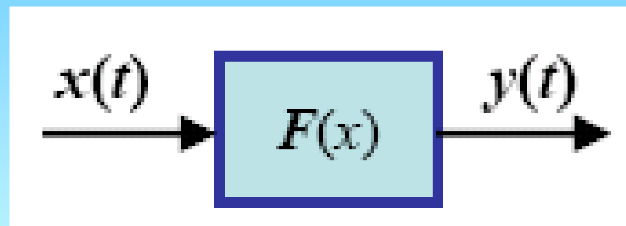
- Time-Invariant

$$If \quad x(t) \to y(t),$$
$$then \qquad x(t-\tau) \to y(t-\tau)$$

- Memoryless

$$y(t) = f(x(t)),$$
$$else \quad called \qquad "dynamic".$$

Nonlinearity Issues



Static model

$$y(t) = F\big(x(t)\big) \qquad \text{nonlinear}$$

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

3rd order polynomial approximation adequate for weak nonlinearities, eg. LNA

Linear part

Dynamic model

$$\dot{y}(t) = F\big(y(t), x(t)\big)$$

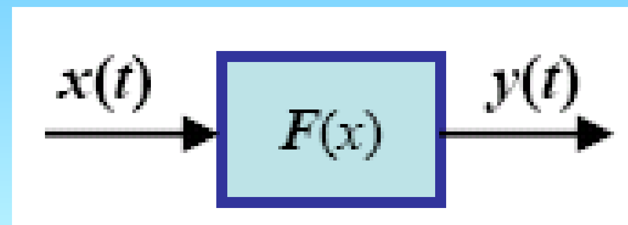
Also prefer polynomial approximation

Nonlinearity issues

- Harmonic distortions
- Gain compression
- Desensitization and Blocking
- Cross Modulation
- Intermodulation distortions
- Volterra series-based model
- Effect of feedback on nonlinearity

Harmonic distortions

$$x(t) = A\cos$$



$$y(t) = \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3 A^3\cos^3\omega t$$

$$= \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}(1+\cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3\cos\omega t + \cos 3\omega t)$$

$$= \frac{\alpha_2 A^2}{2} + (\alpha_1 A + \frac{3\alpha_3 A^3}{4})\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t$$

β_0 β_1 β_2 β_3

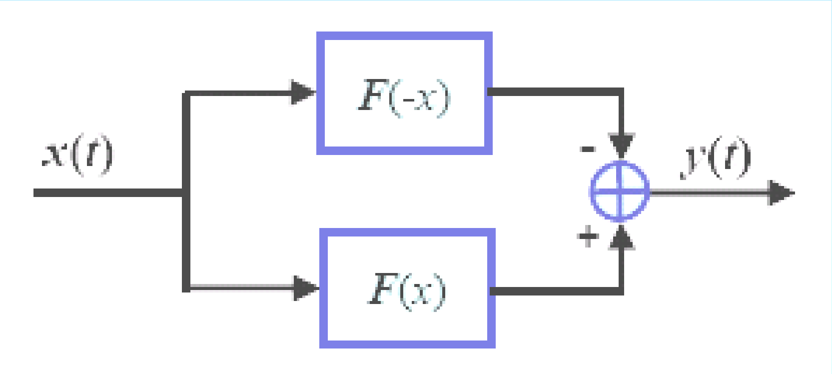
DC component (offset)

Fundamental (*amplitude influenced by nonlinearity*)

Harmonics (*not of major concern in a receiver – as out of band*)

Advantages of differential circuits

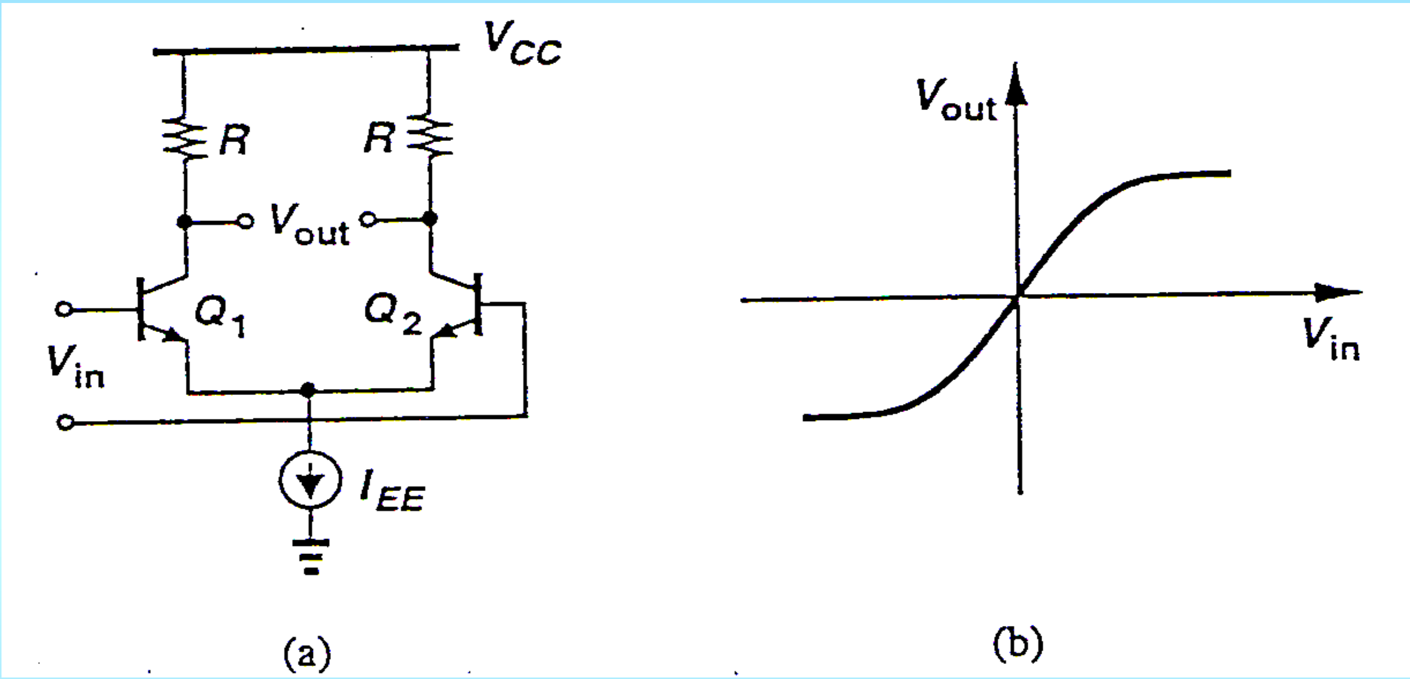
Even harmonics can be suppressed by using differential mode (*as in Differ. Amplifier*):



$$y(t) = \left(2\alpha_1 A + \frac{3\alpha_3 A^3}{2}\right)\cos \omega t + \frac{\alpha_3 A^3}{2}\cos 3\omega t$$

In practice, only partial suppression, mismatch corrupts symmetry between the two signal paths

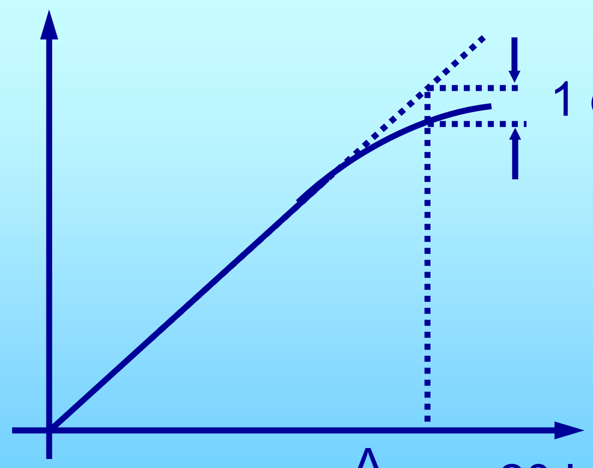
Balanced or Differential system



$$v_{out} = RI_{EE} \tanh \frac{v_{in}}{2V_T}$$

Gain compression

- As A ↑, $\beta_1/\alpha_1 A$ ↓
- When $\beta_1/\alpha_1 A$ = - 1 dB, A = "1-dB compression point"



$$\beta_1 = \alpha_1 A + \frac{3}{4}\alpha_3 A^3, \alpha_3 < 0$$

$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A^2{}_{1-dB}\right| = 20\log|\alpha_1| - 1dB$$

$$A_{1-dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

Desensitization and Blocking

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t,$$

$$y(t) = (\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2) \cos \omega_1 t + ..,$$

$$\text{For} \quad A_1 << A_2 (\text{Interferen} \quad \text{ce}),$$

$$y(t) = (\alpha_1 + \frac{3}{2}\alpha_3 A_2^2) A_1 \cos \omega_1 t + ...$$

$$if \quad \alpha_3 < 0, \quad \alpha_1 + \frac{3}{2}\alpha_3 A_2^2 < \alpha_1$$

Gain of desired signal is reduced by interferer signal.

Nonlinearity issues

- Desensitization and Blocking

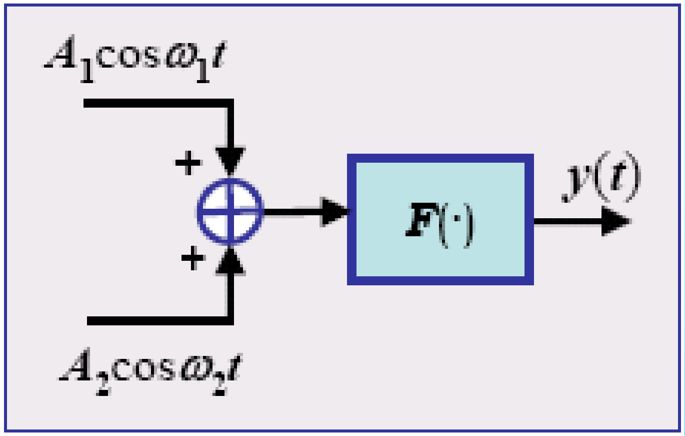
Desired signal: $A_1 \cos \omega_1 t$

Blocker (interferer): $A_2 \cos \omega_2 t$ and $A_2 >> A_1$ *even 1000x*

$$y(t) = \left(\alpha_1 A_1 + \frac{3\alpha_3 A_1^3}{4} + \frac{3\alpha_3 A_1 A_2^2}{2} \right) \cos \omega_1 t + \ \ldots$$

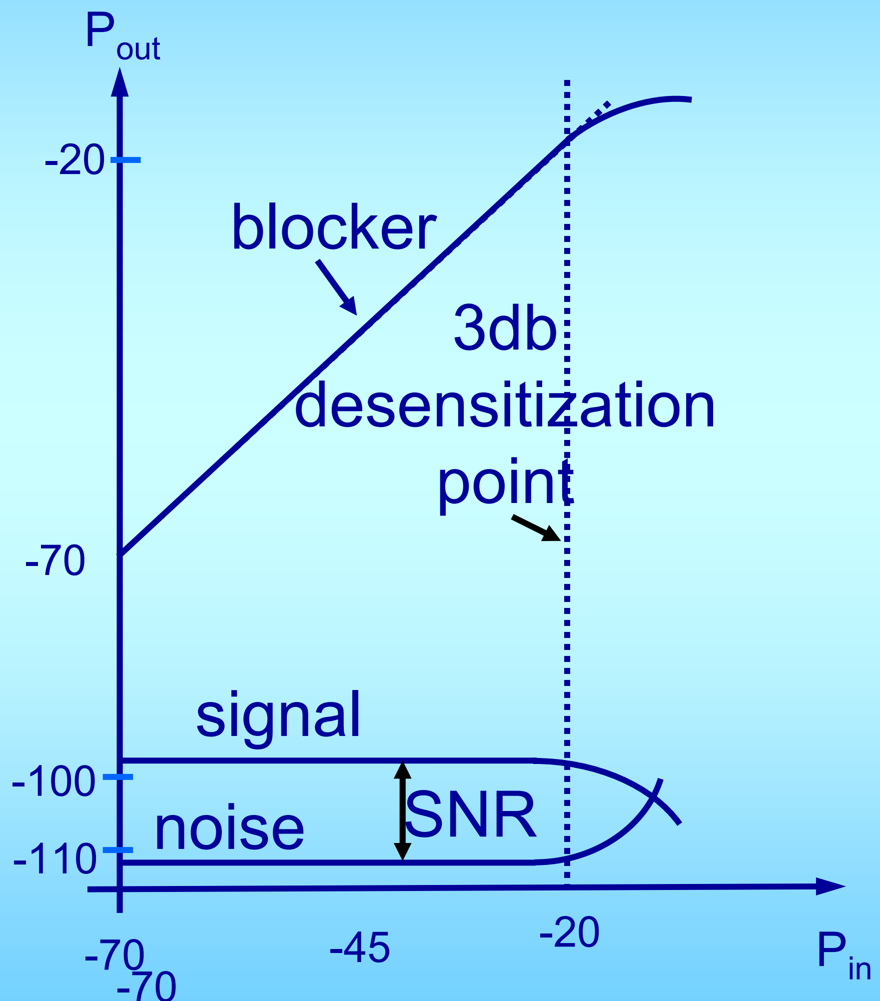
$$\approx \left(\alpha_1 + \frac{3\alpha_3 A_2^2}{2} \right) A_1 \cos \omega_1 t + \ \ldots$$

Gain may drop to zero for large A_2.



