It follows from (2.84) that

$$NF = 4kT(R_S||R_P)\frac{(R_S+R_P)^2}{R_P^2}\frac{1}{4kTR_S} \tag{2.87}$$

$$= 1 + \frac{R_S}{R_P}. \tag{2.88}$$

Thus, the noise figure is minimized by maximizing R_P. Interestingly, the condition for minimum noise figure does not coincide with that for maximum power transfer ($R_S = R_P$).

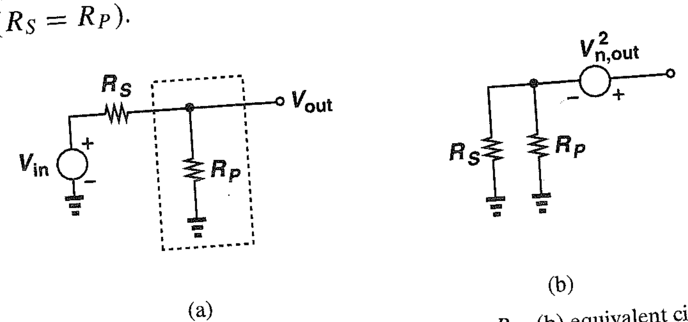

Figure 2.30 (a) Calculation of noise figure of resistor R_P, (b) equivalent circuit of (a).

As another example, let us compute the noise figure of the amplifier shown in Fig. 2.31(a). The circuit consists of a common-source stage and a feedback source follower so as to provide an input resistance equal to R_S. Neglecting body effect, channel length modulation, parasitic capacitances, and the noise of I_1, we utilize the equivalent circuit shown in Fig. 2.31(b) to calculate the total output noise. The condition $R_{in} = R_S$ translates to

$$R_S = \frac{1}{g_{m2}}\frac{1}{1+g_{m1}R_D}. \tag{2.89}$$

The noise current of M_2 flows through $R_S/2$, generating an output noise voltage equal to $(I_{n2}R_S/2)g_{m1}R_D$. The noise current of R_D and M_1 is multiplied by the output resistance of the circuit, which for $R_{in} = R_S$ reduces to

$$R_{out} = \frac{R_D}{2}(1+g_{m2}R_S). \tag{2.90}$$

The total output noise power is therefore equal to

$$V^2_{n,out} = 4kTR_S\left(\frac{1}{4}g^2_{m1}R^2_D\right) + \frac{1}{4}I^2_{n2}R^2_S g^2_{m1}R^2_D$$
$$+ (I^2_{RD} + I^2_{n1})\frac{R^2_D}{4}(1+g_{m2}R_S)^2 \tag{2.91}$$

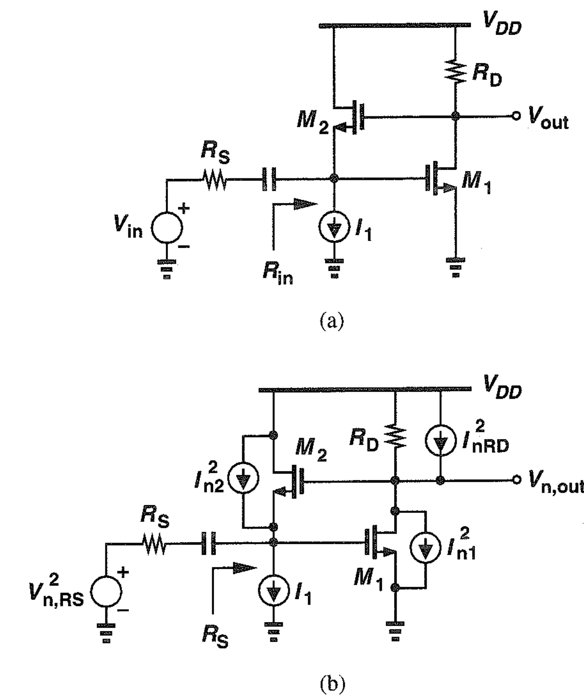

Figure 2.31 (a) Feedback amplifier with input match, (b) noise equivalent circuit of (a).

