S-parameters

S-parameters (II)

Linear networks, or nonlinear networks operating with signals sufficiently small to cause the networks to respond in a linear manner, can be completely characterized by parameters measured at the network terminals (ports) without regard to the contents of the networks. Once the parameters of a network have been determined, its behavior in any external environment can be predicted, again without regard to the contents of the network.

S-parameters are important in RF and microwave design because they are easier to measure and work with at high frequencies thanother kinds of parameters.

Source: HP AP-95

S-parameters (I)

Maxwell's equations
All electromagnetic behaviors can ultimately be explained by Maxwell's four basic equations:

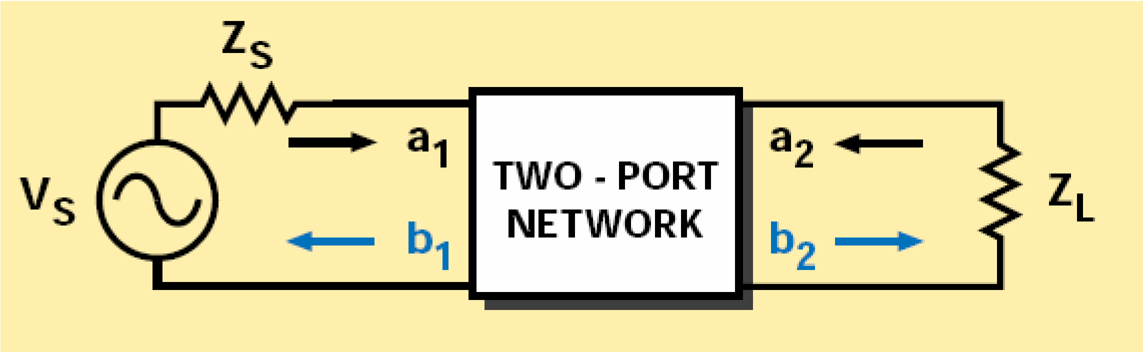
$$\nabla \cdot \mathbf{D} = \rho \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$$

However, it is often possible and efficient to use approximations such as distributed and lumped models.

Source: HP AP-95

S-parameter definitions (I)



Two-port network showing incident waves(a1, a2) and reflected waves (b1, b2) used in S-parameter definitions.

An important advantage of s-parameters stems from the fact that traveling waves, unlike terminal voltages and currents, do not vary in magnitude at points along a lossless transmission line.

Source: HP AP-95

S-parameter definitions (II)

$$b_1 = s_{11} a_1 + s_{12} a_2$$

$$b_2 = s_{21} a_1 + s_{22} a_2$$

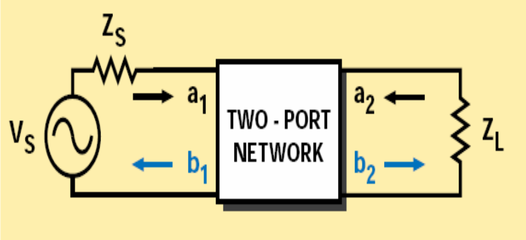
The s-parameters s_{11}, s_{22}, s_{21}, and s_{12} are:

$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$ = Input reflection coefficient with the output port terminated by a matched load ($Z_L=Z_0$ sets $a_2=0$)

$$s_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$ = Output reflection coefficient with the input terminated by a matched load ($Z_S=Z_0$ sets $V_s=0$)

$$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$ = Forward transmission (insertion) gain with the output port terminated in a matched load.

$$s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$ = Reverse transmission (insertion) gain with the input port terminated in a matched load.



Z_o=50 Ohm

Source: HP AP-95

Why 50 Ohm ?

- To maximized the power-handling capabilities of an air-dielectric transmission line of a given diameter we should choose Zo~30 Ohm
- Z_0 ~77 Ohm gives minimum attenuation per length due to resistive losses
- Zo~50 Ohm nice compromise (although power handling capabilities are normally not an issue).

Source: Thomas Lee

Power gain and mismatch: S-parameters

$$\left|s_{11}\right|^2 = \frac{\text{Power reflected from the network input}}{\text{Power incident on the network input}}$$

$$\left|s_{22}\right|^2 = \frac{\text{Power reflected from the network output}}{\text{Power incident on the network output}}$$

$$\left|s_{21}\right|^2 = \frac{\text{Power delivered to a } Z_0 \text{ load}}{\text{Power available from } Z_0 \text{ source}}$$

$$= \text{Transducer power gain with } Z_0 \text{ load and source}$$

$$\left|s_{12}\right|^2 = \text{Reverse transducer power gain with } Z_0 \text{ load and source}$$

Source: HP AP-95

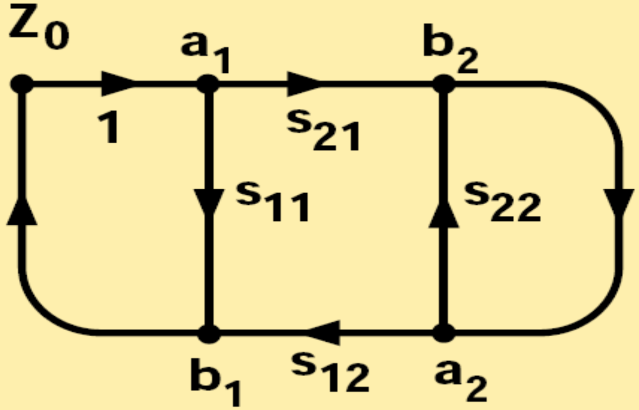
Measurement of S-parameters

S-parameters are measured (e.g. upto 110 GHz is no problem) with network-analysers (single-ended, but there are also diff. analysers.)

