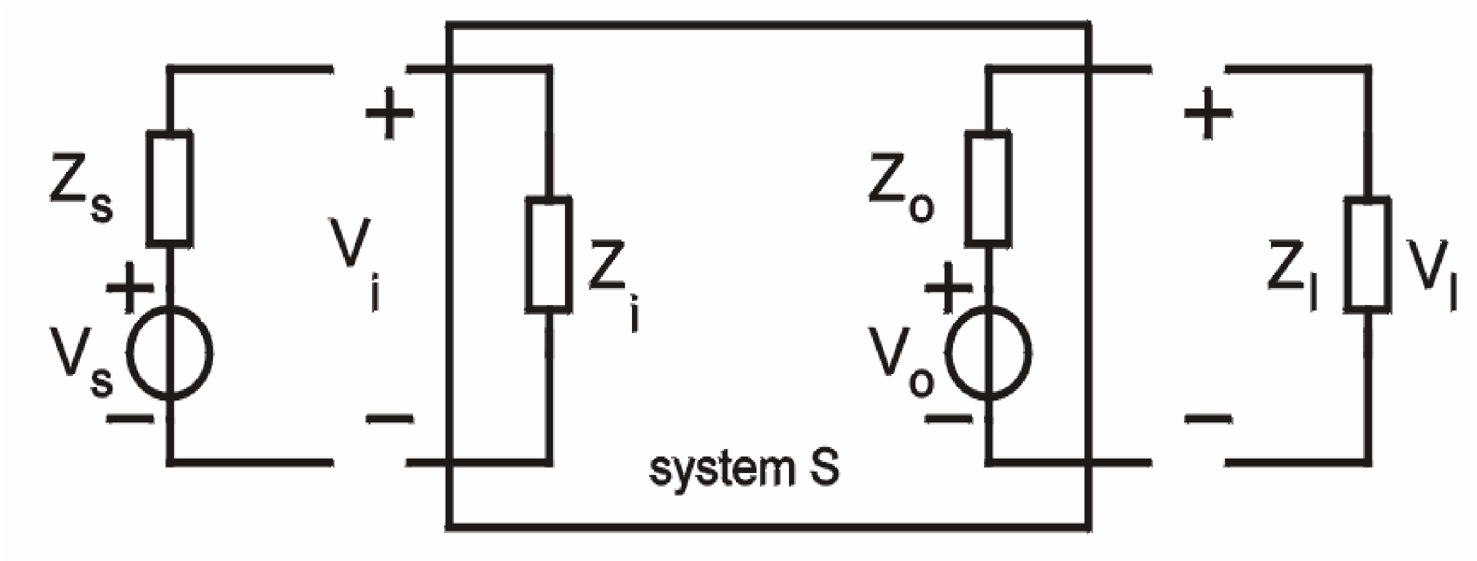
Gain: system model



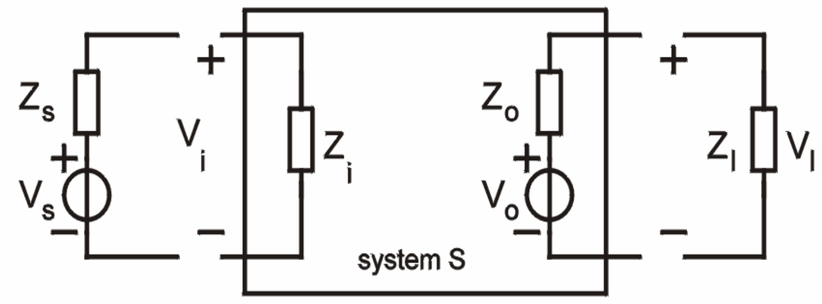
Time-invariant system S with load impedance Z_l . System S has an input impedance Z_i and an output impedance Z_o

Meaning of the word gain

- Means nothing without the correct additional specifications:
  - *What is the load / source impedance*
  - *Voltage or power gain, and which power gain?*

- Analog designers normally use the definition of voltage gain.
- RF / microwave designers are more familiar with several gain definitions in terms of power.

