Linearity, Time-Variation, Phase Modulation and Oscillator Phase Noise

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Preliminaries (to refresh dormant neurons...)

- A system is linear as long as superposition holds.
  - Scaling of a single input is included, since scaling may always be viewed as the result of summing.
  - The response to an impulse then yields sufficient information to deduce the response to any arbitrary input.
  - All real systems may be made to act nonlinearly for some inputs (e.g., the response to 1mV may differ in shape and odor from the response to a gigavolt).
    - Linearity thus holds only over some restricted range of excitations, in practice.
- A system is time-invariant if the only result of time-shifting any input is to shift the response by precisely the same amount.
- If a system is LTI, it may be shown that excitation at f produces a response only at f.

Preliminaries

- If a system is LTV, it is no longer generally true that an excitation at *f* produces a response at the same frequency.
  - Superposition still holds, however, so the response to the sum of two inputs may be deduced from the response to each.
- If a system is nonlinear, the response may also contain spectral components not present in the excitation.
  - Dependency of output on combination of inputs not necessarily linear; this difference can be used as a basis for determining whether spectral shaping is due to time-variation or nonlinearity.

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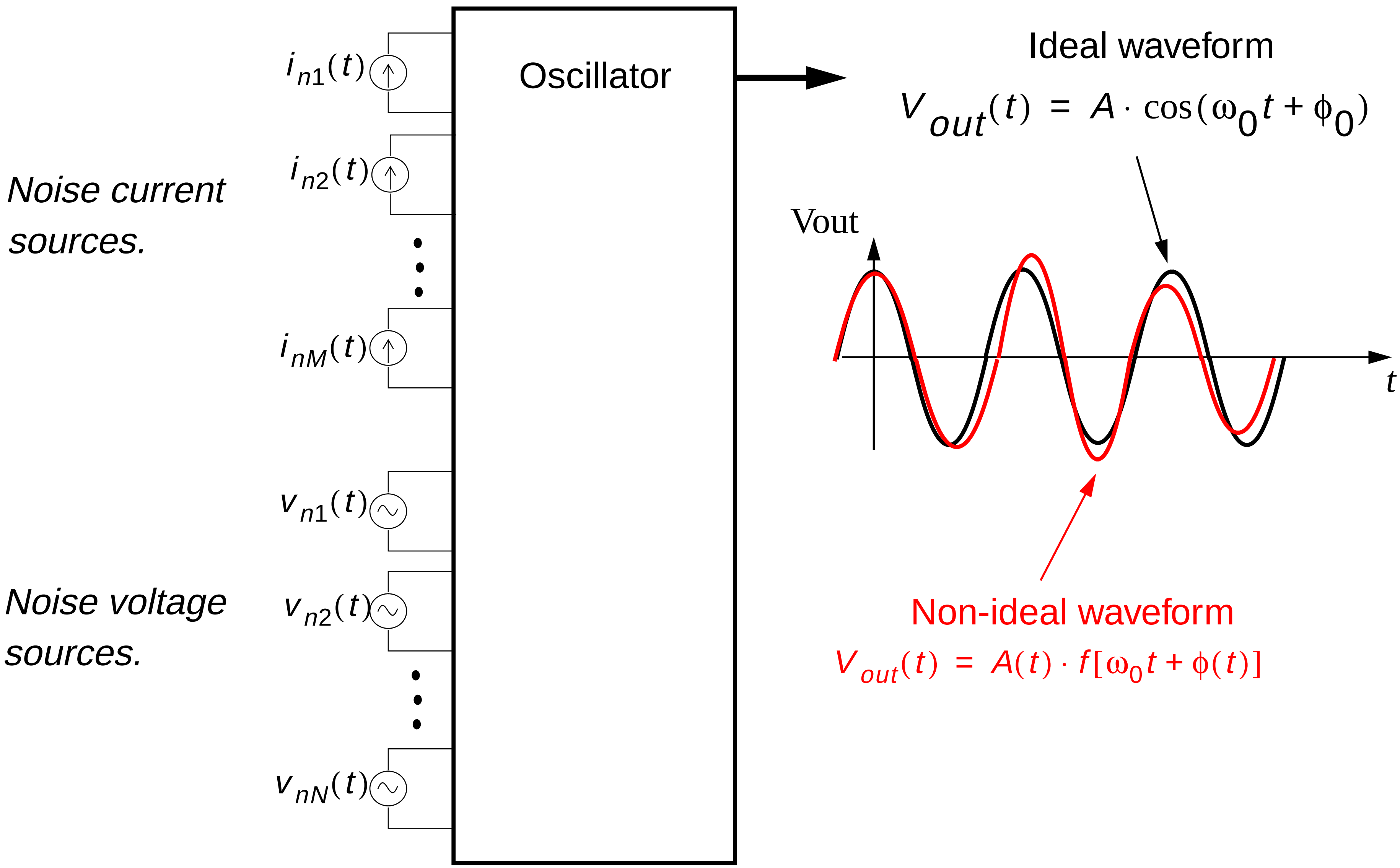
Oscillator with Input Noise Sources



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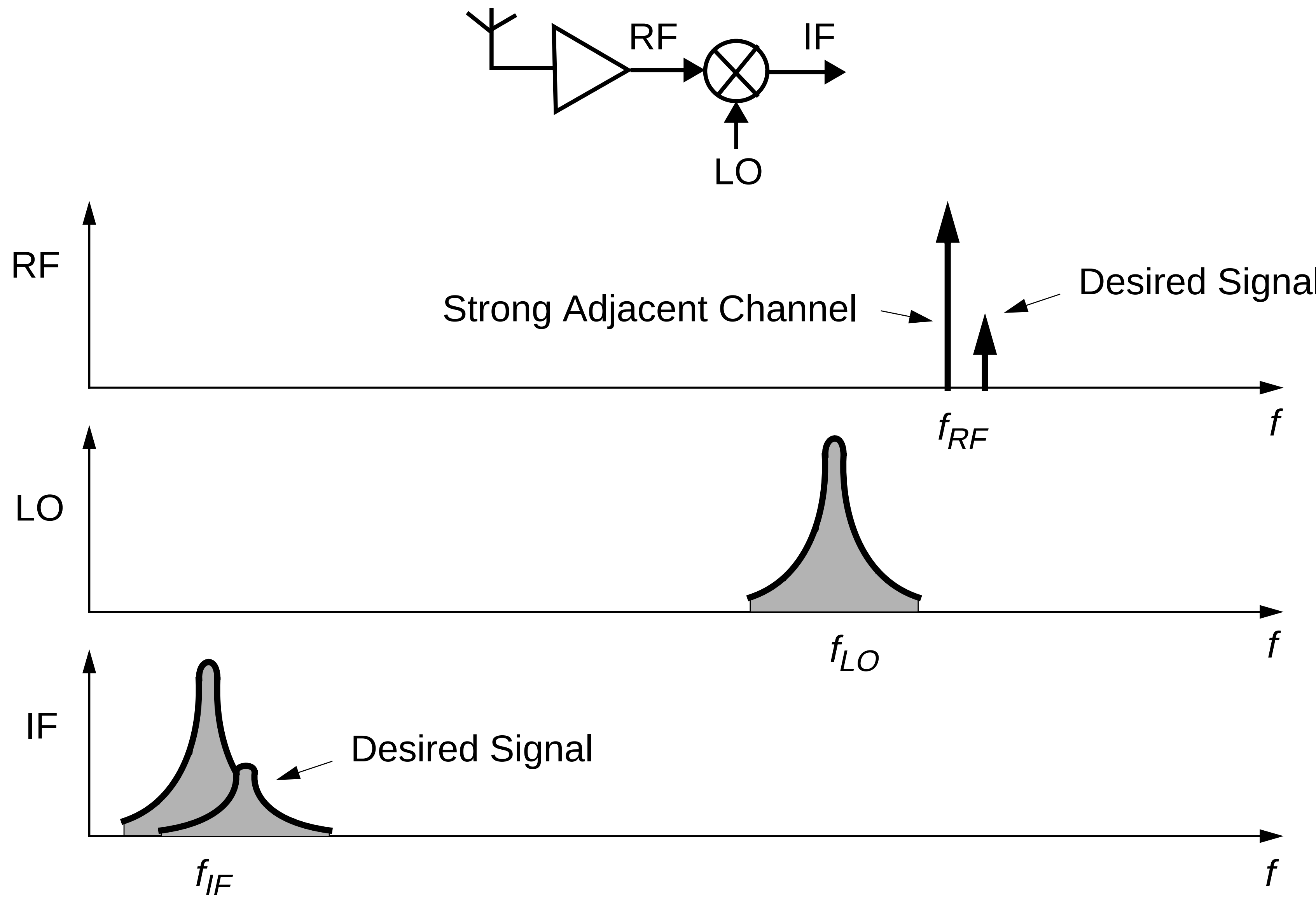
Phase Noise in RF Applications



The desired signal is buried under the phase noise of an adjacent strong channel.

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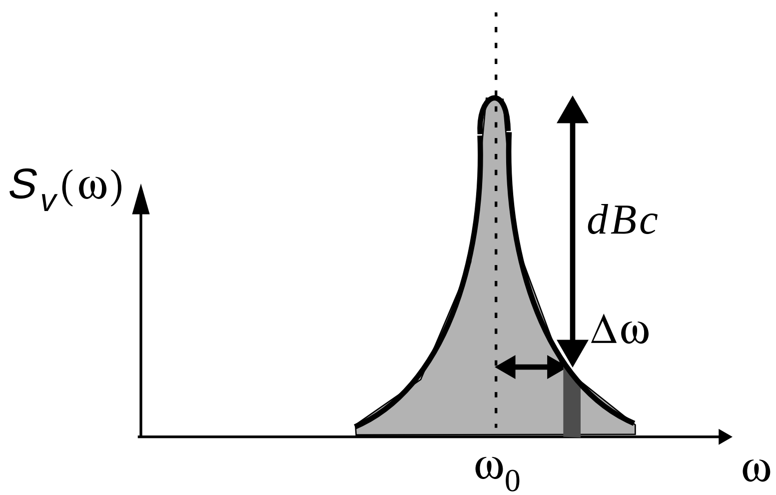
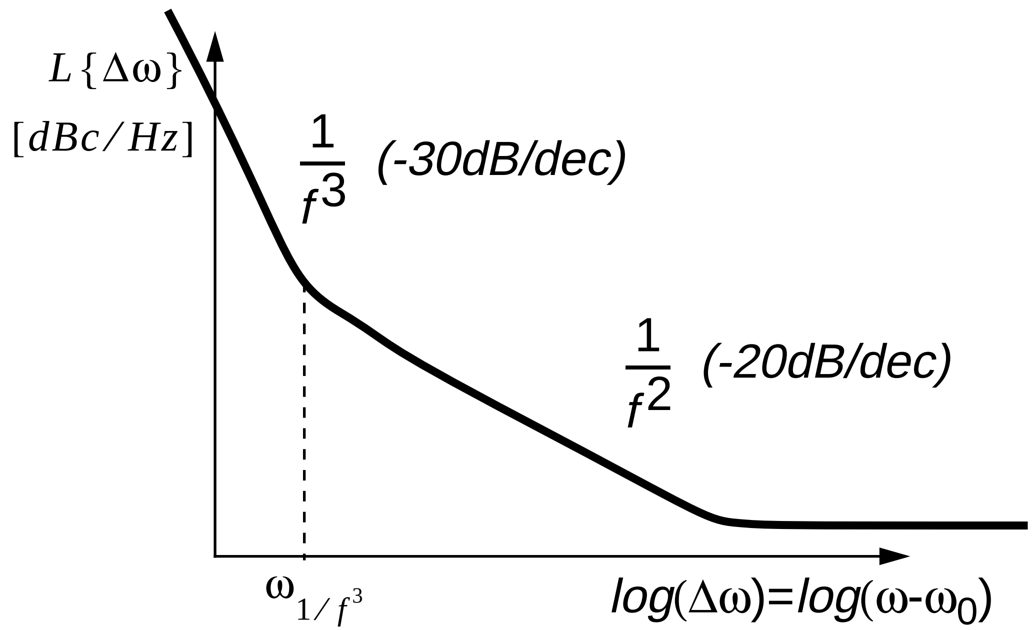
Units of Phase Noise





Measured in dB below carrier per unit bandwidth.

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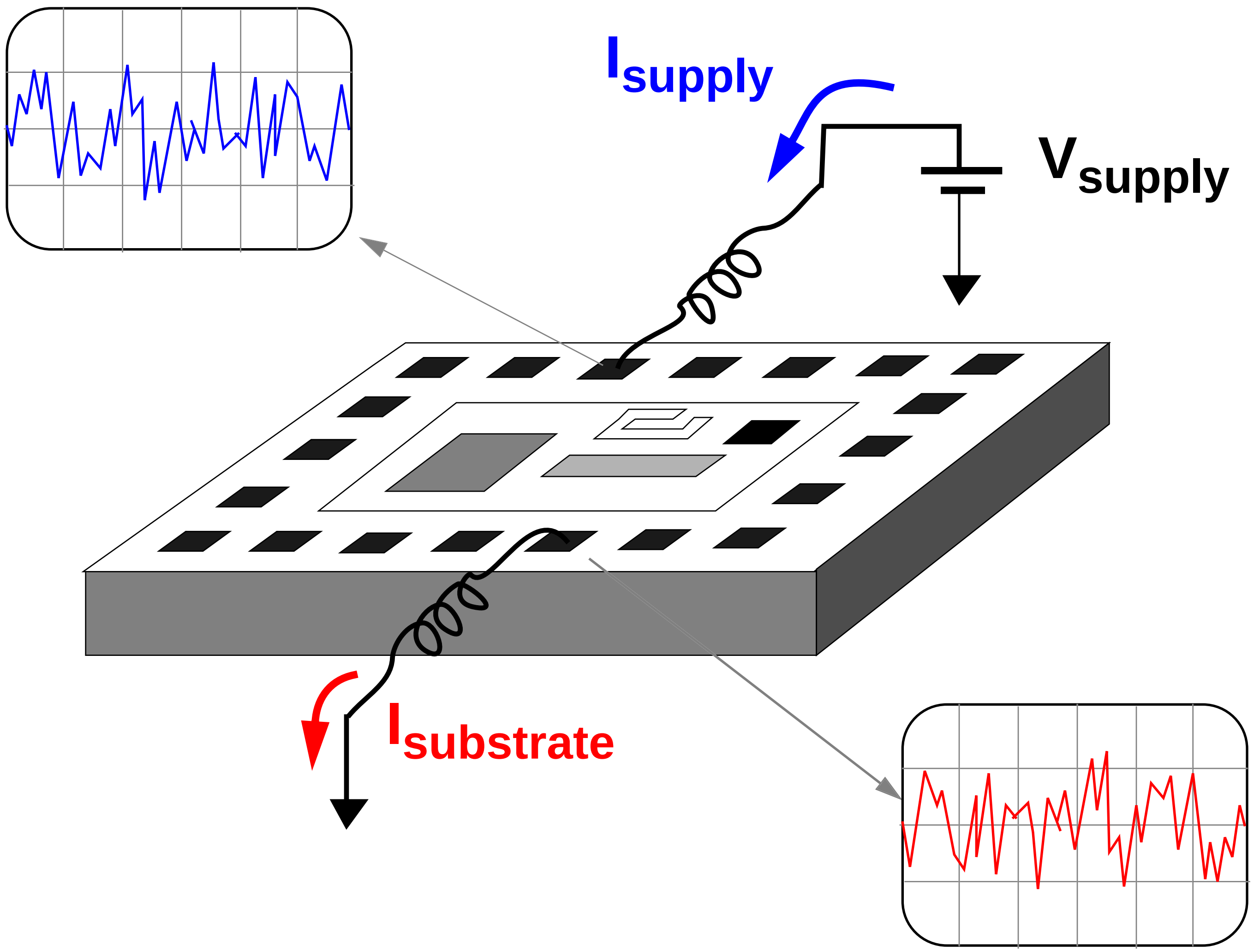
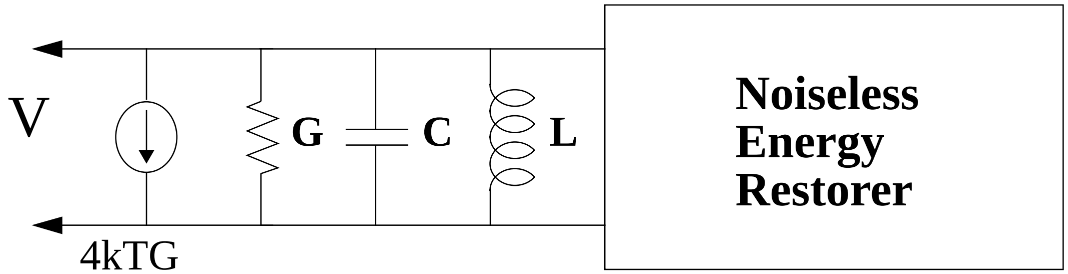
Substrate and Supply Noise



Phase Noise: General Considerations

- Consider simple oscillator: ***RLC* + *noiseless* negative *R***:



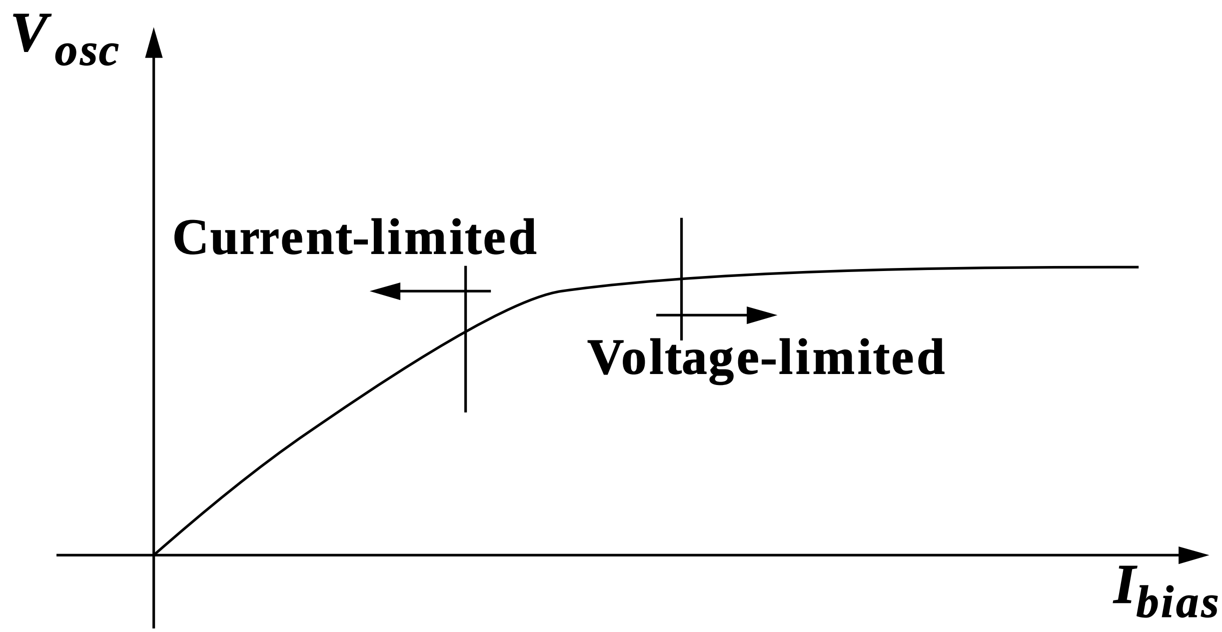
- Can show that noise-to-signal power ratio is

$$\frac{N}{S} = \frac{\overline{V_n^2}}{\overline{V_{sig}^2}} = \frac{kT}{E_{stored}} = \frac{\omega kT}{QP_{diss}}$$

- **Negative-*R* must cancel the tank loss in steady state.**
- **Noise current sees pure *LC* impedance.**

General Considerations

- Practical oscillators operate in one of two regimes:
  - *Current-limited*, in which the oscillation amplitude is linearly proportional to $I_{bias}R_{tank}$.
  - *Voltage-limited*, in which the oscillation amplitude is largely independent of bias current.



General Considerations

- In the voltage-limited regime, increases in bias current do not increase carrier power.
  - Additional dissipation only increases noise power, so CNR degrades (decreases).
- In the current-limited regime, increases in bias current increase signal power faster than noise power.
  - CNR increases until boundary with voltage-limited region is approached.
- Best oscillator performance is typically achieved near the transition point between current- and voltage-limited modes of operation.

Oscillator Phase Noise

- An expression for noise-to-carrier ratio reveals important optimization objectives:
  - $V_{carrier} \propto I_{bias} R_{tank} \Longrightarrow P_{carrier} \propto (I_{bias})^2 R_{tank}$ in the current-limited regime of oscillation.
  - $P_{noise} = kT/C = kT\omega^2 L$, if dominated by tank loss.
  - So, N/C $\propto kT\omega^2 L/(I_{bias})^2 R_{tank}$ to an approximation.
- Generally want to *minimize* L/R to optimize oscillator for a given oscillation frequency and power consumption.
  - This result contradicts much published advice, which advocates maximizing tank inductance.
- N/C is important, but also need to know noise spectrum.

General Considerations

- Assuming all noise comes from tank loss, PSD of tank voltage is given approximately by

$$\frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f} \cdot |Z|^2 = 4kTG\left(\frac{1}{G} \cdot \frac{\omega_0}{2Q\Delta\omega}\right)^2 = 4kTR\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2 .$$

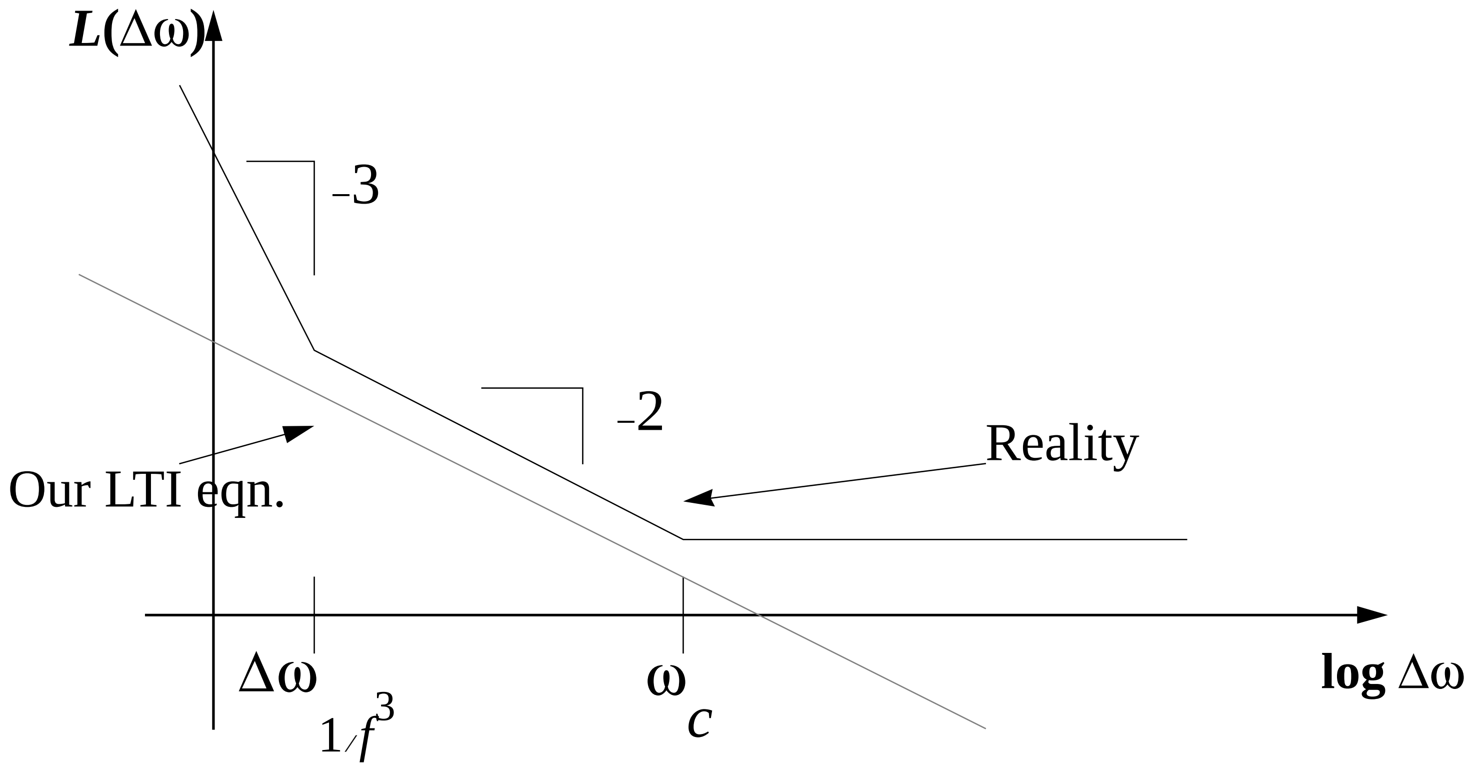
- Noise power splits evenly between phase and amplitude domains. Then, we finally have

$$L\{\Delta\omega\} = 10 \cdot \log\left[\frac{\overline{v_n^2}/\Delta f}{\overline{v_{sig}^2}}\right] = 10 \cdot \log\left[\frac{2kT}{P_{sig}} \cdot \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right] .$$

  - Funny units: Some dBc/hertz at a certain offset frequency. Example: –110dBc/Hz @ 600kHz offset, at 1.8GHz.

