A Comparative Study of state-of the-art High Performance Spectral Test Methods

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Abstract

Spectral test of high performance Analog-to-Digital Converters is very challenging due to several stringent requirements on the test setup. As a result, expensive instrumentation and long test time are needed. Several methods have been proposed to relax some of these requirements to decrease test time and cost. In this article, four such methods are described and compared based on several criteria pertinent to practical implementation in high performance spectral testing. A summary is presented that is aimed at helping engineers determine the best method to use depending on their available resources.

1. Introduction

Spectral analysis is one of the most widely used methods in several areas such as geophysics, oceanography, medical sciences, etc. It provides information about different frequency components present in a signal that are otherwise not observed in time domain. In electronics, it is extensively used to test dynamic parameters of a wide range of semiconductor circuits and systems, such as Analog-to-Digital Converters (ADC).

ADCs are one of the most widely used mixed-signal circuits. Sufficient spectral performance of ADCs is critical for high speed and high resolution applications such as communications [1-2]. To ensure high accuracy in spectral test, the IEEE standards [3-4] recommend the test setup to satisfy a list of stringent requirements.

As ADC resolution and speed become high, the IEEE requirements have become challenging. Furthermore, satisfying these requirements inevitably increases test time and test cost. Relaxing these requirements not only decreases test time/cost but also facilitates on-chip test capability. Several methods have been proposed in the past that relax one or more of these requirements. However, it is not clear which method suits what applications.

We provide a comparative study of four methods that relax stringent conditions for spectral test, based on several practical criteria. This enables the user to select the most suited method for his/her situation.

2. Ideal DFT based Spectral Test

Fig. 1 shows the ideal DFT based spectral test setup. The IEEE recommended requirements include:

1) <u>Input Signal:</u> To test an N-bit ADC, the input signal should be at least N+3 bit pure, a very challenging

task for large N. Filters are necessary when source is not sufficiently pure.

2) <u>Clock Signal:</u> Jitter in clock signals should be less than the noise floor. Equation (1) is used to get the maximum allowable jitter requirement [5]. Here t_j is rms jitter in the system, *f* is input frequency. With increase in ADC resolution, the maximum jitter acceptable by test system needs to be decreased, which is another challenge.

$$SNR = 20\log_{10}\left(\frac{1}{2\pi f t_j}\right) \tag{1}$$

,

3) <u>Sampling</u>: The input signal should be coherently sampled. A signal is said to be coherently sampled if the data record contains an integer number of cycles of the signal. This requires high accuracy signal generators and the required accuracy of signal generators increases with increase in resolution of ADC. Typically, a master clock is used to control frequencies of input and clock signals simultaneously to achieve coherent sampling, thus, increasing the test area and cost.

4) <u>Input Amplitude:</u> The peak-to-peak voltage of the input signal should be within the ADC input range $[F_b F_l]$. If the standard deviation of noise rms is given as σ_N and full scale of ADC as *FS*, the input signal, V_{IN} , should follow equation (2) to avoid clipping. In applications such as on-chip testing, a lot of area is consumed to obtain precise amplitude control, thus increasing the cost.

$$\min(V_{IN}) > F_b + 10\sigma_N$$

$$\max(V_{IN}) < F_t - 10\sigma_N$$
(2)

5) <u>Data record</u>: The number of samples in data record should be selected such that at least five periods of input signal are sampled [4]. A trade-off is made between required noise floor and data acquisition time.

Working Principle

Single-tone test: Let f_{Sig} be the input signal frequency, f_{Samp} be the clock frequency, M be the total number of data points and J be the number of cycles of input signal in data record. The four parameters are related by equation (3), where J_{int} and δ are the integer and non-integer part of J respectively.

$$J = M \frac{f_{Sig}}{f_{Samp}} = J_{int} + \delta$$
(3)



The sampling is said to be coherent if J is an integer that is co-prime with M ($\delta = 0$) and non-coherent if J is not an integer ($\delta \neq 0$).