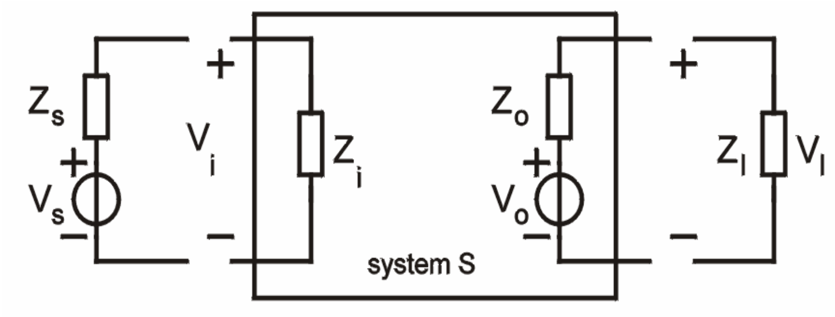
Voltage gain



$$A_v = \frac{V_o}{V_i} \rightarrow \frac{V_l}{V_s}$$

Assuming Z_i is much higher (i.e. $Z_i \rightarrow \infty$) than Z_s and Z_o is much smaller (i.e. $Z_o \rightarrow 0$) than Z_l. In the limiting case no current is flowing into system S.

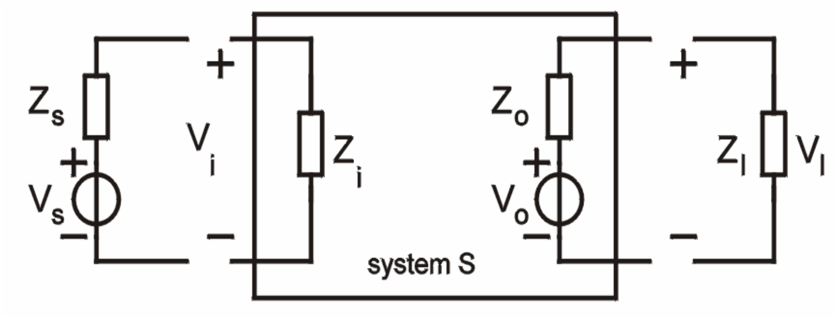
Delivered power at the input



$$P_{i,del} = \frac{V_i^2}{R_i} = \frac{R_i}{(R_i + R_s)^2} \cdot V_s^2$$

This is the power delivered at the input of system S. (Treating the impedances Z as resistors).

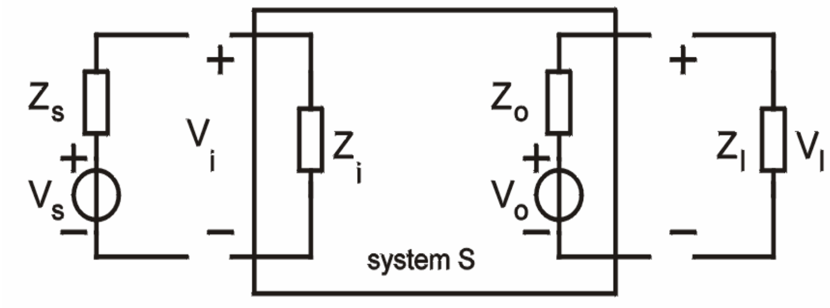
Available power at the input



$$P_{s,av} = \frac{V_s^2}{4R_s}$$

Maximum power that can be delivered to the input of the system S: $Z_s = Z_i^*$ (conjugate input match).

