**CHAPTER 8**

**MULTI-TONE SINE WAVE GENERATION ACHIEVING THE THEORETICAL MINIMUM OF PEAK-TO-AVERAGE POWER RATIO**

Multi-tone signals have been widely used in various applications. One of the bottlenecks is how to maximize the signal power given a certain peak range, namely achieving the minimum peak-to-average power ratio (PAPR). In this chapter, a novel strategy is proposed to achieve the minimum PAPR for multi-tone sine waves. By properly selecting each tone’s frequency and initial phase, the multi-tone sine waves can achieve the minimum amplitude, while maintaining total signal power, without power loss during signal generation. It is rigorously proved that the proposed method can achieve the theoretical minimum of PAPR. Extensive simulation results for various cases are presented that validate the desired property of the generated waveform. Guidelines are provided for practical implementation of the multi-tone sine waves, such as signal and system spectral testing, maximizing power amplifier transmission efficiency, multi-career transmission, orthogonal frequency-division multiplexing (OFDM), and other wireless communication systems.

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1. Introduction

Sine waves are among the most widely used signals in communications, system analysis, and many other applications. Often, single-tone sine waves are used, such as the test stimulus for dynamic testing of data converters, transceivers, power amplifiers (PAs), other devices, systems, etc. [2-5]. They also serve as a career wave in a global system for mobile (GSM) transmissions [6], as well as the fields of material and chemical spectroscopy measurements. For the multi-tone signal, especially the multi-tone sine wave, has attracted a lot of attention during the past decades. For example, characterizing a wideband circuit and system often requires the frequency response across the entire bandwidth, which is challenging for single-tone test [7]. One widely adopted approach is using the multi-tone test signal, since it reduces the test time compared with multiple measurements using a single tone. Moreover, as the performance of the circuit and system tends to vary with frequency, using a multi-tone signal is more practical in real laboratory and production testing. In communications systems, intermodulation distortion (IMD) is a key specification to determine the linearity performance of the systems, which also requires multi-tone signals [8-10, 33]. As the interest in multi-carrier transmission grows, multi-carrier modulation (MCM) and OFDM have been proposed and deployed in many wireless communication standards [11-13], where the carriers and the representation of the transmitted signals in the time domain are often multi-tone sine waves.

Apart from the wide applications for multi-tone sine waves, how to reduce and minimize the PAPR is still a challenging task. One example would be the usage of PAs in the transceivers normally working in a linear region to achieve best power efficiency, and to prevent spectral growth of multi-carriers in the form of intermodulation among subcarriers and out-of-band radiation. If the PAs are operating beyond the linear region due to high PAPR of the input signal, many distortions will be created in the transmitted signal and the drawbacks of high PAPR often outweigh the benefits of multi-carrier transmission systems. Therefore, it is vital to control the signal peak values within the PA’s saturation region, while maximizing the output power for transmission. Another example is in the testing of systems like transceivers, data converters, operational amplifiers, etc., to achieve best Signal-to-Noise Ratio (SNR) possible. The signal’s power must be maximized, so the test stimulus often reaches the full range of the system input. If the multi-tone signal has a high PAPR without clipping the signal at the input, it would compromise the signal power; hence, decrease the measured performance of the systems. Therefore, it is crucial to find solutions that can achieve minimum PAPR for multi-tone signals, especially for multi-tone sine waves.

In the past, many methods have been proposed to deal with the PAPR issue [14-30]. In the application of OFDM, some use amplitude clipping and filtering [16-17]. Coding [18-22], tone reservation, and tone injection [23] are used to reduce PAPR. Selected mapping (SLM) [24-25] and partial transmit sequences (PTS) are also used [26-27]. Based on [14-16], these methods are capable of reducing PAPR, but at the cost of data rate loss, increasing computational complexity, average power increase, etc. A novel circuit for reducing the crest factor of a multi-tone data signal is proposed [28], where the correction signal is subtracted from the original multi-tone signal. In [29], a novel, improved method for generating the reduced peak amplitude high data rate channels is proposed. This consists of several lower rate channels with phase rotated before summed and transmitted. However, none of these methods discussed the application for multi-sine waves amplitude reduction. In [30], an optimal multi-sine design is proposed with either logarithmically or equally-spaced frequencies. It appropriately selected the phases to compress the amplitude; however, it becomes more difficult as the number of tones increases to select the optimal phases. In industry, to create low PAPA multi-tone signals, one of the most widely used approaches is to vary the phase from 0 to 360 degrees between adjacent single tones and it is recommended to vary the tone phases randomly. This approach has been adopted for many years and been used by many researchers and engineers. However, it is time consuming, especially with more tones. Therefore, a new method is necessary and vital to find the optimal PAPR for multi-tone sine waves.