Let x(t) in (4) be the time domain representation of pure analog input signal to ADC.

$$x(t) = V_{OS} + A\cos\left(2\pi f_{Sig}t\right) \tag{4}$$

where, V_{OS} is DC and A is amplitude of signal x.

Let x[n] in (5) be the analog interpretation of n^{th} digital output of ADC whose gain error and offset are calibrated.

$$x[n] = V_{OS} + A\cos\left(\frac{2\pi J}{M}n + \phi\right) + \sum_{h=2}^{H} A_h \cos\left(\frac{2\pi h J}{M}n + \phi_h\right) + w[n]$$
(5)

for n=0,1,2,...,*M*-1. *M* is usually selected to be a power of 2 for faster processing of Fast Fourier Transform (FFT). *H* is total number of harmonics, ϕ is initial phase of fundamental in *x*[*n*], *A_h* and ϕ_h are amplitude and initial phase of *h*th harmonic respectively such that *A_h*<<*A* and $\phi_h \in [0,2\pi)$ for all $2 \le h \le H$. *w*[*n*] corresponds to noise in nth sample. The harmonics in (5) correspond to ADC distortion.

Spectral parameters are obtained by taking DFT of M sampled points. DFT of x[n] is given by (6) and the spectrum is shown in Fig. 2 (Blue).

$$X_{k} = \frac{1}{M} \sum_{n=0}^{M-1} x[n] e^{-j\frac{2\pi k}{M}n}, \quad for \quad k = 0, 1, 2, ..., M-1$$
(6)

where *k* represents the frequency bin's index. Here, k=h*J represents the frequency bin of h^{th} harmonic. X_0 corresponds to DC component in signal *x*. Other values of *k* correspond to noise. The power of fundamental, h^{th} harmonic and noise is accurately estimated as P_1 , P_h and P_{noise} respectively using (7),

$$P_{1} = 2|X_{J}|^{2} = \frac{A^{2}}{2}; P_{h} = 2|X_{hJ}|^{2} = \frac{A_{h}^{2}}{2};$$

$$P_{noise} = \sum_{\substack{k=1\\k \neq J, hJ\\h=2,3,..,H}}^{M-1} |X_{k}|^{2}$$
(7)

From (7), spectral parameters such as THD, SNR, SNDR, ENOB and SFDR are calculated using (8).

$$THD = \frac{\sum_{h=2}^{H} P_{h}}{P_{1}}; \quad SNR = \frac{P_{1}}{P_{noise}}; \quad SNDR = \frac{P_{1}}{\sum_{h=2}^{H} P_{h} + P_{noise}};$$

$$SFDR = \frac{P_{1}}{2*\max_{\substack{k=1,...,(M/2)\\k\neq J}} (|X_{k}|^{2})}; \quad ENOB = \frac{SNDR - 1.76}{6.02}$$
(8)

Multi-tone test: The input signal containing K tones with M sampled points can be given as (9).

$$x(t) = \sum_{i=1}^{K} a_i \cos\left(2\pi f_{Sig,i}t\right) \tag{9}$$

where a_i and $f_{Sig,i}$ are the amplitude and frequency of i^{th} frequency tone respectively.

In multi-tone testing, intermodulation distortion (IMD) occurs due to ADC nonlinearities. The intermodulation frequencies may occur at sum and difference frequencies for all possible integer multiples of input frequency tones such as $(2f_{Sig,I} - f_{Sig,2})$ or $(f_{Sig,2} - f_{Sig,I})$, etc. IMD is measured by estimating the power of bin that corresponds to intermodulation frequency in the spectrum. If the bin corresponding to an intermodulation component is given as *m*, the power of that frequency component is given by (10)

$$P_{IM} = 2\left|X_m\right|^2 \tag{10}$$

Full spectrum test refers to the ability of a method to provide accurate power of each frequency component in the spectrum. As clean spectrum is obtained using *ideal DFT based spectral test* (as shown in Fig. 2), the method performs Full Spectrum test.

In this article the test procedure for single-tone testing is described.

Test Procedure

After satisfying all conditions mentioned above,

- *i)* Acquire M samples
- *ii)* Perform DFT on acquired data
- *iii)* Obtain spectral parameters from DFT.