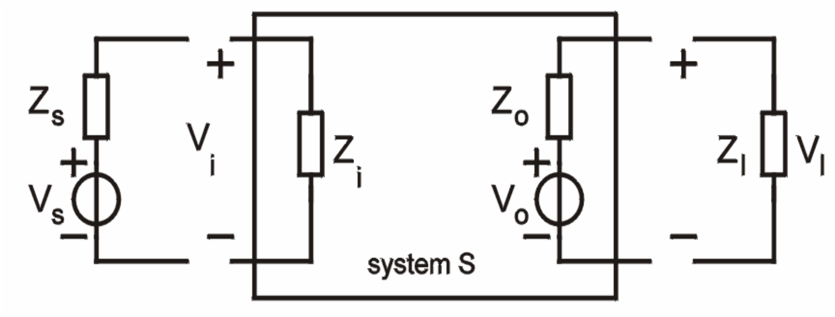
Available power at the output



$$P_{o,av} = \frac{V_o^2}{4R_o} = \left(\frac{R_i}{R_i + R_s}\right)^2 V_s^2 A_v^2 \frac{1}{4R_o}$$

Maximum power that can be delivered to the output of the system S: $Z_o = Z_l^*$ (conjugate output match).

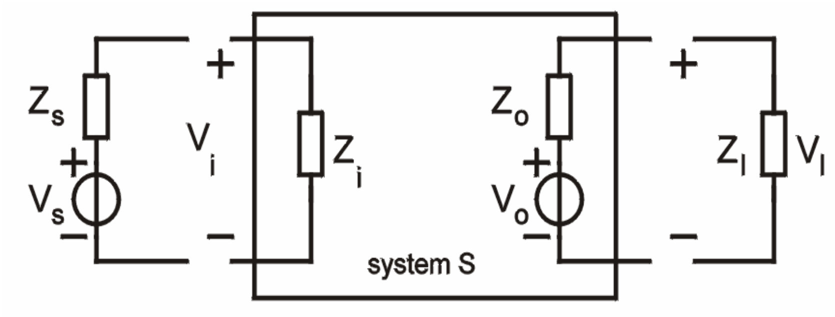
Available gain G_a



$$G_a = \frac{P_{o,av}}{P_{s,av}} = \left(\frac{R_i}{R_i + R_s}\right)^2 A_v^2 \frac{R_s}{R_o} < \frac{P_{o,av}}{P_{i,del}}$$

Mismatch between R_i and R_s reduced G_a. A perfect input match yields $P_{s,av} = P_{i,del}$. G_a assumes a perfect output match.

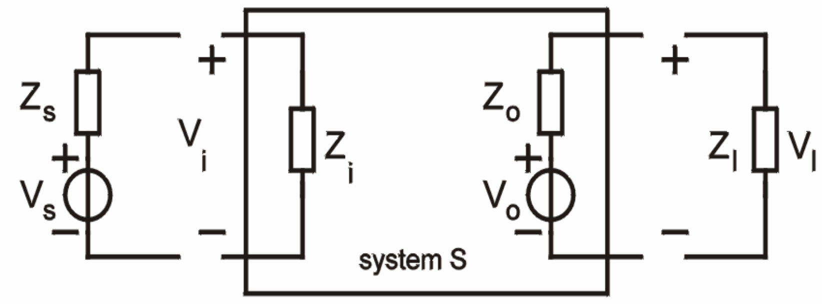
Delivered power at the load



$$P_{l,del} = \frac{V_l^2}{R_L} = V_i^2 A_v^2 \frac{R_L}{\left(R_L + R_o\right)^2}$$

The delivered power at the output of system S, $P_{l,del}$ is lower than the available power $P_{o,av}$.