The remainder of the chapter is arranged as follows. Section II discusses multi-tone sine waves and their challenges to achieve small PAPR. Section III introduces the proposed method to generate multi-tone sine waves with the theoretical minimum PAPR. Section IV presents the simulation results in MATLAB to validate the proposed method to generate multi-tone sine waves with minimum PAPR. Section V concludes this chapter.

1. Multi-tone Sine Waves and PAPR

In this section, the definition of multi-tone sine waves and PAPR are provided. The problems of a high PAPR for multi-tone sine waves are illustrated.

OFDM signals can be written as:

, (8.1)

where *N* is the number of symbols, *X* is the block of N symbols: in the frequency domain with each symbol modulating one set of subcarriers: , , and *T* is the original symbol period.

This low-pass signal can be either real or complex. Real valued low-pass equivalent signals are typically transmitted at baseband, such as wireline applications. For wireless applications, the transmitted signal is up converted to carrier frequency, *fc*. In general, the transmitted signal can be written as:

. (8.2)

It can also be represented as:

. (8.3)

The transmission signal can be viewed as a summation of N-tone sine waves with different frequencies, namely multi-tone sine waves. For simplicity in this chapter, each considered tone has an equal amplitude, but arbitrary initial phase.

In the time domain, the N-tone sine wave with normalized amplitude is given by:

, (8.4)

whereandare each tone’s frequency and initial phase, respectively.

The PAPR is defined as the peak amplitude squared divided by the rms value squared:

. (8.5)

It can also be expressed in dB:

. (8.6)

For a single-tone sine wave, whose amplitude is normalized to 1, the rms value is, while its peak value is 1. Then, the PAPR isor 3.01dB. For multi-tone sine waves, the rms value is and the peak value at worst case scenario becomes *N*. Then, the PAPR comes to *2N* orin dB, which worsens as the number of tones increases. For the best case where the power remains the same, while the peak reaches the theoretical minimum of, the PAPR reaches the minimum like for the single-tone case: or 3.01dB. Therefore, the goal essentially is to find the minimum peak value of the multi-tone sine wave, while maintaining the signal power.

In recent years, such multi-tone signals are achieved by Arbitrary Waveform Generators (AWGs) and Digital-to-Analog Converters (DACs). In Automatic Test Equipment (ATE) the signal generation pattern is by digitized or sampled signal with low-pass or band-pass filtering. In the future, it can be envisioned that all waveforms will be digitally-generated or synthesized.

The sampled or discrete multi-tone sine wave is given by:

, (8.7)

where, M is the total data record length, *Ts* is the sampling period, *fs* is the sampling rate and, *w[n]* contains signal noise, input referred noise from the digitizer, and quantization noise of the digitizer. Since it is not related to this chapter’s focus, for simplification purposes the noise term is ignored in the following derivations.

If the coherent sampling condition is met [31-33], the integer number waveform cycles in the data record is *J*, the input and sampling frequency, and the total data record length satisfies the following relationship:

 . (8.8)

Therefore, frequency selection is essentially selecting the *J* and M is usually selected as a power of 2 for faster processing of the Fast Fourier Transform (FFT). Usually, *fi* is selected not to be a sub-harmonic of *fs*, then the quantization error is random and uncorrelated with *fi*. If this condition is not satisfied, it will cause the quantization noise energy to be concentrated at harmonics of the fundamental frequency, thereby producing distortion—an artifact of the sampling process rather than nonlinearity of the Analog-to-Digital Converter (ADC) or signal [3]. *J* is preferred chosen to be an odd number. In addition, if the input frequency exceeds half of the sampling frequency, aliasing will occur on the spectrum and the fundamental tone will be reflected back according to |*n.fs-fi*| at frequency range: [0 *fs*/2], viewed as a low frequency fundamental. For this reason, in the following analysis, only the non-aliasing input frequency is selected, meaning *J* is smaller than M/2. From this discussion, it can be seen that to satisfy these conditions, *J* cannot be arbitrarily selected. Hence, the frequency for each tone cannot be arbitrarily selected either.



Figure 8.1. 8-tone sine wave maximum and minimum peak values.