Summary of Ideal DFT based Spectral test Method

Ideal DFT based spectral test method is the golden reference. If such test setup is available, it is recommended to use it for accurate results. The method is *Universal* (independent of ADC resolution), can perform Full spectrum test and multi-tone test and is also fast. However, it cannot be used when the conditions on test setup are not satisfied.

3. Accurate Spectral Testing with Relaxed Constraints

Achieving coherent sampling is one of the most challenging constraints. If coherent sampling is not obtained ($\delta \neq 0$), the spectrum could contain huge leakage as shown in Fig 2 (red).

If precise amplitude control is not attained (for on-chip testing), the input signal might exceed the ADC input range and result in clipped output. Since clipping introduces non-linearities, the FFT of such data shows large power in several frequency bins as shown in Fig. 2 (Magenta). Both the effects provide inaccurate results.

Here, four methods are presented that relax one or both these conditions. Such methods help faster spectral test (quick test setup) and make on-chip spectral testing practical. It should be noted that the setup still needs to satisfy the other two conditions (Pure input and less jitter).

Consider an *N*-bit ADC as the DUT with input voltage range $[F_b F_t]$, where any input voltage below F_b is clipped at code 0 and above F_t is clipped at code $(2^{N}-1)$. Let the input and sampled output of DUT be given by (4) and (5) respectively. Each of the following methods take (5) as the input, process and provide spectral results.

I. WINDOWING

Windowing is one of the most widely recommended methods in both industry and academia to perform spectral test when the data is non-coherently sampled [4,6-7]. The non-coherently sampled data is multiplied with a window function. FFT is performed on this windowed data to obtain spectral parameters. The test procedure for the method is given below.

Test Procedure

- *i)* Collect M samples
- *ii) Generate a window*
- *iii) Obtain windowed data*
- *iv)* Take DFT of windowed data and estimate spectral parameters

II. FOUR PARAMETER SINE FIT (FPSF)

Four Parameter Sine Fit (FPSF) method is another approach that is widely used for spectral test. It can be used when the data is non-coherently sampled [4,8] or clipped, thus, relaxing two conditions for spectral test (coherent sampling and in-range amplitude). The method includes estimating amplitudes of fundamental and harmonic components and evaluating the spectral parameters. The test procedure using FPSF method is given.



Fig. 2: Spectrums of coherently sampled (Blue), non-coherently sampled (Red) and clipped (Magenta) data.

Test Procedure

- *i)* Acquire M samples and truncate to contain integer cycles
- *ii)* Obtain initial estimates of J, A, ϕ , V_{OS}
- *iii)* Obtain final estimates of J, A, ϕ , V_{OS} using nonlinear least squares.
- *iv) Remove DC and fundamental from initial data to obtain residue*
- *v) Perform linear least squares on residue to estimate harmonics*
- vi) Remove harmonics in residue to obtain noise power
- vii) Calculate spectral parameters using estimates
- III. FUNDAMENTAL IDENTIFICATION AND REPLACEMENT (FIRE)

The FIRE method was recently proposed to provide accurate and robust spectral results for any value of δ , thus eliminating the need for coherent sampling[9]. The method includes estimating the non-coherent fundamental from frequency domain and replacing it with a coherently sampled fundamental. Detailed description of FIRE method along with measurement results is provided in [9]. The test procedure to perform FIRE method is given below.

Test Procedure

- i. Acquire M points and take DFT.
- *ii.* Obtain initial estimates of J, A, ϕ .
- iii. Using newton method, estimate final values of J, A, ϕ .
- iv. Replace non-coherent fundamental with coherent fundamental to obtain x_{New} .
- v. Take DFT of x_{New} and estimate all spectral parameters.