Simple LTI Model vs. Reality

- Previous expression doesn't quite describe real phase noise spectra:



Leeson Model

- Leeson provided empirical fix to remove discrepancies:

$$L\{\Delta\omega\} = 10 \cdot \log\left[\frac{2FkT}{P_{sig}} \cdot \left\{1 + \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right\} \cdot \left(1 + \frac{\Delta\omega_{1/f^3}}{|\Delta\omega|}\right)\right]$$

- Factor *F* accounts for excess noise in all regions.
- $\Delta\omega_{1/f^3}$ accounts for $1/f^3$ region close to carrier.
- First additive factor of 1 accounts for noise floor.
- Problem: Can't compute these fudge factors *a priori*; they are basically *post hoc* fitting parameters.
- Need to revisit unstated assumptions.
  - Is an oscillator truly a linear, time-invariant system?

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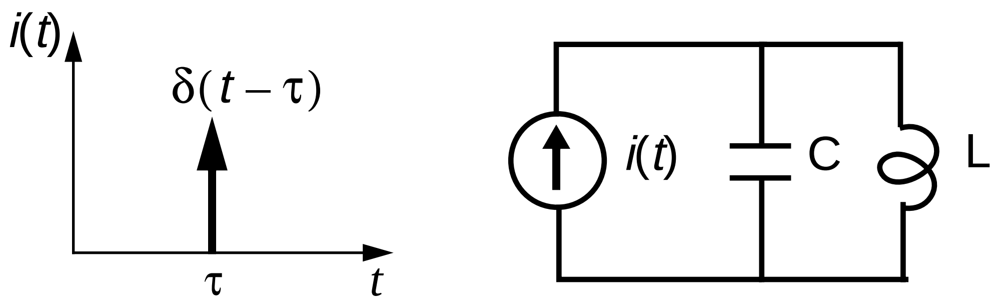
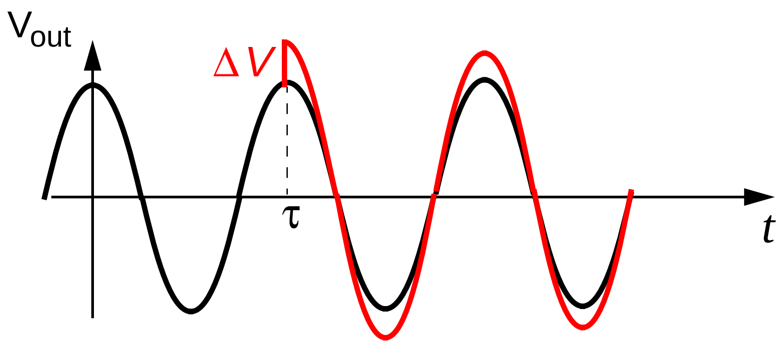
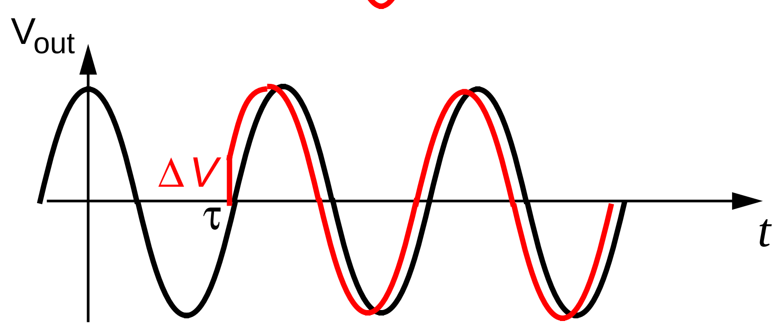
Oscillators Are Time-Variant Systems





Impulse injected at the peak of amplitude.



Impulse injected at zero crossing.

Even for an ideal LC oscillator, the phase response is *Time Variant.*

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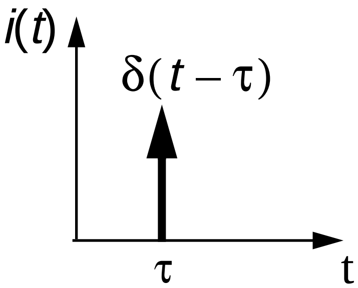
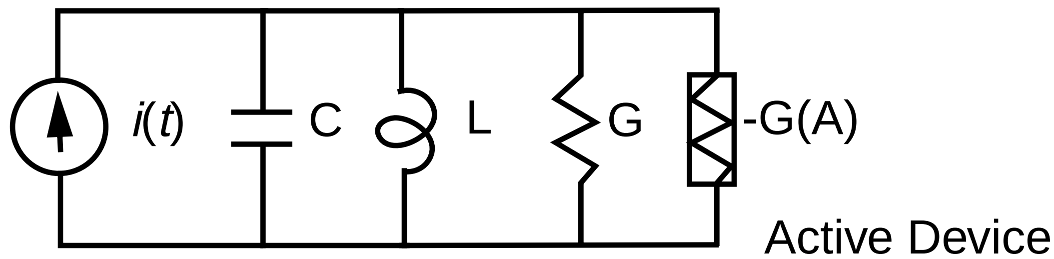
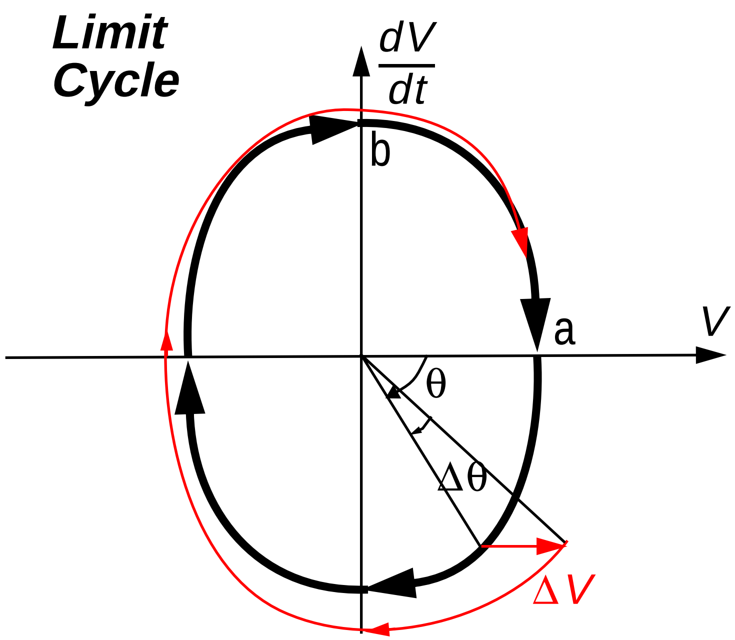
Amplitude Restoring Mechanism





Once Introduced, phase error persists indefinitely.

Non-linearity quenches amplitude changes over time.



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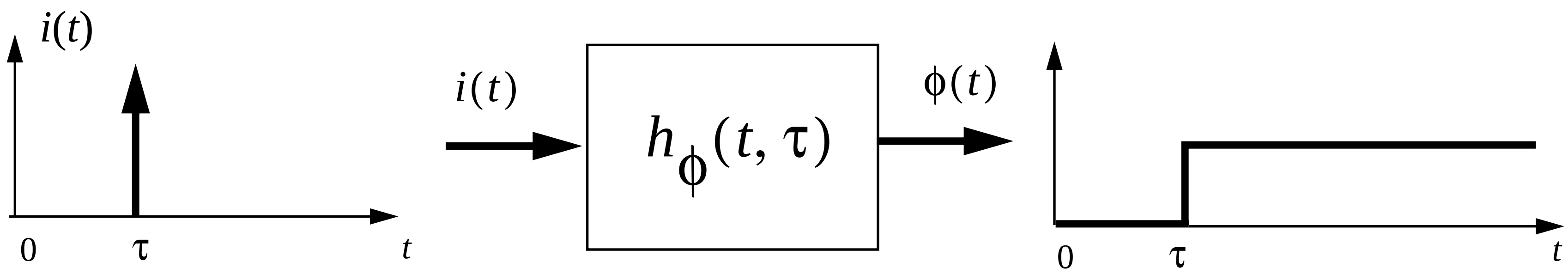
Phase Impulse Response

The phase impulse response of an arbitrary oscillator is a time varying step.



The unit impulse response is:

$$h_\phi(t, \tau) = \frac{\Gamma(\omega_o \tau)}{q_{max}} u(t - \tau)$$

$\Gamma(x)$ is a dimensionless function periodic in 2π, describing how much phase change results from applying an impulse at time: $t = T\frac{x}{2\pi}$

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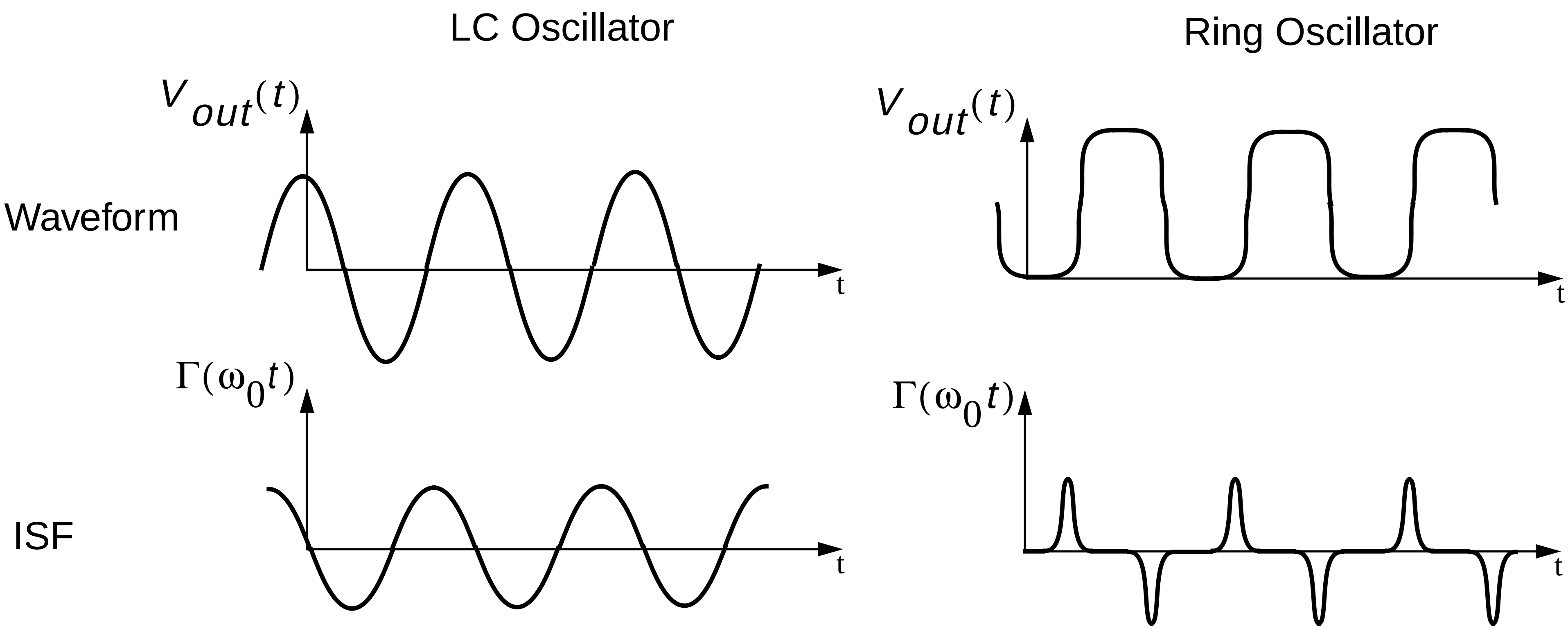
Impulse Sensitivity Function (ISF)



The ISF quantifies the sensitivity of every point in the waveform to perturbations.

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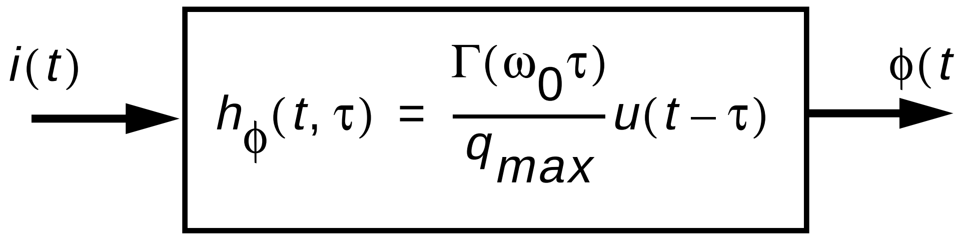
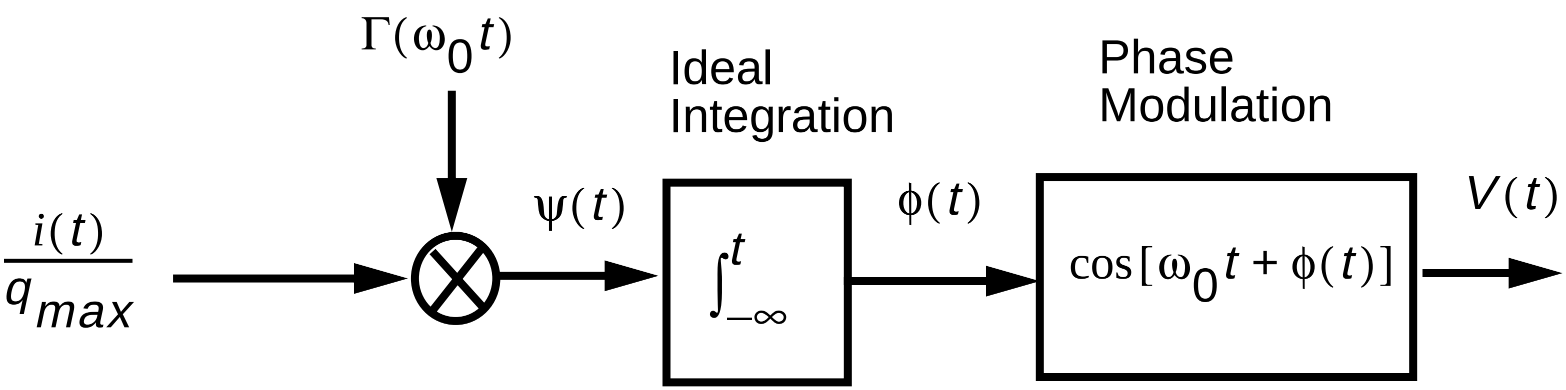
Phase Response to an Arbitrary Source



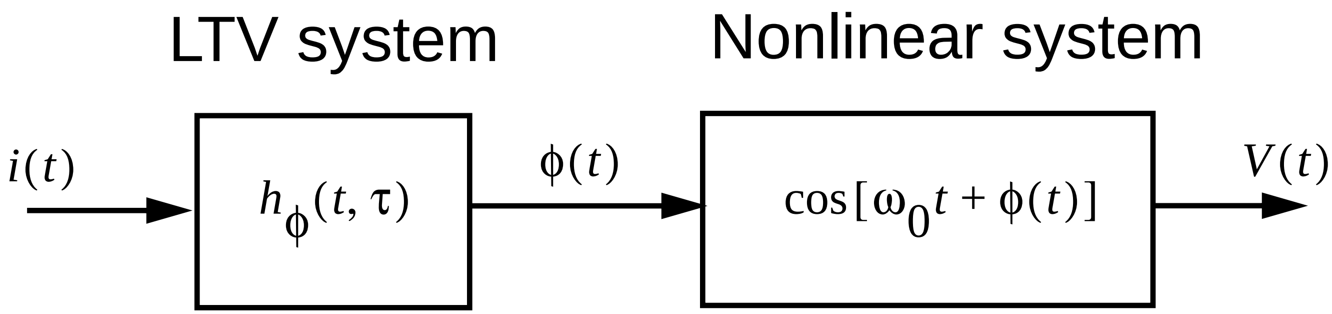
Superposition Integral:

$$\phi(t) = \int_{-\infty}^{\infty} h_{\phi}(t,\tau)i(\tau)d\tau = \frac{1}{q_{max}}\int_{-\infty}^{t} \Gamma(\omega_0\tau)i(\tau)d\tau$$

Equivalent representation:



Phase Noise Due to White Noise



For a white input noise current with the spectral density of $\frac{\overline{i_n^2}}{\Delta f}$

The phase noise sideband power below carrier at an offset of $\Delta\omega$ *is:*

$$L\{\Delta\omega\} = \frac{\Gamma_{rms}^2}{q_{max}^2} \cdot \frac{\overline{i_n^2}/\Delta f}{2\Delta\omega^2}$$

Γ_{rms} *is the rms value of the ISF.*

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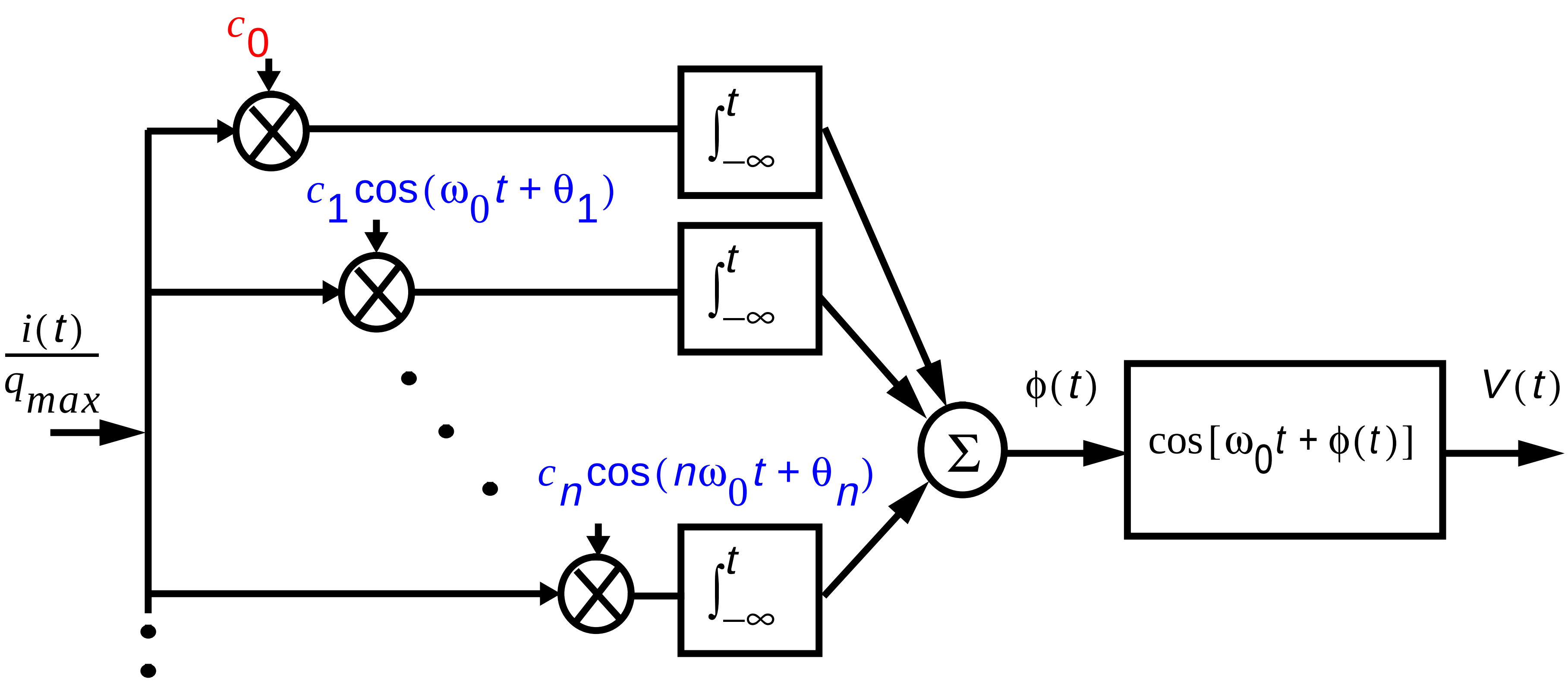
ISF Decomposition

ISF is a periodic function:

$$\Gamma(\omega_o t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0\tau + \theta_n)$$

Phase can be written as:

$$\phi(t) = \frac{1}{q_{max}}\left[c_0 \int_{-\infty}^{t} i(\tau)d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} i(\tau)\cos(n\omega_0\tau)d\tau\right]$$



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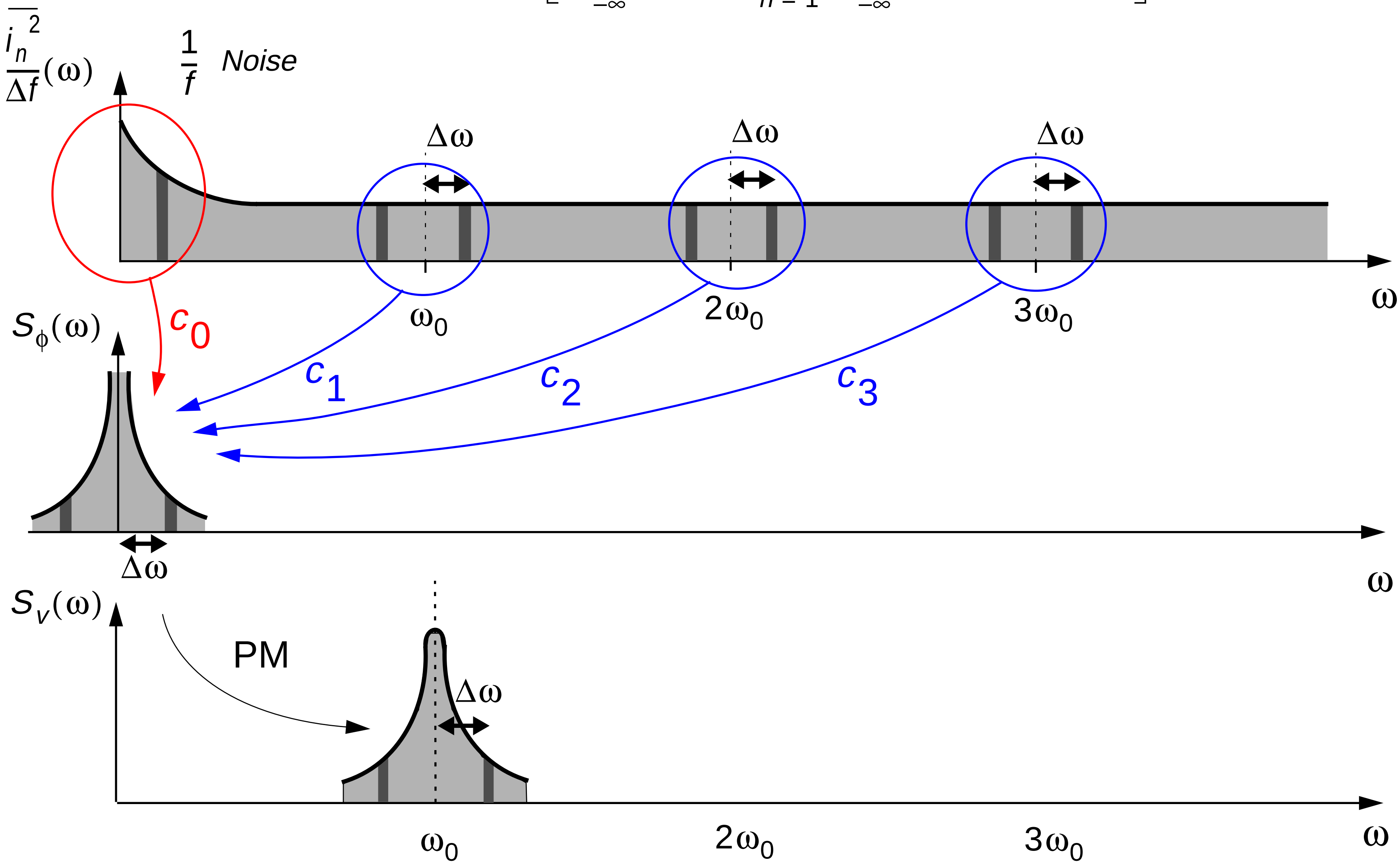
Noise Contributions from $n\omega_o$

$$\phi(t) = \frac{1}{q_{max}}\left[c_0 \int_{-\infty}^{t} i(\tau)d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} i(\tau)\cos(n\omega\tau)d\tau\right]$$



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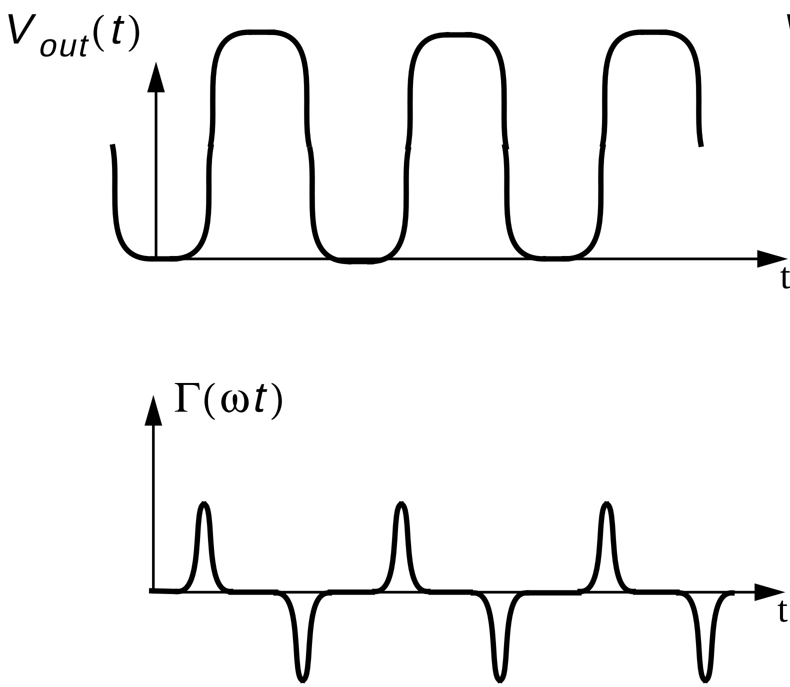
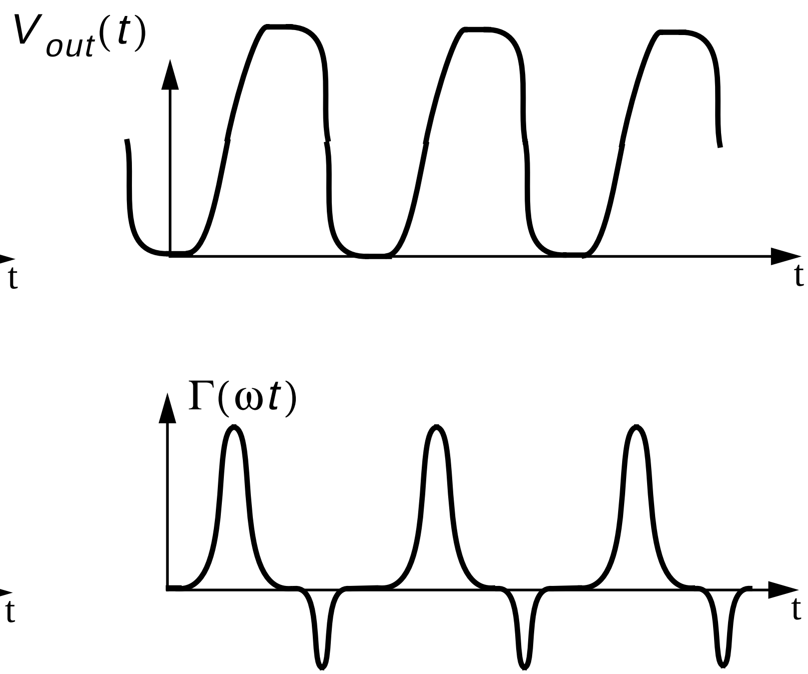
Effect of Symmetry

$$c_0 = \frac{1}{2\pi}\int_0^{2\pi} \Gamma(x)\,dx$$

Symmetric rise and fall time



Asymmetric rise and fall time



The dc value of the ISF is governed by rise and fall time symmetry, and controls the contribution of low frequency noise to the phase noise.