Blocking:

$$A_{block} = \sqrt{\frac{2}{3} \left| \frac{\alpha_1}{\alpha_3} \right|}$$



As block power become excessive, signal gain is reduced, equivalent noise is increase.

GSM blocking test:

Wanted signal is -99 dBm

At 3dB desensitization point, SNR=9dB

Nonlinearity issues

Cross Modulation

If the blocker's amplitude varies in time the amplitude modulation occurs at the output:

$$y(t) = \left(\alpha_1 + \frac{3\alpha_3 A_2^2(t)}{2} \right) A_1 \cos \omega_1 t + \ \ldots$$

Extra harmonics are produced.
Similar effect for large noise imposed.

Typical of multi-channel transmitters with a common amplifier

If the interferer is amplitude modulated:

$$A_2\cos(w_2 t) = A_2(1+m(t))\cos(w_2 t)$$

Where m(t) is the message being transmitted.

The cross modulation component becomes:

$$\left(\alpha_1 + \tfrac{3}{2}\alpha_3 A_2^2 (1 + 2m(t) + m^2(t))\right) A\cos(\omega_1 t)$$

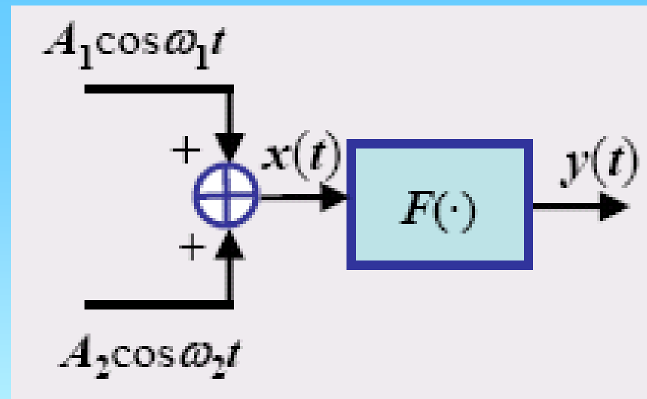
The component that can be heard in our channel is:

$$\left(\alpha_1 + \tfrac{3}{2}\alpha_3 A_2^2 (1 + 2m(t))\right) A_1 \cos(\omega_1 t)$$

$$= (\alpha_1 + \tfrac{3}{2}\alpha_3 A_2^2)\left(1 + \frac{3\alpha_3 A_2^2}{\alpha_1 + \tfrac{3}{2}\alpha_3 A_2^2} m(t)\right) A_1 \cos(\omega_1 t)$$

$$\approx (\alpha_1 + \tfrac{3}{2}\alpha_3 A_2^2)\left(1 + \frac{3\alpha_3 A_2^2}{\alpha_1} m(t)\right) A_1 \cos(\omega_1 t)$$

The modulation index ➔ that of int.ch. When: $A_2 = (\alpha_1/3\alpha_3)$^0.5



- Intermodulation

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t,$$

$$y(t) = \alpha_1 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)$$

$$+ \alpha_2 (\cos \omega_1 t + A_2 \cos \omega_2 t)^2$$

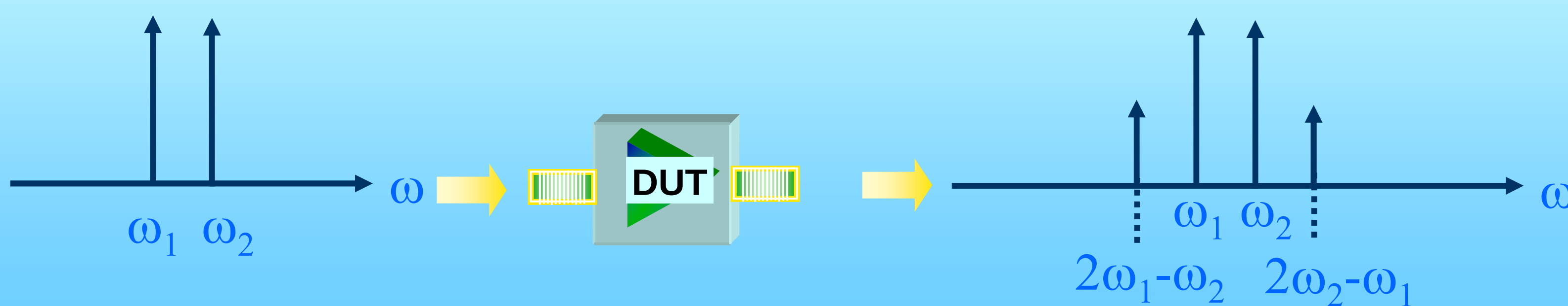
$$+ \alpha_3 (\cos \omega_1 t + A_2 \cos \omega_2 t)^3$$

$$\omega_1 : \alpha_1 A_1 + \frac{3}{4} \alpha_3 A_1^3 + \frac{3}{2} \alpha_3 A_1 A_2^2$$

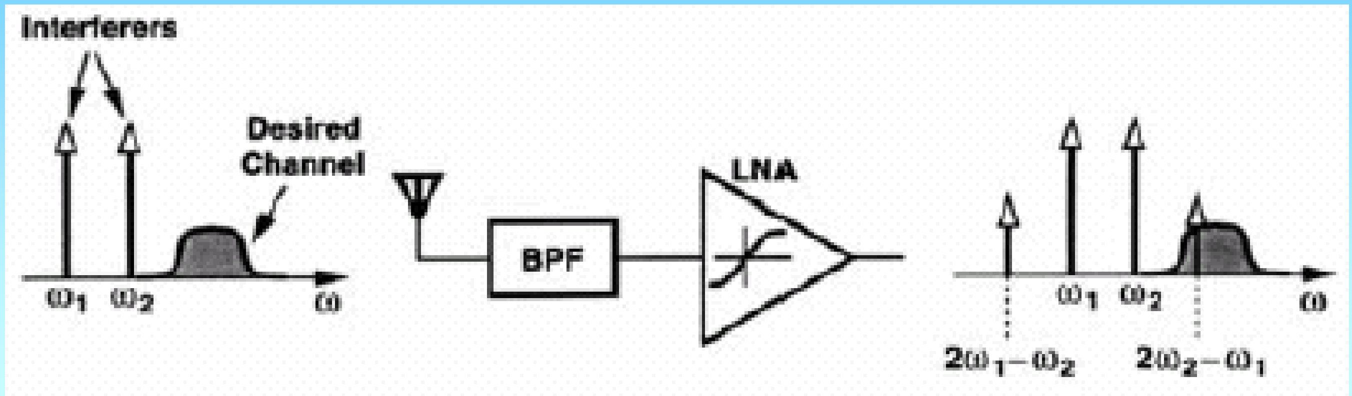
$$\omega_2 : \alpha_1 A_2 + \frac{3}{4} \alpha_3 A_2^3 + \frac{3}{2} \alpha_3 A_2 A_1^2$$

$$2\omega_1 - \omega_2 : \frac{3}{4} \alpha_3 A_2 A_1^2$$

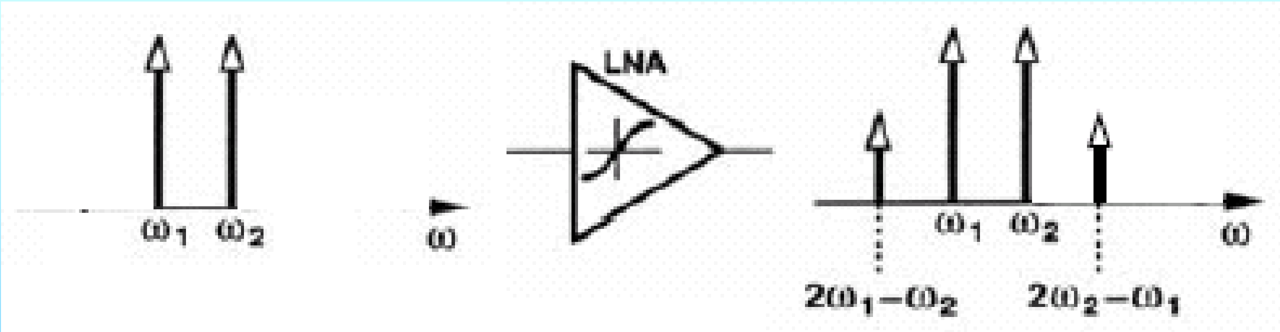
$$2\omega_2 - \omega_1 : \frac{3}{4} \alpha_3 A_1 A_2^2$$



Intermodulation distortions



Corruption of a signal due to intermodulation between two interferers (of big concern)



Two-tone test for nonlinearity: A_1=A_2 and $\omega_1 \approx \omega_2$

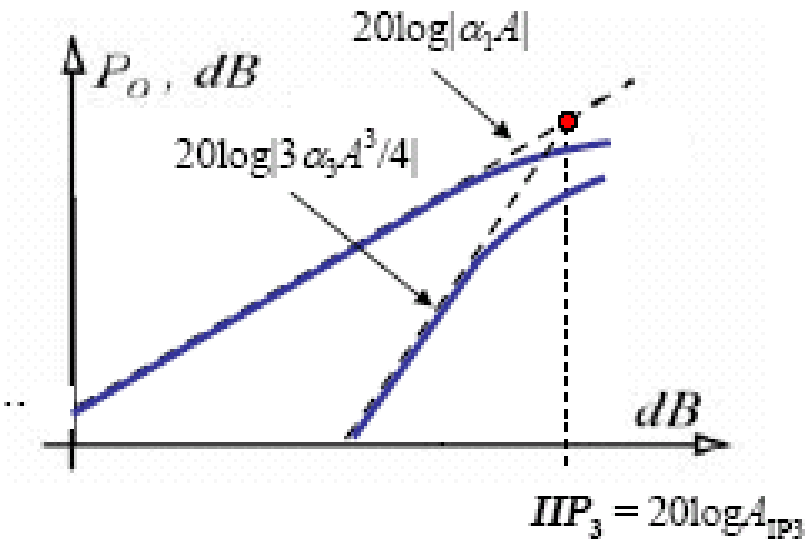
Intermodulation distortions

Third intercept point IP3

$$x(t) = A\cos\omega_1 t + A\cos\omega_2 t$$

$$y(t) = \left(\alpha_1 A + \frac{9\alpha_3 A^3}{4}\right)(\cos\omega_1 t + \cos\omega_2 t)$$

$$+\frac{3\alpha_3 A^3}{4}\left(\cos(2\omega_1 - \omega_2)t + \cos(2\omega_2 - \omega_1)t\right) + \ \ldots$$



For small amplitudes the fundamental rises linearly with *A*, and the 3rd order intermodulation terms with A^3

At IP3, $|\alpha_1 A| = |\frac{3}{4}\alpha_3 A^3|$

Two-tone Distortion

- 3rd Order Intermodulation Intercept Point (IP3)

$$Let \quad A_1 = A_2,$$

$$y(t) = (\alpha_1 + \frac{9}{4}\alpha_3 A^2)A\cos\omega_1 t$$

$$+ (\alpha_1 + \frac{9}{4}\alpha_3 A^2)A\cos\omega_2 t$$

$$+ \frac{3}{4}\alpha_3 A^3\cos(2\omega_1 - \omega_2)t$$

$$+ \frac{3}{4}\alpha_3 A^3\cos(2\omega_2 - \omega_1)t + \ldots$$

If $\alpha_1 >> \alpha_3$

$$|\alpha_1| A_{IP3} = \frac{3}{4}|\alpha_3| A_{IP3}^3 \Rightarrow A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$P_{IP3} = 20\log A_{IP3}$$