$$= kTR_S g^2_{m1}R^2_D + \frac{2}{3}kTg_{m2}R^2_S g^2_{m1}R^2_D$$
$$+ \left(\frac{4kT}{R_D} + \frac{8}{3}kTg_{m1}\right)\frac{R^2_D}{4}(1+g_{m2}R_S)^2. \tag{2.92}$$

Thus, the noise figure is

$$NF = \frac{V^2_{n,out}}{A^2_v}\frac{1}{4kTR_S} \tag{2.93}$$

$$= 1 + \frac{2}{3}g_{m2}R_S + \left(\frac{1}{R_D} + \frac{2}{3}g_{m1}\right)\frac{(1+g_{m2}R_S)^2}{g^2_{m1}R_S}, \tag{2.94}$$

subject to the condition $g_{m2}R_S = (1 + g_{m1}R_D)^{-1}$.

Noise Figure of Cascaded Stages For a cascade of stages, the overall noise figure can be obtained in terms of the NF and gain of each stage. Consider the system shown in Fig. 2.32, where input noise generators and input and output resistances of each stage are shown. Note that reactive components of the impedances are nulled and A_{v1} and A_{v2} denote the *unloaded* voltage gain

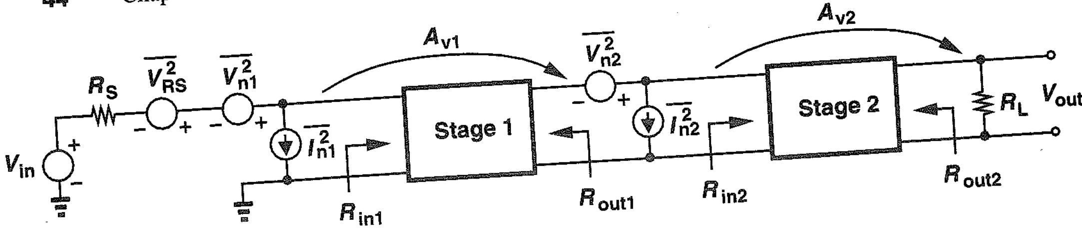

Figure 2.32 Cascaded noisy stages.

of the two stages. The total noise power at the input of the first stage can be written as

$$\overline{V^2_{n,in1}} = \overline{[I_{n1}(R_S||R_{in1}) + V_{n1}\frac{R_{in1}}{R_{in1}+R_S}]^2} + \overline{V^2_{RS}}\frac{R^2_{in1}}{(R_{in1}+R_S)^2}. \quad (2.95)$$

We also note that the total noise power at the input of the second stage is

$$\overline{V^2_{n,in2}} = \overline{V^2_{n,in1}}A^2_{v1}\left(\frac{R_{in2}}{R_{out1}+R_{in2}}\right)^2 + \overline{\left[I_{n2}(R_{out1}||R_{in2}) + V_{n2}\frac{R_{in2}}{R_{in2}+R_{out1}}\right]^2}. \quad (2.96)$$

Thus, the total output noise power of the cascade equals $A^2_{v2}\overline{V^2_{n,in2}}R^2_L/(R_L + R_{out2})^2$. Since the total voltage gain from V_{in} to V_{out} equals

$$A_{v,tot} = \frac{R_{in1}}{R_S+R_{in1}}A_{v1}\frac{R_{in2}}{R_{out1}+R_{in2}}A_{v2}\frac{R_L}{R_{out2}+R_L}, \quad (2.97)$$

the overall noise figure is

$$NF_{tot} = \frac{1}{A^2_{v,tot}}A^2_{v2}\left(\frac{R_L}{R_L+R_{out2}}\right)^2\overline{V^2_{n,in2}}\cdot\frac{1}{4kTR_S}. \quad (2.98)$$