IV. FUNDAMENTAL ESTIMATION, REMOVAL AND RESIDUE INTERPOLATION (FERARI)

Recently, another method that performs accurate and robust spectral test with simultaneous non-coherent sampling and amplitude clipping was presented, thus relaxing the constraints on Sampling and input Amplitude [10]. The method includes estimating non-coherent, overrange fundamental and removing it from raw data to obtain residue. A coherently sampled fundamental is generated and the information of harmonics and noise at each coherently sampled point is obtained by interpolating the residue. Detailed description of the method along with measurement results is provided in [10]. The test procedure is given below.

Test Procedure

- *i)* Acquire M samples
- ii) Estimate A, V_{OS}
- iii) Take DFT of data and get initial estimates of J, ϕ
- iv) Obtain accurate estimates of J, ϕ using least squares
- v) Remove estimated fundamental from data to obtain residue. Prepare LUT with residue vs. ADC code
- vi) Generate coherently sampled signal with amplitude equal to ADC's full range, \mathbf{x}_{cl}
- vii) Using LUT, interpolate residue onto each code in \mathbf{x}_{c1} to get information of harmonics and noise. Add this information to \mathbf{x}_{c1} to get \mathbf{x}_{Final}
- viii) Perform DFT on x_{Final} and estimate spectral parameters

4. Comparative Study

In this section, a comparative study on four methods is presented based on different criteria related to spectral test. The comparison is done with a default condition that the data is non-coherently sampled.

Computation time

1. <u>Windowing:</u> The time complexity of windowing is $O(Mlog_2M)$ as DFT evaluation is the major time consuming step in the method.

2. <u>FPSF</u>: The method involves estimation of parameters using non-linear and linear least squares method, which typically has a time complexity of $O(G^2T)$, where *G* is the number of parameters and *T* is the number of equations. The method is highly computation intensive as non-linear least squares is performed until convergence is achieved to estimate the fundamental and linear least squares is performed to estimate each harmonic.

3. <u>FIRE</u>: The time complexity of FIRE is $O(Mlog_2M)$, as the major time consuming step is

evaluating the DFT. Other steps in the process consume negligible time compared to that of DFT.

4. <u>FERARI</u>: Of all the steps involved in performing this method, DFT evaluation is the most time consuming step and hence time complexity is given as $O(Mlog_2M)$. However, due to several calculations involved such as least squares and interpolation, FERARI takes considerable computation time.

Table 1 presents the computation time of all four algorithms using non-coherently sampled data. Table 2 provides computation time of FERARI and FPSF methods using non-coherently sampled and clipped data. Both FIRE and windowing methods have less computation time followed by FERARI and FPSF methods.

Table 1: Computation time of four methods with noncoherent sampling (J = 6537.14, 18-bit ADC, $M=2^{16}$)

Method	Computation Time		
Windowing (blackmanharris)	0.003s		
FPSF	0.12s (fundamental) 0.83s(20harmonics)		
FIRE	0.0028s		
FERARI	0.041s		

Table 2: Computation time using non-coherently sampled and clipped data (J = 6537.14, Clipping = 2%, 18-bit ADC, $M=2^{16}$)

Method	time		
FERARI	0.08s		
FPSF	0.14s (fundamental) 1s(20harmonics)		

The comparison above is provided for a given data record length. However, if frequency resolution is of interest, windowing requires more data points compared to that of other methods (as explained later). In such cases, windowing consumes more time.

Universal Applicability

A method is said to be *Universal* if it can be readily used without any information about the DUT.

1. <u>Windowing:</u> The primary requirement when using windowing method is that the power of secondary lobes in the chosen window should be less than that of the noise floor of DUT's spectrum. Hence, windowing cannot be termed as a *Universal* method as its application (window selection) depends on resolution of the DUT.

2. <u>FPSF, FIRE and FERARI</u>: All three methods can be termed as Universal methods as they are not dependent on the DUT's information to provide accurate test results.

Frequency Resolution

1. <u>Windowing</u>: When windowing is used for spectral test, each frequency is split into several bins in the spectrum due to the presence of primary lobe as shown in Fig. 3. This reduces the frequency resolution that is achievable. To test a high resolution ADC, a window with low secondary lobes' power is selected. However, from Fig. 3, such windows have lower frequency resolution due to presence of more bins in primary lobe. In order to obtain sufficient frequency resolution, the data record should be increased. This increases data acquisition time and computation time.