Let $A_{\omega 1 \omega 2}$ be the amplitude of ω_1 and ω_2
Let A_{in} be the input level at each frequency
Let A_{IM3} be the amplitude of the IM3 products

$$y(t) = (\alpha_1 + \frac{9}{4}\alpha_3 A^2) A \cos \omega_1 t$$
$$+ (\alpha_1 + \frac{9}{4}\alpha_3 A^2) A \cos \omega_2 t$$
$$+ \frac{3}{4}\alpha_3 A^3 \cos(2\omega_1 - \omega_2) t$$
$$+ \frac{3}{4}\alpha_3 A^3 \cos(2\omega_2 - \omega_1) t + \ldots$$

$$\frac{A_{\omega 1,\omega 2}}{A_{IM3}} \approx \frac{|\alpha_1| A_{in}}{3|\alpha_3| A_{in}^3 / 4} = \frac{4|\alpha_1|}{3|\alpha_3|} \frac{1}{A_{in}^2}$$

$$A_{IP3} = \sqrt{\frac{4}{3} \left| \frac{\alpha_1}{\alpha_3} \right|}$$

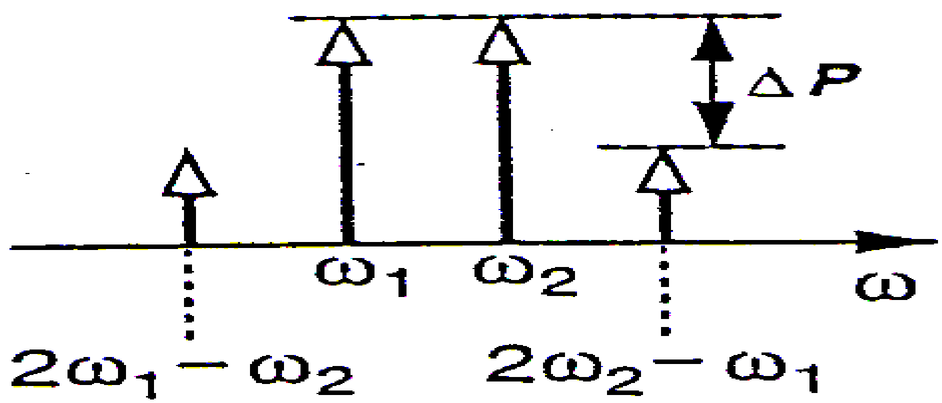
$$\frac{A_{\omega 1,\omega 2}}{A_{IM3}} \approx \frac{A_{IP3}^2}{A_{in}^2}$$

$$\frac{A_{\omega 1,\omega 2}}{A_{IM3}} \approx \frac{A_{IP3}^2}{A_{in}^2}$$

The IP3 can be measured with single input.

$$20\log A_{\omega 1,\omega 2} - 20\log A_{IM3} = 20\log A_{IP3}^2 - 20\log A_{in}^2$$

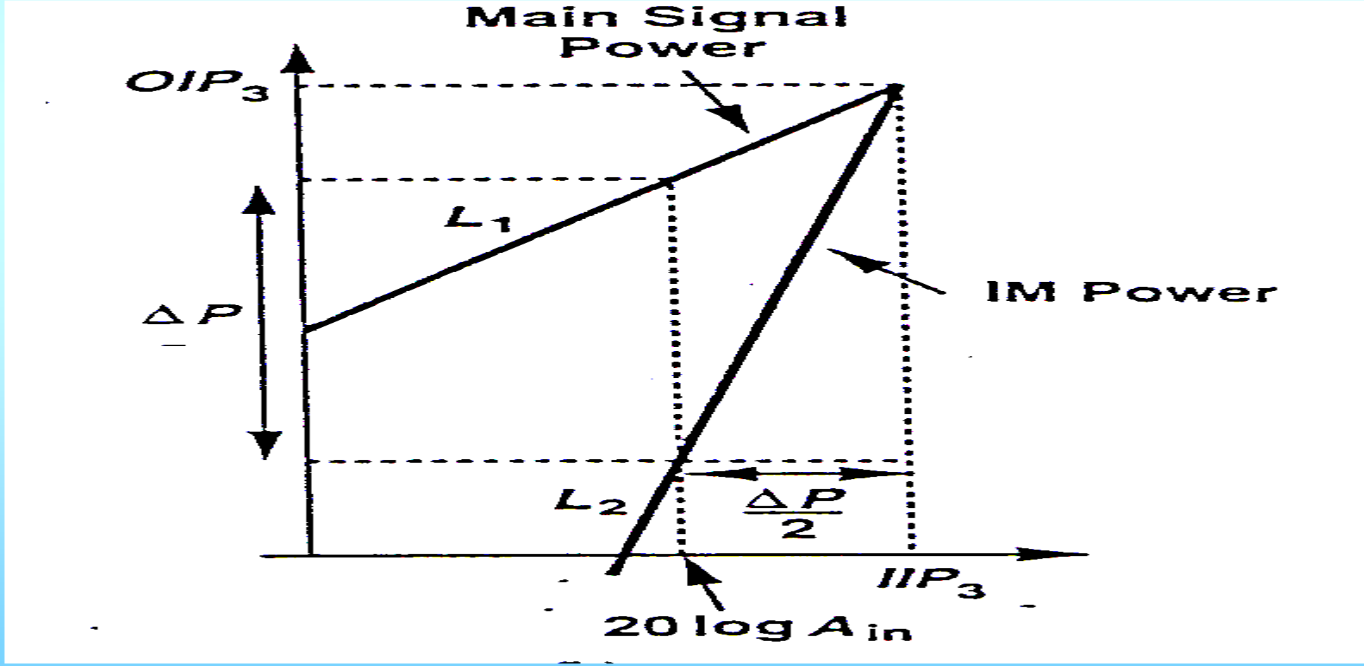
$$20\log A_{IP3} = \tfrac{1}{2}(20\log A_{\omega 1,\omega 2} - 20\log A_{IM3}) + 20\log A_{in}$$



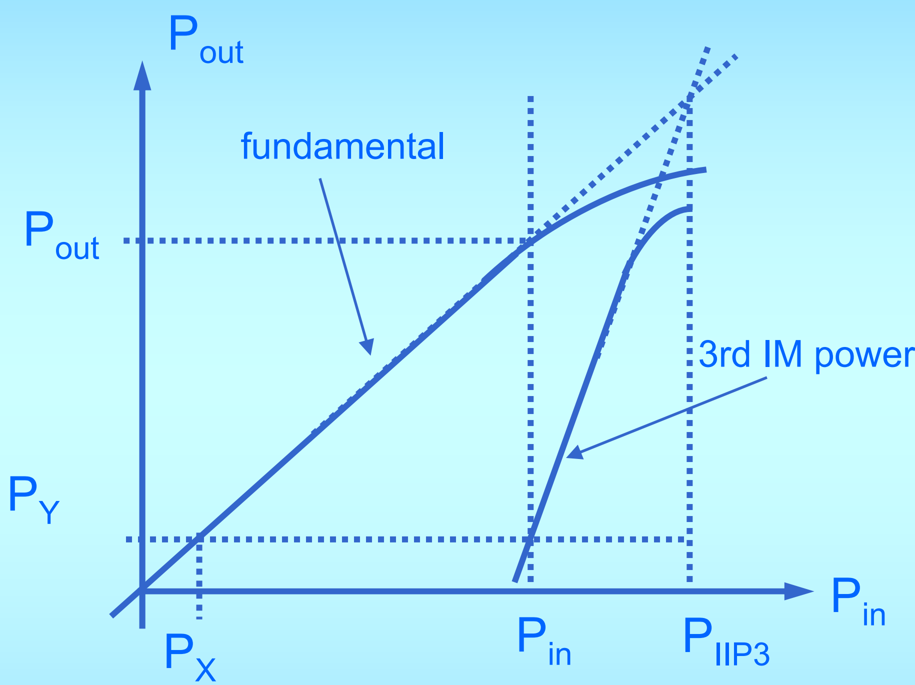
$$IIP_3\big|_{dBm} = \frac{\Delta P\big|_{dB}}{2} + P_{in}\big|_{dBm}$$

$$20\log A_{\omega 1,\omega 2} - 20\log A_{IM3} = 20\log A_{IP3}^{2} - 20\log A_{in}^{2}$$

$$20\log A_{IP3} = \tfrac{1}{2}(20\log A_{\omega 1,\omega 2} - 20\log A_{IM3}) + 20\log A_{in}$$



3rd Order IM Intercept Point (IP3)



$$P_{IIP3} = P_{in} + \frac{P_{out} - P_Y}{2}$$

$$= P_{in} + \frac{(P_{in} + G) - (P_X + G)}{2}$$

$$= \frac{3P_{in} - P_X}{2}$$

$$P_{in} = \frac{2P_{IIP3} + P_X}{3}$$

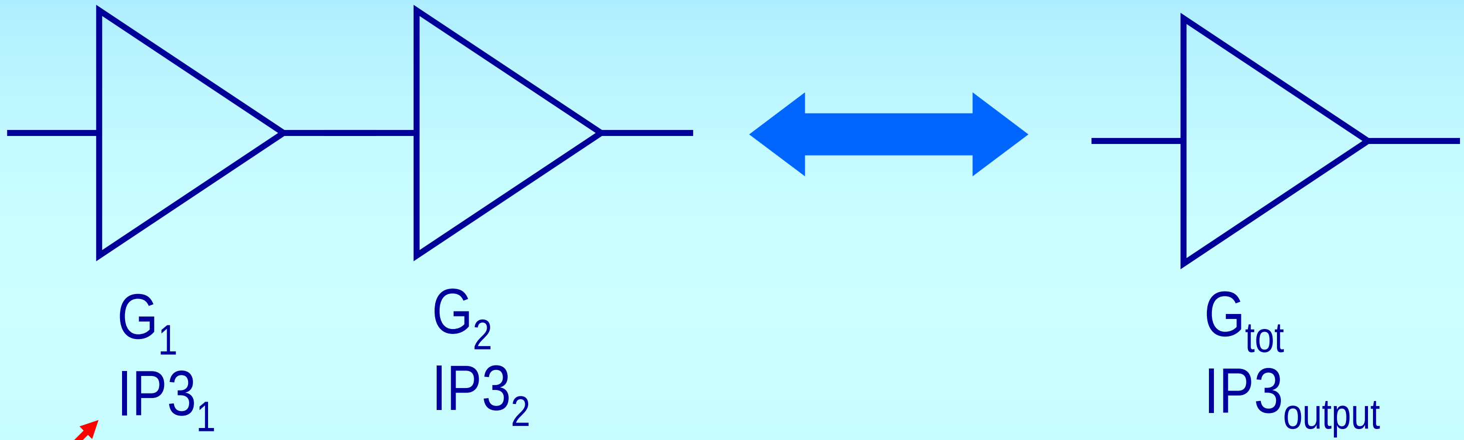
If P_X is the minimum receivable input power, and if the corresponding output P_Y is not to be exceeded by IM3, then P_{in} must stay below the above expression

IM2

- 2nd order intermodulation (IM2) can be similarly defined and computed
- Main cause:
  - – Coupling from ant to LO (self mixing)
  - – Distortion in amp and mixer
  - – Mismatch in differential topologies
- Main effect is on DC
- Can be minimized with fully differential circuit

IP3 of Cascade Networks

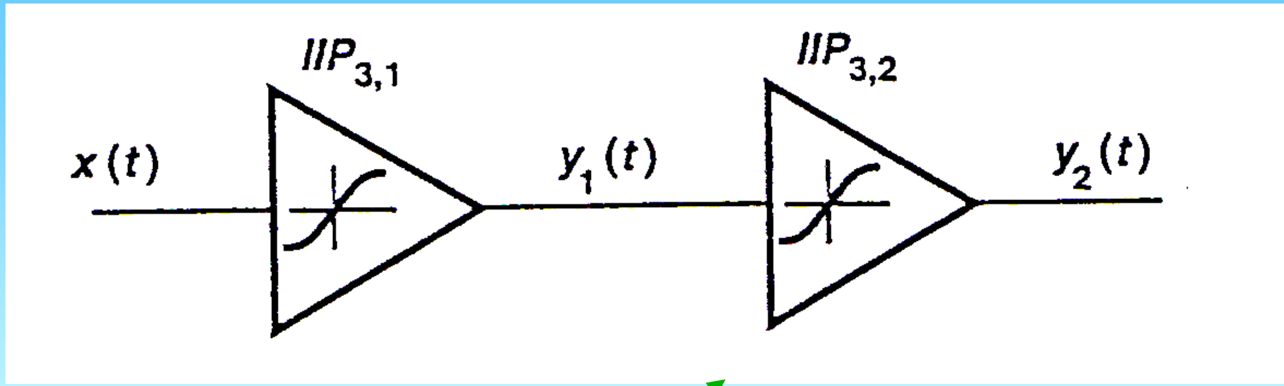
- IP3 of Cascade Networks with odd nonlinearity



$$A_{IP3}^2 G^2$$

$$IP3_{output} = \frac{1}{\sum_i \frac{1}{IP3_i G_{i+1}^2 G_{i+2}^2 \cdots G_n^2}} (mW)$$