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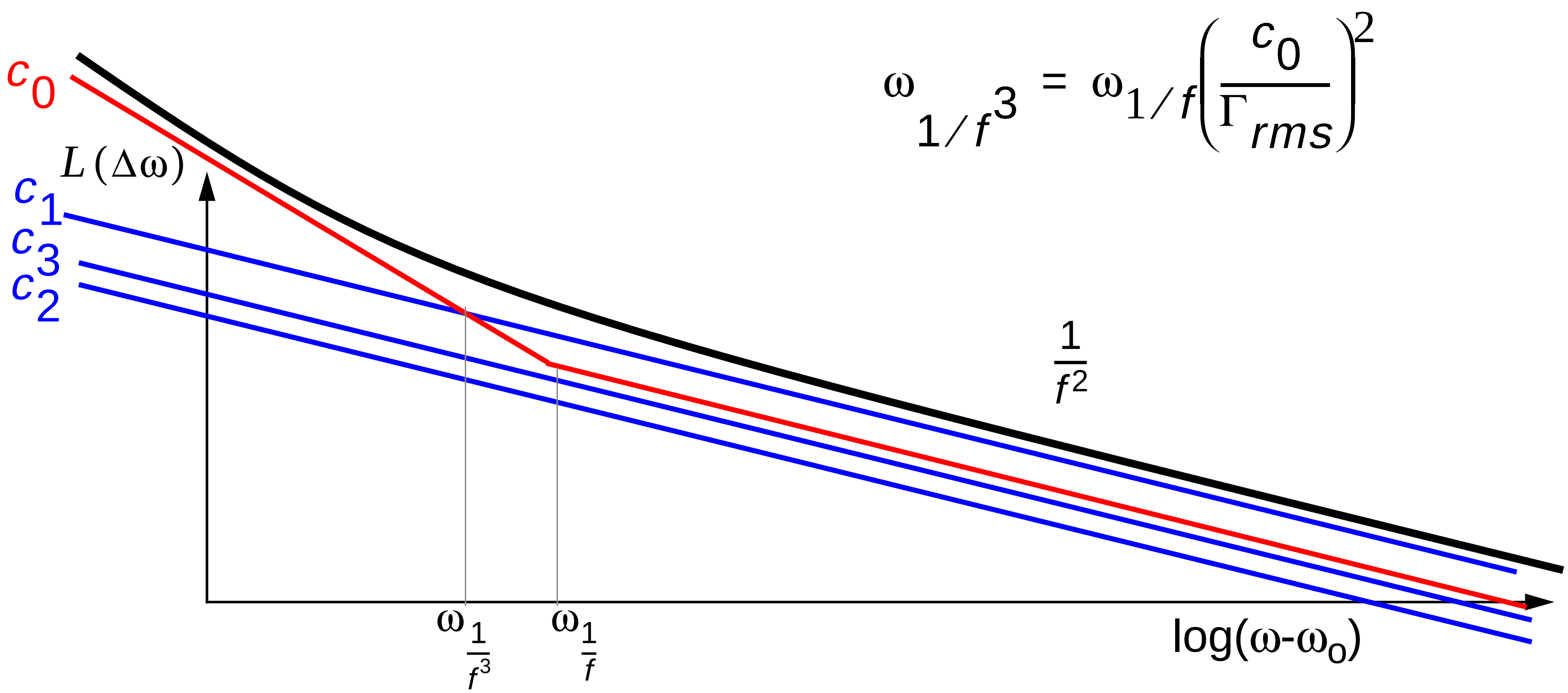
$1/f^3$ Corner of Phase Noise Spectrum

The $1/f^3$ corner of phase noise is NOT the same as 1/f corner of device noise



By designing for a symmetric waveform, the performance degradation due to low frequency noise can be minimized.

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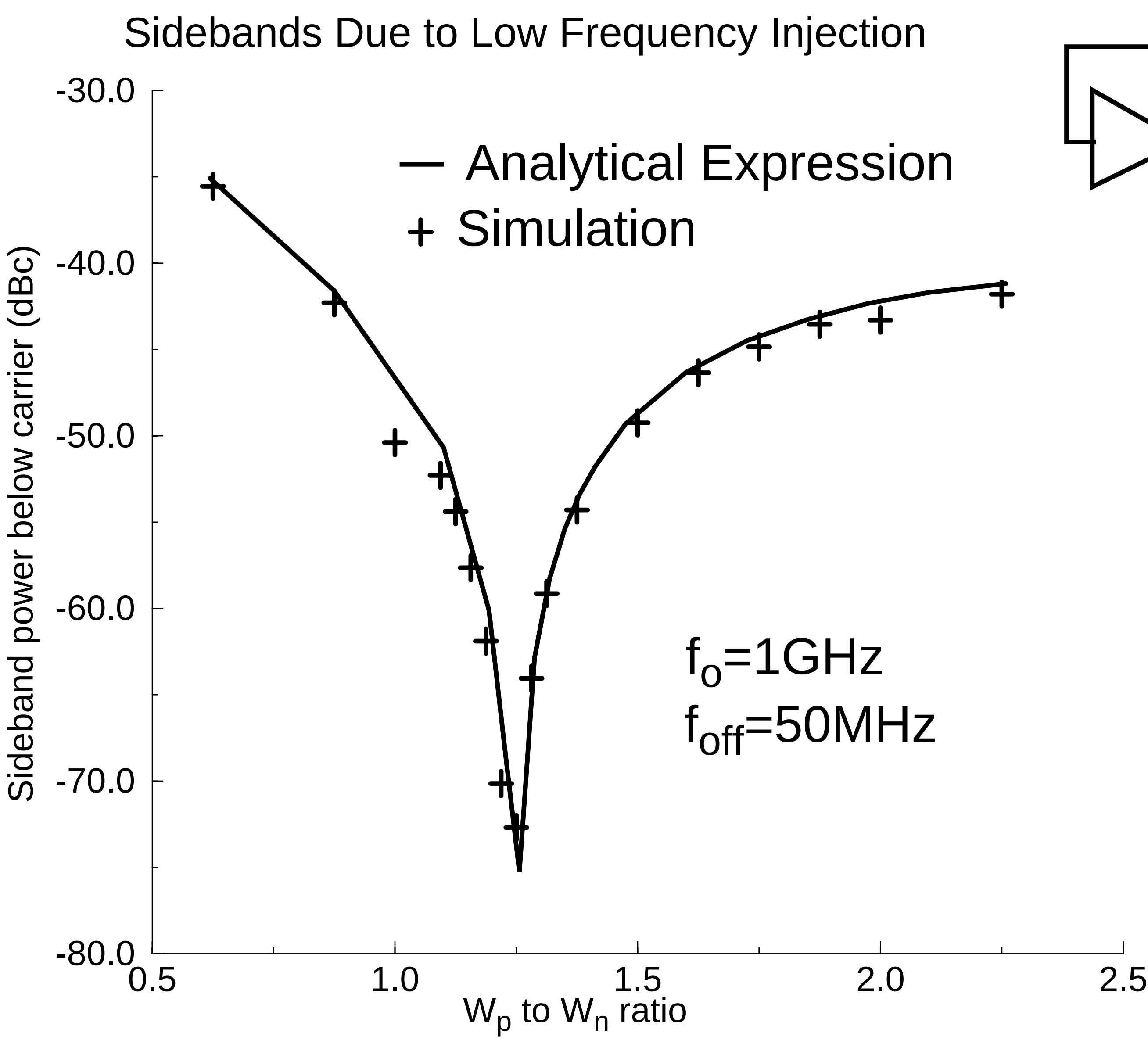
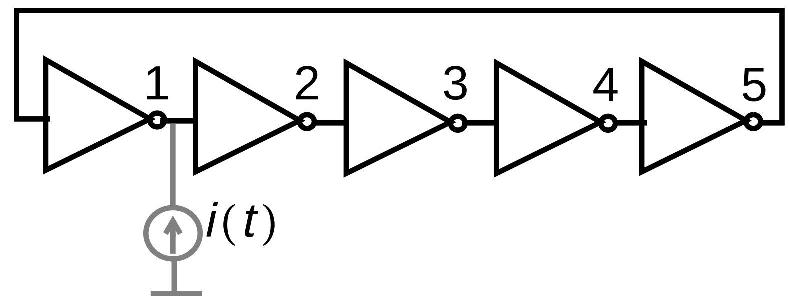
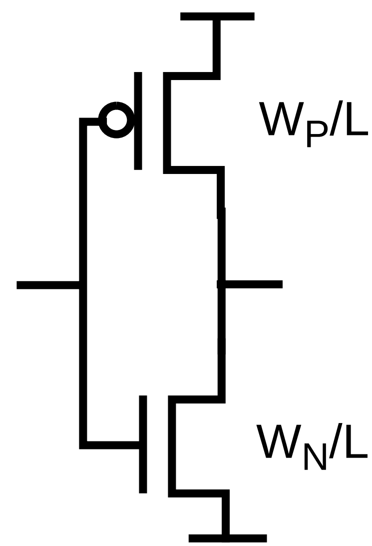
Effect of Rise and Fall Time Symmetry







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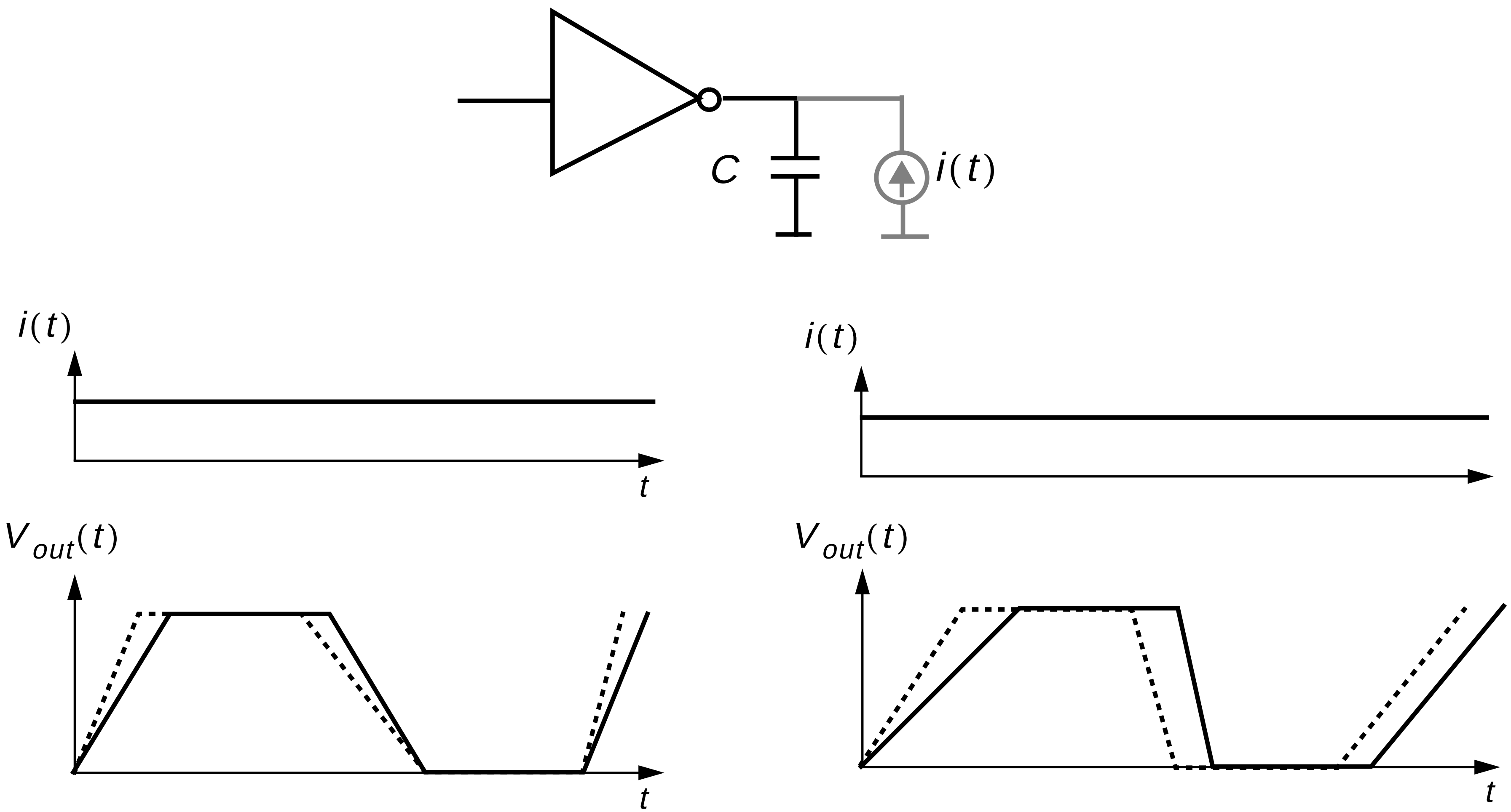
Effect of Symmetry



A low frequency current induces a frequency change for the asymmetric waveform.

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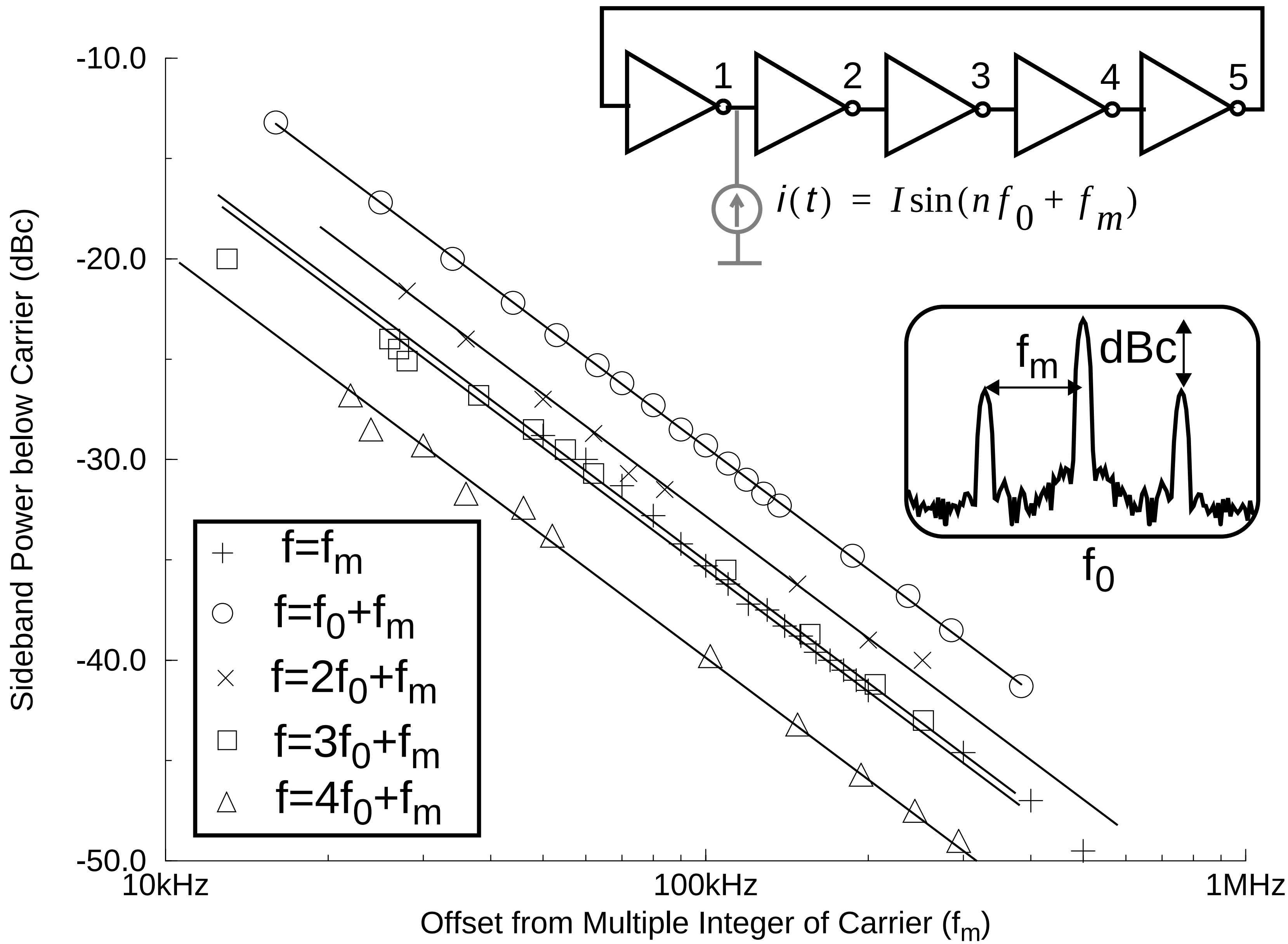
Injection at Integer Multiples of f_0



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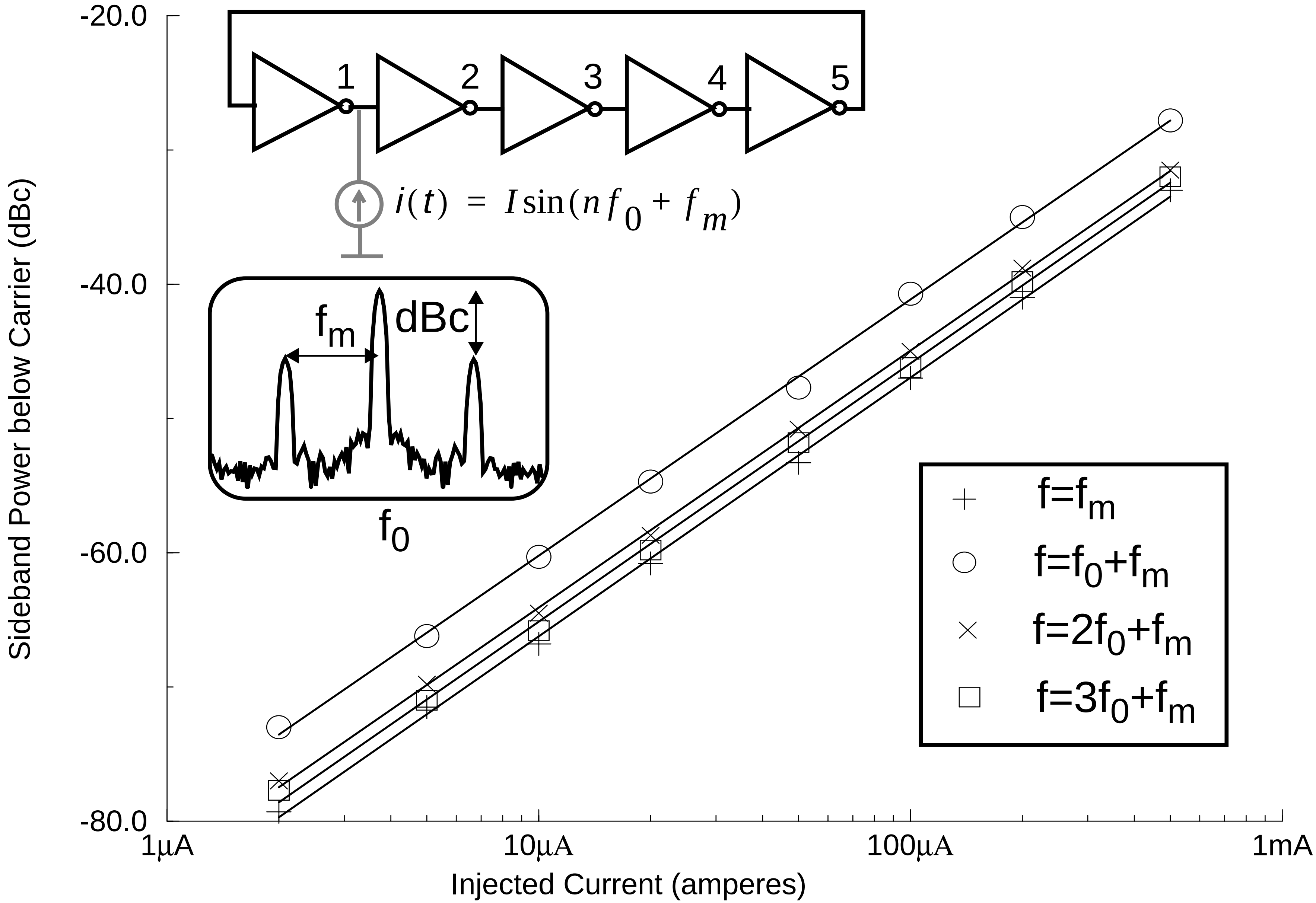
Sideband Power vs. Injection Current