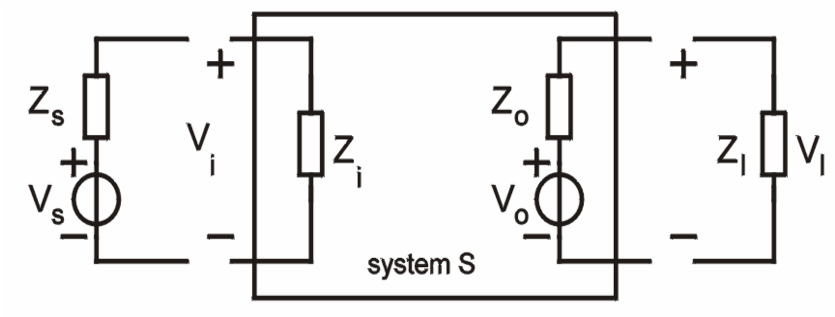
(Delivered) power gain



$$G_P = \frac{P_{l,del}}{P_{i,del}} = A_v^2 \frac{R_L R_i}{(R_L + R_o)^2} < \frac{P_{o,av}}{P_{i,del}}$$

Mismatch between R_L and R_o reduced G_p. A perfect output match yields $P_{o,av} = P_{l,del}$. G_p assumes a perfect input match.

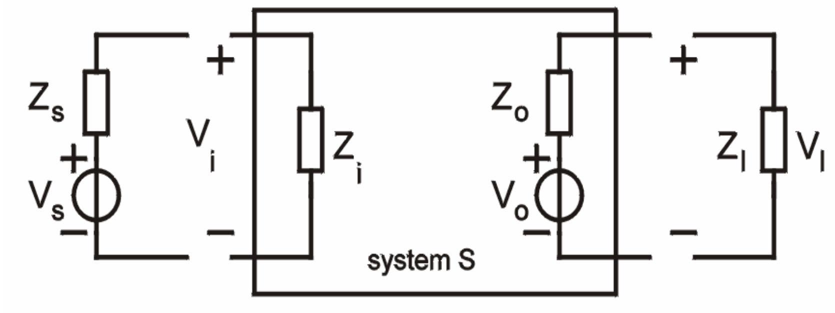
Maximum gain



$$G_{\max} = \frac{P_{o,av}}{P_{i,del}} = \frac{P_{l,del}}{P_{s,av}} = A_v^2 \frac{R_i}{4R_o}$$

The maximum gain for system S is obtained when there is a perfect input and output match.
In equations: $Z_s = Z_i^*$ and $Z_o = Z_l^*$

Transducer gain



$$G_t = \frac{P_{l,del}}{P_{s,av}} = \left(\frac{R_i}{R_i + R_s}\right)^2 A_v^2 \frac{4R_L R_s}{(R_L + R_o)^2}$$

The transducer gain (lowest gain) is obtained assuming both at input and output a mismatch. When R_L is R_o, G_t becomes G_a. When R_i=R_s, G_p is obtained.

Five different gain definitions

$$A_v = \frac{V_o}{V_i} \rightarrow \frac{V_l}{V_s}$$

$$G_a = \frac{P_{o,av}}{P_{s,av}} = \left(\frac{R_i}{R_i + R_s}\right)^2 A_v^2 \frac{R_s}{R_o} < \frac{P_{o,av}}{P_{i,del}}$$