Reflection coefficients (and flow charts)

- All gain definition (see previous slides) can all be formulated in terms of S-parameters and source and load reflection coeffients, Γ_s and Γ_l

$$b_S = \frac{V_S\sqrt{Z_0}}{Z_S + Z_0}$$



$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

No reflection for a perfect match: no return loss (20*log |Γ|): all power is “absorbed” in the load.

Source: HP AP-95

Some gain definitions in S-parameters and Γ

$$G_T = \frac{|s_{21}|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|(1 - s_{11}\Gamma_S)(1 - s_{22}\Gamma_L) - s_{21}s_{12}\Gamma_L\Gamma_S|^2}$$

$$A_V = \frac{b_2 (1 + \Gamma_L)}{a_1 (1 + s'_{11})} = \frac{s_{21}(1 + \Gamma_L)}{(1 - s_{22}\Gamma_L)(1 + s'_{11})}$$

With S`$_{11}$: **Input reflection coefficient with arbitrary Z_L**

$$s'_{11} = s_{11} + \frac{s_{12}\, s_{21}\, \Gamma_L}{1 - s_{22}\Gamma_L}$$

Matching

Matching

- There are a number of reasons for matching of a source impedance Z_s to an input impedance Z_i, (or an output impedance to a load impedance) :
  - Minimization of reflections (long lines)
  - Maximization of power transfer (power match, 50 ohm on/off chip connections)
  - Minimization of noise figure F (noise match)

When do reflections occur

- Signals reflect when the distance between to points is more than ¼*λ. The wavelength λ = c/f (e.g. λ is 30 cm for f=1 GHz in free space, but can reduce to a few cm under realistic conditions).

For example, the distance between antenna and LNA is normally larger than ¼*λ, and reflections take place.

For short on-chip connections reflections normally (depending on λ) not occur, and matching is not required (not that the voltage across a load is always halved under 50 ohm power matching conditions).

Smith Chart

The Smith-chart

The Smith chart appeared in 1939 (Ref. 1) as a graph-based method of simplifying the complex math (that is, calculations involving variables of the form $x + jy$) needed to describe the characteristics of microwave components. Although calculators and computers can now make short work of the problems the Smith chart was designed to solve, the Smith chart, remains a valuable tool.

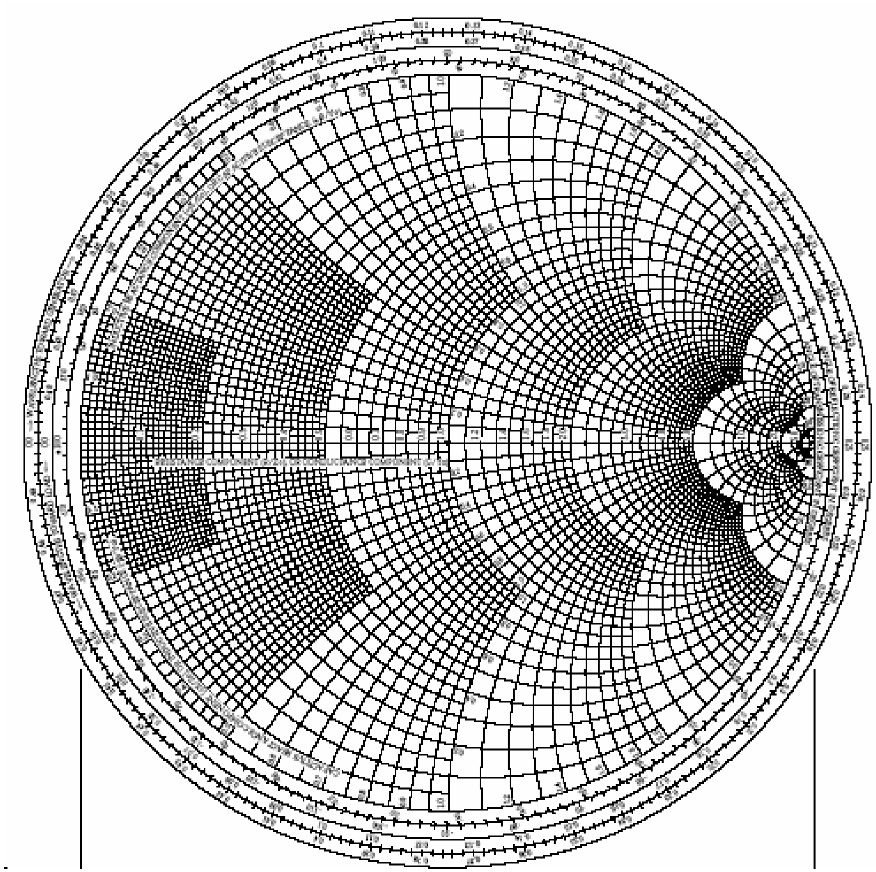
Ref 1. "Philip Smith, Electrical Engineer," an oral history conducted in 1973 by Frank A. Polkinghorn, IEEE History Center, Rutgers University, New Brunswick, NJ. www.ieee.org/organizations/history_center/oral_histories/transcripts/smith3.html.

Source: Rick Nelson

Smith charts are useful to

- Simplify design of matching networks
- Display constant-gain circles
- Display constant noise circles
- Display stability circles
- Analyze transmission lines
- …

Looks rather complicated, this Smith chart

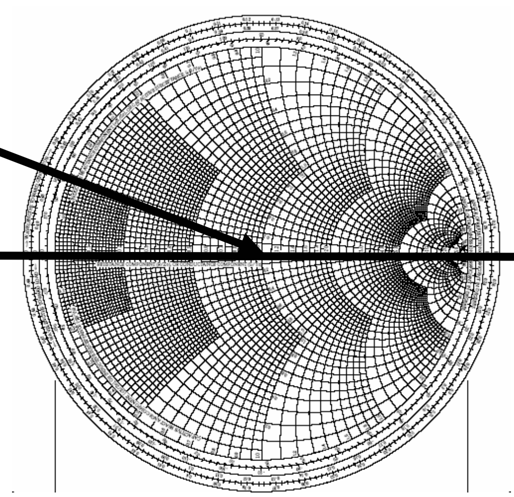


What is it?