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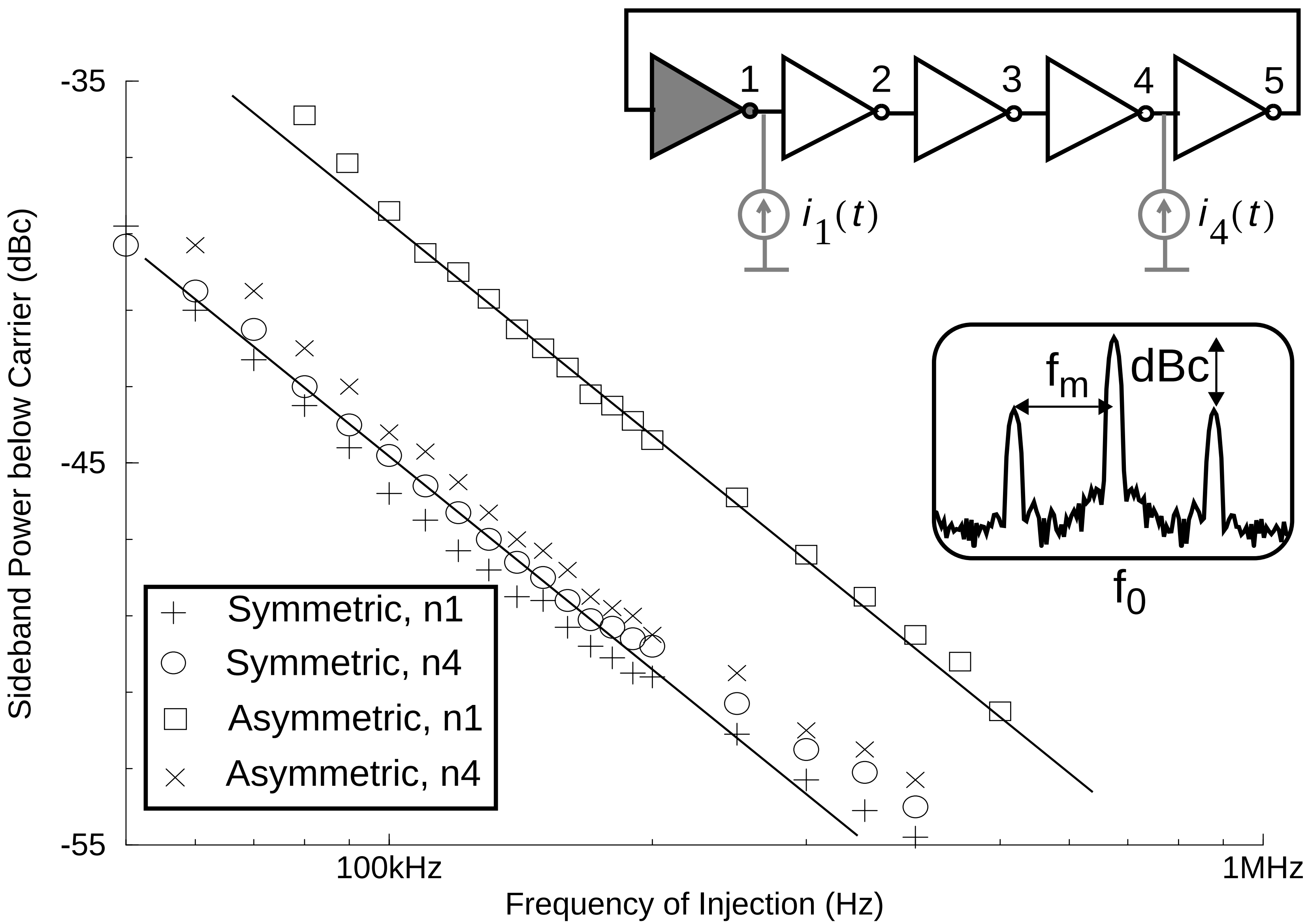
Symmetric vs. Asymmetric Ring Oscillator



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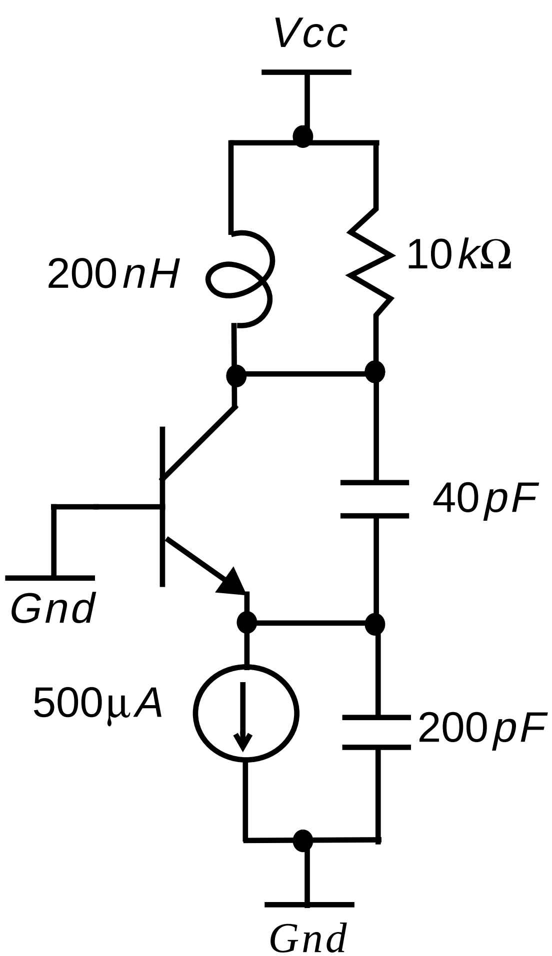
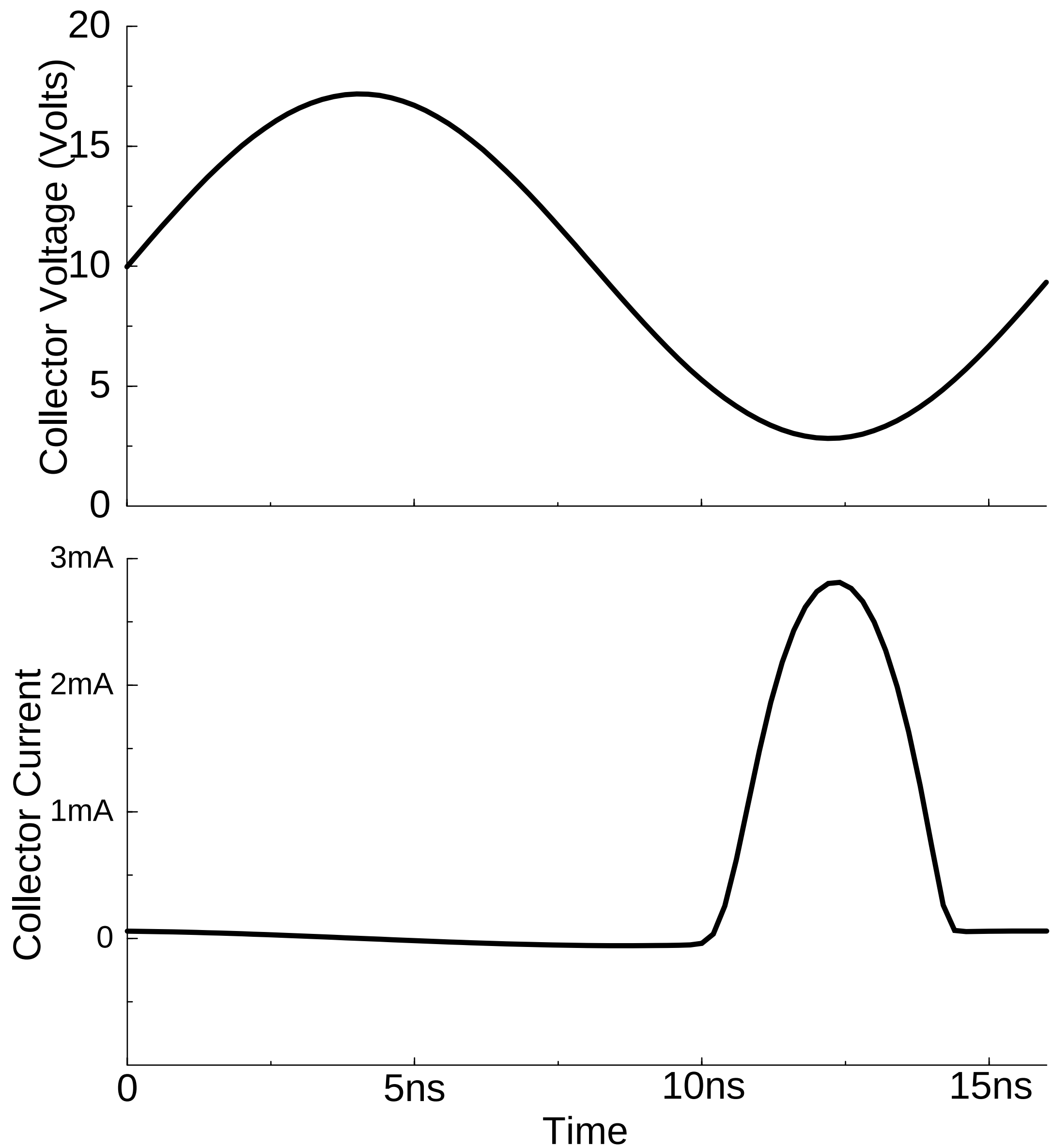
Time Varying Current in Colpitts Oscillator





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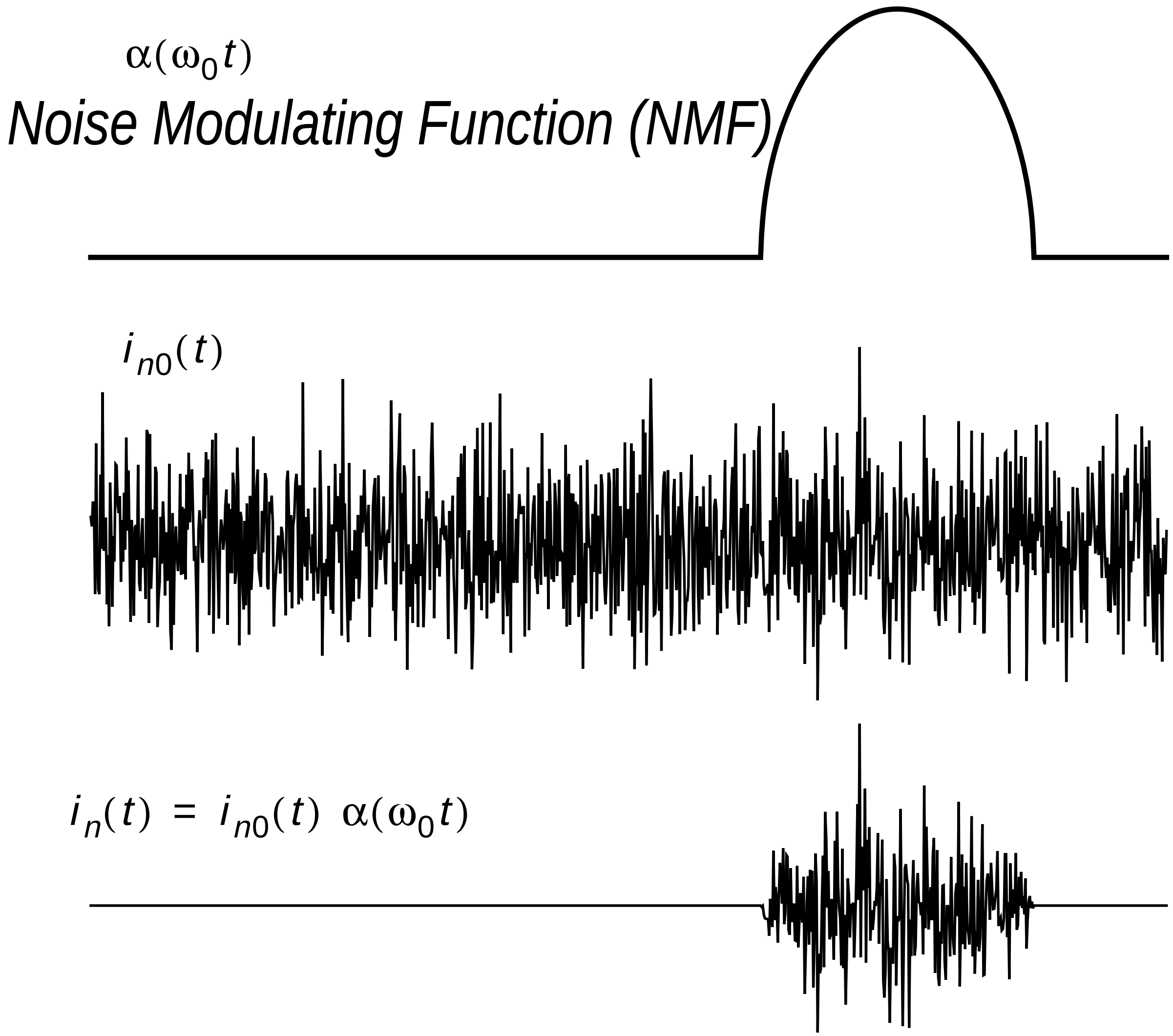
Cyclostationary Properties, Time Domain



$$\phi(t) = \int_{-\infty}^{t} i_n(\tau)\frac{\Gamma(\omega_0\tau)}{q_{max}}d\tau$$

$$= \int_{-\infty}^{t} i_{n0}(\tau)\frac{\alpha(\omega_0\tau)\Gamma(\omega_0\tau)}{q_{max}}d\tau$$

Effective ISF:

$$\Gamma_{eff}(x) = \Gamma(x)\cdot\alpha(x)$$

A cyclostationary source can be modeled as stationary with a new ISF.

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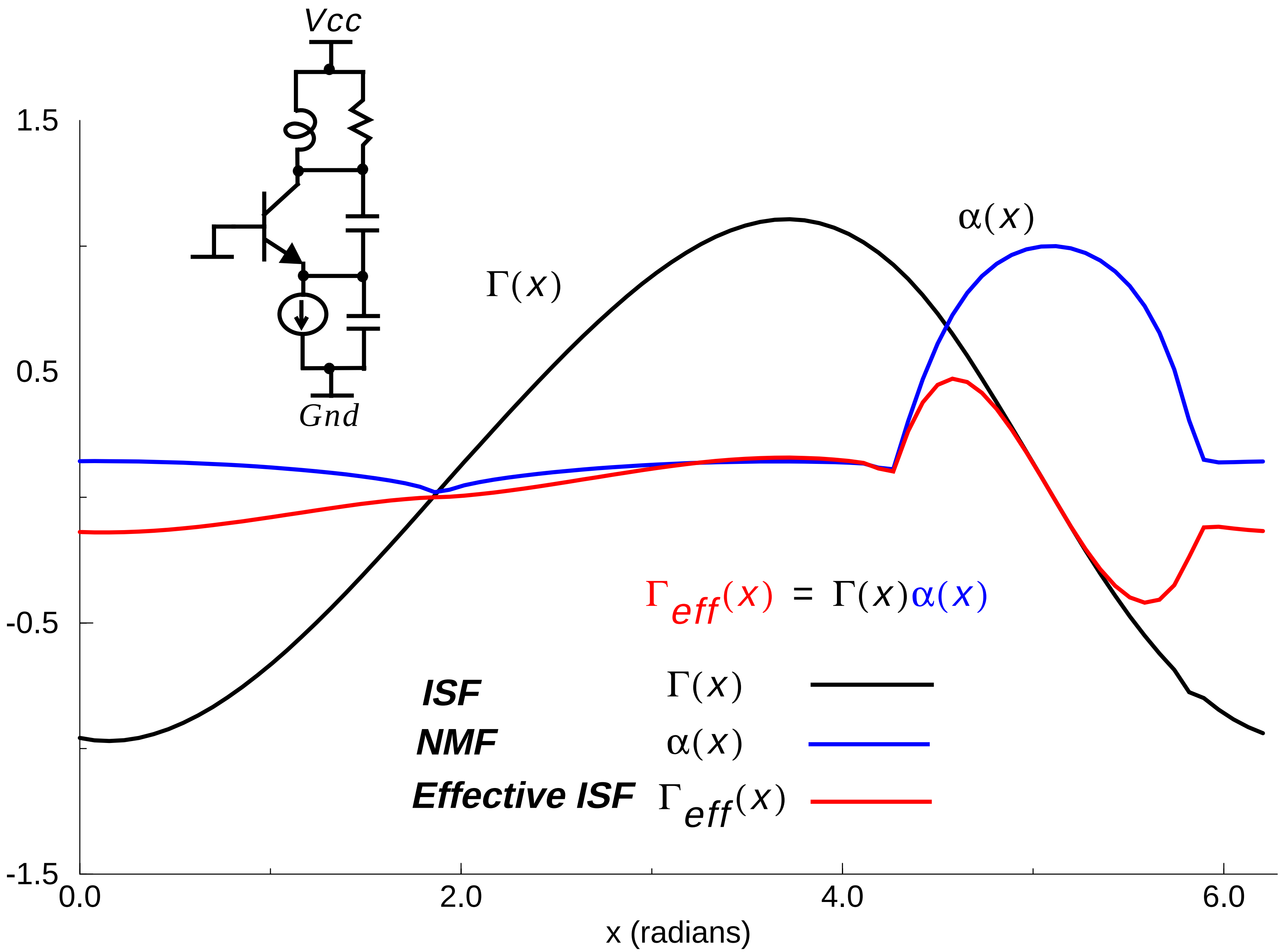
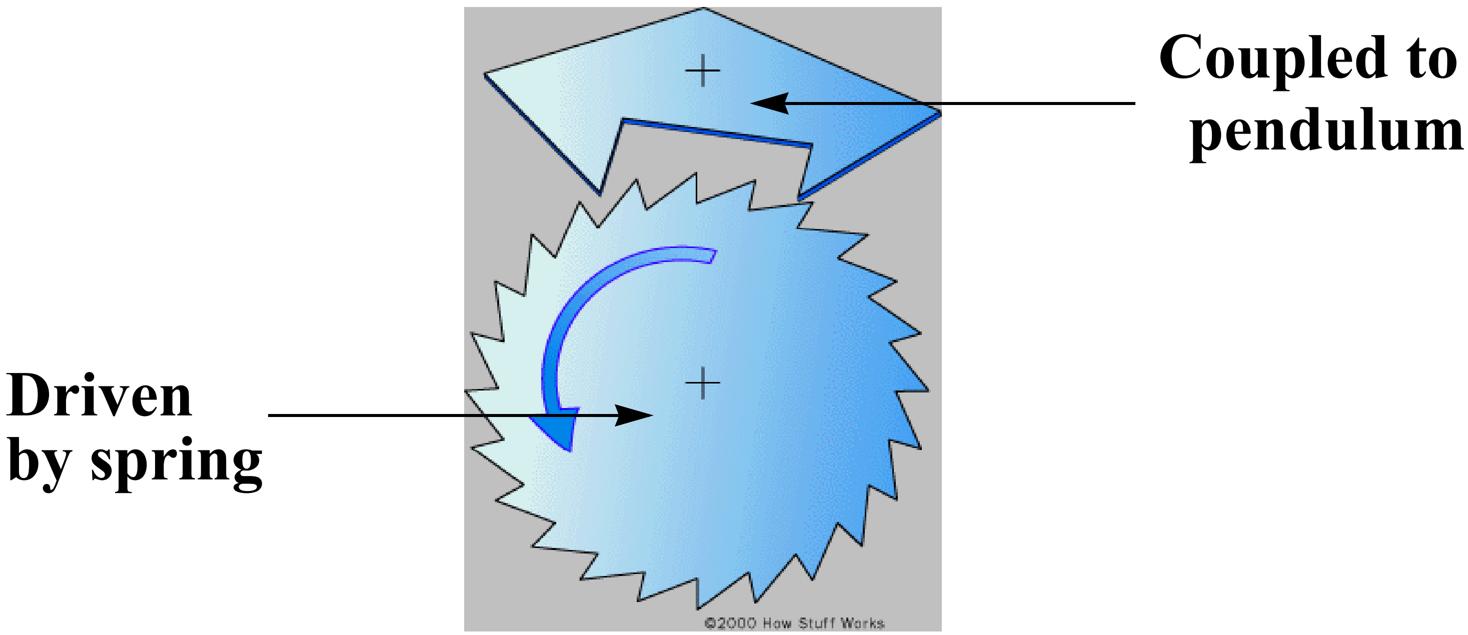
Colpitts Oscillator



Plus ça change...

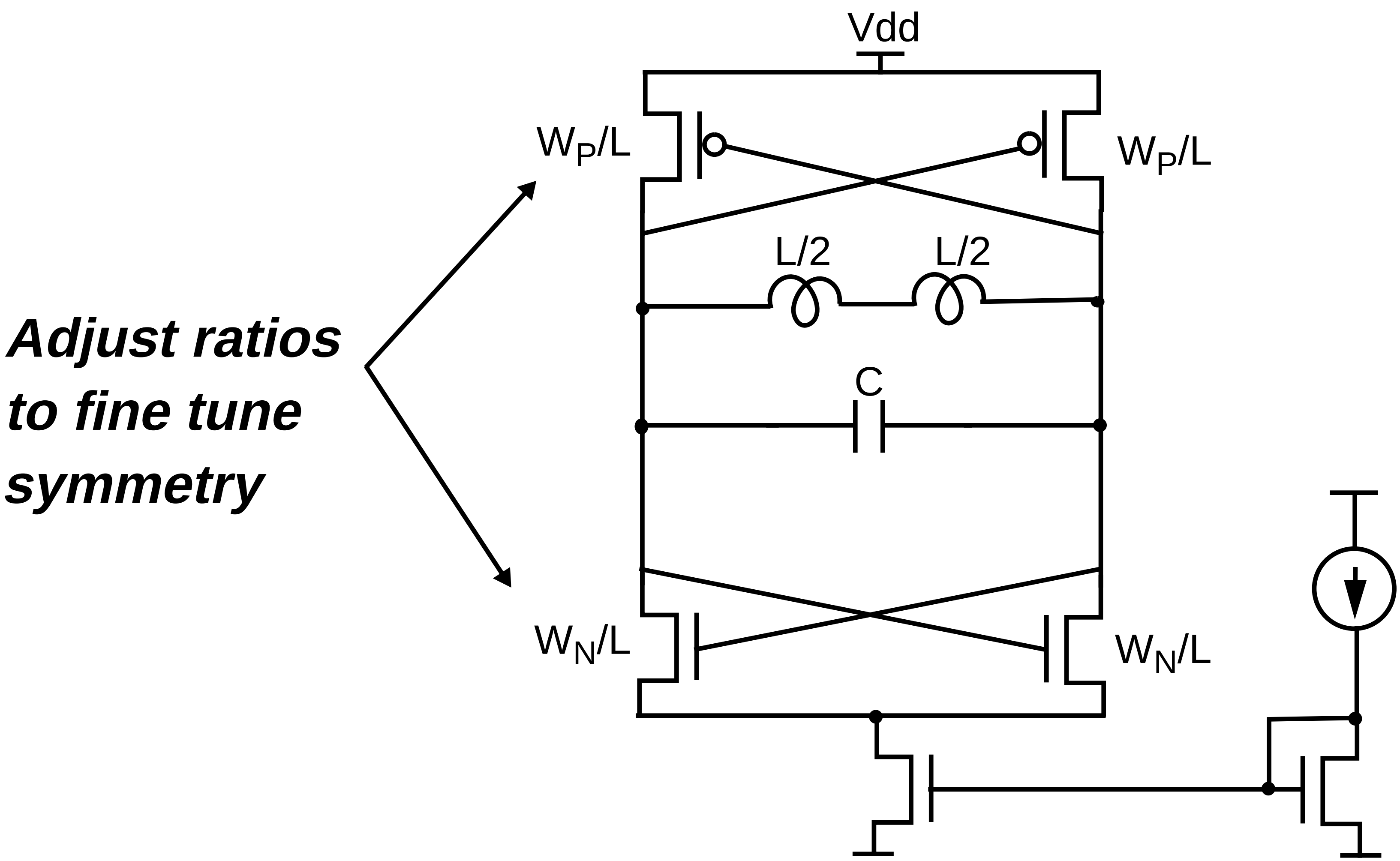
- To exploit cyclostationary effects, arrange to supply energy to the tank impulsively, where the ISF is a minimum.
- This idea is actually very old; mechanical clocks use an *escapement* to deliver energy from a spring, to a pendulum in impulses.



Phase Noise

- In the best implementations, impulses are delivered at or near the pendulum's velocity maxima. The escapement thus restores energy without disturbing the period of oscillation. [Airy, 1826]
  - See: www.database.com/~lemur/dmh-airy-1826.html (my thanks to Byron Blanchard for finding this reference.)
- Similarly, the optimal moments for an *LC* oscillator are near the voltage maxima.