2. <u>FPSF</u>: This method only estimates the power of frequencies that are integral multiples of fundamental as part of test. If there is a particular frequency of interest that is known beforehand, the method can estimate the power of that frequency using least squares. Estimating power of several frequency components using this method becomes very time consuming. Hence, it is not preferred if unknown non-harmonic frequencies need to be estimated.

3. <u>FIRE</u>, <u>FERARI</u>: Since clean and accurate spectrums are obtained using both methods, the frequency resolution obtained using FIRE and FERARI is similar to that obtained using ideal DFT based spectral test.

Harmonic Power Calculation

To estimate the power of a frequency component, it is conventional to add power of a set of bins on either side of the bin that is closest to the frequency of interest. When a signal is non-coherently sampled, the bin that is closest to h^{th} harmonic is dependent on δ and is given as $B_{h,a}$ in (11). The accurate power of h^{th} harmonic is obtained by adding power of *L* bins on either side of $B_{h,a}$ as given in (13).

1. <u>Windowing:</u> In windowing, the value of δ is not known and hence, the estimated bin that is closest to h^{th} harmonic is given as $B_{h,e}$ in (12). The h^{th} harmonic power is estimated as $P_{h,e}$ in (14) by adding power of L bins on either side of $B_{h,e}$. For large δ , windowing calculates harmonic power inaccurately. An example to illustrate this effect is provided using Table 3 and Fig. 4. Fig. 4 shows the spectrum of a 16-bit ADC that is non-coherently sampled and windowed with J = 1031.48 and M = 65536. Table 3 provides the values of $B_{h,a}$ and $B_{h,e}$ with estimated error. Windowing provides inaccurate estimate of 12^{th} harmonic power and results in wrong spectral parameters.

Hence, even after selecting a window with minimum secondary lobes' power, windowing cannot provide accurate spectral result if δ is large. However, if δ is small (of the order of 0.01), it provides accurate results.

$$B_{h,a} = round \left(h^* (J_{int} + \delta)\right) = h J_{int} + round (h\delta)$$
(11)

$$B_{h,e} = h J_{\rm int} \tag{12}$$

$$P_{h,a} = 2 \sum_{i=B_{h,a}-L}^{B_{h,a}+L} |X_i|^2 \qquad (13); \ P_{h,e} = 2 \sum_{i=B_{h,e}-L}^{B_{h,e}+L} |X_i|^2 \qquad (14)$$



Fig. 3: DFT of different window functions. The presence of primary lobe and the power of secondary lobes is visible.



Fig. 4: Spectrum showing the center-bin of fundamental and harmonic lobes of a windowed data using 7-tern blackman-harris window.

Table 3: Table showing the error in estimating the centerbin of harmonics' lobe using Windowing for J = 1031.48

Harmonic	Actual center bin B_i	Estimated center-bin using windows B ₁	Error $B_{h,a}$ - $B_{h,e}$
1	1021	1021	0
1	1031	1031	
3	3094	3093	1
12	12378	12372	6

2. <u>FPSF</u>: The power of harmonics is accurately estimates using T equations.

3. <u>FIRE</u>: Since accurate value of δ is known in FIRE, the bin closest to h^{th} harmonic is accurately identified and spectral parameters are accurately estimated.

4. <u>FERARI</u>: The information of harmonics and noise is present in the residue after removing the overranged, non-coherent fundamental. Residue interpolation on to each point of the coherent fundamental ensures that the harmonic power is correctly estimated from final DFT.

SNR Calculation:

1. <u>Windowing:</u> Since the primary lobe of the window convolutes with the non-coherently sampled spectrum, windowing modulates the noise power and results in inaccurate SNR values

2. <u>FPSF</u>: If the data only contains fundamental and harmonics apart from noise, this method can provide accurate SNR values. However, if there are non-harmonic components in the data, the method considers them as noise and provides inaccurate estimate of SNR.