The Smith chart in words

The plot of the constant-resistance and constant reactance circles (normalized to Z_0) for all values of Z such that $Re(Z) \geq 0$ in a graph is known as the Smith chart. (Because there is a one to one correspondence between the Z plane and the Γ plane).

$Z/Z_0=1$, $\Gamma=0$

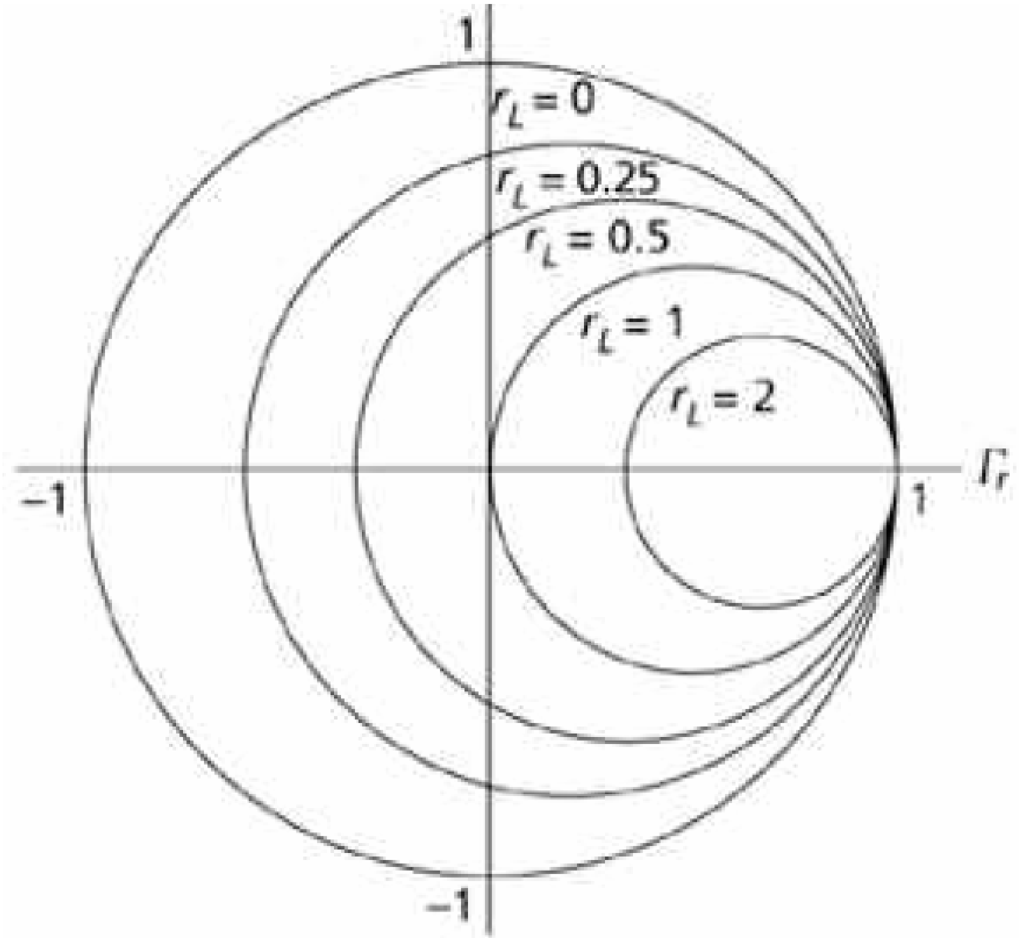


Upper-half: positive reactance; Inductance

Lower-half: negative reactance; capacitance

(Z-Smith chart)

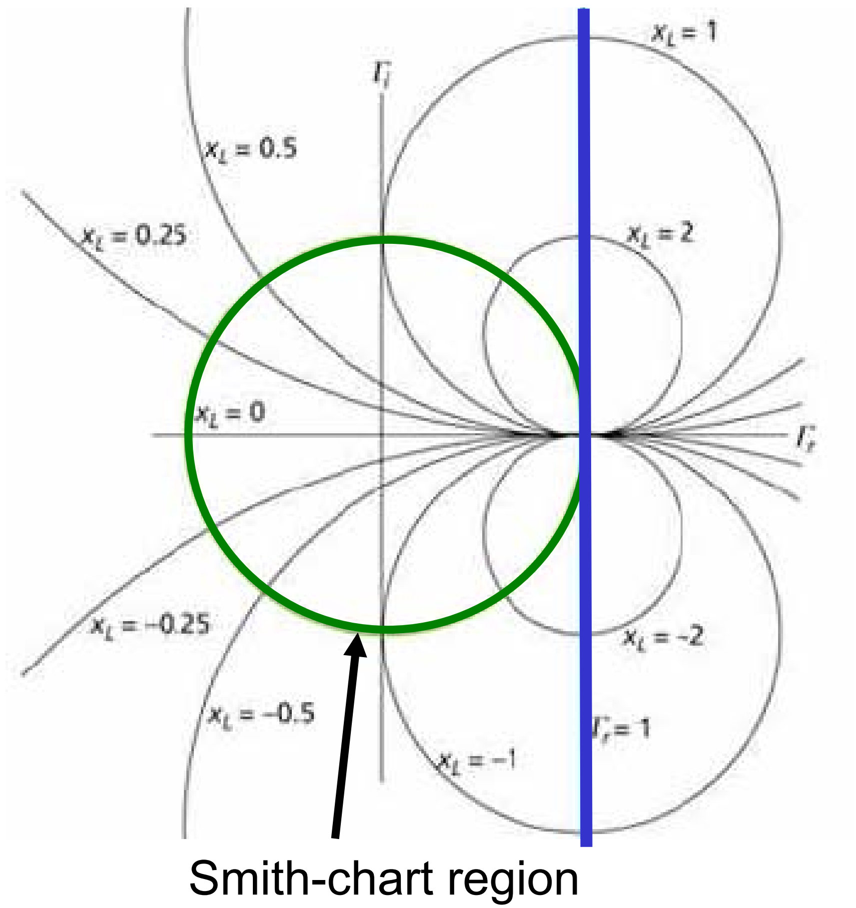
Points of constant resistance form circles on the complex reflection-coefficient plane