2 stage: $$IP3_{output} = \frac{1}{\frac{1}{IP3_1 \cdot G_2^2} + \frac{1}{IP3_2}} = \frac{IP3_2 \cdot IP3_1 \cdot G_2^2}{IP3_2 + IP3_1 G_2^2}$$



$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

G1=α1

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t)$$

G2=β1

$$y_2(t) = \beta_1[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \beta_2[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 + \beta_3[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3$$

$$y_2(t) = \alpha_1 \beta_1 x(t) + (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) x^3(t) + \ldots\ldots$$

$$A_{IP3} = \sqrt{\frac{4}{3} \left| \frac{\alpha_1 \beta_1}{\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3} \right|}$$

$$\frac{1}{A_{IP3}^2} = \frac{3}{4}\frac{|\alpha_3\beta_1|+|2\alpha_1\alpha_2\beta_2|+|\alpha_1^3\beta_3|}{|\alpha_1\beta_1|}$$

$$= \frac{1}{A_{IP3,1}^2} + \frac{3\alpha_2\beta_2}{2\beta_1} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$

$$\frac{1}{A_{IP3}^2} = \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$

In general:

$$\frac{1}{A_{IP3}^2} = \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2\beta_1^2}{A_{IP3,3}^2} + \ldots\ldots$$

Harmonics Distortion

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_2 x^3(t) + \cdots$$

$$x(t) = A\cos\omega t$$

$$\begin{aligned} y(t) &= \alpha_0 + \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3 A^3\cos^3\omega t + \cdots \\ &= \beta_0 + \beta_1\cos\omega t + \beta_2\cos 2\omega t + \beta_3\cos 3\omega t + \cdots \end{aligned}$$

$$\beta_0 = \alpha_0 + \frac{\alpha_2 A^2}{2}, \beta_1 = \alpha_1 A + \frac{3\alpha_3 A^3}{4}, \beta_2 = \frac{\alpha_2 A^2}{2}, \beta_3 = \frac{\alpha_3 A^3}{4}$$

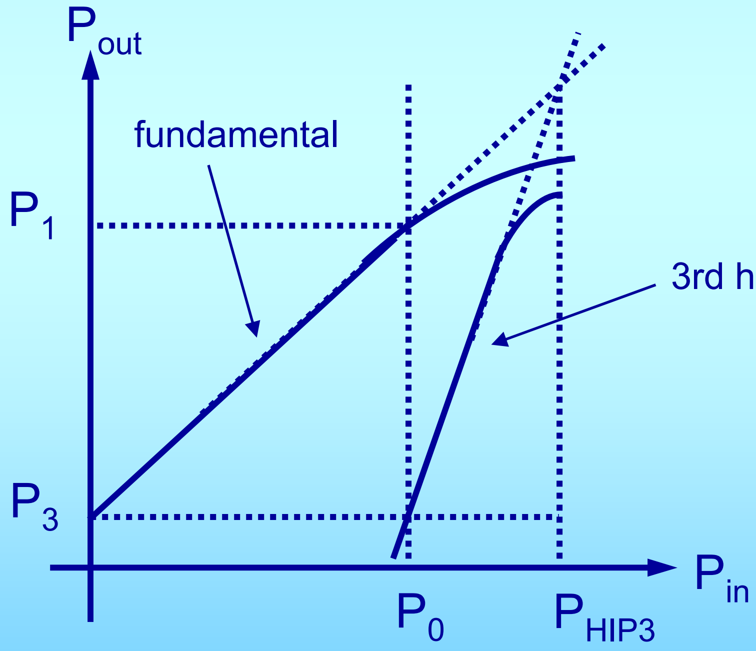
$$HD_2 \equiv \left|\frac{\beta_2}{\beta_1}\right| \approx \frac{1}{2}\frac{\alpha_2}{\alpha_1}A,$$

$$HD_3 \equiv \left|\frac{\beta_3}{\beta_1}\right| \approx \frac{1}{4}\frac{\alpha_3}{\alpha_1}A^2$$

$$THD = \frac{\sqrt{\beta_2^2 + \beta_3^2 + \cdots}}{\beta_1}$$

Third Order Harmonic Intercept

- As A $\uparrow$, HD's increase faster than basic
- At one point, HD3 becomes 1.



$$P_{HIP3} = P_{sig} + \frac{P_1 - P_3}{2} = P_{sig} + HD_3 / 2$$

$$|\beta_1| = |\alpha_1 A| = \left|\frac{1}{4}\alpha_3 A^3\right| = |\beta_3|$$

$$\alpha_1 = \frac{1}{4}\alpha_3 A_{HIP\,3}^2$$

$$A_{HIP\,3} = \sqrt{4\left|\frac{\alpha_1}{\alpha_3}\right|}$$

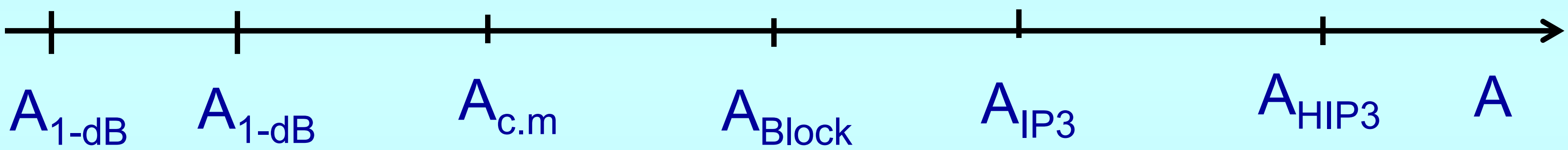
$$P_{HIP\,3} = 20 \log A_{HIP\,3}$$

Comparison of these points

$$A_{-1dBC} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$A_{c.m.} = \sqrt{\frac{1}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$



$$A_{-3dBD} = \sqrt{0.195\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$A_{block} = \sqrt{\frac{2}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$A_{HIP3} = \sqrt{4\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$P_{IIP3} = 10*\log(3) + P_{sig} + HD_3/2$

Dynamic Range

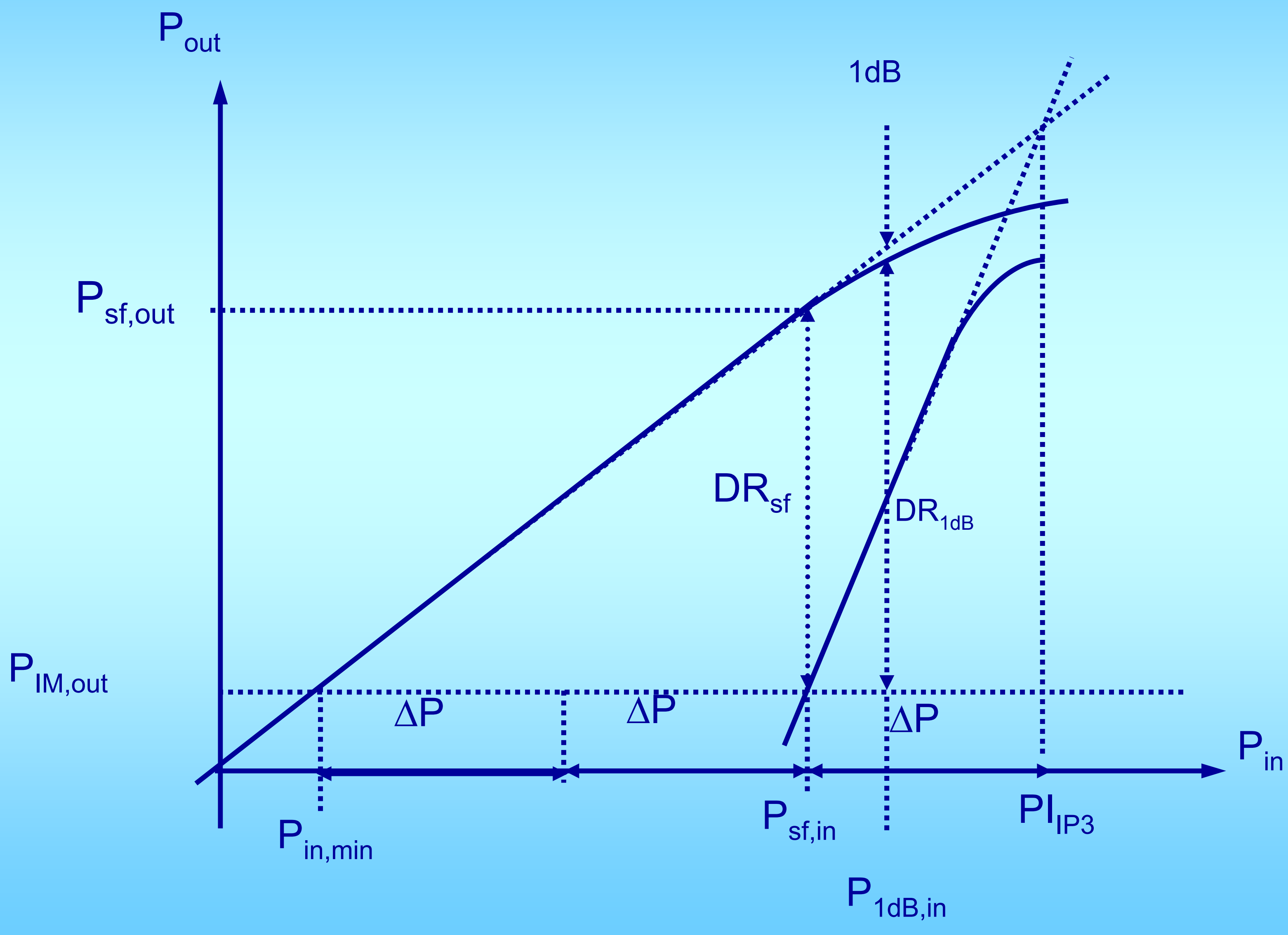
- Could be measured at either input or output
- Could be based on various nonlinearity measures

Examples: $DR_{1dB} = P_{1dB,in} - P_{in,\min}$

$$DR_{sf} = P_{sf,in} - P_{in,\min}$$

$$= \frac{2P_{IIP3} + P_{IM,in}}{3} - P_{in,\min} = \frac{2(P_{IIP3} - P_{in,\min})}{3}$$

Dynamic Range



Nonlinearity issues

- Volterra series-based model

$$y(t) = \int_{-\infty}^{\infty} h_1(\tau_1)\cdot x(t-\tau_1)d\tau_1$$

← Linear component

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2)\cdot x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_3(\tau_1,\tau_2,\tau_3)\cdot x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3$$

$$+ \ \ldots$$

} Nonlinear components

Harmonic input: $x(t) = A\cos\omega_0 t = \frac{A}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$

$$\int_{-\infty}^{\infty} h_1(\tau)\cdot e^{j\omega_0(t-\tau)}d\tau = e^{j\omega_0 t}\int_{-\infty}^{\infty} h_1(\tau)\cdot e^{-j\omega_0\tau}d\tau = e^{j\omega_0 t}H_1(\omega_0)$$

← 1st order component

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2)\cdot e^{j\omega_0(t-\tau_1)}e^{j\omega_0(t-\tau_2)}d\tau_1 d\tau_2 = e^{j2\omega_0 t}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2)\cdot e^{-j\omega_0\tau_1}e^{-j\omega_0\tau_2}d\tau_1 d\tau_2 = e^{j2\omega_0 t}H_2(\omega_0,\omega_0)$$

← 2nd order component

Volterra series-based model

$H1(\omega)$
$H2(\omega 1, \omega 2)$
$H3(\omega 1, \omega 2, \omega 3)$
....