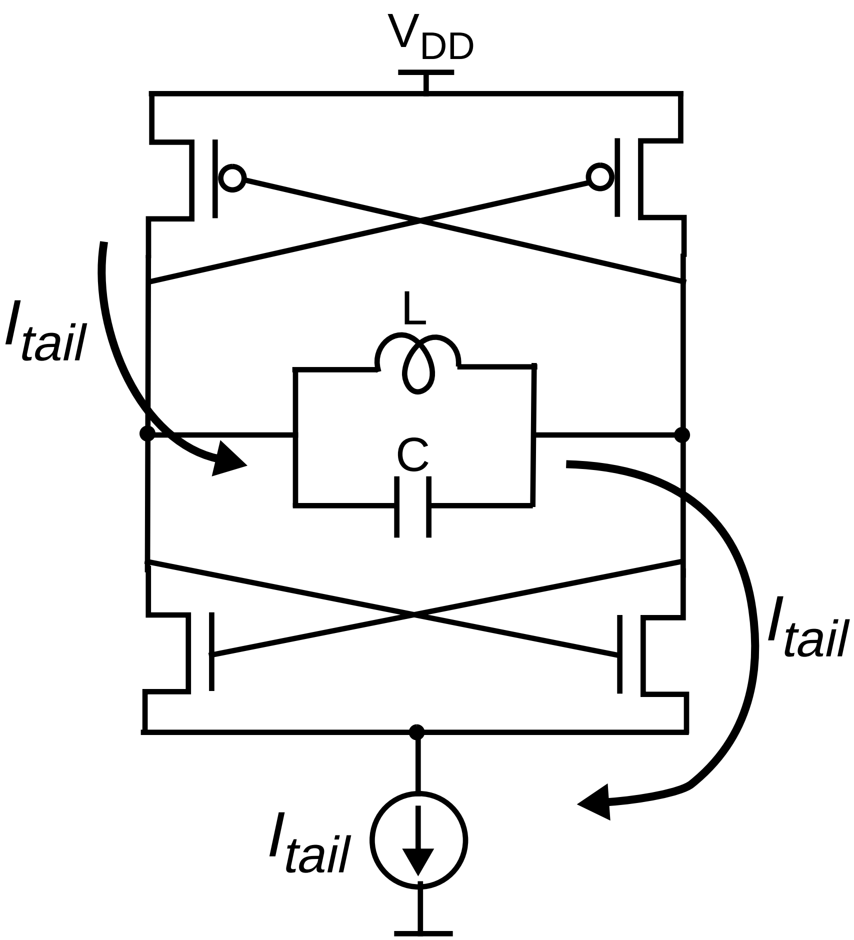
A Symmetric LC Oscillator



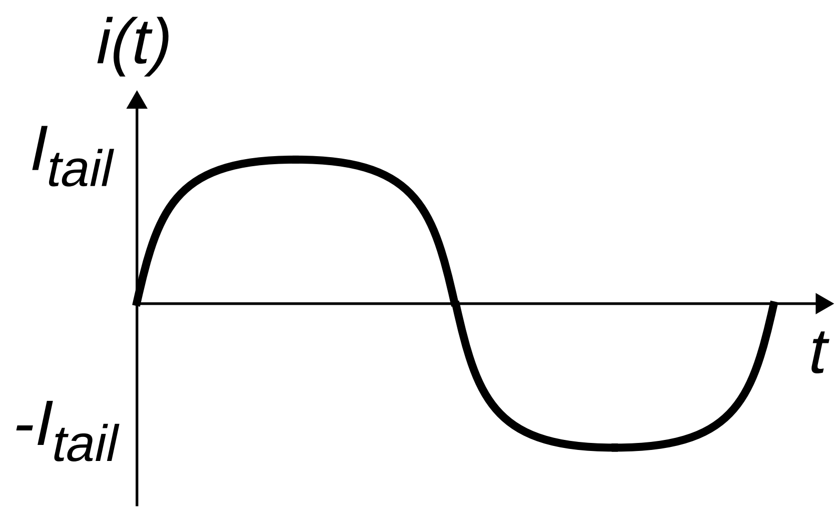
Uses the same current twice for high transconductance.

[Also appears in: J.Craninckx, *et al*, Proceedings of CICC 97.]

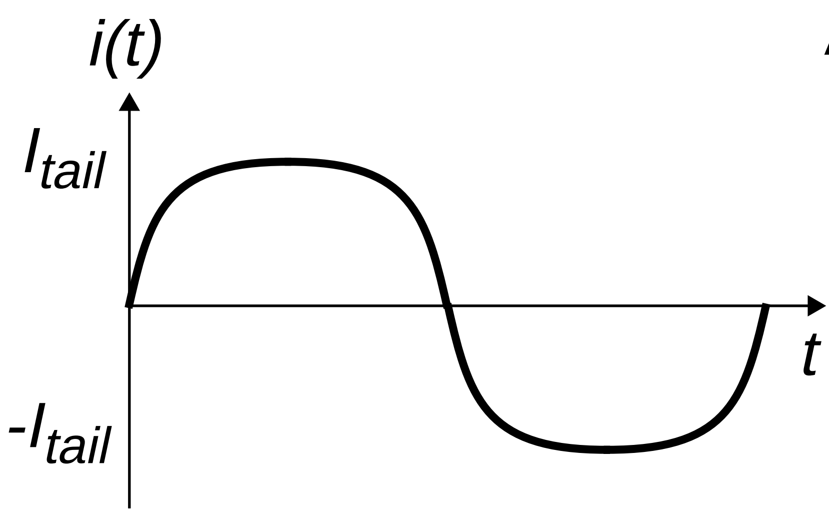
Tank Voltage Amplitude



Assuming fast switching of the differential pair, the current can be approximated as:



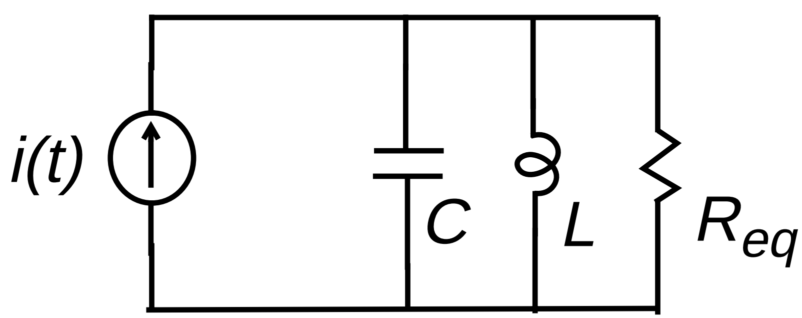
Tank Voltage Amplitude



Assuming rectangular waveform:

$$V_{max} = \frac{4}{\pi} I_{tail} R_{eq}$$

Effectively, the current waveform is closer to sinusoidal, therefore:

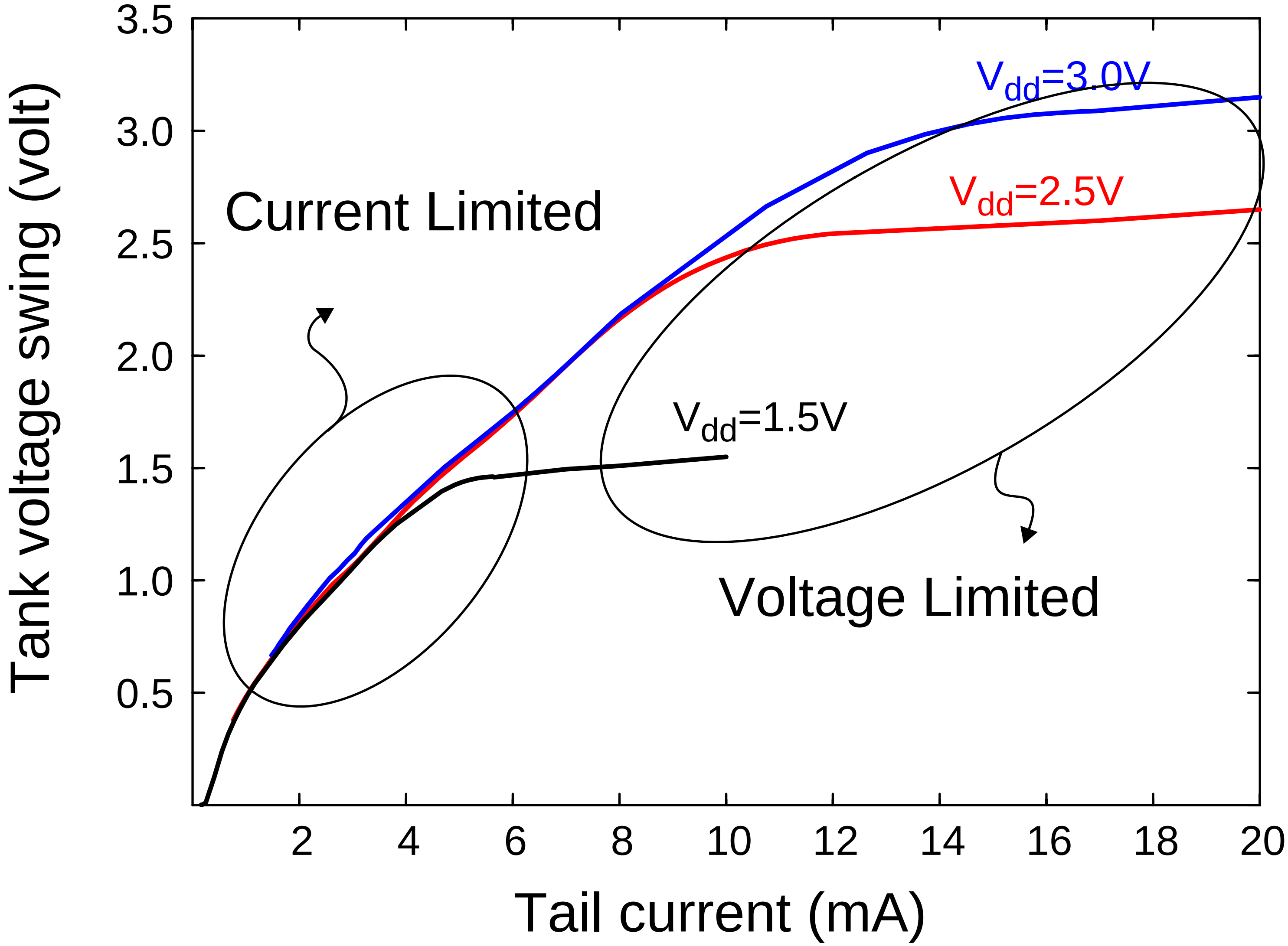


$$V_{max} \approx I_{tail} R_{eq}$$

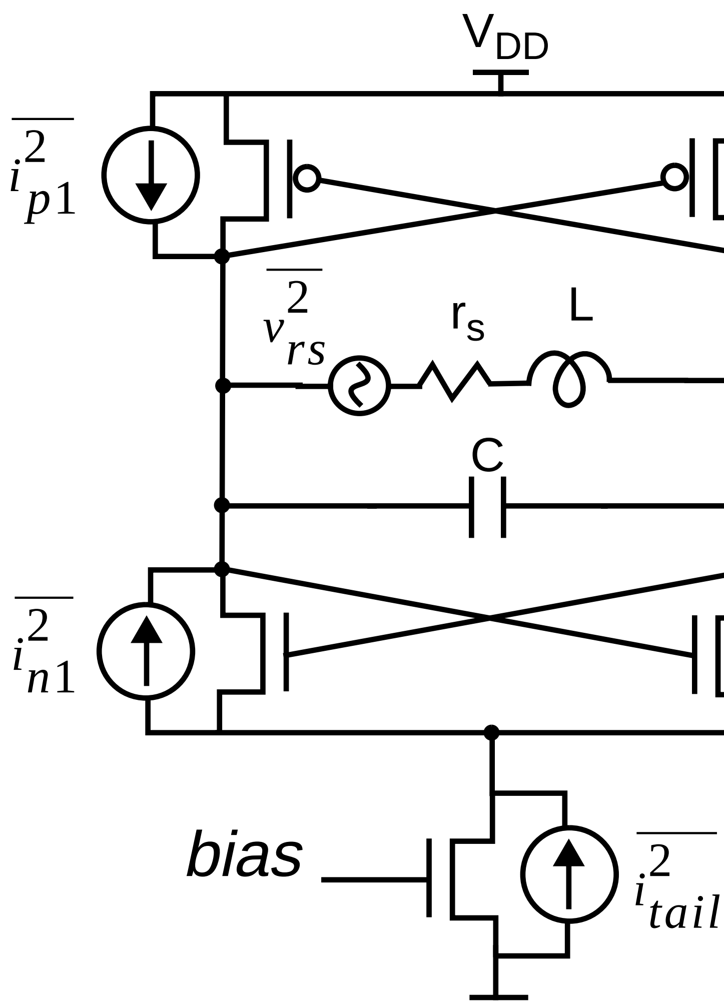
"Current limited" mode.

Modes of Amplitude Limiting

Complementary cross-coupled LC oscillator



Major Noise Sources



Different noise sources affect phase noise differently.

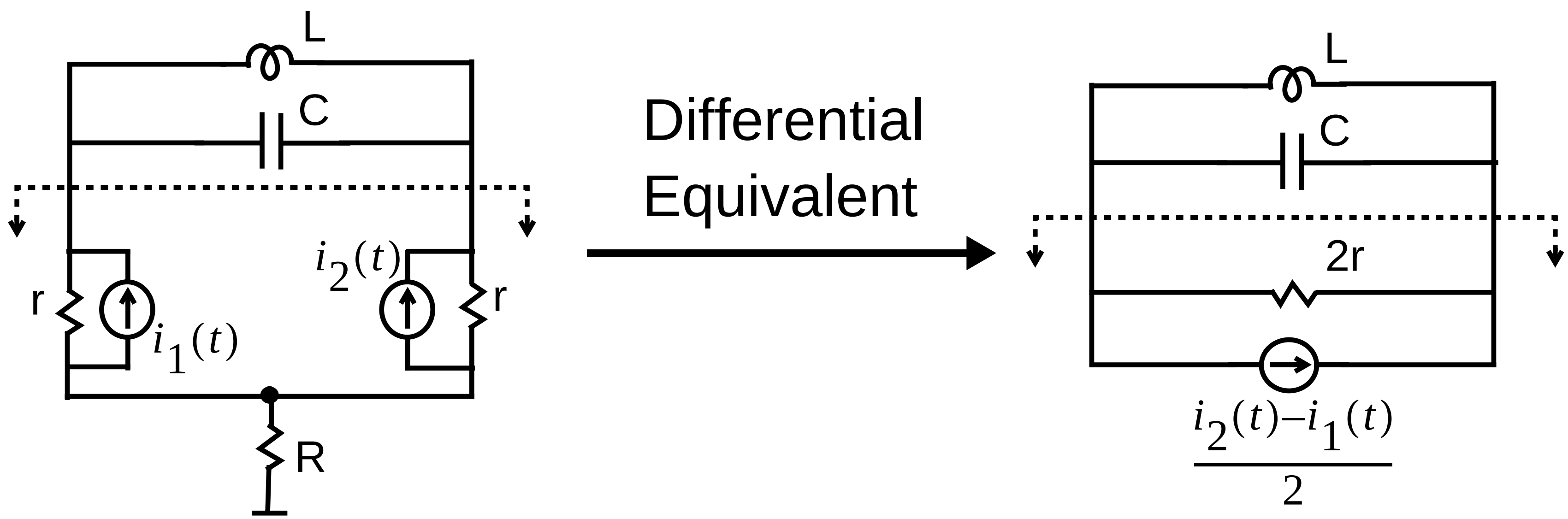
$$\frac{\overline{i_n^2}}{\Delta f} = 4kT\gamma\mu C_{ox}\frac{W}{L}(V_{GS} - V_T)$$

Valid in both long and short channel regimes.

Inductor Noise:

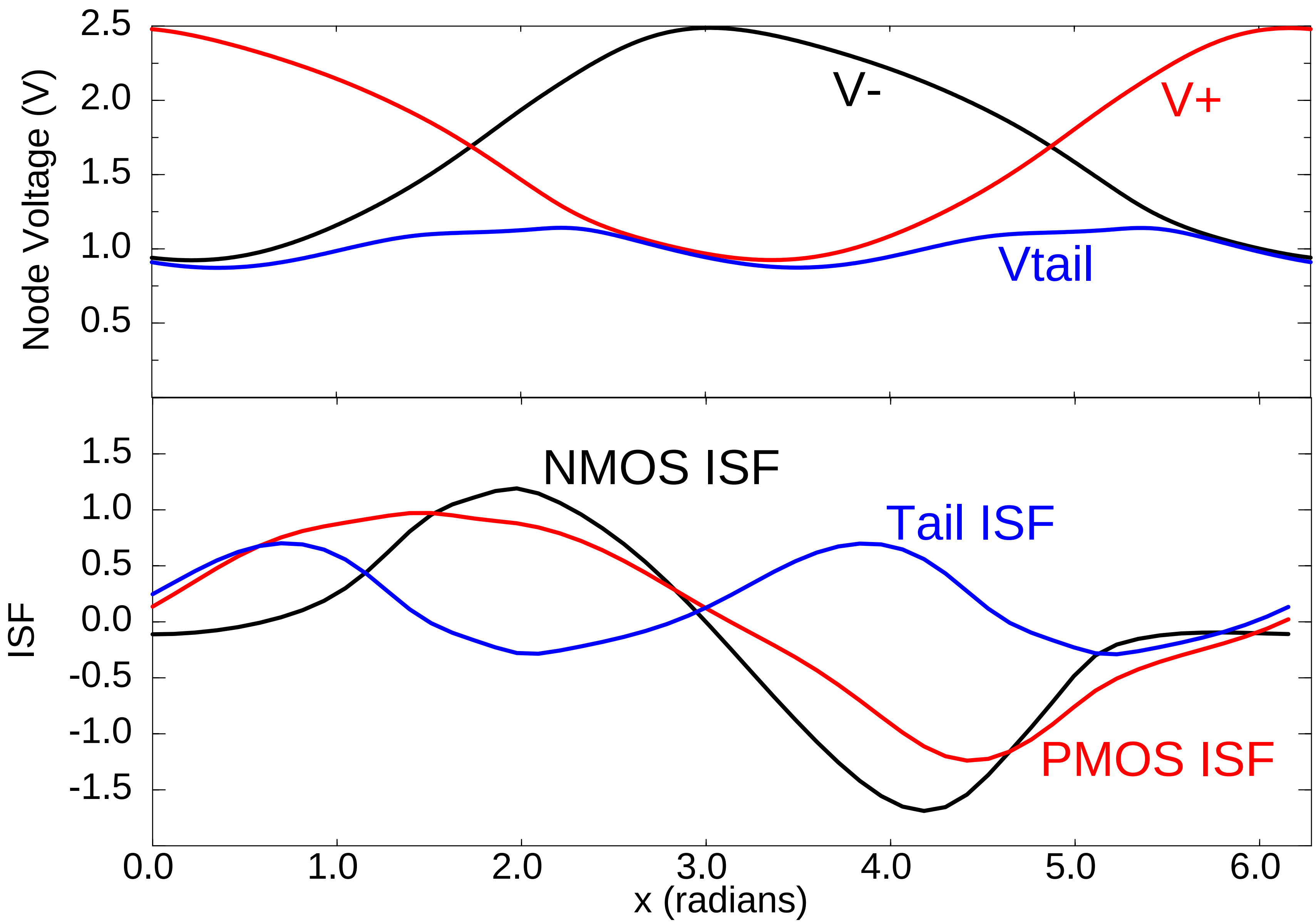
$$\frac{\overline{v_n^2}}{\Delta f} = 4kTr_s$$

Equivalent Circuit for Differential Sources

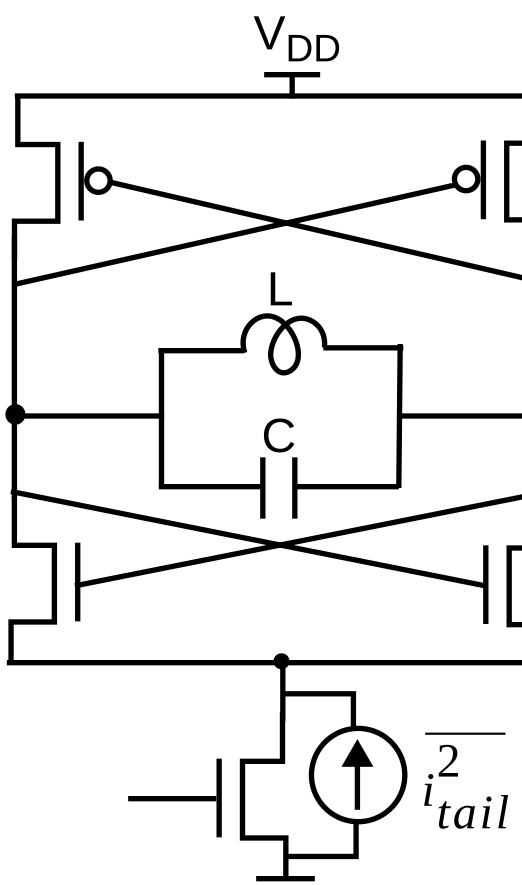


$$\left.\frac{\overline{i_n^2}}{\Delta f}\right|_{diff-pair} = \frac{1}{4}\left(\frac{\overline{i_{n1}^2}}{\Delta f} + \frac{\overline{i_{n2}^2}}{\Delta f} + \frac{\overline{i_{p1}^2}}{\Delta f} + \frac{\overline{i_{p2}^2}}{\Delta f}\right) = \frac{1}{2}\left(\frac{\overline{i_n^2}}{\Delta f} + \frac{\overline{i_p^2}}{\Delta f}\right)$$

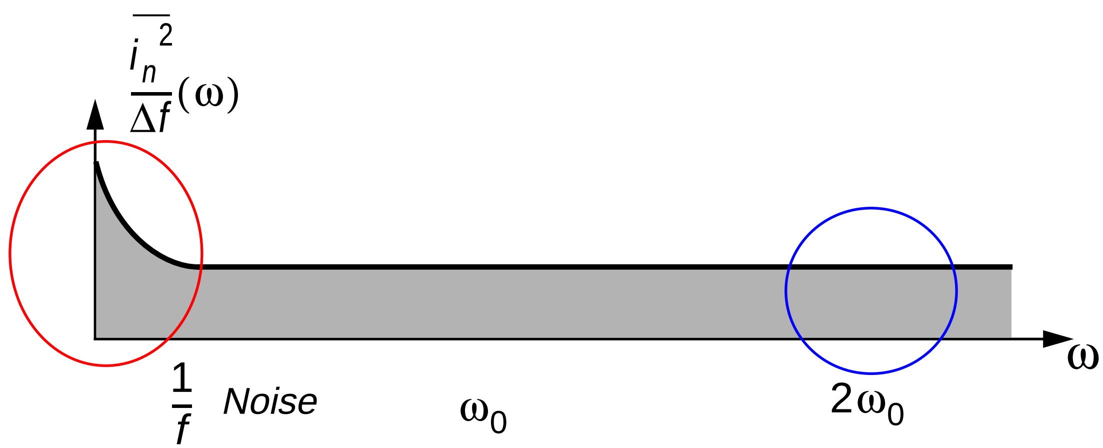
Waveform and ISF



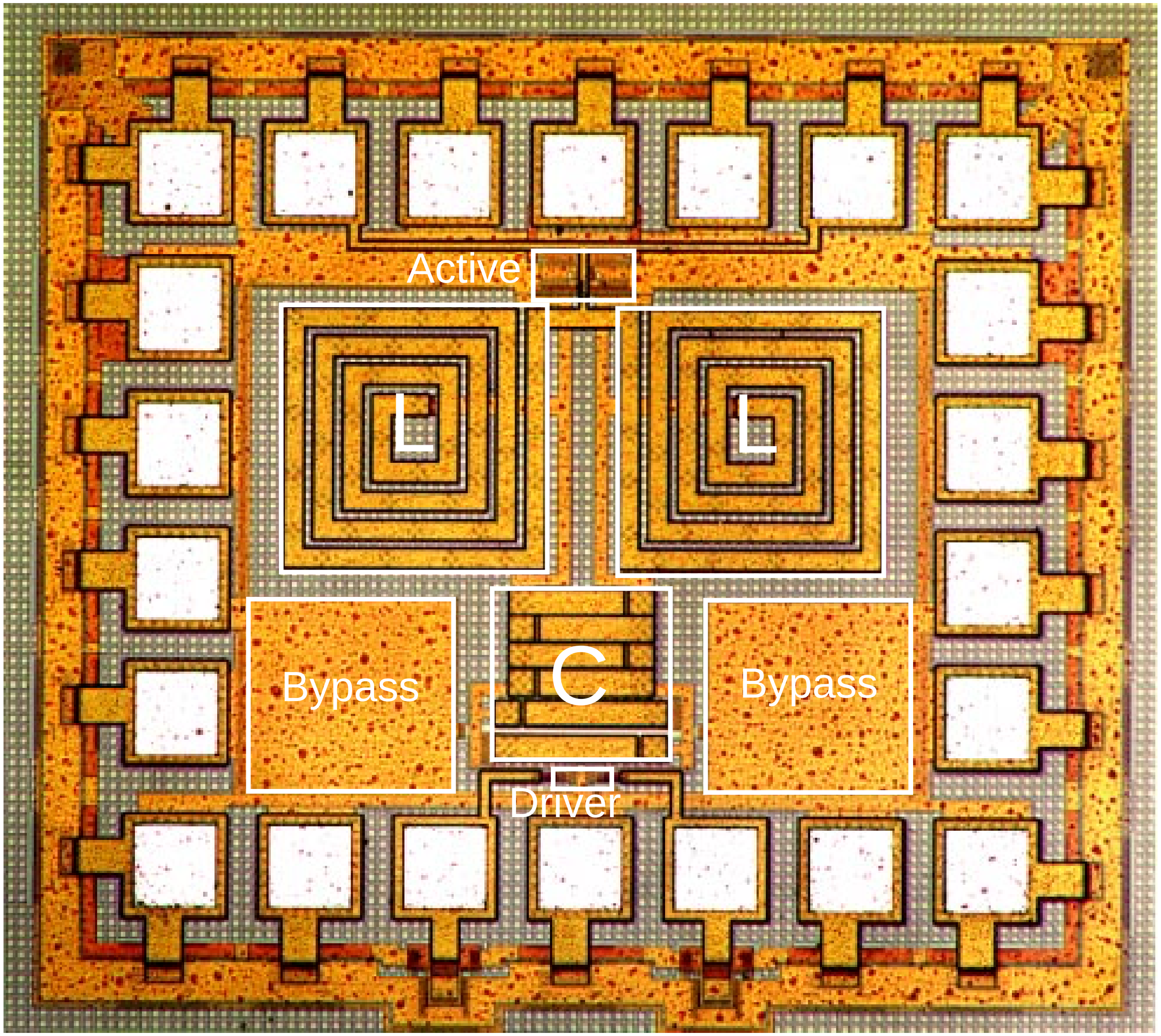
Effect of Tail Current Source



For the tail current source, only low frequency noise and noise in the vicinity of even harmonics of the tail current source affect phase noise.