Source: Rick Nelson

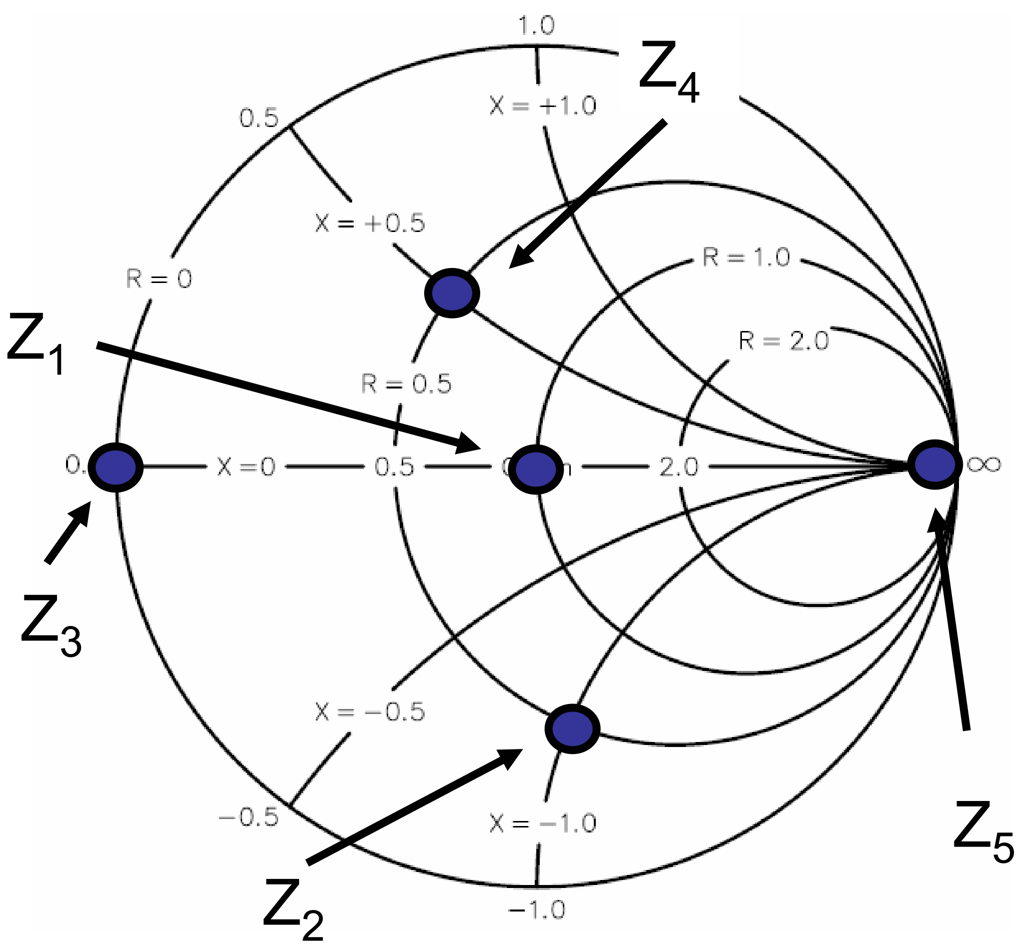
Values of constant imaginary load impedances make up circles centered at points along the blue vertical line

The segments lying in the top half of the complex impedance plane represent inductive reactances; those lying in the bottom half represent capacitive reactances. Only the circle segments within the green circle have meaning for the Smith chart.



Source: Rick Nelson

Some impedances in the Smith chart



All impedances normalized to Z_o

$Z_1=1$ (match)

$Z_2=0.5-j$

$Z_3=0$ (short)

$Z_4=0.5+j0.5$

$Z_5=\infty$ (open)

Further reading: S-parameters, Smith chart and matching

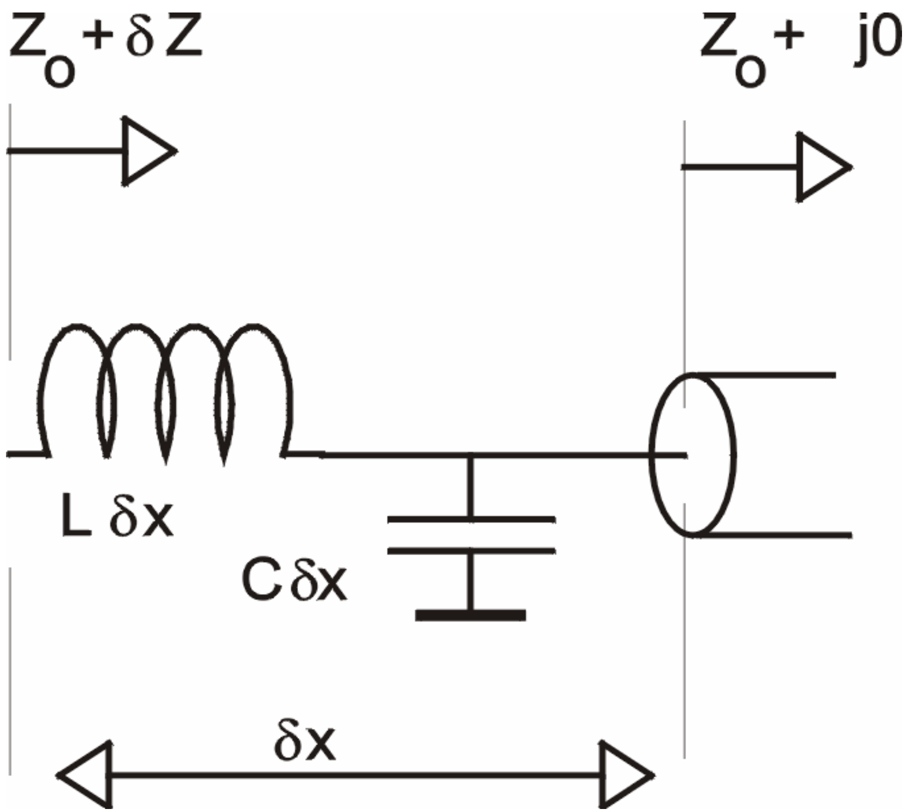
- HP Application Note 95-1
- HP Application Note 154
- Microwave Transistor Amplifiers, *Analysis and Design*, Guillermo Gonzalez, ISBN 0-13-254335-4