Frequency domain transfer functions (esp. useful in analysis of mixers)

- Can be calculated from nonlinear model equations by "*harmonic balance*" approach provided the nonlinearity is "weak"
- Represent gains for harmonic distortions when all $\omega_k = \omega_0$, eg. for the second harmonic (assuming symmetry):

$$y_2(t) = 2A^2 \left| H_2(\omega_0, \omega_0) \right| \cos(2\omega_0 t + \arg H_2(\omega_0, \omega_0)) + 2A^2 H_2(\omega_0, -\omega_0)$$

- Represent gains for intermodulation distortions for arguments with different ω_k respectively

Example results (details skipped)

$$HD_2 = \frac{A_1}{2}\left|\frac{H_2(j\omega_1, j\omega_1)}{H_1(j\omega_1)}\right|, \quad HD_3 = \frac{A_1^2}{4}\left|\frac{H_3(j\omega_1, j\omega_1, j\omega_1)}{H_1(j\omega_1)}\right|$$

$$IM_2 = A_1\left|\frac{H_2(j\omega_1, j\omega_2)}{H_1(j\omega_1)}\right|, \quad IM_2 = \frac{3}{4}A_1^4\left|\frac{H_3(-j\omega_1, j\omega_2, j\omega_2)}{H_1(j\omega_1)}\right|$$

$$HIP_3 = 2\sqrt{\left|\frac{H_1(j\omega_1)}{H_3(j\omega_1, j\omega_1, j\omega_1)}\right|}, \quad IP_3 = \frac{2}{\sqrt{3}}\sqrt{\left|\frac{H_1(j\omega_1)}{H_3(-j\omega_1, j\omega_2, j\omega_2)}\right|}$$

Noise Issues

- Randomness of noise
- Signal to Noise ratio
- Time/frequency domain description
- Circuit noise
- Noise figure
- Sensitivity
- Spurious Free Dynamic Range

Noise Issues

Randomness of noise

Additive White Gaussian Noise (**AWGN**)

– random processes

Classical model:

Additive: $v(t) + x(t)$ (signal + noise) typical of LNA's and Mixers

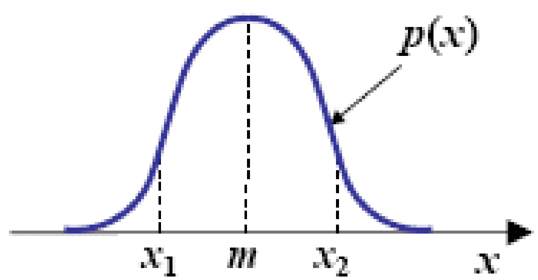
Gaussian Probability Distribution Function (**PDF**)

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\frac{-(x-m)^2}{2\sigma^2}$$

x – amplitude,
m – mean value,
σ -standard deviation

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} p(x)dx$$

$$(m-3\sigma) < x < (m+3\sigma) \;\Rightarrow\; P = 0.99$$



Randomness of noise

Time dependence: $\boldsymbol{x = x(t)}$

For stationary noise the mean (and "mean square") values over the probability domain, and over the time domain are same:

$$\int_{-\infty}^{\infty} x \cdot p(x)dx = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(t)dt$$

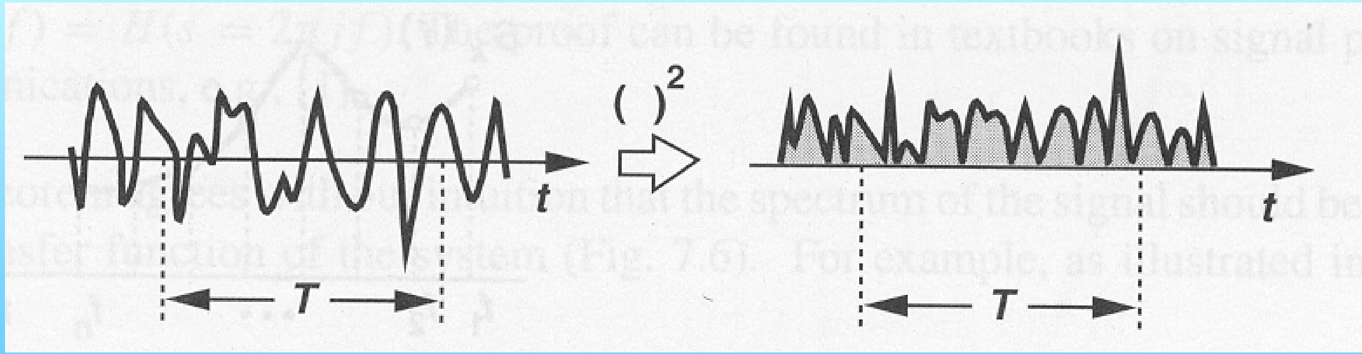
$$\int_{-\infty}^{\infty} x^2 p(x)dx = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x^2(t)dt$$

Signal and noise power

$$x(t) = s(t) + n(t)$$

$$P_s = \frac{1}{T}\int_0^T s^2(t)\,dt, \quad S(rms) = S_{rms} = \sqrt{P_s}$$

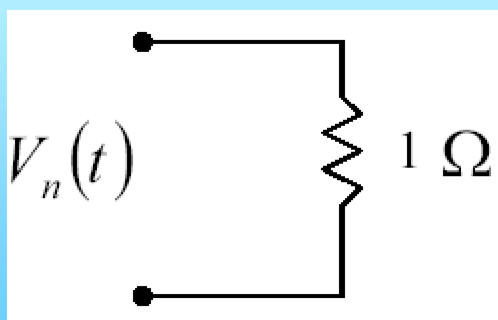
$$P_n = \frac{1}{T}\int_0^T n^2(t)\,dt, \quad N(rms) = N_{rms} = \sqrt{P_n}$$



Physical interpretation

If we apply a signal (or noise) as a voltage source across a one Ohm resistor, the power delivered by the source is equal to the signal power.

Signal power can be viewer as a measure of normalized power.



$$p_{diss} = \frac{V_n^2(rms)}{1\Omega} = V_n^2(rms)$$

A measure of Normalized power

Signal to noise ratio

$$SNR = 10\log_{10}\left(\frac{P_s}{P_n}\right) = 20\log_{10}\left(\frac{S_{rms}}{N_{rms}}\right)$$

SNR = 0 dB when signal power = noise power

Absolute noise level in dB:
w.r.t. 1 mW of signal power

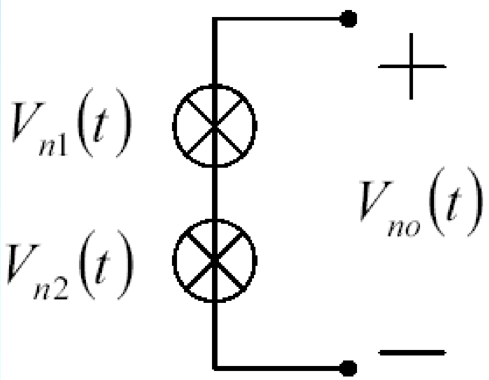
$$P_n \text{ in } \mathrm{d}\mathbf{B}_m = 10\log\frac{P_n}{1\,\mathrm{mW}}$$

$$= 30\,\mathrm{dB} + 10\log(P_n)$$

SNR in bits

- A sine wave with magnitude 1 has power = 1/2.
- Quantize it into $N=2^n$ equal levels between -1 and 1 (with step size = $2/2^n$)
- Quantization error uniformly distributed between $\pm 1/2^n$
- Noise (quantization error) power
  $=1/3\ (1/2^n)^2$
- Signal to noise ratio
  $= 1/2 \div 1/3\ (1/2^n)^2 = 1.5(1/2^n)^2$
  = 1.76 + 6.02n dB or n bits

Adding Noises



$$V^2_{no(rms)} = \frac{1}{T}\int_0^T \left[V_{n1}(t) + V_{n2}(t)\right]^2 dt$$

$$= V^2_{n1(rms)} + V^2_{n2(rms)} + \frac{2}{T}\int_0^T \left[V_{n1}(t)V_{n2}(t)\right]^2 dt$$

Correlation between V_{n1} and V_{n2}

$$C = \frac{\frac{1}{T}\int_0^T V_{n1}(t)V_{n2}(t)\,dt}{V_{n1(rms)}V_{n2(rms)}}$$

(Correlation Coefficient)

$$V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)} + 2CV_{n1(rms)}V_{n2(rms)}$$

Frequency domain description of noise

Given n(t) stationary, its autocorrelation is:

$$R_n(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} n(t)n(t+\tau)\,dt$$

The power spectral density of n(t) is:

$$PSD_n(f) = S_n(f) = F(R_n(\tau))$$

$$P_n = \int_{-\infty}^{+\infty} PSD_n(f)\,df$$

For real signals, PSD is even. ➔ can use single sided spectrum: 2x positive side

$$P_n = \int_{0}^{+\infty} PSD_n(f)\,df$$

↑ single sided PSD

Time-frequency domain relation

Parseval's Theorem:

If $x(t) \Leftrightarrow X(f)$

$$\Downarrow$$

$$\int_{-\infty}^{+\infty} |x(t)|^2 \, dt = \int_{-\infty}^{+\infty} |X(f)|^2 \, df$$