Die Photo of the Complementary Oscillator

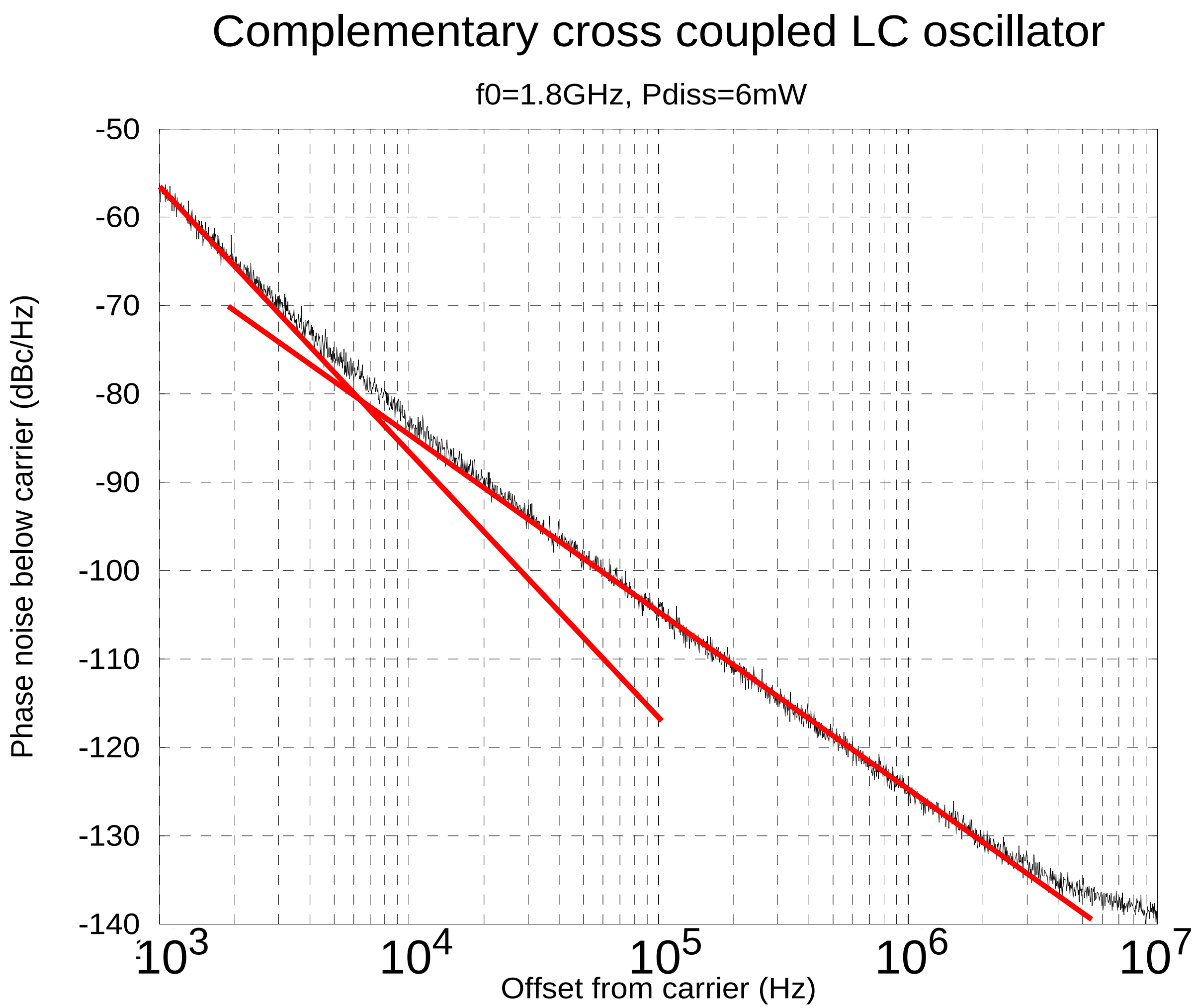


0.25µm process

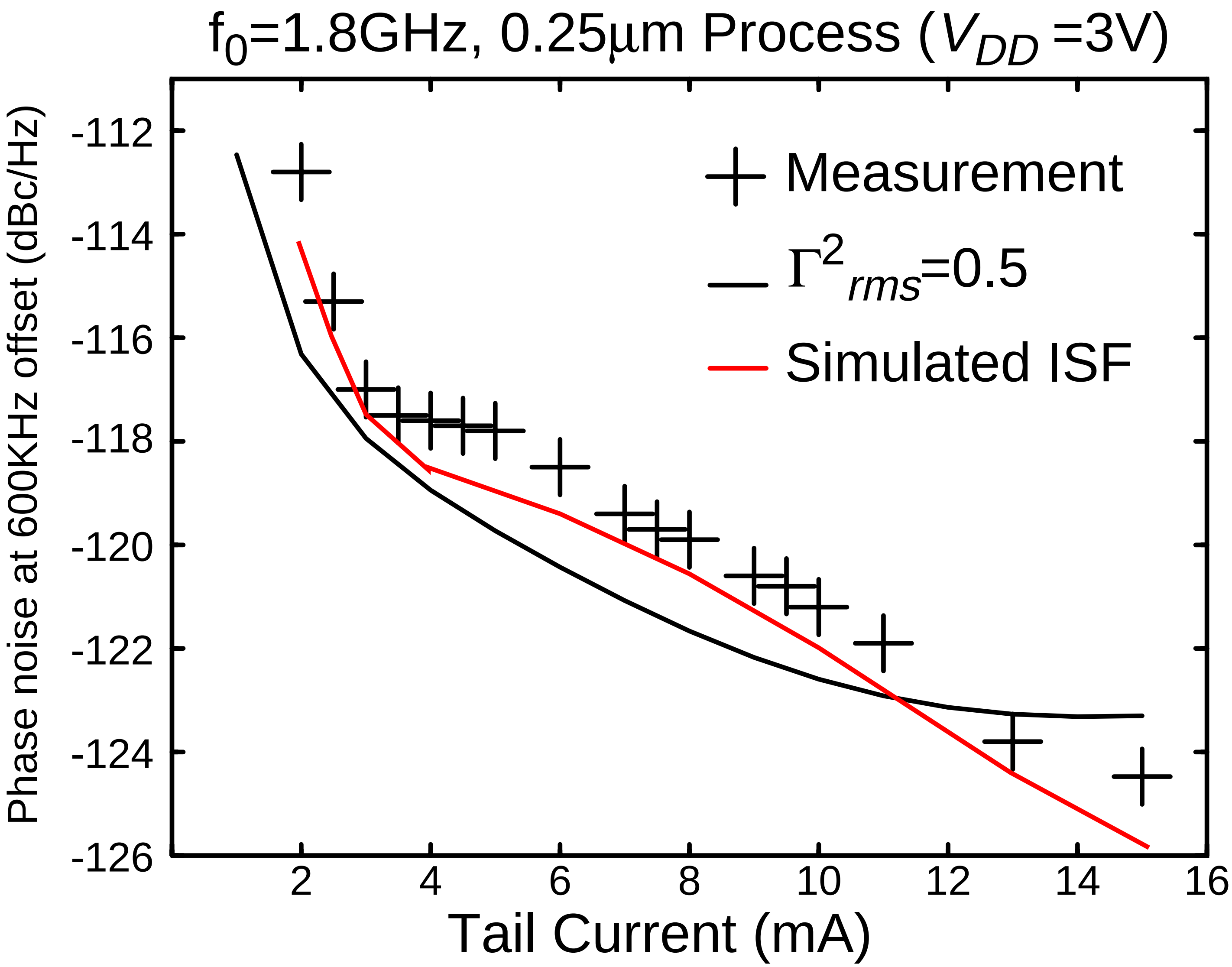
700µm x 800µm

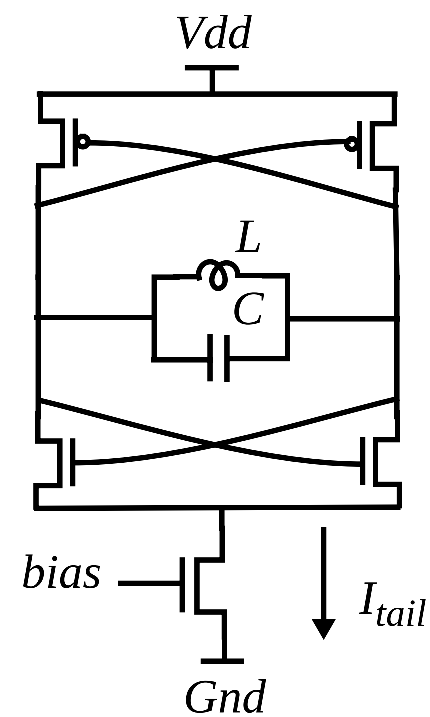
Pad limited

Phase Noise vs. Offset from Carrier



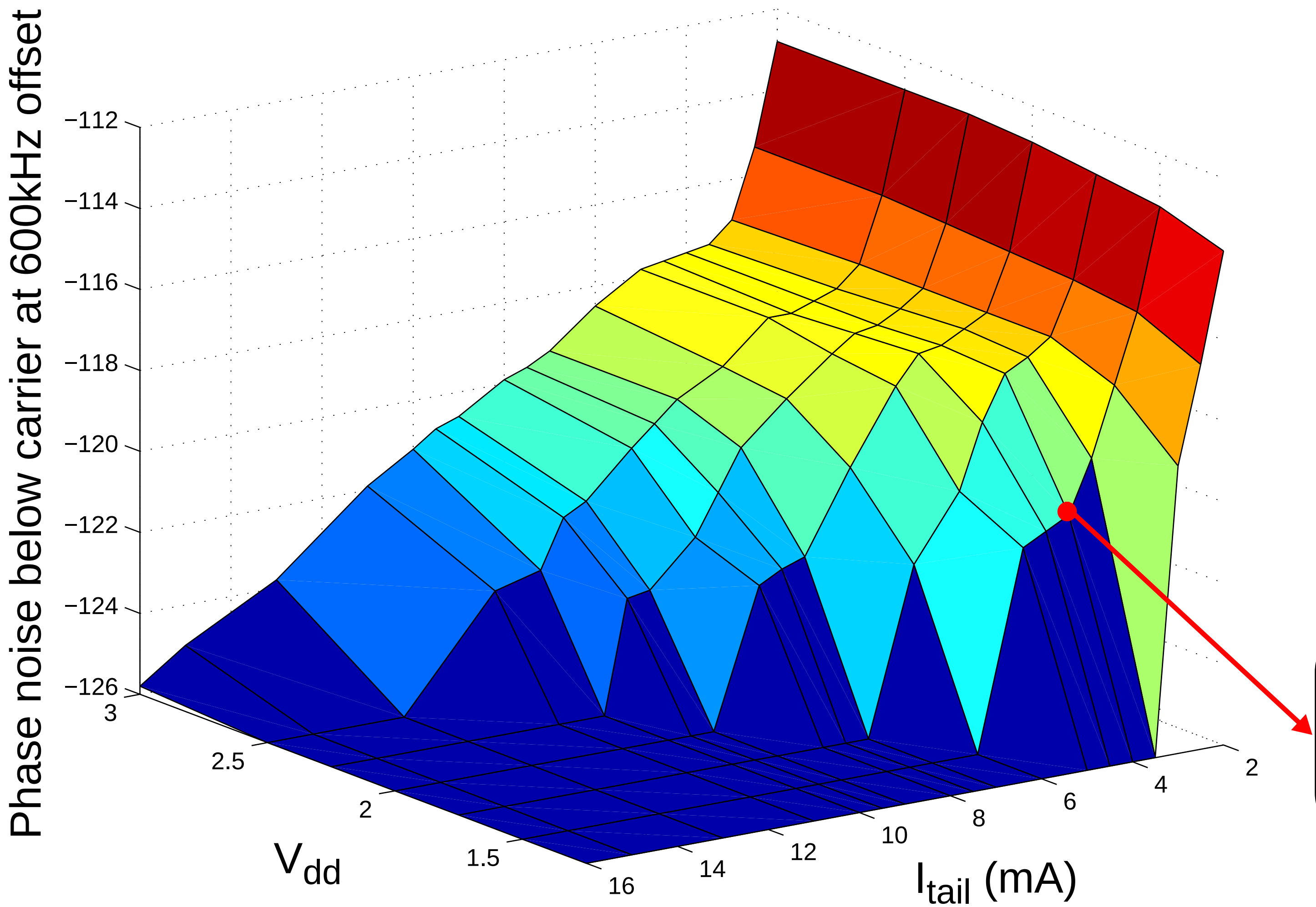
Complementary Cross-Coupled LC Oscillator

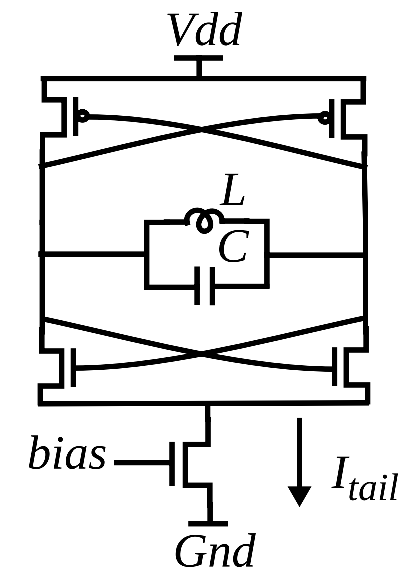




Complementary Cross-Coupled VCO

f_0=1.8GHz, 0.25µm Process

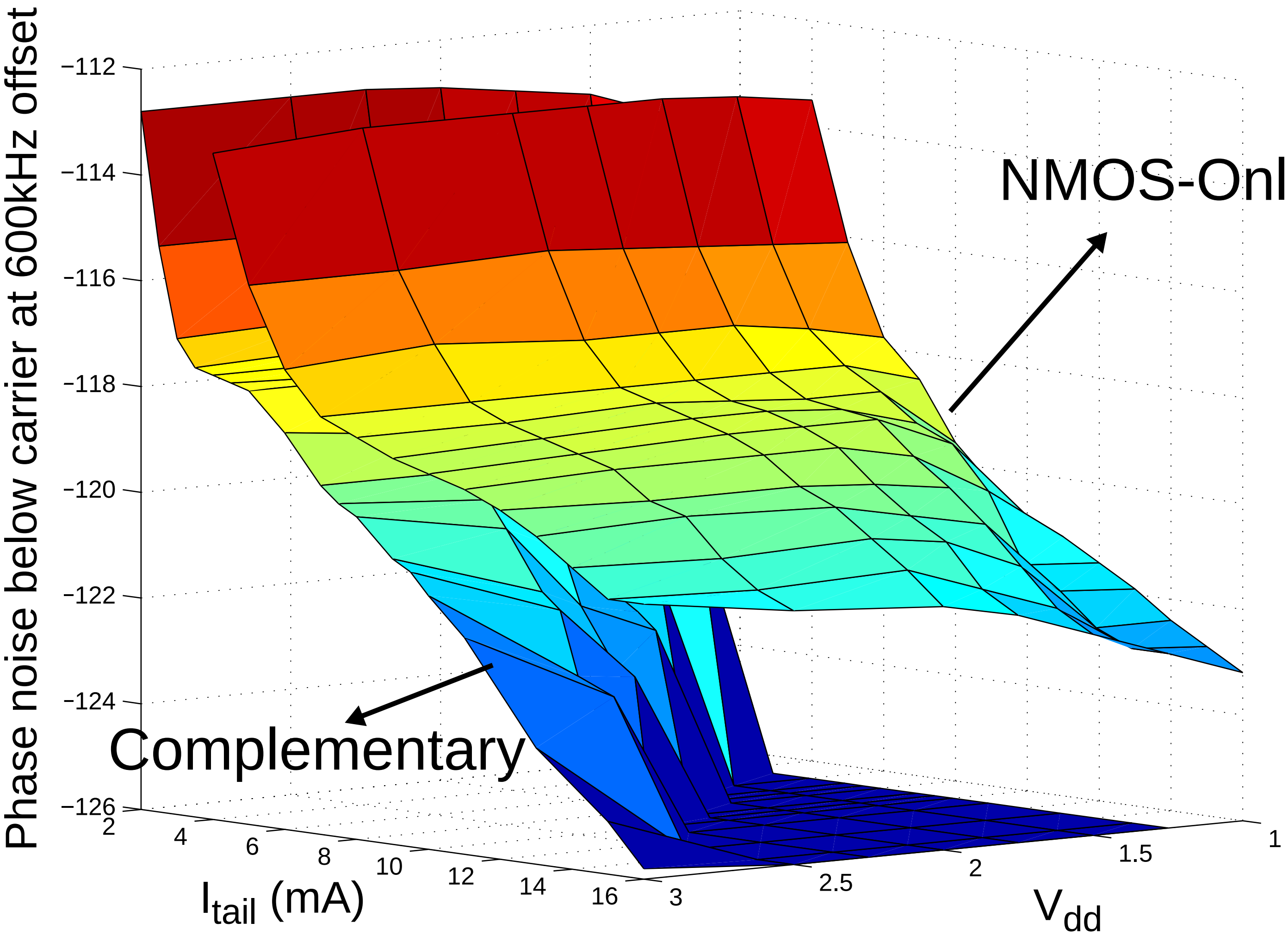


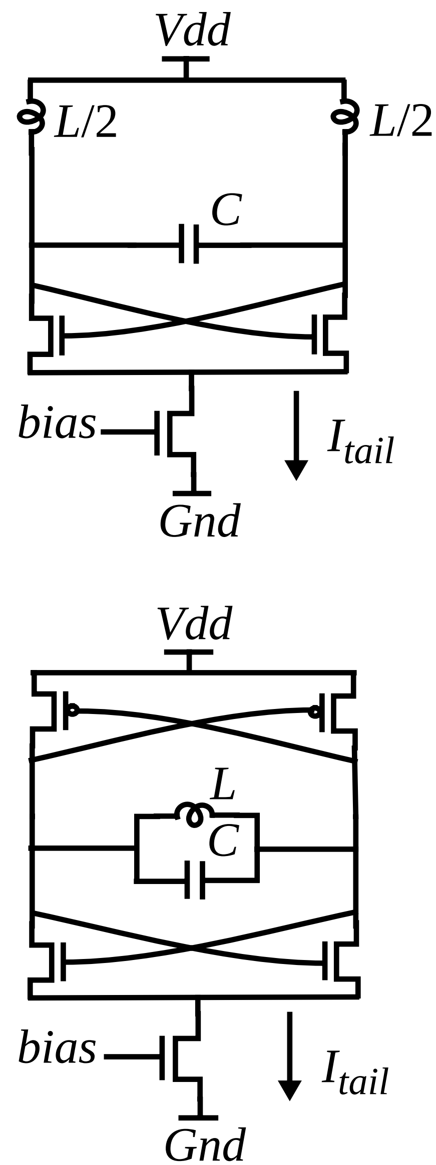


f_0=1.8GHz
P=6mW
-121dBc/Hz@600kHz

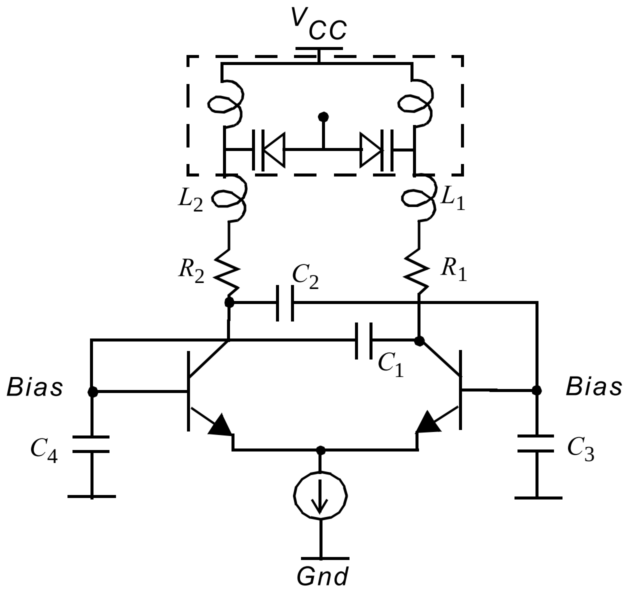
Complementary vs. NMOS-Only VCO

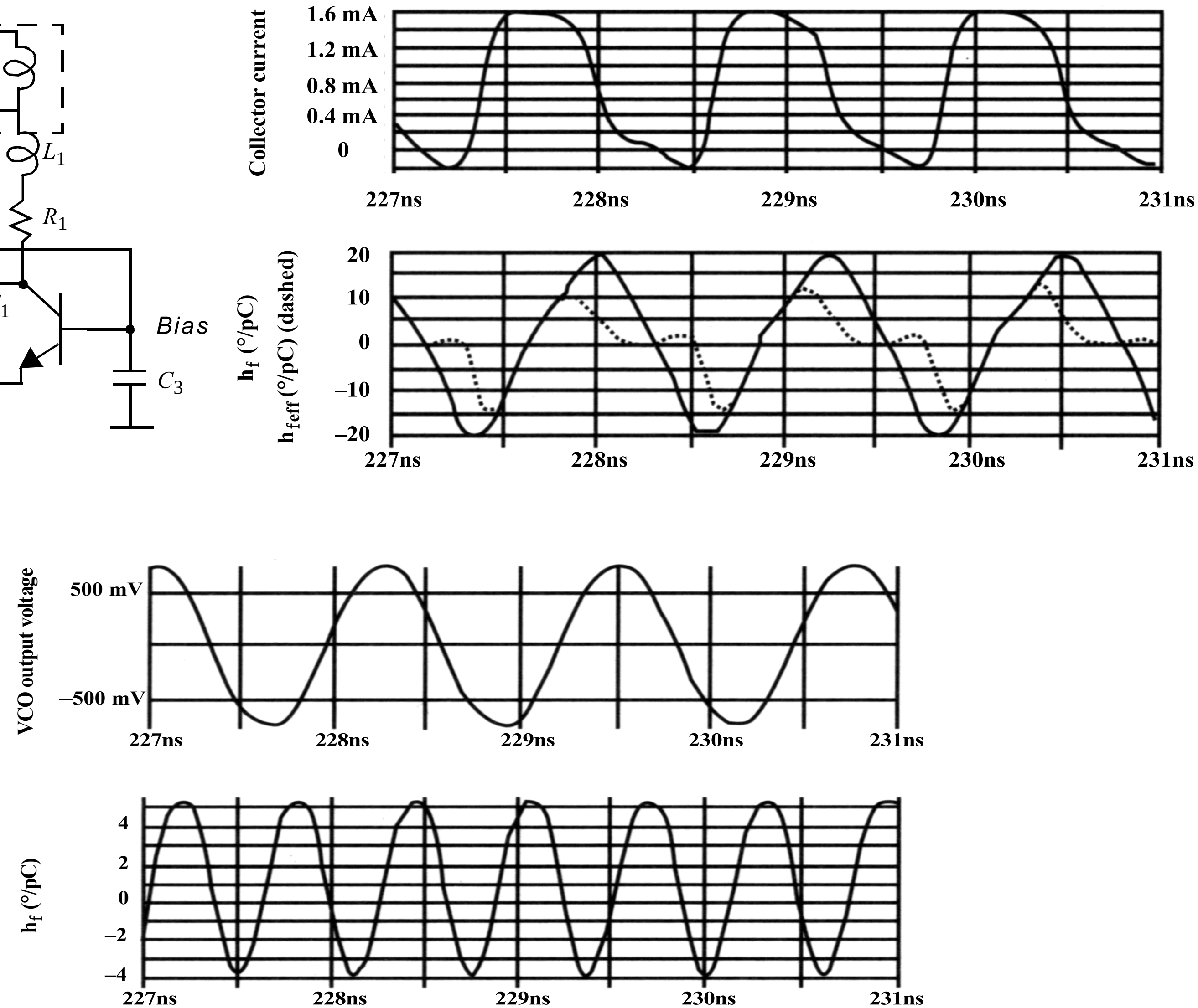
f_0=1.8GHz, 0.25μm Process



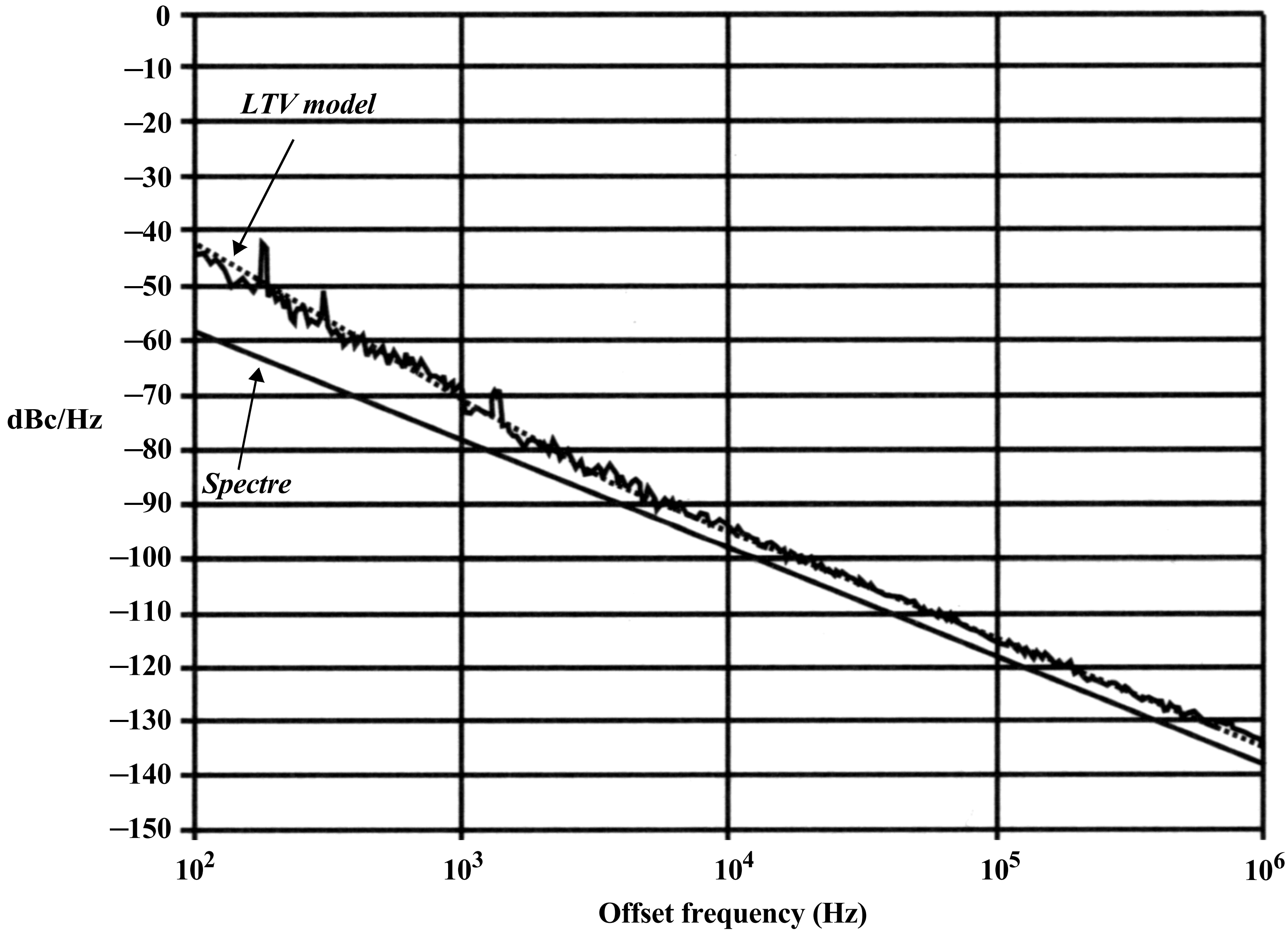


Another example (*Margarit et al.*)