Transmission lines

Transmission lines (I)

- Analog designers consider their circuits to be built up with lumped components connected by lines with zero-length.
- This assumptions only holds for low frequen-cies: the wavelength is large compared to the circuit dimensions
- When this is not the case: wires must be treated as transmission lines

Infinitesimal section of a transmission line



$$Z_0 = \sqrt{L/C}$$

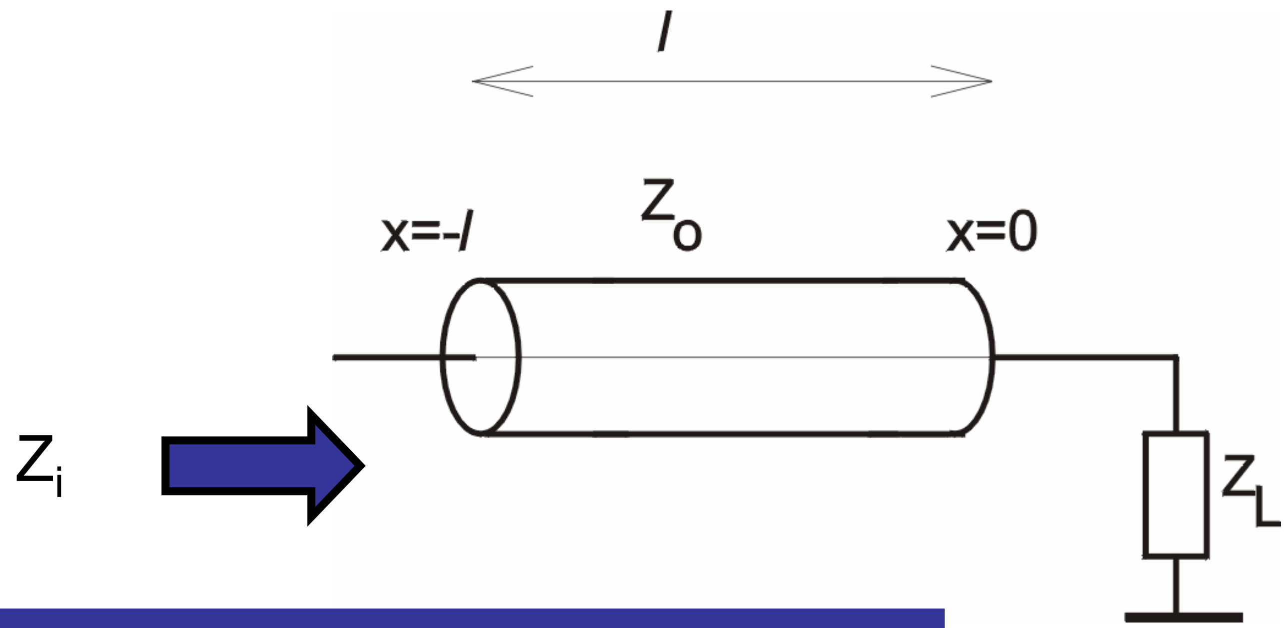
$$V_x = V_0 \exp(-\alpha x - j\beta x)$$

With:

α the attenuation factor
ω/β=v, the wave velocity

α=0 when the line is lossless

Impedance matching using transmission lines



$$Z_i = Z_0 \frac{Z_L + jZ_0 \tan(k_p l)}{Z_0 + jZ_L \tan(k_p l)}$$

With: $K_{p=}2\pi/\lambda$,

The propagation constant

Remarks / implications

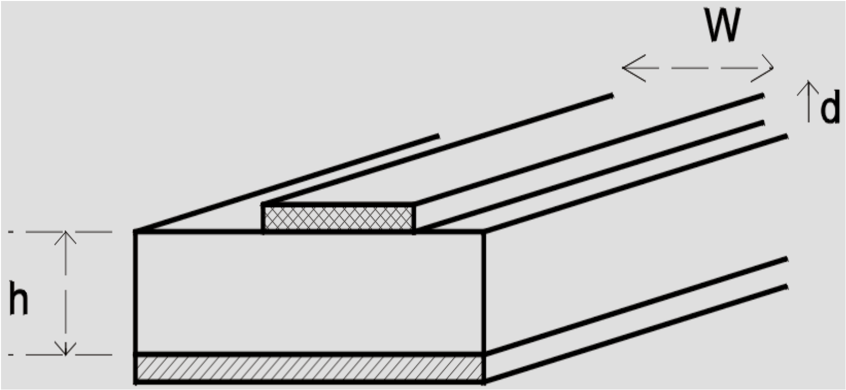
- If the line is terminated with the characteristic impedance (Z_L is Z_0), then Z_i=Z_o

$$Z_i = Z_0 \frac{Z_L + jZ_0 \tan(k_p l)}{Z_0 + jZ_L \tan(k_p l)}$$

- If the line is shorted at x=0, Z_i=jZ_0tan(ωl/v). Hence depending on l the input impedance is inductive or capacitive (for a narrow frequency range)
- If the line is shorted at x=0, Z_i=Z_0/(jtan(ωl/v)). Hence depending on l the input impedance is capacitive of inductive (for a narrow frequency range)
- Suppose l=λ/4, then Z_i/Z_0=Z_0/Z_l: The load impedance is inverted, Furthermore, Z_0=√($Z_i$$Z_l$). The characteristic impedance can be chosen for a power match