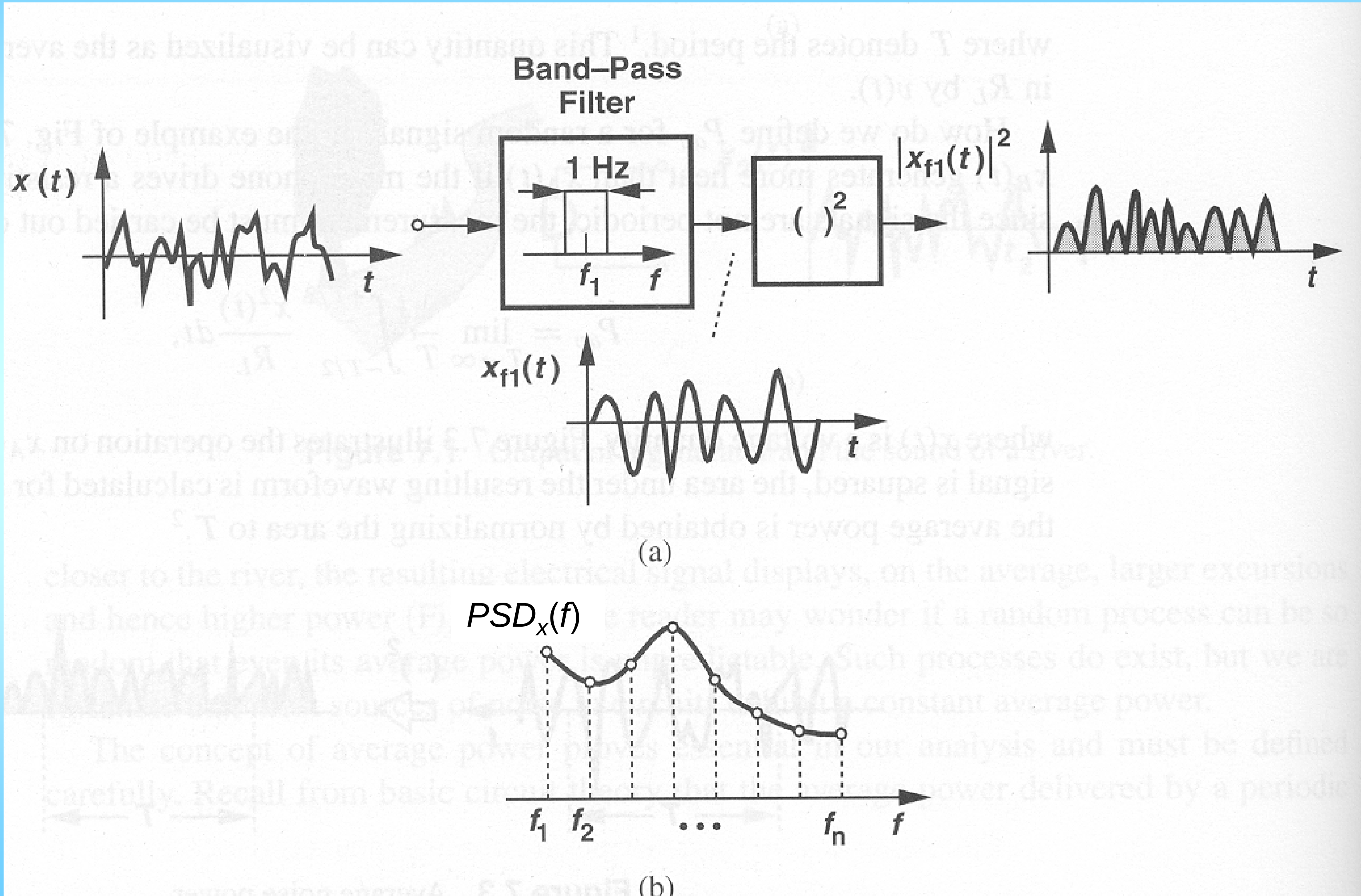
If *x*(*t*) stationary,

$$R_X(\tau) \Leftrightarrow PSD_X(f)$$

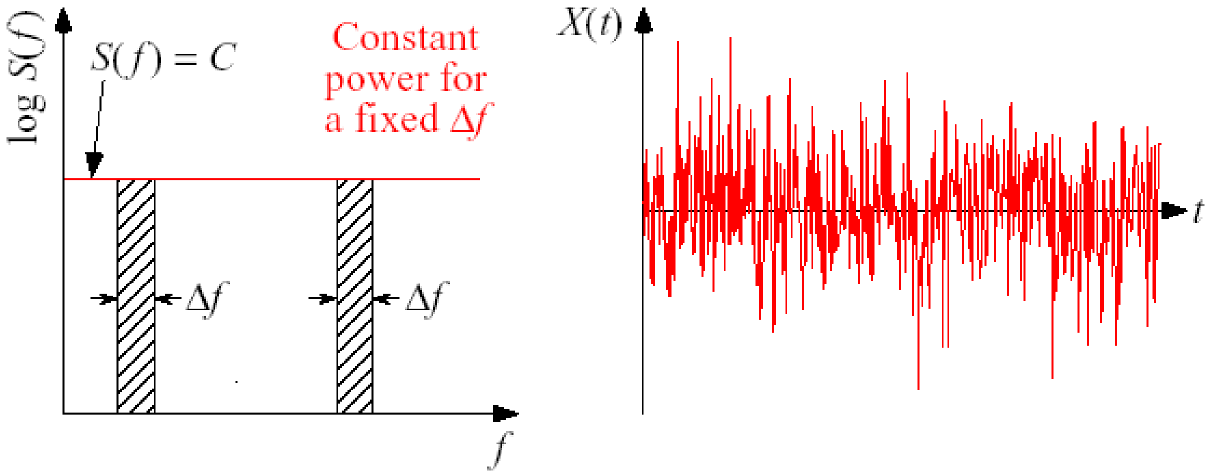
$$\Downarrow$$

$$\lim_{T \to \infty} \int_{-T}^{+T} |x(t)|^2 \, dt = R_x(0) = \int_{-\infty}^{+\infty} PSD_x(f) \, df$$

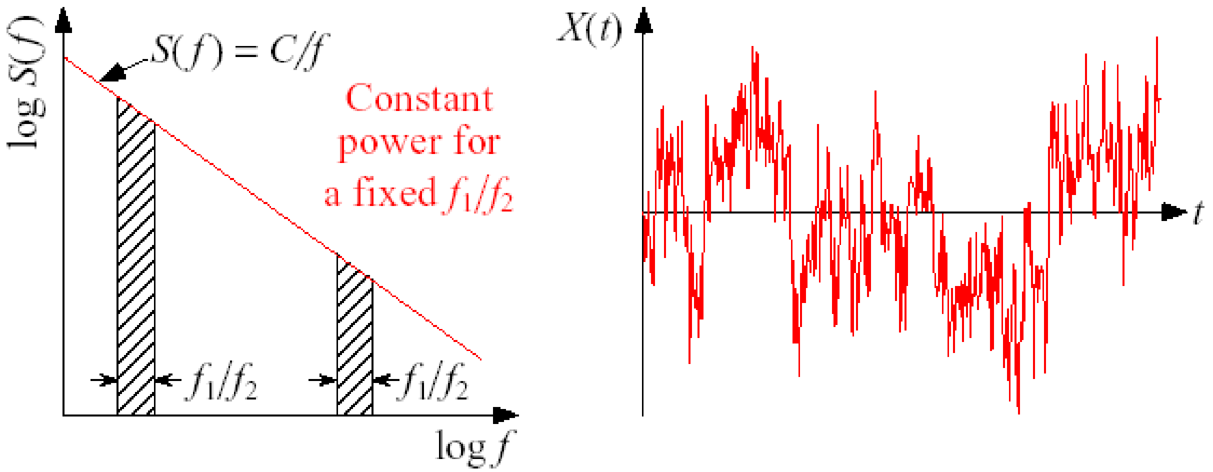
Interpretation of PDS



- White noise: Noise power is spread uniformly across the spectrum (cf. white light).



- Pink noise (a.k.a. flicker or 1/f noise): Noise power is concentrated at lower frequencies (cf. pink light).



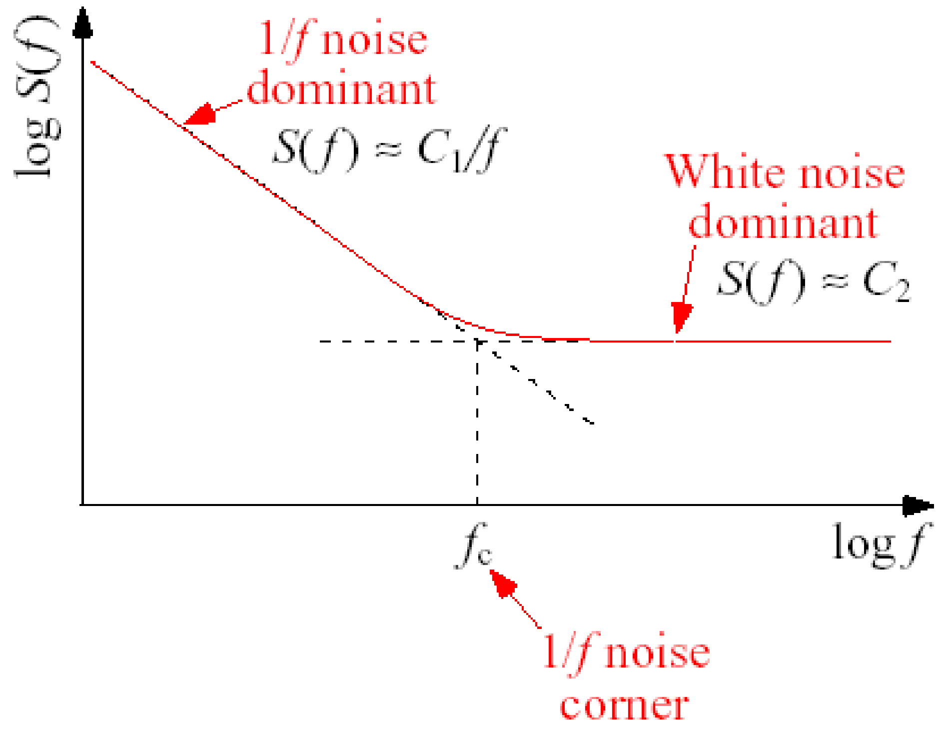
- Usually, the noise in a device is a mixture of white noise and $1/f$ noise, where the two noise processes are independent.

PSD

$$S(f) = \frac{C_1}{f} + C_2$$

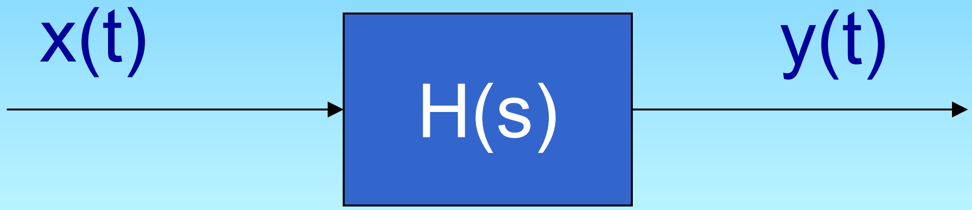
$1/f$ noise component

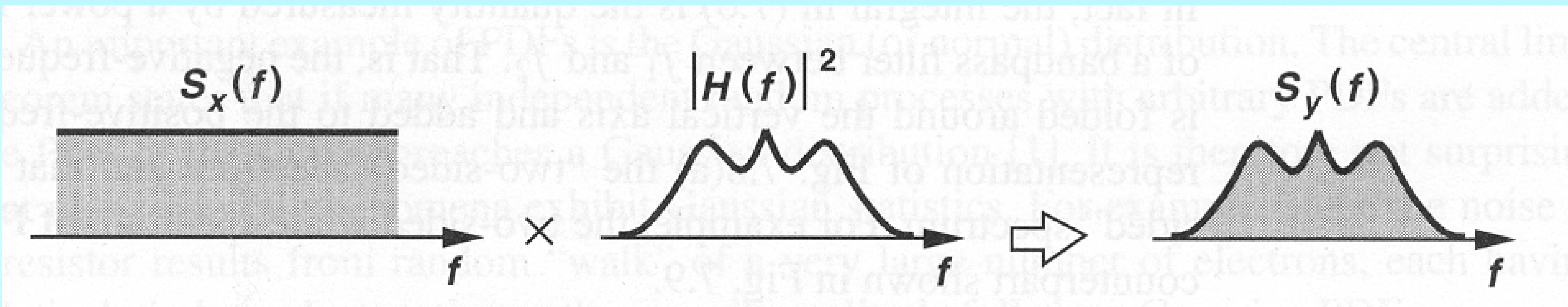
white noise component



- In general, the $1/f$ noise corner frequency is highly process and bias dependent.

Filtering of noise

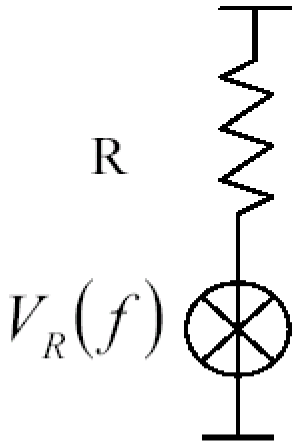




$$|H(f)|^2 = H(s)\big|_{s=j2\pi f}\, H(s)\big|_{s=-j2\pi f}$$

Total output noise = transferred noise + circuit generated noise

Circuit noise



$$V_R^2(f) = 4kTR$$

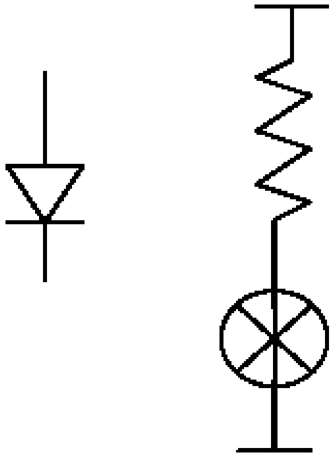
k : Boltzmann's consta
T : temperature (300ºK
R: 1k(ohm)

$$V_R(f) = \sqrt{\frac{R}{1k}} \times 4.06nV/\sqrt{Hz}$$

at 27º C

k : Boltzmann's constant ($1.38 \times 10^{-23} JK^{-1}$)
T : temperature (300ºK)
R: 1k(ohm)

Example:
R = 1kΩ, B = 1MHz, 4μV rms or 4nA rms



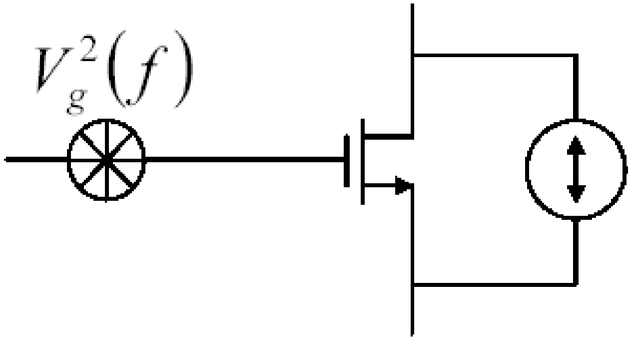
$$r_d = \frac{kT}{qI_D}$$

$$V_d^2(f) = 2kTr_d = 2kT\frac{kT}{qI_D}$$

Example:
ID= 1mA, B = 1MHz, 17nA rms

MOS Noise Model

Dominant source : flicker and thermal noise



$$I_d^2(f) = 4kT\left(\frac{2}{3}\right)g_m \quad \textbf{: active}$$

$$I_d^2(f) = 4kT / r_{ds} \quad \textbf{: triode}$$

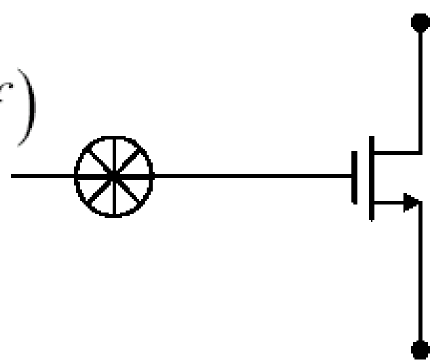
$$V_g^2(f) = \frac{k}{WLC_{OX}f}$$

k → **constant**

↓ **Flicker : dominate at low frequency**

$$r_{ds} = \frac{1}{\mu C_{OX}\frac{W}{L}\left(V_G - V_{Th}\right)}$$

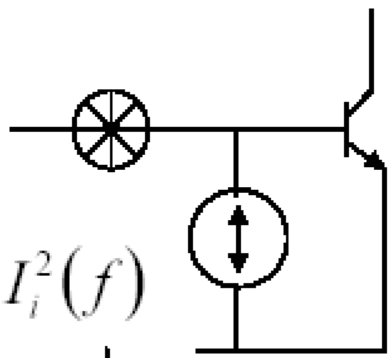
p-mos has less noise than n-mos
(holes are less traped)