Using (2.95) and (2.96) and simplifying the result, we have

$$NF_{tot} = \frac{4kTR_S + \overline{(I_{n1}R_S+V_{n1})^2}}{4kTR_S} + \frac{\overline{(I_{n2}R_{out1}+V_{n2})^2}}{A^2_{v1}}\frac{1}{\left(\frac{R_{in1}}{R_S+R_{in1}}\right)^2}\frac{1}{4kTR_S}. \quad (2.99)$$

The first term on the right-hand side can be identified as the NF of the first stage with respect to a source impedance R_S. The second term, on the other hand, is not as straightforward. In the special case where $R_S = R_{in1} = R_{out1} = R_{in2}$, we have

$$NF_{tot} = NF_1 + \frac{\overline{(I_{n2}R_S+V_{n2})^2}}{A^2_{v1}}\frac{1}{4kTR_S} \quad (2.100)$$

$$= NF_1 + \frac{NF_2 - 1}{A^2_{v1}}, \quad (2.101)$$

where NF_2 is the noise figure of the second stage with respect to a source impedance R_S.

In the general case, we simplify (2.99) using the concept of "available power gain," A_P. This type of gain is defined as the available power at the output (the power that the circuit would deliver to a conjugate-matched load) divided by the available source power (the power that the source would deliver to a conjugate-matched circuit.) The available output power of stage 1 in Fig. 2.32 is

$$P_{out,av} = \overline{V^2_{in}}\left(\frac{R_{in1}}{R_S+R_{in1}}\right)^2 A^2_{v1}\cdot\frac{1}{4R_{out1}}, \quad (2.102)$$

and the available source power is

$$P_{source,av} = \frac{\overline{V^2_{in}}}{4R_S}. \quad (2.103)$$

Thus,

$$A_P = \left(\frac{R_{in1}}{R_S+R_{in1}}\right)^2 A^2_{v1}\frac{R_S}{R_{out1}}. \quad (2.104)$$

Noting that the noise figure of stage 2 with respect to a source impedance R_{out1} is

$$NF_{2,Rout1} = 1 + \frac{\overline{(I_{n2}R_{out1}+V_{n2})^2}}{4kTR_{out1}}, \quad (2.105)$$

we can write (2.99) as

$$NF_{tot} = NF_{1,RS} + \frac{NF_{2,Rout1} - 1}{A_P}, \quad (2.106)$$

where $NF_{1,RS}$ denotes the noise figure of stage 1 with respect to a source impedance R_S.

Similarly, for m stages,

$$NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{p1}} + \cdots + \frac{NF_m - 1}{A_{p1}\cdots A_{p(m-1)}}, \quad (2.107)$$

where the NF of each stage is calculated with respect to the source impedance driving that stage. This is called the Friis equation [11]. Expressing the overall noise figure in terms of the noise figure of each stage, this relation proves especially useful if a receiver employs various off-the-shelf building blocks that are characterized independently by manufacturers.

The Friis equation indicates that the noise contributed by each stage decreases as the gain preceding the stage increases, implying that the first few stages in a cascade are the most critical. Conversely, if a stage exhibits attenuation (loss), then the noise figure of the following circuit is "amplified" when referred to the input of that stage. This occurs, for example, if a narrowband lossy filter is interposed between the antenna and the low noise amplifier in a receiver to reject out-of-band interferers (Chapter 5).

Noise Figure of Lossy Circuits Passive filters used in RF receivers have a finite in-band loss. In addition to attenuating the desired signal, lossy circuits in general contribute noise as well, a fact that may not be obvious if we consider an ideal LC filter [Fig. 2.33(a)] as an example. Recall, however, that many RF circuits are required to have well-defined *resistive* input and output impedances for proper matching. For example, the filter placed between the antenna and the LNA can be viewed as depicted in Fig. 2.33(b). Our goal is to find the relationship between the noise figure and the loss of a passive circuit with resistive input and output impedances.