Using Eq. (8.8), replace *fi* with *Ji*, Eq. (8.7) can be re-written as:

. (8.9)

In the following derivations, the frequency selection essentially becomes selecting the proper *J*. Using Eq. (8.9) as a starting point, derive all the following multi-tone cases. To illustrate the issue of high PAPR, namely the high peak value, an 8-tone sine wave is generated. Each tone’s *J* is selected for coherent sampling and odd numbers are randomly selected among [0 M/2]. A total of more than 1800 test cases were performed. Once the frequencies are selected, their initial phases are randomly generated from  for a total of 10,000 runs to generate 1000 8-tone sine waves with different peak values. This is the conventional method mentioned in Section I. Figure 8.1 shows the maximum and minimum peak values for these 10,000 sine waves at each test case. Even though each case has 10,000 runs with different initial phases, for many test cases the maximum peak values are near 8. However, minimum peak values never reach the theoretical minimum of. This implies without the careful selection of the tone frequency and their initial phases, it would be extremely difficult to calculate the minimum PAPR for multi-tone sine waves.

1. Proposed Strategy

The statement in the previous section leads to the need for the proposed method to generate the multi-tone sine wave with minimum PAPR, which will be described in detail in this section. Since the rms of the signal needs to maintain the same level ofwithout losing power to reach the theoretical minimum PAPR, the generated multi-tone signal amplitude must be. Therefore, the proposed method seeks a way to generate a minimum peak value ofwithout losing power.

## **2z Multi-Tones**

The first part of the proposed method focuses on finding the minimum PAPR for multi-tone sine waves, using 2z number of tones, since this number of tones is commonly used in communication applications such as OFDM.

### 2-tone

Starting with a 2-tone sine wave *x2[n]*:

, (8.10)

where and .

*x2[n]* can be written as:

. (8.11)

We propose the 2-tone has the relationship given by. *x2[n]* is then given by:

. (8.12)

Since the goal is to find the peak value for the first cos term, the peak value is 1. The second cos term, given a different n, can be categorized into four groups:

. (8.13)

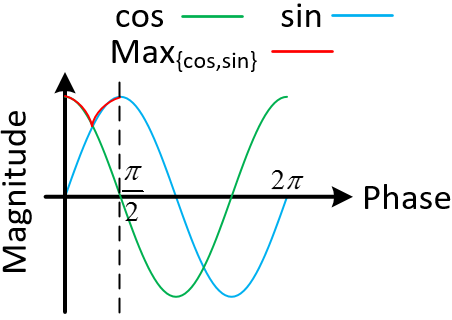


Figure 8.2. Illustration of Eq. (8.13) for minimum peak value

For the peak value, it is evident for the cos term or the sin term, the value is 1. Since we are looking for the minimum peak value, due to its symmetry in Eq. (8.13) without loss of generality within one period , assume and . The peak value is illustrated in Figure 8.2 along the red curve, whose minimum value occurs at , which is . By symmetry, the minimum peak also occurs at, meaning that if , . Since the sine waves are periodic, for simplicity in the following derivations, assume the first tone’s initial phase is 0:. Thencan be. A 2-tone sine wave will have a minimum peak value of, if each tone’s amplitude is 1.

### 4-tone

For the 4-tone case:

. (8.14)

Similarly, we propose two pairs of frequencies: and, whose initial phases sum to either 0 or. If, then assume, *x4[n]* is given by:

, (8.15)

which can be further simplified to.

If *n* is an even number, *x4[n]* becomes. If *n* is an odd number, x4[n] becomes. Therefore, the proposed 4-tone sine wave can achieve a minimum peak value of 2.

### 8-tone

For the 8-tone sine wave, the *J*s will have four pairs. Each sums to M/2:, ,and. For their initial phases, we propose such a relationship: ,,. Using this information, the 8-tone sine wave is given by:

. (8.16)

To further simplify the frequency terms, we propose:

, (8.17)

. (8.18)

Then, Eq. (8.16) can be written as:

. (8.19)

Further simplification leads to:

. (8.20)

The 8-tone sine wave can be categorized into four groups:

. (8.21)

It is evident each of these four groups will lead to a peak value offor the proposed 8-tone sine wave.

### 16-tone

A similar theory is applied to the 16-tone sine wave, where we have eight pairs of *J*s. Each sums to M/2. For initial phases, we propose: 

,,.