$$V_i^2(f) = 4kT\left(\frac{2}{3}\right)\frac{1}{g_m} + \frac{k}{WLC_{OX}f}$$

BJT Noise

- Shot noise in I_C & I_B
- Flicker noise in I_B
- Thermal noise in r_b (base resistance)



$$V_i^2(f)$$

$$4kT\left(r_b + \frac{1}{2g_m}\right)$$

r_b: Thermal noise

$\frac{1}{2g_m}$: Collector current shot noise

$$I_i^2(f)$$

$$I_i^2(f) \cong 2q\left[I_B + \frac{kI_B}{f}\right]$$

I_B: Base current shot

$\frac{kI_B}{f}$: base current flicker

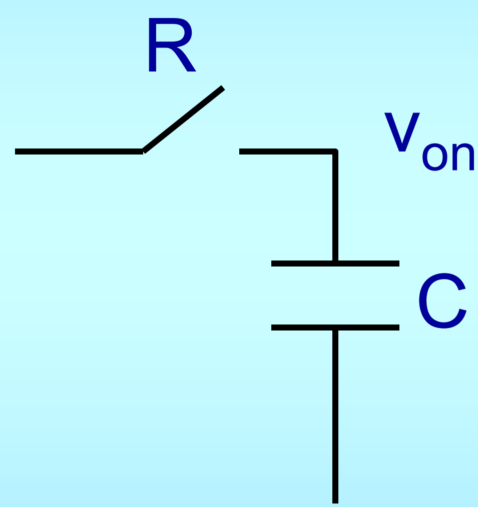
Sampling Noise

- Commonly called "kT/C" noise
- Applications: ADC, SC circuits, …

$$\overline{v_{on}^2(f)} = 4k_BTR\left|\frac{1}{1+sRC}\right|^2$$

$$\overline{v_{oT}^2} = \int_0^\infty \overline{v_{on}^2(f)}\,df = \frac{k_BT}{C}$$

Used: $$\int_0^\infty \left|\frac{1}{1+\frac{s}{\omega_o}}\right|^2 df = \frac{\omega_o}{4}$$



NF: Noise Figure (dB) Noise Factor (value)

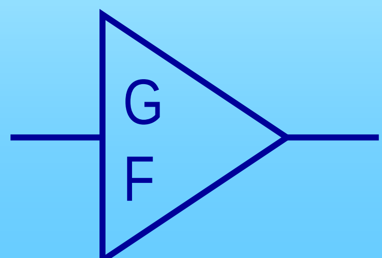
$$NF = \frac{SNR_{in}}{SNR_{out}}$$

Might be for 1-Hz bandwidth at a given frequency

For a system with no inherent noise *NF* =1 (0 dB)
Noisy system degrades *SNR* and *NF* > 1

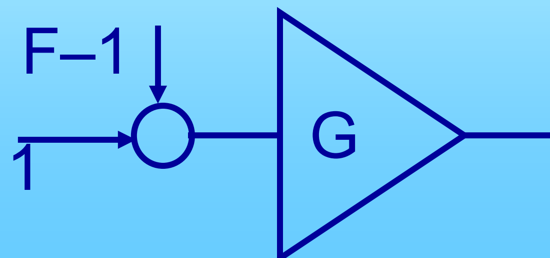
$$NF = \frac{S_{in} / N_{in}}{S_{out} / N_{out}} = \frac{N_{out}}{G \cdot N_{in}}$$

$$= \frac{GN_{in} + N_{out}^{inh}}{G \cdot N_{in}} = 1 + \frac{N_{out}^{inh}}{G \cdot N_{in}} = 1 + \frac{N_{out,i-r}^{inh}}{N_{in}}$$

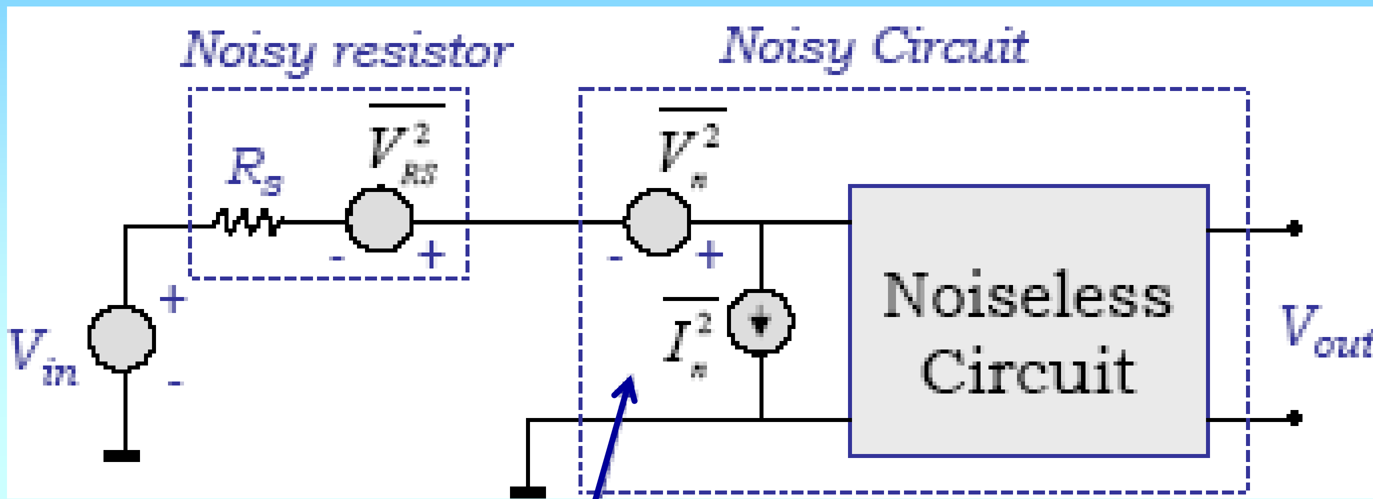


$N_{out} = F*G*N_{in}$

$N^{inh} = (F - 1)*N_{in}$



Noise Figure Example



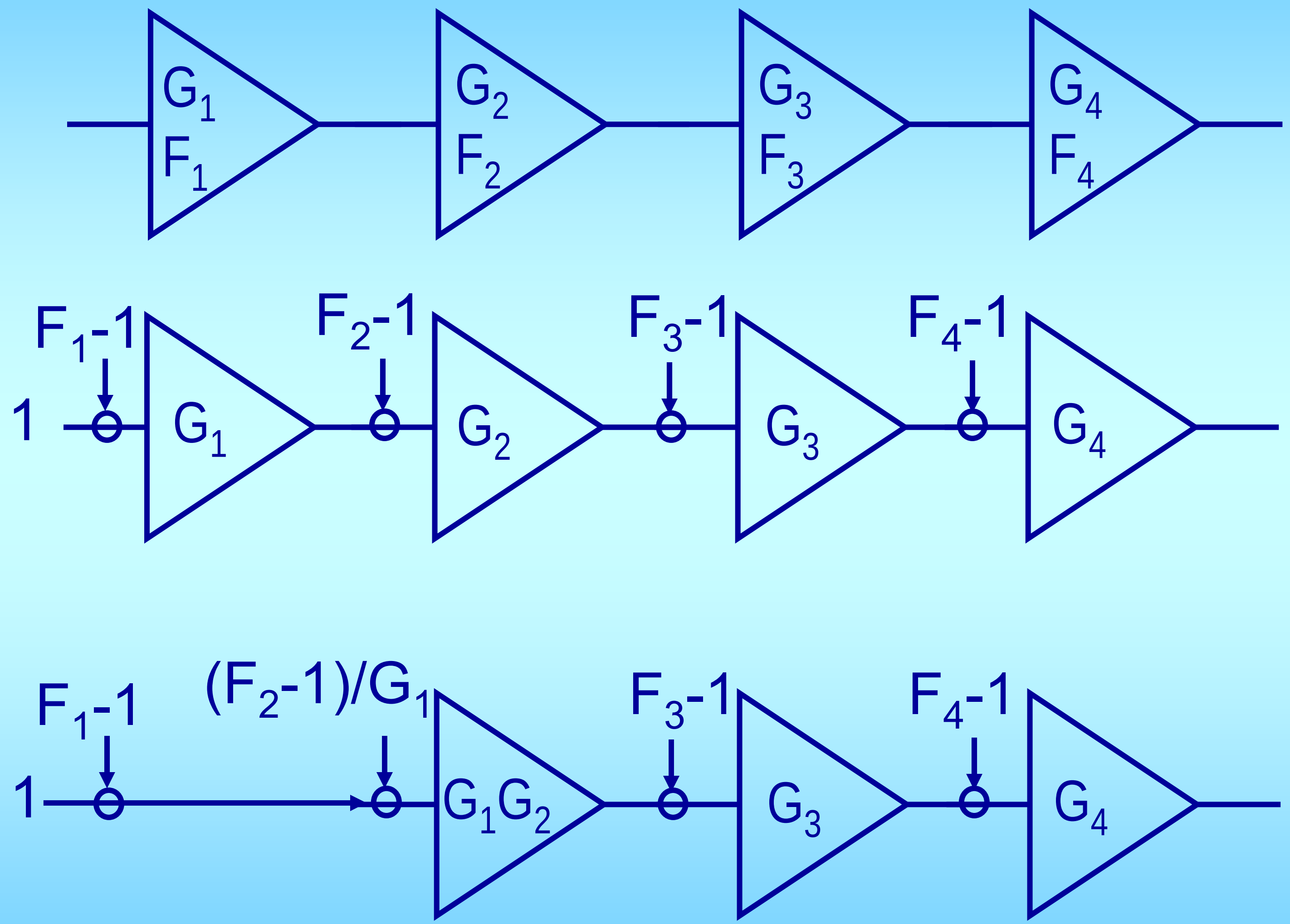
Input referred noise

$N_{IN} = \overline{V_{R_S}^2}$

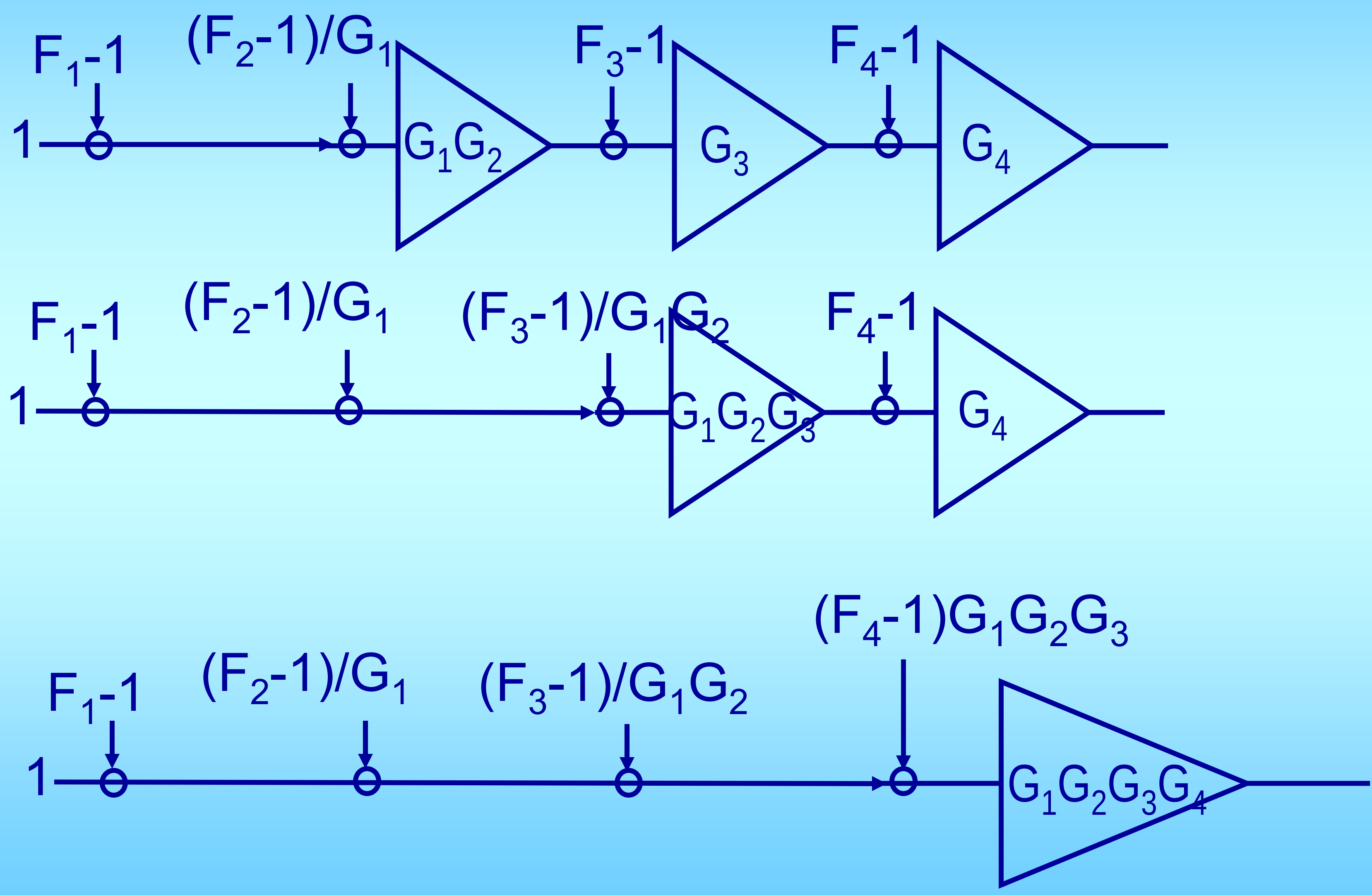
$$N_{out,i-r}^{inh} = \overline{\left(V_n + R_S I_n\right)^2}$$