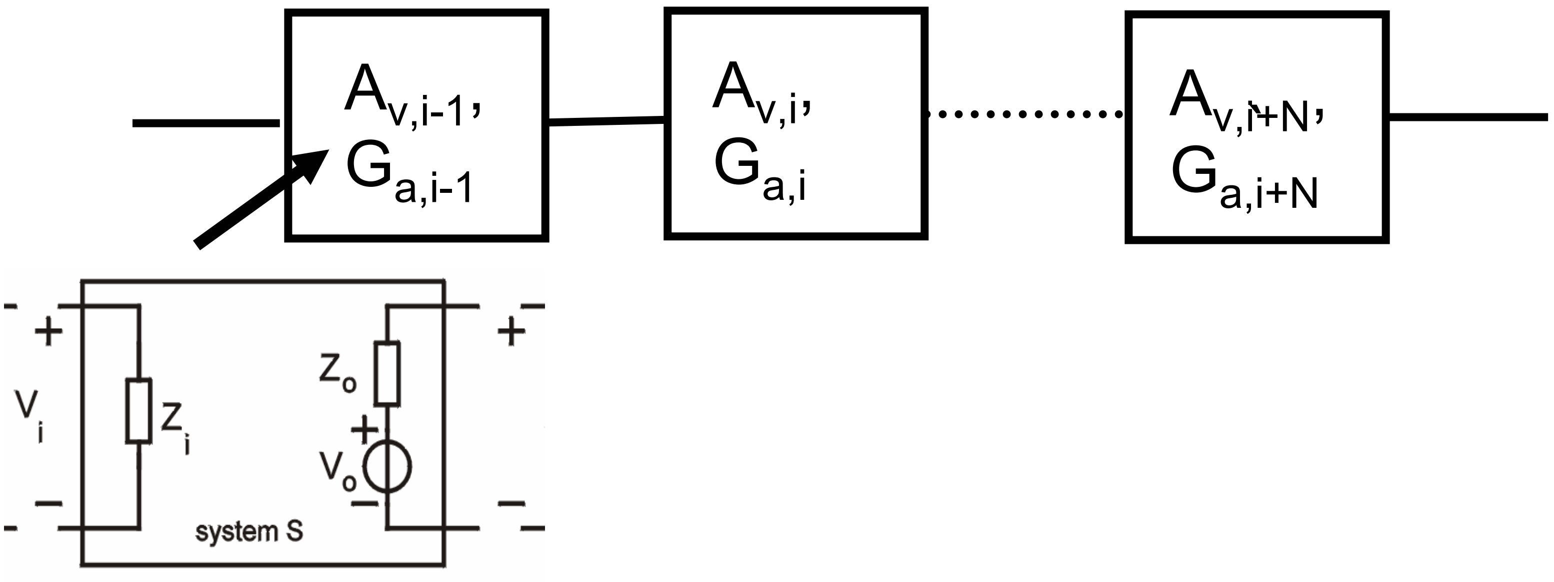
$$G_P = \frac{P_{l,del}}{P_{i,del}} = A_v^2 \frac{R_L R_i}{(R_L + R_o)^2} < \frac{P_{o,av}}{P_{i,del}}$$

$$G_{\max} = \frac{P_{o,av}}{P_{i,del}} = \frac{P_{l,del}}{P_{s,av}} = A_v^2 \frac{R_i}{4R_o}$$

$$G_t = \frac{P_{l,del}}{P_{s,av}} = \left(\frac{R_i}{R_i + R_s}\right)^2 A_v^2 \frac{4R_L R_s}{(R_L + R_o)^2}$$

Hence it is important to specify the gain definition one is using

Question ?



What is the relation between available power gain and (unloaded) voltage gain Av for a cascade of N stages?

G_a and A_v of i^{th} stage of N cascaded stages

Assume perfect input and output matching (by definition since we talking about available gain)

$$G_{a,i} = \frac{P_{o,av,i}}{P_{s,av,i}} \qquad P_{s,av,i} = \frac{V_{o,i-1}^2}{4R_{o,i-1}}$$

$$P_{o,av,i} = \frac{V_{o,i}^2}{R_{o,i}} = \left(\frac{R_{i,i}}{R_{i,i}+R_{o,i-1}}\right)^2 V_{o,i-1}^2 A_{v,i}^2 \frac{1}{R_{o,i}}$$

No factor 4 compared to single stage G_a equation: power dissipated in the output resistance of the two port has already been taken into consideration (in the previous stage)

So $Av_{,i}$ can be calculated if we know $Ga_{,I}$ (or the other way around)

$$G_{a,i} = \frac{P_{o,av,i}}{P_{s,av,i}} = A_{v,i}^{2} \frac{R_{o,i-1}}{R_{o,i}} = A_{v,i}^{2} \frac{R_{i,i}}{R_{o,i}}$$

As mentioned: this equation assumes that the input of the Ith stage is matched to the output of the previous stage (the (i-1)th stage)