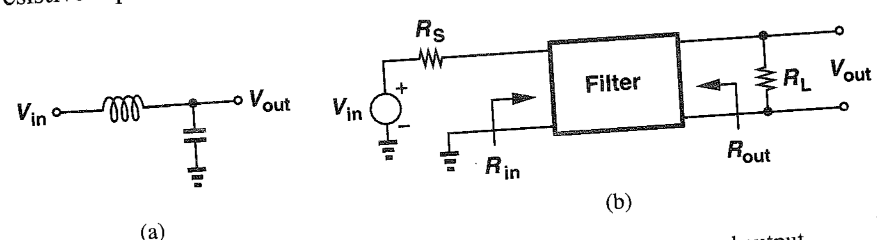

Figure 2.33 (a) LC attenuator, (b) lossy circuit matched at input and output.

Consider a linear time-invariant passive reciprocal network as shown in Fig. 2.34(a) with real input and output impedance. It can be proved [5] that if the output resistance is R_{out}, then the noise Thevenin equivalent circuit is as depicted in Fig. 2.34(b), with the PSD of the voltage source given by $4kTR_{out}$. Note that R_{out} and hence $\overline{V_n^2}$ generally depend on the source impedance, R_S.

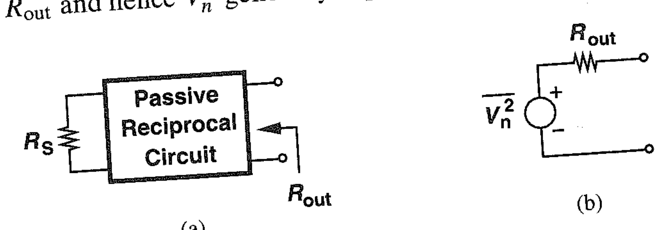

Figure 2.34 (a) Passive reciprocal network, (b) Thevenin noise model of (a).

Now consider the circuit of Fig. 2.35(a). In analogy with the concept of available power gain, we define the power loss L as P_{in}/P_{out}, where P_{in} is the available source power and P_{out} the available power at the output. To calculate the signal loss, the circuit can be modeled by the Thevenin equivalent shown in Fig. 2.35(b). Note that this circuit is the equivalent for the *signal*, whereas that in Fig. 2.34(b) is for the noise. Since $P_{in} = V_{in}^2/(4R_S)$ and $P_{out} = V_{TH}^2/(4R_{out})$, we have

$$L = \frac{V_{in}^2}{V_{TH}^2}\frac{R_{out}}{R_S}. \tag{2.108}$$

To compute the noise figure, we find the output noise voltage and the voltage gain. With a load R_L, Fig. 2.34(b) implies that

$$V_{n,out}^2 = 4kTR_{out}\frac{R_L^2}{(R_L + R_{out})^2}. \tag{2.109}$$

The voltage gain from V_{in} to V_{out} in Fig. 2.35(b) is

$$A_v = \frac{V_{TH}}{V_{in}}\frac{R_L}{R_L + R_{out}}. \tag{2.110}$$

Thus,

$$NF = 4kTR_{out}\frac{V_{in}^2}{V_{TH}^2}\frac{1}{4kTR_S} \tag{2.111}$$

$$= L. \tag{2.112}$$

We conclude that for a passive reciprocal network the noise figure is equal to the loss if the latter is defined as above.

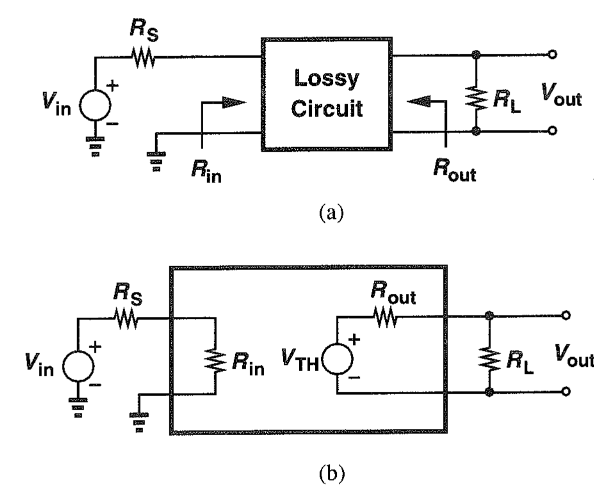

Figure 2.35 (a) Circuit for noise figure calculation, (b) equivalent circuit of (a).