$$NF = 1 + \frac{N_{out,i-r}^{inh}}{N_{in}} = 1 + \frac{\overline{\left(V_n + R_S I_n\right)^2}}{\overline{V_{R_S}^2}}$$

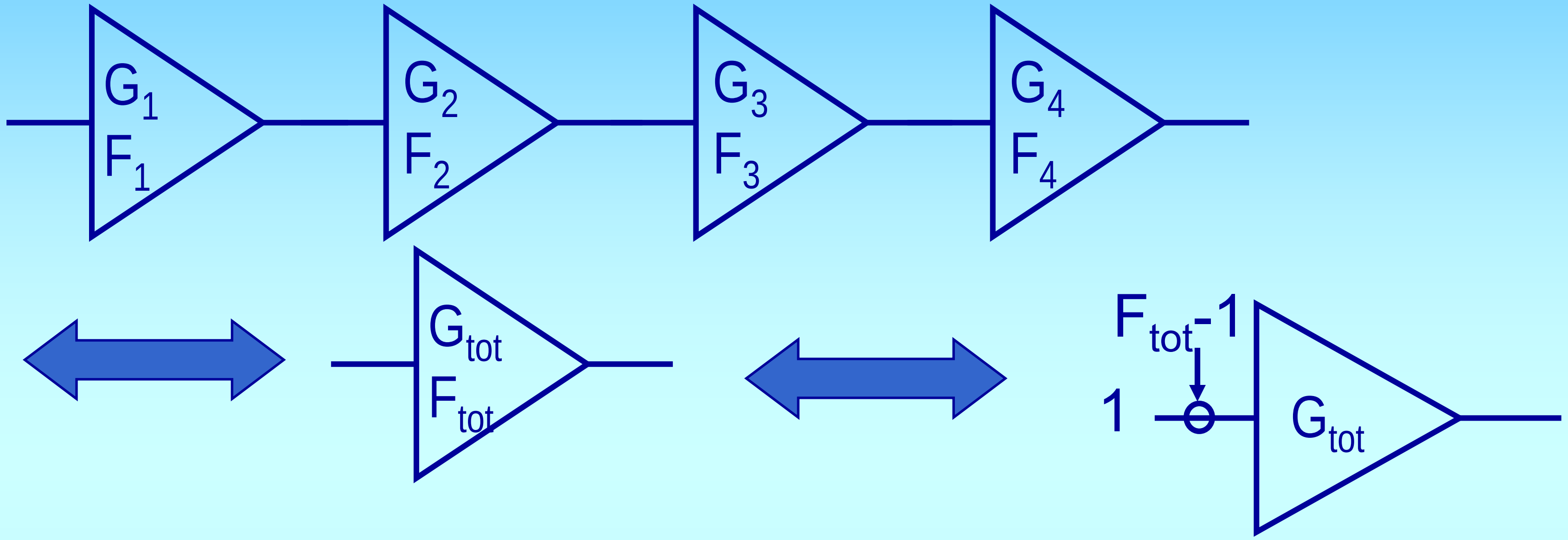
Noise Figure of Cascade Networks



Noise Figure of Cascade Networks



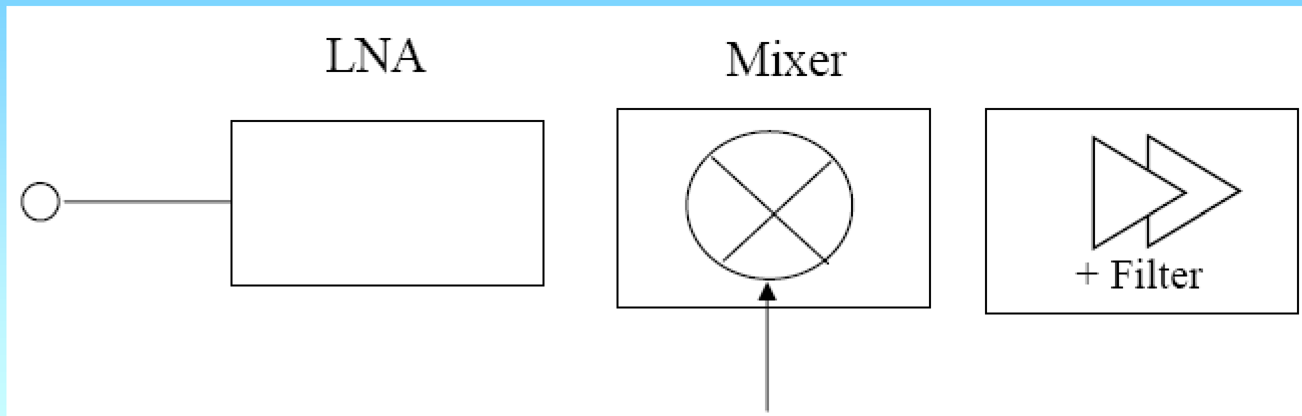
Noise Figure of Cascade Networks



$$F_{tot} = 1 + \frac{F_1 - 1}{1} + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \cdots$$

Note: here G is power gain. If A_i is voltage gain, $G_i = A_i$^2

EXAMPLE



If the NF and gain of the blocks are:

LNA:	20dB gain,	3 dB NF	(A1, F1)
Mixer:	10dB gain,	10 dB NF	(A2, F2)
VGA + Filter:	80dB gain,	20 dB NF	(A3, F3)

$$F_{tot} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 3dB + \frac{10dB - 1}{20dB} + \frac{20dB - 1}{20dB * 10db}$$

$$= 1.995 + \frac{9}{100} + \frac{99}{100 * 10} = 2.184 = 3.39dB$$

Sensitivity

$$\mathrm{NF} = \frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}}$$

$$SNR_{in} = \frac{P_{sig-in}}{P_{N_{in}}}$$

P_{sig}: Input signal power

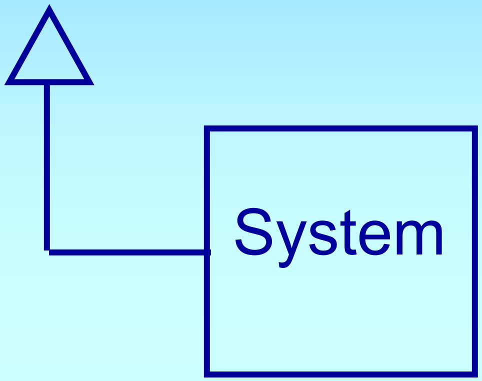
P_{IN}: Input noise power

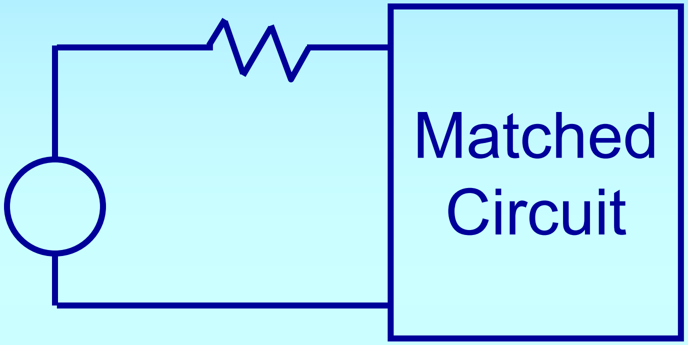
$$P_{sig-in} = P_{N_{in}} \cdot NF \cdot SNR_{out} = P_{N_{in}/Hz} \cdot BW \cdot NF \cdot SNR_{out}$$

$$P_{sig-in.\min}\Big|_{dBm} = P_{N_{in}}\Big|_{dBm/Hz} + NF\Big|_{dB} + SNR_{\min}\Big|_{dB} + 10\log BW$$

The minimum required signal power at the input terminal in order for the circuit to achieve a specific SNR at the output, which determines the Bit Error Rate (BER)

Input Noise Power





Noise voltage: $(4kTR)^{0.5}$ /HZ^0.5

Power delivered to matched load:

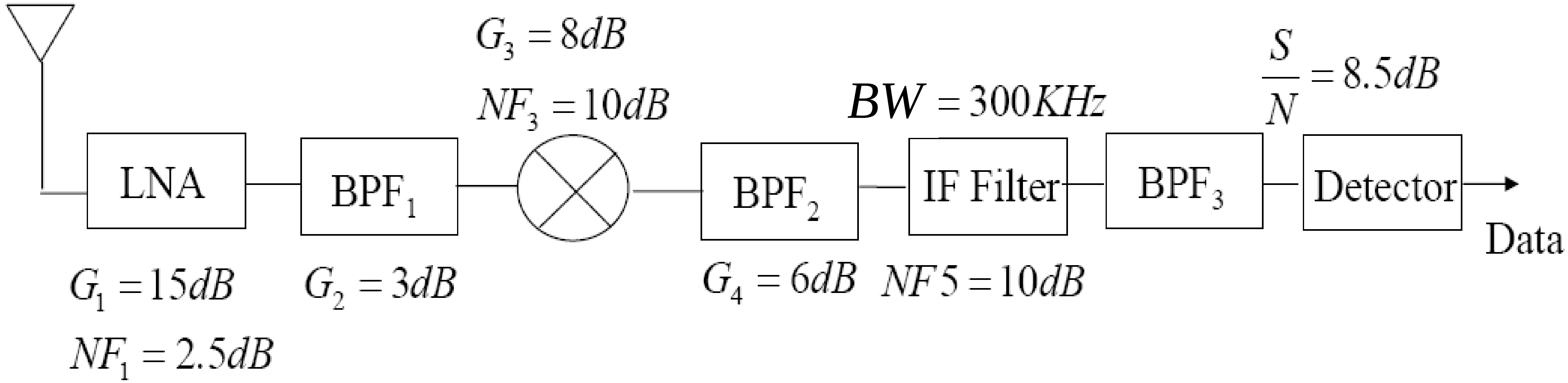
$$P = \frac{1}{4} V^2 / R$$

Input noise power: $P_{N\text{-}in} = kT$ /Hz

At room temp: $P_{N\text{-}in} = -204$ dBWatt /Hz

= –174 dBm /Hz

Noise power over a bandwidth: = –174 dBm + 10log(BW)



$$F_{tot} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \frac{F_5 - 1}{G_1 G_2 G_3 G4} = 2.42 = 3.8dB$$

$$P_{sig-in.\min}\Big|_{dBm} = P_{N_{in}}\Big|_{dBm/Hz} + NF\Big|_{dB} + SNR_{\min}\Big|_{dB} + 10\log BW$$

$$= -174dBm + 2.48dB + 8.5dB + 10\log(3 * 10^5)$$

$$= -106.9dBm$$

If the received signal power is below this, it cannot be detected correctly

Example: GSM receiver

- Baseband required SNR
  - 6 dB for static conditions
  - 9 dB with fading
- Channel bandwidth typically 170kHz
  - 10log(170000)=52.3 dB
- GSM specification required sensitivity:
  - <= – 102 dBm