Predicted vs. measured



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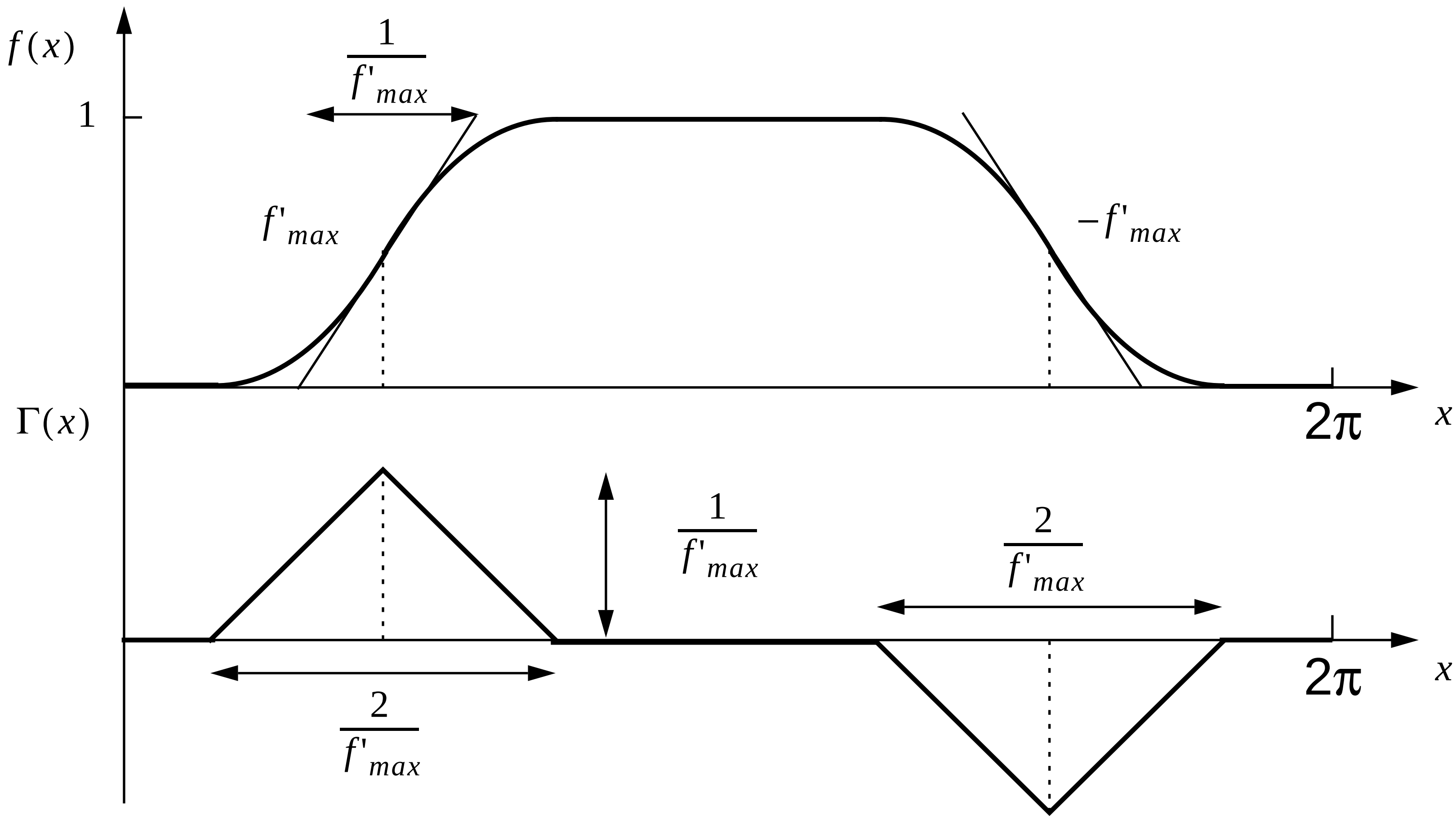
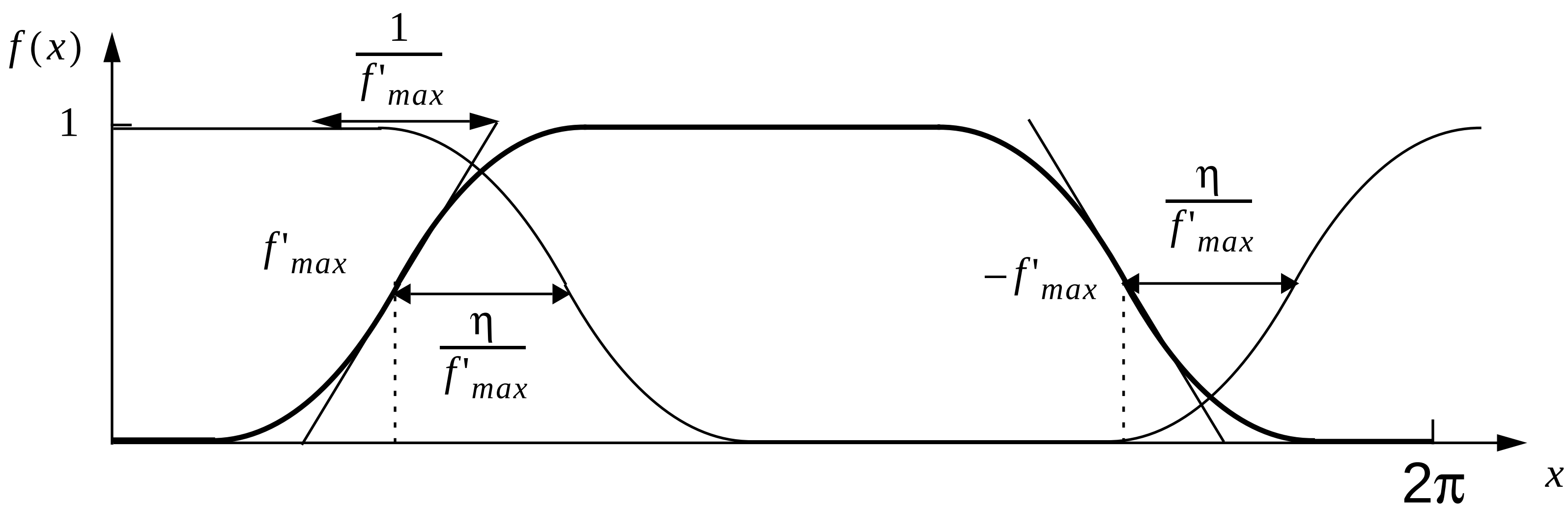
Approximate ISF for Ring Oscillators

The peak of the ISF is inversely proportional to the maximum slope of the normalized waveform.



$$\Gamma_{rms}^2 = \frac{1}{2\pi}\int_0^{2\pi}\Gamma^2(x)dx = \frac{4}{2\pi}\int_0^{1/f'_{max}} x^2 dx = \frac{2}{3\pi}\left(\frac{1}{f'_{max}}\right)^3$$

Risetime and Delay Relationship



Stage Delay: $t_D = \frac{\eta}{f'_{max}}$

Period: $2\pi = 2Nt_D = \frac{2N\eta}{f'_{max}} \Rightarrow \frac{1}{f'_{max}} = \frac{\pi}{N\eta}$

ISF RMS: $$\Gamma^2_{rms} = \frac{2\pi^2}{3\eta^3}\frac{1}{N^3}$$

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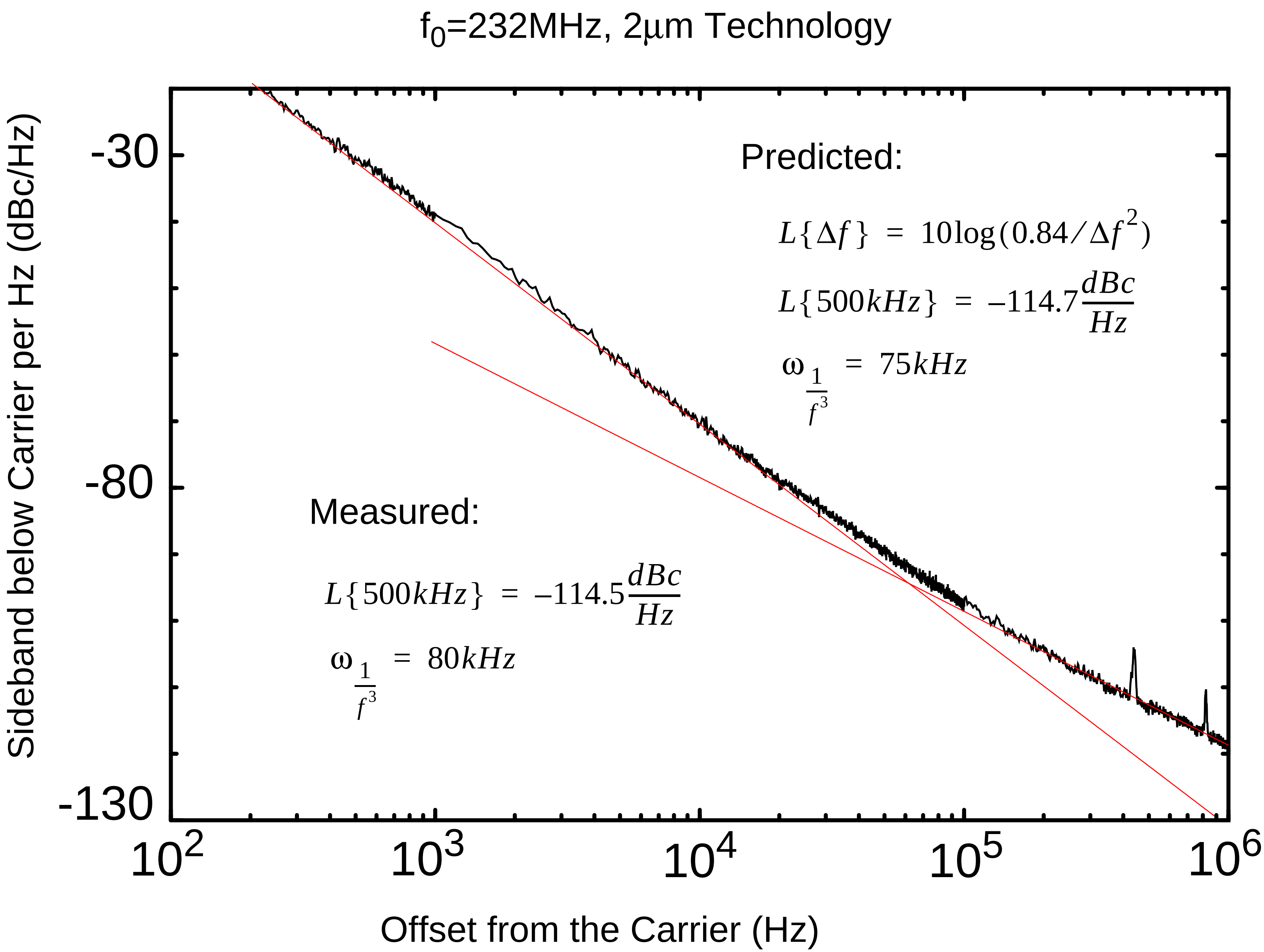
5-Stage Single-Ended Ring Oscillator



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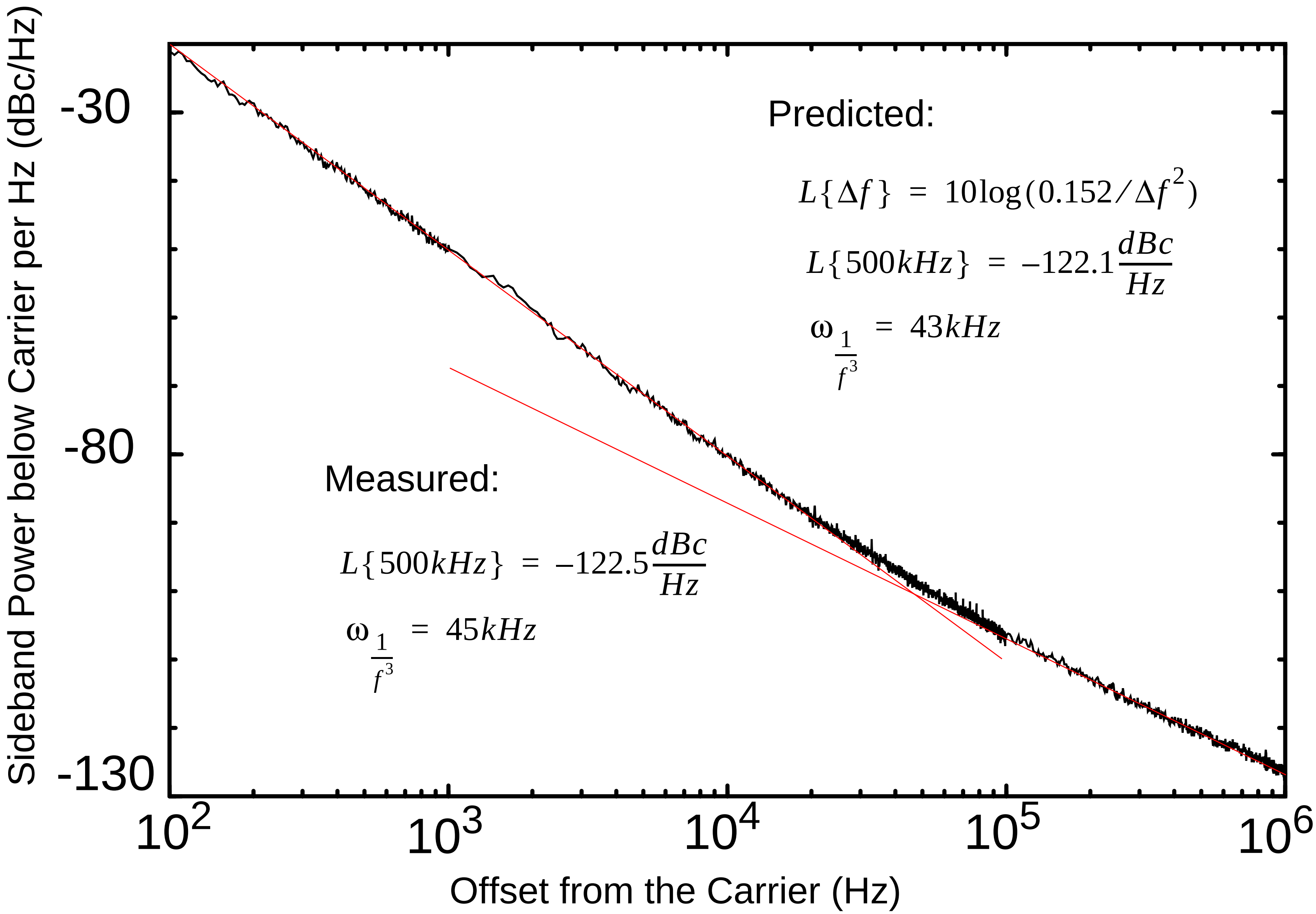
11-Stage Single-Ended Ring Oscillator

f_0=115MHz, 2μm Process



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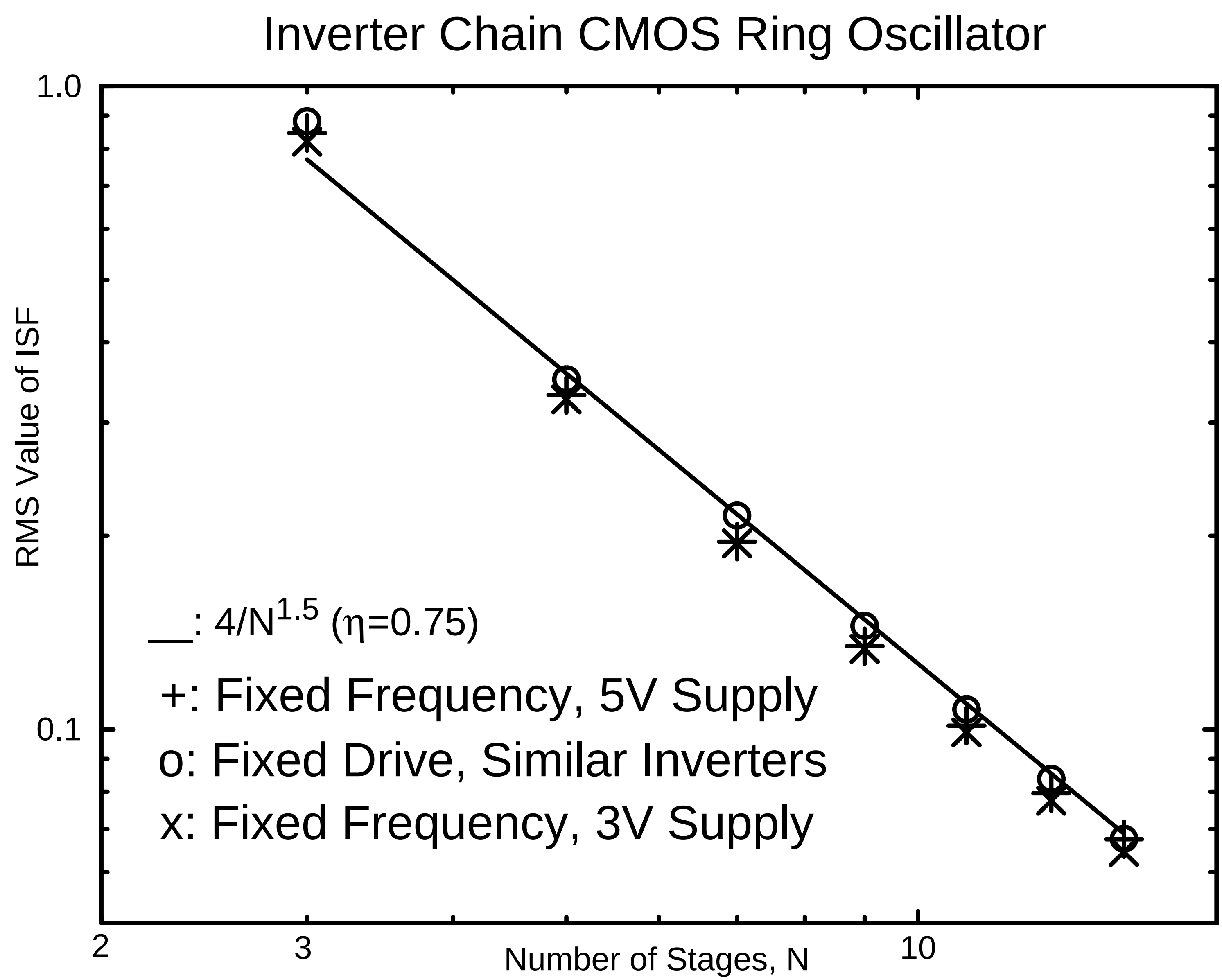
ISF RMS vs. Number of Stages



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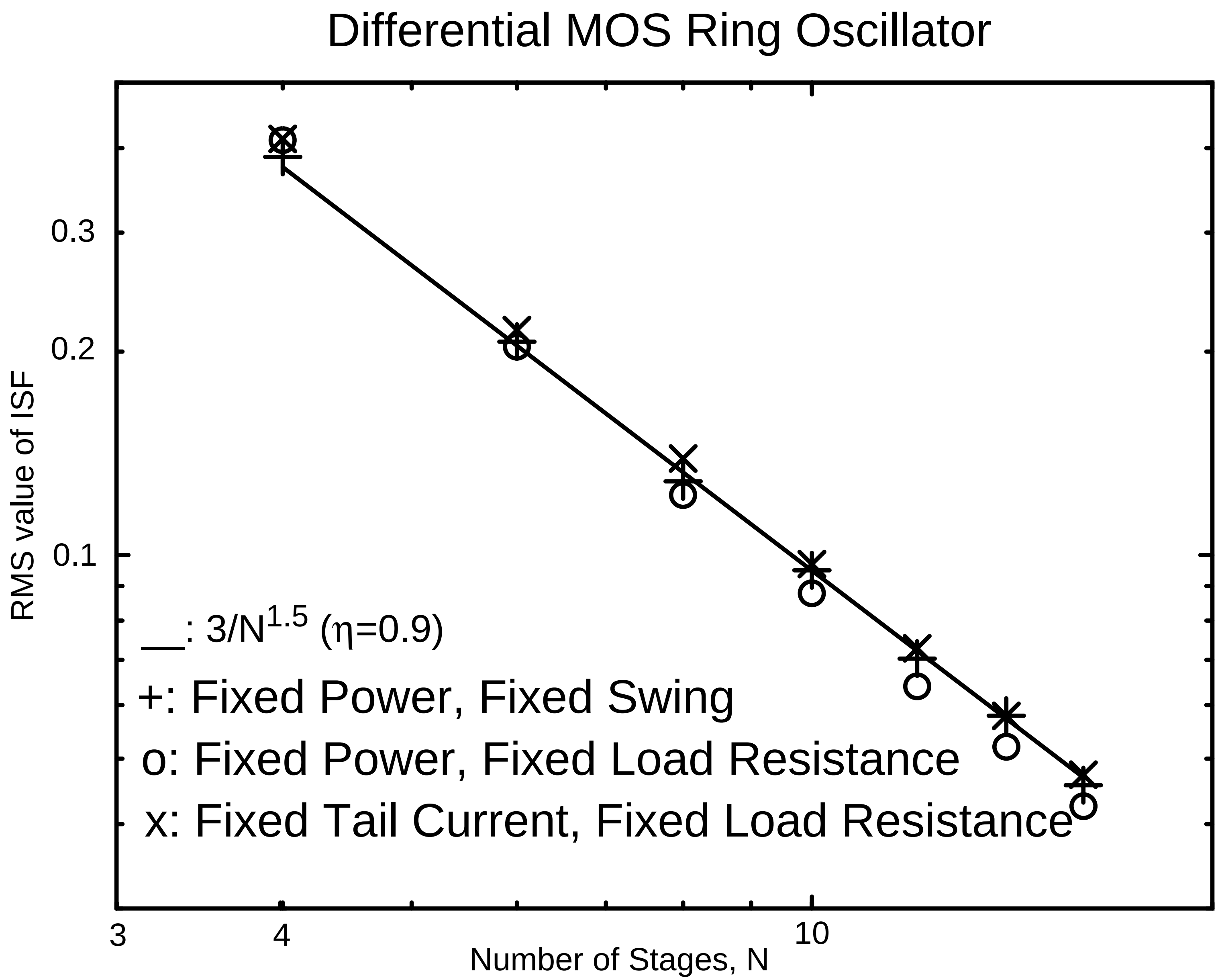
ISF RMS vs. Number of Stages