The 16-tone sine wave can be given by:

. (8.22)

If only the first four terms are examined:

. (8.23)

If *J2* is related to *J1* given by:

, (8.24)

Then Eq. (8.23) can be simplified to:

 . (8.25)

It is clear to see that its peak value only equals 4, if *n=0,4,8…,* and the remainder of the *n*s will result in 0. Similarly, the second four terms will produce the peak value of 4 with a different frequency, if *n=1,5,9….* Thus, *x16[n]* is simplified to:

. (8.26)

By dividing the 16 tones into four groups, each group is only non-zero at 1 of every 4 *n*s. There are no non-zero values summing among different groups, which is similar to the idea of interleaving among the four different sine waves. Moreover, as shown in Eq. (8.26), there are no relationships among the four group frequencies andwill have many selection choices. This adds flexibility to the proposed strategy, as more frequency choices can be selected that will lead to the minimum peak value of 4 for the 16-tone sine wave.

### 32-tone

Then for the 32-tone case, we can use a similar approach as for the 8-tone case: with term . Find  that can add a peak value of. Since we know from the 16-tone that it can be divided into four groups, each group has a peak value of 4 and only shows every 1 of 4 *n*. We can again divide the 32-tone into four groups, each with eight tones. Assume the first group already has 4-tone equivalent to Eq. (8.25). Another 4-tone configuration is needed to obtain the term. Then, like Eq. (8.25), the proposed additional 4-tone is. Expanding this term leads to:

. (8.27)

Hence, another four sine waves in this group can be determined with their *J*s related to *J1* given by:

, (8.28)

 . (8.29)

These four sine waves can be paired to two pairs. Each sums to M/2 like all previous cases: and. With all known information, the first group of 8-tone sine waves can be given by:

. (8.30)

This only equals toat *n=0,4,…* The remainder are 0. From Eq. (8.30), the initial phases can be obtained: ,.

The remaining three groups can be constructed in a similar way. The 32-tone sine wave is therefore given by:

. (8.31)

Therefore, the proposed 32-tone sine wave can generate the minimum peak value of.

### 64-tone

Based on the 4-tone and 16-tone results, use the same idea by adding another term to: and making it only non-zero at 1 of every 8 *n*. Thus, for 64 tones, by dividing them into eight groups, each group is only non-zero at 1 of every 8 *n*, with no non-zero values summing among the different groups. The proposed solution in the first group can be:

. (8.32)

By expanding Eq. (8.32), the end result is a group of eight tones:

. (8.33)

Then, we propose their frequency relationships similar to previous cases with each pair sums to M/2: , ,and. In addition, an offset of M/4 and M/8 are among *J2*, *J3* and *J4*, represented by Eqs. (8.24), (8.28)-(8.29). Their initial phases are all 0 for this group. Then, the remaining of seven groups can be constructed similarly, which leads to the proposed 64-tone sine waves:

. (8.34)

From Eq. (8.34), it is clear to see the proposed 64-tone sine wave can achieve a minimum peak value of 8.

From these results, we hypothesis:

1. For 4y number of tones where *y=1,2,3…,* we categorize them into 2y groups according to different *n*s. They are non-zero only at certain locations, none of the groups show as non-zero at the same time. This can be accomplished by multiplying the term to Eq. (8.32). With proper frequency and phase selection, this will guarantee the multi-tone sine waves with a minimum peak value of 2y.

2. For other cases 2.4y, such as 8, 32…, they can be separated into 2y groups. With proper frequency and phase selection, they sum similar to the 2-tone case in the form of to achieve a minimum peak value of.

## **Low Order Multi-Tones**

In addition to multi-tones with 2z number of tones, other low order multi-tone sine waves are studied, such as the commonly used 3-tone and 5-tone.

### 3-tone

The 3-tone can be derived from the 2-tone case. Two tones are paired with, and the third tone’s *J3* has an offset of M/4 with *J1*, which can be represented by Eq. (8.17). Their initial phases include: first two tones sum to  and the third tone has an offset of  with the first tone. Assuming the first tone hasinitial phase, then. Thus, the 3-tone is given by:

. (8.35)

It can be categorized into four groups:

. (8.36)

From Eq. (8.36), it is evident each group can be combined into a single sine wave, with the peak value of. Therefore, the proposed 3-tone sine wave can achieve the minimum peak value ofgiven proper frequency and phase selections.