To reinforce the above concepts, we calculate the overall noise figure of a lossy filter followed by a low-noise amplifier (Fig. 2.36). From the Friis equation,

$$NF_{\text{tot}} = NF_{\text{filt}} + \frac{NF_{LNA} - 1}{L^{-1}} \quad (2.113)$$

$$= L + (NF_{LNA} - 1)L \quad (2.114)$$

$$= L \cdot NF_{LNA}, \quad (2.115)$$

where the noise figure of the LNA is calculated with respect to the output resistance of the filter.

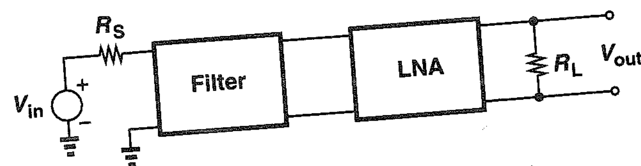

Figure 2.36 Cascade of lossy filter and LNA.

2.4 SENSITIVITY AND DYNAMIC RANGE

Sensitivity The sensitivity of an RF receiver is defined as the minimum signal level that the system can detect with acceptable signal-to-noise ratio. To calculate the sensitivity, we write

$$NF = \frac{SNR_{\text{in}}}{SNR_{\text{out}}} \quad (2.116)$$

$$= \frac{P_{\text{sig}}/P_{RS}}{SNR_{\text{out}}}, \quad (2.117)$$

where P_{sig} denotes the input signal power and P_{RS} the source resistance noise power, both per unit bandwidth. It follows that

$$P_{\text{sig}} = P_{RS} \cdot NF \cdot SNR_{\text{out}}. \quad (2.118)$$

Since the overall signal power is distributed across the channel bandwidth, B, the two sides of (2.118) must be integrated over the bandwidth to obtain the total mean square power. Thus, for a flat channel,

$$P_{\text{sig,tot}} = P_{RS} \cdot NF \cdot SNR_{\text{out}} \cdot B. \quad (2.119)$$

Equation (2.119) predicts the sensitivity as the minimum input signal that yields a given value for the output SNR. Changing the notation slightly and expressing the quantities in dB or dBm, we have

$$P_{\text{in,min}}|_{\text{dBm}} = P_{RS}|_{\text{dBm/Hz}} + NF|_{\text{dB}} + SNR_{\text{min}}|_{\text{dB}} + 10\log B, \quad (2.120)$$

where $P_{\text{in,min}}$ is the minimum input level that achieves SNR_{min} and B is expressed in hertz. Note that (2.120) does not depend on the gain of the system.

Assuming conjugate matching at the input, we obtain P_{RS} as the noise power that R_S delivers to the receiver:

$$P_{RS} = \frac{4kTR_S}{4}\frac{1}{R_{\text{in}}} \quad (2.121)$$

$$= kT \quad (2.122)$$

$$= -174\ \text{dBm/Hz} \quad (2.123)$$

at room temperature. We thus simplify (2.120) as

$$P_{\text{in,min}} = -174\ \text{dBm/Hz} + NF + 10\log B + SNR_{\text{min}}. \quad (2.124)$$

Note that the sum of the first three terms is the total integrated noise of the system and is sometimes called the "noise floor." Since $P_{\text{in,min}}$ is a function of the bandwidth, a receiver may appear very sensitive simply because it employs a narrowband channel (but of course at the cost of low information rate.)