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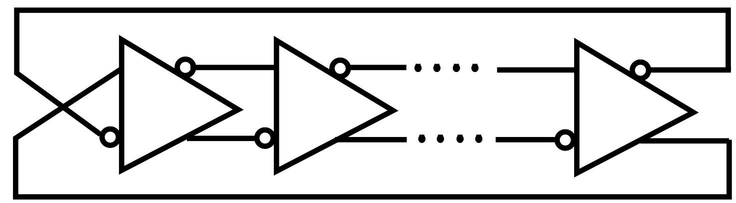
Phase Noise in Differential Ring Oscillator

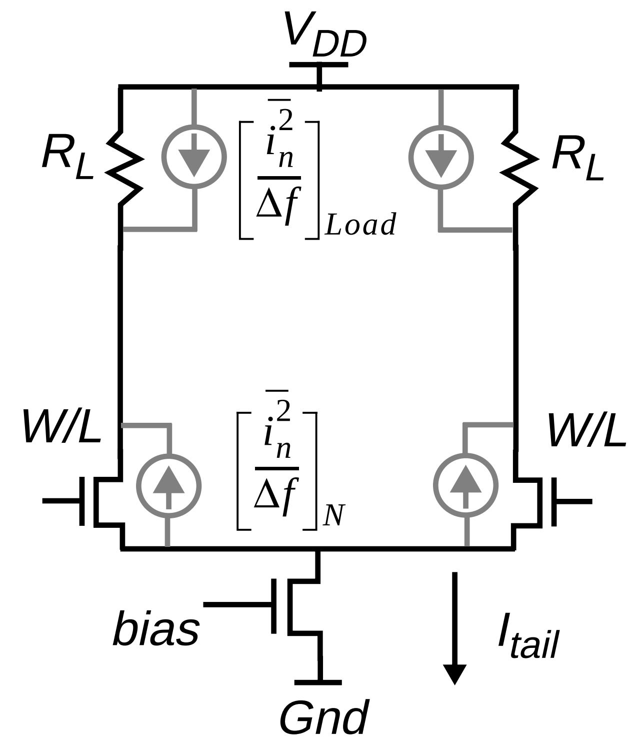
N Stages



Power Dissipation: $P = NI_{tail}V_{DD}$

Frequency: $f_0 = \frac{1}{2Nt_D} \approx \frac{1}{2\eta N t_r} \approx \frac{I_{tail}}{2\eta N q_{max}}$

Noise: $\frac{\overline{i_n^2}}{\Delta f} = \left(\frac{\overline{i_n^2}}{\Delta f}\right)_N + \left(\frac{\overline{i_n^2}}{\Delta f}\right)_{Load} = 4kTI_{tail}\left(\frac{1}{V_{char}} + \frac{1}{R_L I_{tail}}\right)$



$$L_{min}\{\Delta f\} \approx \frac{8}{3\eta} \cdot N \cdot \frac{kT}{P} \cdot \left(\frac{V_{DD}}{V_{char}} + \frac{V_{DD}}{R_L I_{tail}}\right) \cdot \left(\frac{f_0}{\Delta f}\right)^2$$

Short channel: $V_{char} = E_c L/\gamma$

Long channel: $V_{char} = (V_{GS} - V_T)/\gamma$

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Effect of Number of Stages on Phase Noise

For a given power and frequency, phase noise degrades with number of stages, *N*, in differential ring oscillators.

$$L_{min}\{\Delta f\} \approx \frac{8}{3\eta} \cdot N \cdot \frac{kT}{P} \cdot \left(\frac{V_{DD}}{V_{char}} + \frac{V_{DD}}{R_L I_{tail}}\right) \cdot \frac{f_0^2}{\Delta f^2}$$

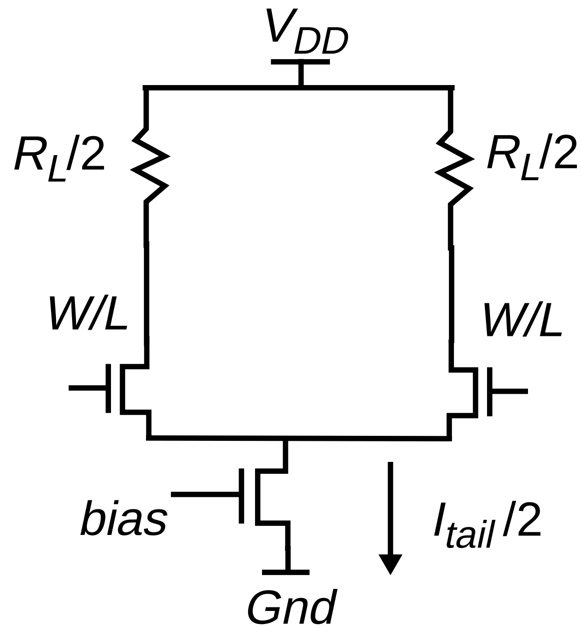
Doubling the number of stages:

R_L is divided by 2 to keep the frequency constant,

I_{tail} is divided by 2 to keep the power constant,

Therefore:

Maximum charge swing, q_{max}, is 4 times smaller.



This is ***NOT*** the case for single-ended ring, since the swing is constant.

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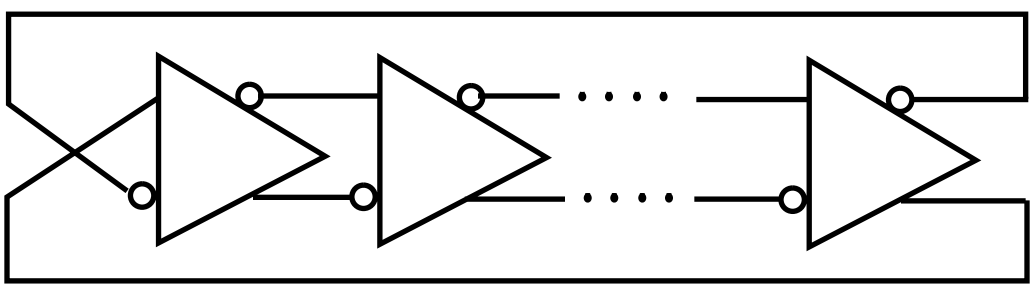
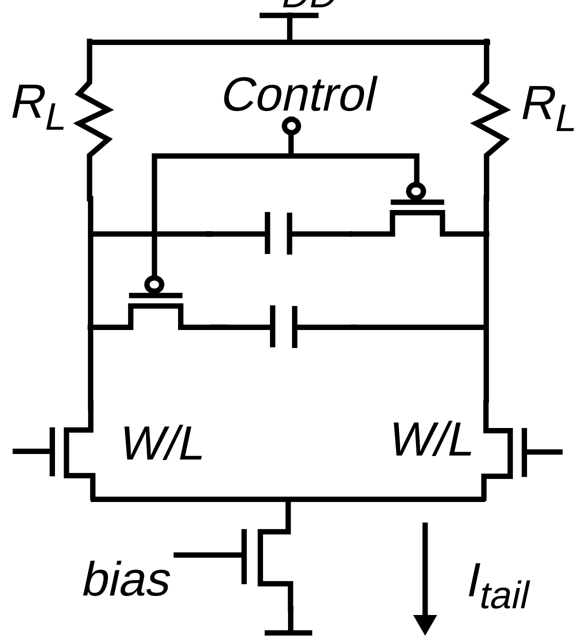
Differential Ring Oscillators

Effective channel length: L_{eff}=0.25μm.

Unsilicided poly load resistors.

Maximum frequency of 5.4GHz.





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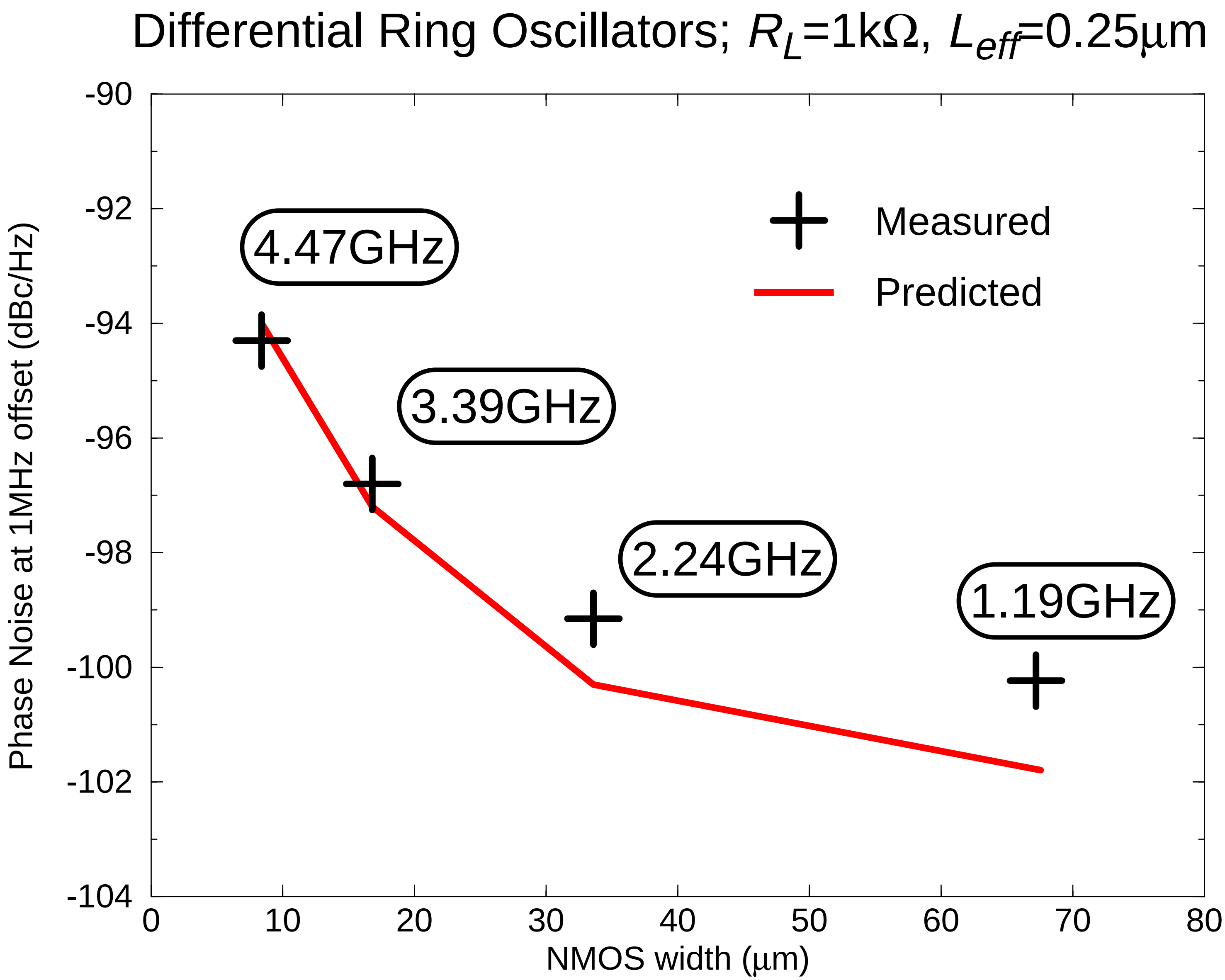
Predicted and Measured Phase Noise



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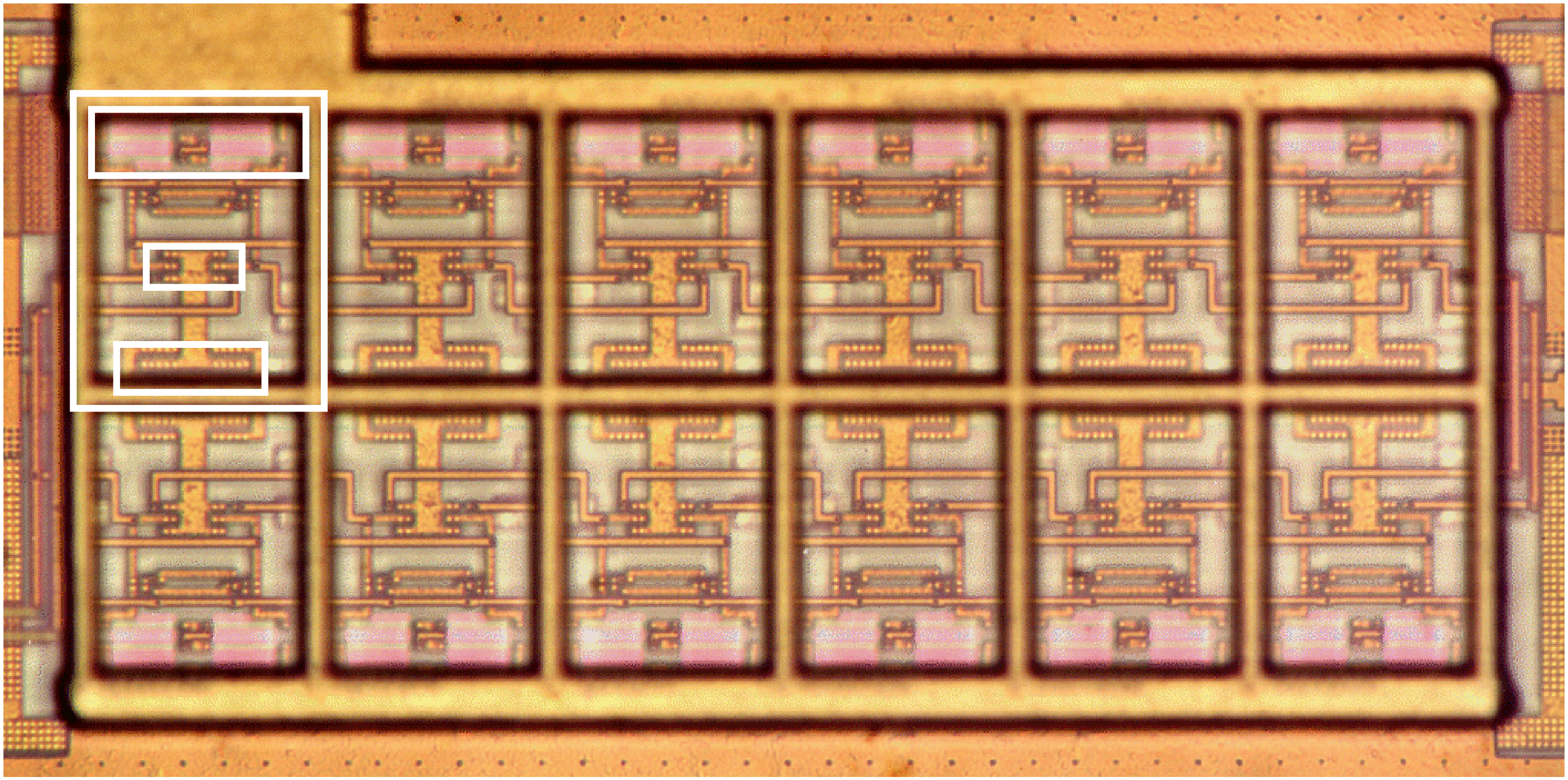
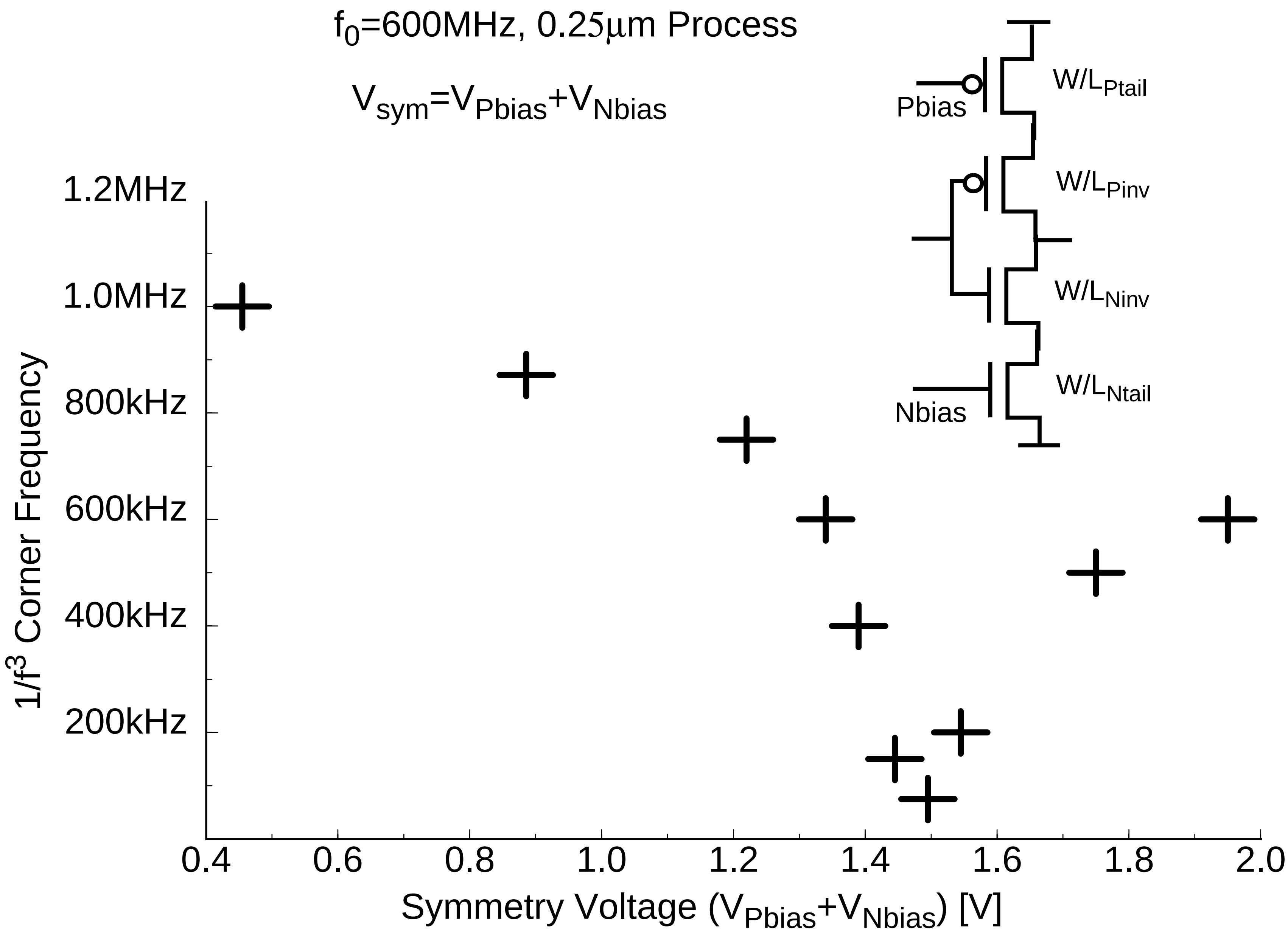
Die Photo of 12-Stage Differential Ring Osc.



9-Stage Current-Starved Single-Ended VCO

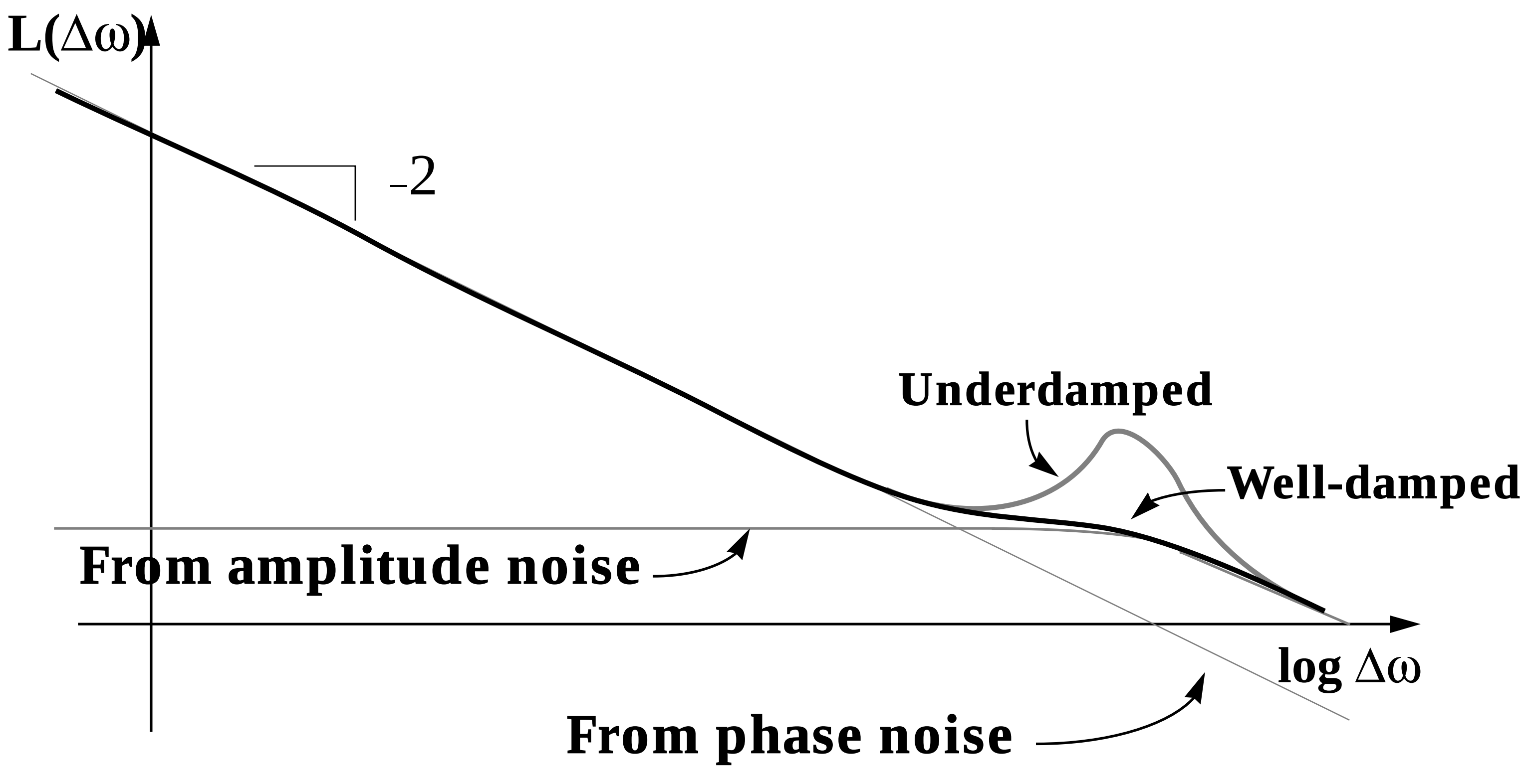


Amplitude Noise

- Phase noise generally dominates close-in spectrum. Amplitude noise typically dominates far-out spectrum.
- Effect of amplitude noise may be accommodated with the same general approach: Investigate impulse response.
  - If amplitude control mechanism acts as a first-order system (e.g., if it is well damped), amplitude impulse response will die out with a time constant equal to the inverse bandwidth of the control loop.
    - For an *LC* tank, this bandwidth is the tank bandwidth, ω_0/Q.
    - Corresponding contribution to noise spectrum is flat to frequency offset equal to that bandwidth, then rolls off; produces pedestal in overall response.
  - If amplitude control is underdamped (e.g., behaves as 2nd order), can get peaking in the spectrum.

Amplitude Response

- Possible responses corresponding to these control dynamics look roughly as follows:



Summary and Conclusions

- LTI theories say:
  - Maximize signal power and resonator Q and operate at edge of current-limited regime, with minimum ratio L/R consistent with oscillation.
  - Can't do anything about $1/f^3$ corner frequency.
    - Corner frequency is strictly technology-limited.
- LTV theory says:
  - Continue to maximize signal power, resonator Q, and R/L.
    - Use tapped tanks (à la Clapp, e.g.).
  - Maximize symmetry (in the ISF sense) to reduce $1/f^3$ corner frequency.
  - Choose topologies and bias conditions so that energy is returned to tank impulsively.

Acknowledgments

- Prof. Ali Hajimiri of Caltech, who developed this theory while a Ph.D. student at Stanford.
- David Leeson, for graciously encouraging us to build on his theory.
- Web URL is, again: http://www-smirc.stanford.edu