### 5-tone

The 5-tone case is derived from the 4-tone case, where it has two pairs of tones, and, whose initial phases add to either 0 or . If, then we assume. If we assumeand, with initial phase of, the 5-tone can then be given by:

. (8.37)

It has two categories:

. (8.38)

When *n* is even, the first term in Eq. (8.38) can be re-written as:

, (8.39)

which will lead to and eventually becomes one sine term, with peak value of. When *n* is odd, we know:

. (8.40)

Substitute Eq. (8.40) into the second term in Eq. (8.38), which will be:

. (8.41)

By finding the maximum value when, it can be proven the maximum value isand smaller than the peak value ofwhen *n* is even. Therefore, the proposed 5-tone sine wave can achieve the minimum peak value of.

## **Discussion**

In this section, several examples of the commonly used multi-tone sine waves are studied, and the strategies to achieve the theoretical minimum peak value are provided and proven mathematically. This is accomplished by appropriately selecting each tone’s frequency and the initial phase, instead of randomly varying the tone phases randomly and selecting the one with the minimum peak value. Since they are generated digitally, such multi-tone sine waves can be easily generated by a DAC or AWG. This will reduce most of the computation time and complexity to generate such multi-tone sine waves. In addition, there are many frequency and initial phase choices that meet the proposed strategy’s criteria, which provide much better flexibility to implement the proposed strategy, if the user wants to generate multi-tones with evenly spaced frequencies that can also be completed using the proposed strategy. Since the proposed tone frequencies are not fixed, simple calculations and validation on the multi-tone sine wave spectrum can lead to frequency selection with evenly spaced frequencies. One exception in our example is the 5-tone case, due to its unique frequency relationship of the five tones. All discussions are validated in section IV. In addition, the user can also select the logarithmically-distributed multi-tone sine wave, it can be achieved with the flexibility of the proposed strategy. Detailed discussions are not in the scope of this chapter.

Although this section provided several examples of multi-tone sine wave generation, based on these strategies a more detailed study regarding other low order multi-tones can also be done in the future.

Table 8.1 summarizes the proposed strategy for generating different multi-tone sine waves, assuming the first tone has 0 initial phase.

Table 8.1. Proposed strategy for multi-tone sine wave with minimum PAPR

|  |  |  |
| --- | --- | --- |
| Tone | Frequency | Initial Phase |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 8 |  |  |
| 16 |  |  |
| 32 |  |  |
| 64 |  |  |

1. Simulation Results

In this section, simulations results are shown to verify the generated multi-tone sine waves by the proposed strategy. Both time and frequency domains of the generated multi-tone sine waves are shown to verify their minimum peak values as well as their powers on the spectrum. Figures 8.3–8.10 show the results for each of the multi-tone sine waves mentioned in Section III, from 2-tone through 64-tone. It is clearly shown in the time domain, each of the generated sine waves achieved the theoretical minimum peak value of. The spectrum in the lower part of each figure shows they are all near the 0dB level, meaning each tone in the generated multi-tone sine wave has the same power and there is no power loss in any case. These demonstrate the proposed strategy generates multi-tone sine waves with minimum PAPR. In addition, every spectrum is shown to have evenly spaced tones except for Figure 8.6, the 5-tone case, as mentioned in Section III.C. So, by properly selecting the frequencies, the proposed strategy can generate evenly spaced multi-tones. Additionally, the user can generate other multi-tone sine waves with different frequencies based on the proposed strategy, which offers more flexibility in generating multi-tone sine waves with minimum PAPR.



Figure 8.3. Proposed 2-tone sine wave in both time and frequency domain.



Figure 8.4. Proposed 3-tone sine wave in both time and frequency domain.



Figure 8.5. Proposed 4-tone sine wave in both time and frequency domain.



Figure 8.6. Proposed 5-tone sine wave in both time and frequency domain.



Figure 8.7. Proposed 8-tone sine wave in both time and frequency domain.



Figure 8.8. Proposed 16-tone sine wave in both time and frequency domain.



Figure 8.9. Proposed 32-tone sine wave in both time and frequency domain.



Figure 8.10. Proposed 64-tone sine wave in both time and frequency domain.

1. Conclusion

In this chapter, a new strategy for generating multi-tone sine waves with theoretical minimum PAPR is described. By properly selecting each tone’s frequency and initial phase, the multi-tone sine waves can achieve the theoretical minimum amplitude without signal power loss. Rigorous mathematical analysis and simulation results both validated the effectiveness of the proposed strategy, which showed the proposed strategy can generate different numbers of multi-tone sine waves. Moreover, it also generalized the strategy of generating a high order of multi-tone sine waves beyond 64 tones and offers much flexibility in its frequency selection. Such strategy can be readily implemented into various fields of applications, such as signal and system testing, PA power efficiency improvement, OFDM, and other wireless communication systems.

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