Dynamic Range Dynamic range (DR) is generally defined as the ratio of the maximum input level that the circuit can tolerate to the minimum input level at which the circuit provides a reasonable signal quality. This definition is quantified in different applications differently. For example, in analog circuits such as op amps and analog-to-digital converters the dynamic range is defined as the ratio of the "full-scale" (FS) input level to the the input level for which $SNR = 1$. The full scale is typically the input level beyond which a hard saturation occurs and can be easily found by examining the circuit, and the minimum input level is determined by the noise floor.

In RF design, on the other hand, the situation is more complicated. Consider a simple common-source stage. How do we define the input full scale for such a circuit? It is possible to define the FS as the input voltage for which the transistor is at the edge of triode region. However, if a sinusoid with full-scale swing is applied to the circuit, the output exhibits substantial distortion. Also, the minimum signal must provide an SNR greater than unity, for example SNR_{min} in Eq. (2.120). For these reasons, we base the definition of the upper end of the dynamic range on the intermodulation behavior and the lower end on the sensitivity. Such a definition is called the "spurious-free dynamic range" (SFDR).

The upper end of the dynamic range is defined as the maximum input level in a two-tone test for which the third-order IM products do not exceed the noise floor. Expressing all of the quantities in dBm, we can rewrite (2.33) as

$$P_{IIP3} = P_{\text{in}} + \frac{P_{\text{out}} - P_{IM,\text{out}}}{2}, \quad (2.125)$$

where $P_{IM,\text{out}}$ denotes the power of IM_3 components at the output. Since $P_{\text{out}} = P_{\text{in}} + G$ and $P_{IM,\text{out}} = P_{IM,\text{in}} + G$, where G is the circuit's power gain in dB and $P_{IM,\text{in}}$ is the input-referred level of the IM_3 products, we have

$$P_{IIP3} = P_{\text{in}} + \frac{P_{\text{in}} - P_{IM,\text{in}}}{2} \tag{2.126}$$

$$= \frac{3P_{\text{in}} - P_{IM,\text{in}}}{2}, \tag{2.127}$$

and hence

$$P_{\text{in}} = \frac{2P_{IIP3} + P_{IM,\text{in}}}{3}. \tag{2.128}$$

The input level for which the IM products become equal to the noise floor is thus given by

$$P_{\text{in,max}} = \frac{2P_{IIP3} + F}{3} \tag{2.129}$$

where $F = -174\ \text{dBm} + NF + 10\log B$.

The SFDR is the difference (in dB) between $P_{\text{in,max}}$ and $P_{\text{in,min}}$:

$$SFDR = \frac{2P_{IIP3} + F}{3} - (F + SNR_{\text{min}}) \tag{2.130}$$

$$= \frac{2(P_{IIP3} - F)}{3} - SNR_{\text{min}}. \tag{2.131}$$

For example, if a receiver with $NF = 9$ dB, $P_{IIP3} = -15$ dBm, and $B = 200$ kHz requires an $SNR_{\text{min}} = 12$ dB, then $SFDR \approx 53$ dB.

The spurious-free dynamic range represents the maximum relative level of interferers that a receiver can tolerate while producing an acceptable signal quality from a small input level.

2.5 PASSIVE IMPEDANCE TRANSFORMATION

At radio frequencies, we often resort to passive circuits to transform impedances—from high to low and vice versa or from complex to real and vice versa. While active devices may also seem a plausible choice for such operations, in some cases only passive components can achieve the required performance. For example, a 3-V power amplifier can deliver a maximum of $P = 3^2/(2 \times 50) = 90$ mW to a 50-Ω load. To increase the output power, a circuit must be interposed between the PA and the load so as to "amplify" the voltage swings while requiring no higher supply voltage (Chapter 9). This is performed by passive matching circuits.

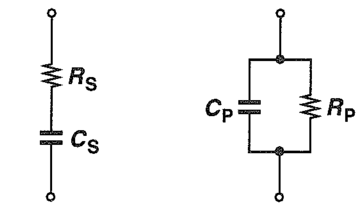

Figure 2.37 Equivalent series and parallel RC circuits.