Microstrip lines

- Transmission lines are often realized using microstrip lines or coplanar strip lines

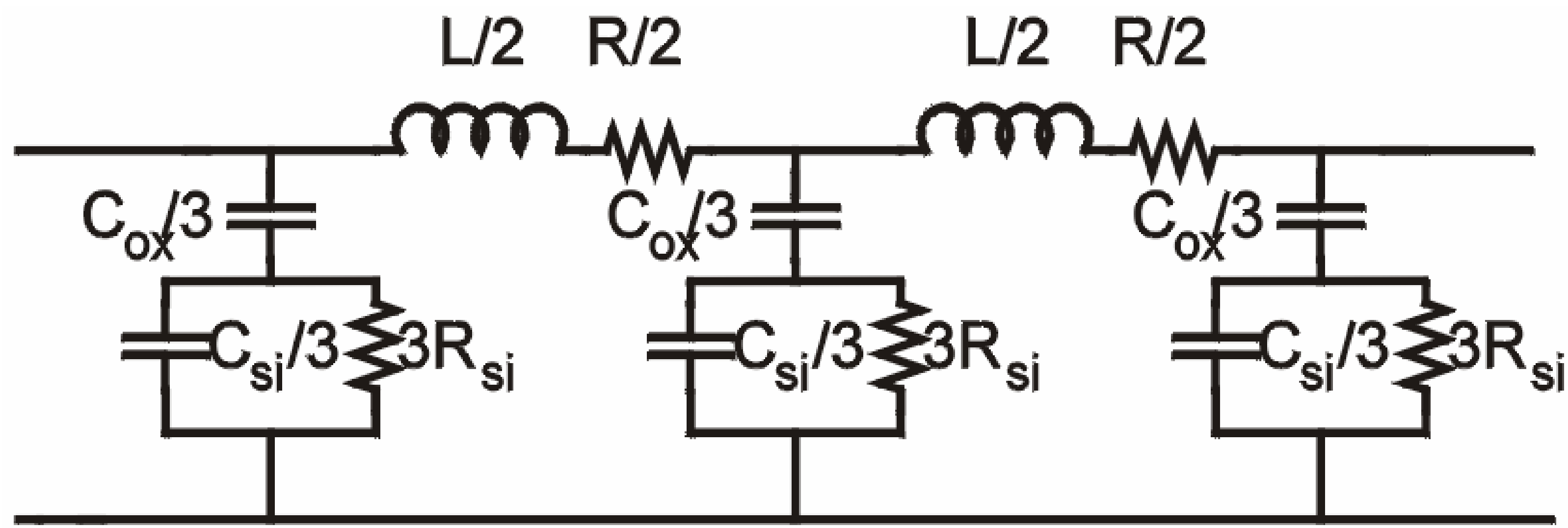


$$Z_0 = \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left(8\frac{h}{W} + 0.25\frac{W}{h}\right)$$

If d/h < 0.005 (negligible thickness d

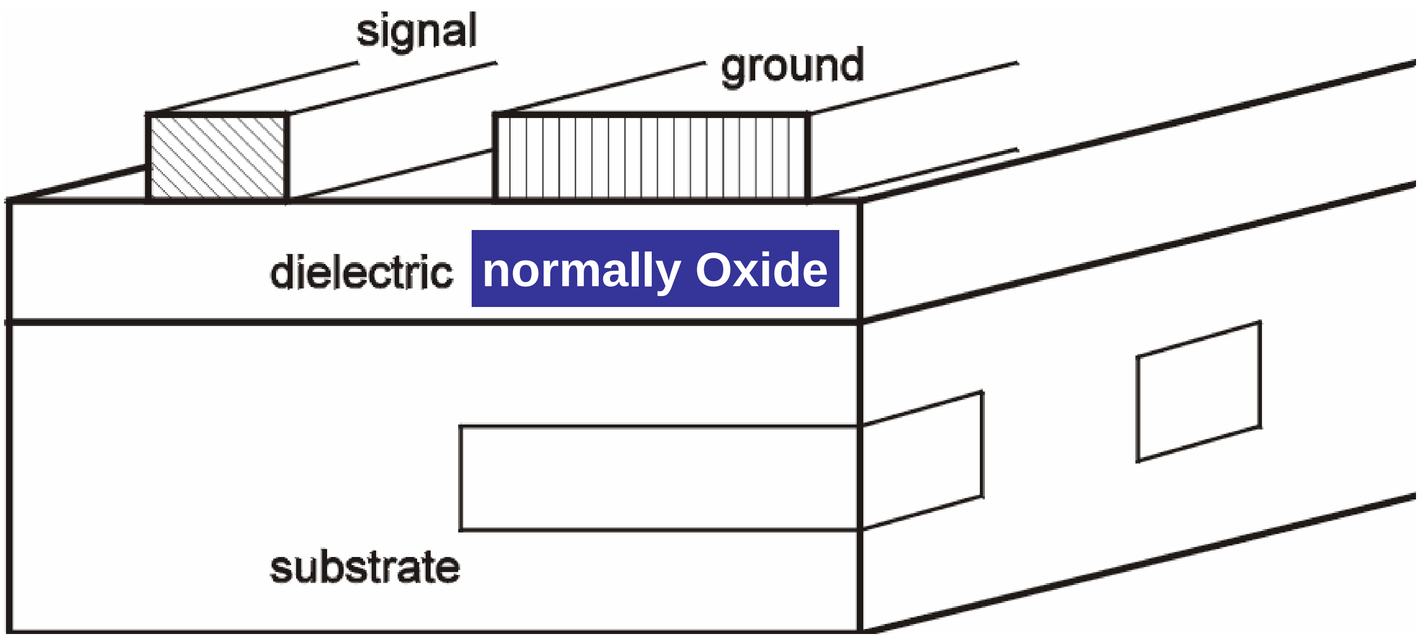
A microstrip is a transmission line consisting of a metal strip and a ground plane with a dielectric medium in between. All EM fields are not completely in the substrate (not a pure TEM wave). The concept of effective dielectric constant ε_{eff}, takes this into account.

Microstrip modeling (metal over substrate)



Distributed equivalent circuit model with lumped components taking into account substrate losses.

Coplanar strip lines



Electromagnetic waves vary only in the horizontal plane

Antennas

Antennas

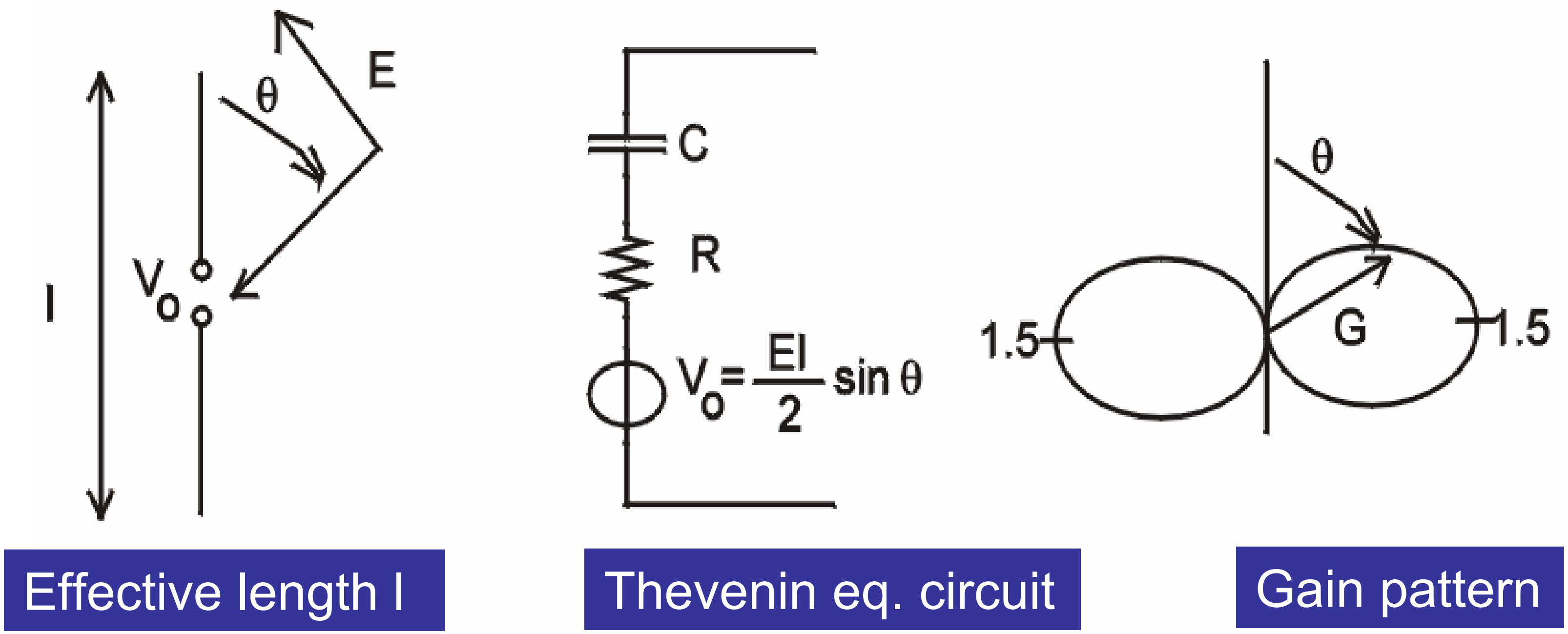
- Good to know something of antennas when you are making trans-ceivers. You don't want to make only chips: you want to make working systems / applications.

- Antennas on-chip possible but because of efficiency and cost (lot of area) often not a good idea.

- An efficient transmit antenna converts most power into EM waves and only a small portion is dissipated in the antenna due to resistive losses. Vice versa for a receive antenna. Normally one antenna in a transceiver, and for example a filter to split receive and transmit signal in case of frequency division duplex.

Antenna example

- Dipole. Angle θ is the angle from the dipole axis for the incident electric field E



Example commercially available patch antennas in ISM bands

A specific and very popular low-profile antenna type, is the microstrip patch. This antenna commonly consists of a metal patch on a dielectric substrate. Size is proportional to $1/\sqrt{\varepsilon_r}$, with ε_r the dielectric constant of the substrate. Consequently, low-profile can be accomplished by selecting a substrate with a high ε_r. As a drawback, increasing ε_r leads to reduced efficiency and bandwidth. Therefore, various methods have been developed to cope with these issues, such as applying a multilayered structure.[8] Here, multiple patches are stacked on top of each other. Each of them can have variable thickness and dielectric constant. Table 6.3 lists some properties of commercially available patch antennas.