Before studying transformation techniques, let us consider the RC circuits of Fig. 2.37. The quality factor Q of the series combination, defined as the impedance of the capacitor divided by the resistor, is equal to $1/(R_S C_S \omega)$, approaching infinity as R_S goes to zero. Similarly, the Q of the parallel combination is equal to $R_P C_P \omega$. If Q is relatively high (greater than approximately 5) and the band of interest relatively narrow, then one network can be converted to the other. The two circuits are equivalent if

$$\frac{R_P}{R_P C_P s + 1} = \frac{R_S C_S s + 1}{C_S s}, \tag{2.132}$$

or, for $s = j\omega$, $R_P C_P = 1/(R_S C_S \omega^2)$ and $R_P C_P + R_S C_S - R_P C_S = 0$. Assuming $R_P \gg R_S$, we have $C_P \approx C_S$ and

$$R_P \approx \frac{1}{R_S (C\omega)^2}, \tag{2.133}$$

where $C = C_P \approx C_S$. Thus, the conversion changes the value of the resistance according to (2.133) while keeping the value of the capacitance nearly constant. We can also write $R_P \approx Q_S^2 R_S$, where Q_S is the Q of the series network. Similar results can easily be derived for RL counterparts.

Transformation of impedance can be accomplished by transformers. An ideal transformer with a turn ratio of m scales an impedance by a factor m^2. In reality, however, high-frequency transformers exhibit loss, capacitive coupling between the primary and the secondary, and even unwanted resonances, thus complicating the design and requiring careful modeling. For this reason, we study other approaches to impedance transformation.

Consider the network shown in Fig. 2.38(a), where the capacitive divider is utilized to transform R_P to a higher value. With the assumptions of high Q and narrow bandwidth, the parallel combination of C_P and R_P can be converted to the series circuit shown in Fig. 2.38(b), where $C_S \approx C_P$ and $R_S \approx 1/[R_P (C_P \omega)^2]$. Combining C_1 and C_S into C_{eq}, we arrive at the circuit of Fig. 2.38(c), which can be converted to the parallel network of Fig. 2.38(d), with $C_{tot} \approx C_1 C_P/(C_1 + C_P)$ and $R_{tot} \approx 1/[R_S (C_{eq}\omega)^2] = (1 + C_P/C_1)^2 R_P$. Thus, the capacitive divider "boosts" the value of R_P by a factor $(1 + C_P/C_1)^2$.

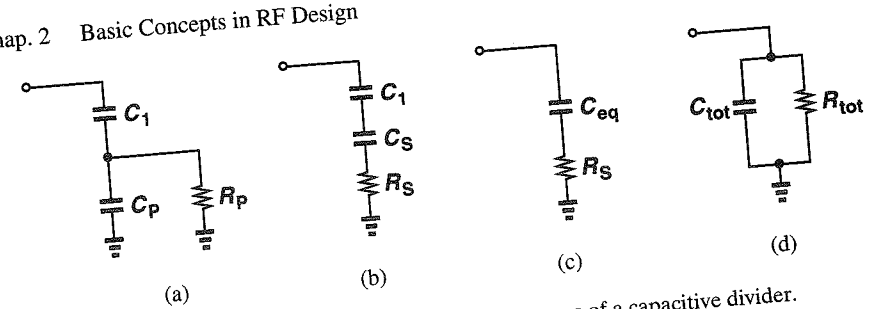

Figure 2.38 Impedance transformation by means of a capacitive divider.

Fig. 2.39(a) depicts a similar transformation circuit using inductive voltage division. For the equivalent circuit of Fig. 2.39(b), we have $L_{tot} \approx L_1 + L_P$ and $R_{tot} \approx (1 + L_1/L_P)^2 R_P$ if the Q is high and the band of interest narrow.