Operation frequency	Max. gain	Size	Bandwidth	Max. VSWR
434MHz[23]	−2dBi	$30 \times 7 \times 1.8$mm	25MHz	Not specified
434MHz[26]	−5dBi	$28 \times 13.7 \times 1.6$mm	10MHz	< 1.7 typical
868MHz[26]	−2dBi	$28 \times 13.7 \times 1.6$mm	20MHz	< 1.7 typical
868MHz[23]	1dBi	$30 \times 7 \times 1.8$mm	60MHz	Not specified
870MHz[25]	1.5dBi	$16.5 \times 14 \times 0.9$mm	100MHz	2
2450MHz[25]	2.5dBi	$7.8 \times 3.6 \times 0.9$mm	$>$ 100MHz	2
2442MHz[24]	−1dBi	$7 \times 7 \times 0.7$mm	85MHz	7
2442MHz[24]	−3dBi	$7 \times 7 \times 0.5$mm	85MHz	8.5
2442MHz[24]	2dBi	$12 \times 12 \times 4.0$mm	85MHz	3
2450MHz[26]	0.8dBi	$6.5 \times 2.2 \times 1.0$mm	180MHz	3

Source: Guy Thoonen

Additional Slides

Conversion between S-parameters and other parameters

S-parameters and z-parameters

s-parameters in terms of z-parameters	z-parameters in terms of s-parameters
$s_{11} = \frac{(z_{11}-1)(z_{22}+1) - z_{12}z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}}$	$z_{11} = \frac{(1+s_{11})(1-s_{22}) + s_{12}s_{21}}{(1-s_{11})(1-s_{22}) - s_{12}s_{21}}$
$s_{12} = \frac{2z_{12}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}}$	$z_{12} = \frac{2s_{12}}{(1-s_{11})(1-s_{22}) - s_{12}s_{21}}$
$s_{21} = \frac{2z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}}$	$z_{21} = \frac{2s_{21}}{(1-s_{11})(1-s_{22}) - s_{12}s_{21}}$
$s_{22} = \frac{(z_{11}+1)(z_{22}-1) - z_{12}z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}}$	$z_{22} = \frac{(1+s_{22})(1-s_{11}) + s_{12}s_{21}}{(1-s_{11})(1-s_{22}) - s_{12}s_{21}}$

Source: HP AP-95

S-parameters and y-parameters

s-parameters in terms of y-parameters	y-parameters in terms of s-parameters
$s_{11} = \frac{(1-y_{11})(1+y_{22})+y_{12}y_{21}}{(1+y_{11})(1+y_{22})-y_{12}y_{21}}$	$y_{11} = \frac{(1+s_{22})(1-s_{11})+s_{12}s_{21}}{(1+s_{11})(1+s_{22})-s_{12}s_{21}}$
$s_{12} = \frac{-2y_{12}}{(1+y_{11})(1+y_{22})-y_{12}y_{21}}$	$y_{12} = \frac{-2s_{12}}{(1+s_{11})(1+s_{22})-s_{12}s_{21}}$
$s_{21} = \frac{-2y_{21}}{(1+y_{11})(1+y_{22})-y_{12}y_{21}}$	$y_{21} = \frac{-2s_{21}}{(1+s_{11})(1+s_{22})-s_{12}s_{21}}$
$s_{22} = \frac{(1+y_{11})(1-y_{22})+y_{12}y_{21}}{(1+y_{11})(1+y_{22})-y_{12}y_{21}}$	$y_{22} = \frac{(1+s_{11})(1-s_{22})+s_{12}s_{21}}{(1+s_{11})(1+s_{22})-s_{12}s_{21}}$

Source: HP AP-95

S-parameters and h-parameters

s-parameters in terms of h-parameters	h-parameters in terms of s-parameters
$s_{11} = \frac{(h_{11}-1)(h_{22}+1) - h_{12}h_{21}}{(h_{11}+1)(h_{22}+1) - h_{12}h_{21}}$	$h_{11} = \frac{(1+s_{11})(1+s_{22}) - s_{12}s_{21}}{(1-s_{11})(1+s_{22}) + s_{12}s_{21}}$
$s_{12} = \frac{2h_{12}}{(h_{11}+1)(h_{22}+1) - h_{12}h_{21}}$	$h_{12} = \frac{2s_{12}}{(1-s_{11})(1+s_{22}) + s_{12}s_{21}}$
$s_{21} = \frac{-2h_{21}}{(h_{11}+1)(h_{22}+1) - h_{12}h_{21}}$	$h_{21} = \frac{-2s_{21}}{(1-s_{11})(1+s_{22}) + s_{12}s_{21}}$
$s_{22} = \frac{(1+h_{11})(1-h_{22}) + h_{12}h_{21}}{(h_{11}+1)(h_{22}+1) - h_{12}h_{21}}$	$h_{22} = \frac{(1-s_{22})(1-s_{11}) - s_{12}s_{21}}{(1-s_{11})(1+s_{22}) + s_{12}s_{21}}$

Source: HP AP-95

Parameter denormalization

The h–, y–, and z–parameters listed in previous tables are all normalized to Z_0. If h′, y′, z′ are the actual parameters, then:		
$z'_{11} = z_{11} Z_0$	$y'_{11} = y_{11} / Z_0$	$h'_{11} = h_{11} Z_0$
$z'_{12} = z_{12} Z_0$	$y'_{12} = y_{12} / Z_0$	$h'_{12} = h_{12}$
$z'_{21} = z_{21} Z_0$	$y'_{21} = y_{21} / Z_0$	$h'_{21} = h_{21}$
$z'_{22} = z_{22} Z_0$	$y'_{22} = y_{22} / Z_0$	$h'_{22} = h_{22} / Z_0$

Z_0

Parameter Normalization

The various scattering parameters are all normalized by the reference impedance, Z_0. This impedance is usually the characteristic impedance of the transmission line in which the network of interest is embedded. Normalizing the scattering parameters makes the Smith Chart readily applicable to transmission lines of any impedance. In addition, impedance and admittance values can be plotted on the same chart.

Source: HP AP-95