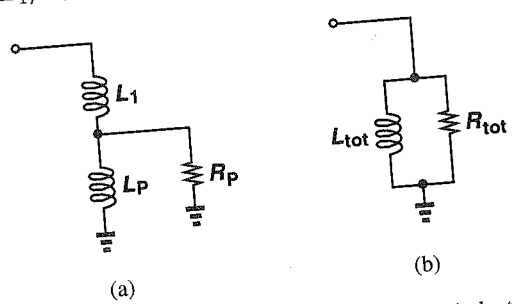

Figure 2.39 Impedance transformation by means of an inductive divider.

A network often employed to transform a resistance to a lower value is illustrated in Fig. 2.40(a). Converting C_P and R_P to the series combination shown in Fig. 2.40(b), we have $C_S \approx C_P$ and $R_S \approx 1/[R_P(C_P\omega)^2]$. In the vicinity of resonance, L_1 and C_S resonate and the network is approximately equivalent to a resistor equal to $1/(C_P^2\omega^2 R_P)$.

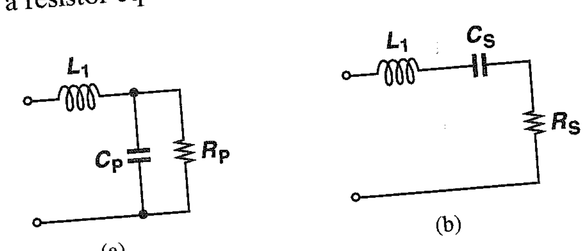

Figure 2.40 Transformation of a resistance to a lower value.

We should note that the assumption of high Q is not always valid, particularly if on-chip inductors are used. Thus, the accuracy of the foregoing ...ications must be checked in such cases.

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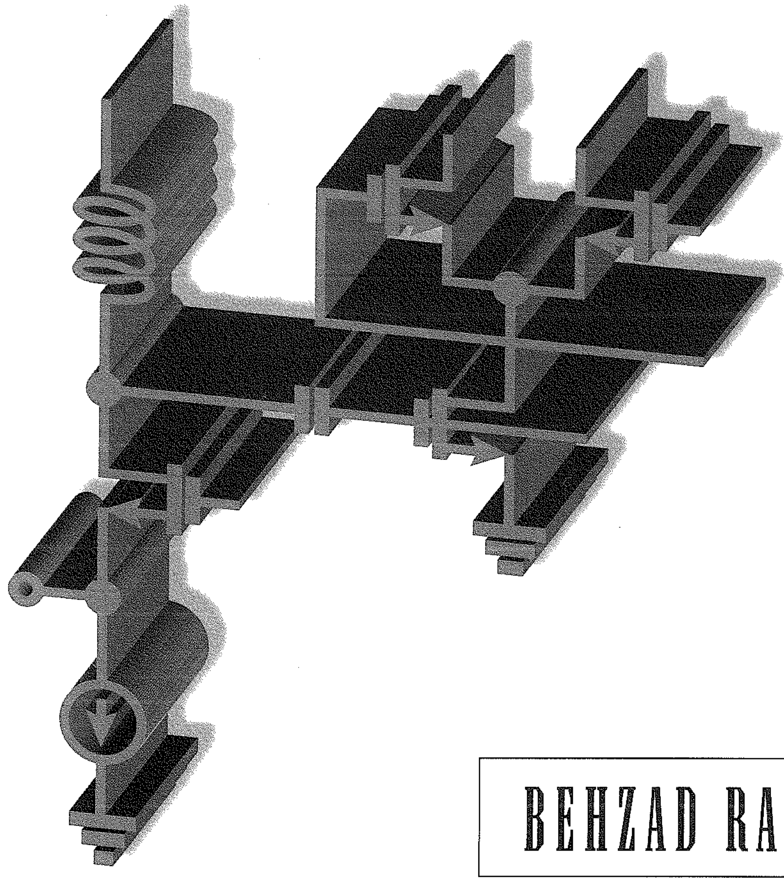

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