3. <u>FIRE:</u> Accurate estimates of SNR are provided using FIRE as the method does not affect noise.

4. <u>FERARI</u>: Since the method includes interpolation of residue, the total noise power estimated from the final spectrum will be less than the actual noise power. This is because interpolation smoothens the noise power. However, the estimated error in SNR is less than +1.0 dB.

Multi-tone test

1. <u>Windowing</u>: When the number of tones is less and the tones are not close to each other in the spectrum, windowing can perform Multi-tone test on non-coherently sampled data. However, if significant frequency (IMD or Harmonic) components fall in the primary lobes, windowing should not be used.

2. <u>FPSF</u>: The method cannot be readily used to perform multi-tone test. It is because, when estimating the fundamental, the method assumes other frequency components are of negligible power. Though the method can be modified to estimate K tones (K should be known), estimating tones in time domain is very time consuming.

3. <u>FIRE</u>: This method can be repetitively used to perform Multi-tone test provided there are less number of tones and the tones are not close to each other in the spectrum. Modifications can be made to use FIRE method if there are many tones, however, with those modifications it could be termed as another new method.

4. <u>FERARI</u>: The method cannot be readily used to perform multi-tone testing.

So, *ideal DFT based spectral test* is still considered as the state-of the-art method to perform multi-tone test.

Convergence

1. <u>Windowing, FERARI</u>: Both the methods do not have convergence issues as they do not use non-linear estimation.

2. <u>FPSF</u>: As the method includes estimating parameters from a set of non-linear equations, it is necessary for the algorithm to converge to a global minimum. This convergence is strongly dependent on the initial values chosen for the parameters. Hence, special care must be taken when selecting initial estimates.

3. <u>FIRE</u>: In this method, non-linear least squares is used to identify the fundamental. As the initial estimates are selected such that they are very close to the actual values, FIRE method has no problem with convergence and always converges to the accurate values.

Full Spectrum test

In order to estimate SFDR, it is required to find the power of maximum spur in the spectrum, be it either a harmonic spur or a non-harmonic spur. In cases such as time interleaved ADCs, the maximum spur could be nonharmonic as shown in Fig.5. Full spectrum test is required to provide accurate results in such cases.

1. <u>Windowing</u>: Windowing can be used to perform Full spectrum test if the bin containing the spur does not coincide with the primary lobe bins.

2. <u>FPSF</u>: This method cannot provide accurate estimate of SFDR if there is a non-harmonic spur that contributes to SFDR. However, if the maximum spur is a harmonic component, this method can be used accurately. Hence, FPSF cannot be used for Full Spectrum test.

3. <u>FIRE, FERARI</u>: Both the methods provide a clean spectrum with information about each frequency component in the spectrum, thus facilitating full spectrum test.

Clipped Data

As mentioned in the test procedures for all methods, only FPSF and FERARI methods have the ability to test a data when it is simultaneously clipped and non-coherently sampled. Windows method cannot be used with clipped data as it is not designed for such applications. FIRE method cannot be used with clipped data as the present method inaccurately estimates the fundamental.

5. Summary

If there is a test setup that satisfies all the requirements for *ideal DFT based test*, it is strongly recommended to use this test as it is the fastest, easiest and most accurate. Also, it is the only method that can provide accurate results for multi-tone test.



Fig. 5: Spectrum of a measured time-interleaved ADC showing non-harmonic spur contributing to SFDR.

Table 6 provides the summary about situations when a particular method can be used. The default condition is that the data is non-coherently sampled. The first column indicates the test setup requirement or capability. The green box (T) indicates the method can be used and the red box (F) indicates the method cannot be used. The yellow box in window and FIRE columns indicate the methods cannot work robustly in such cases, while the yellow box in FERARI column indicates the method can be used, but FIRE method is preferable as it is way faster than FERARI.

Test Setup	Window	FPSF	FIRE	FERARI
Needs fast test	Т	F	Т	Т
Can obtain small δ	Т	Т	Т	Т
Cannot control δ	F	Т	Т	Т
Knows DUT Resolution	Т	Т	Т	Т
Does not know DUT's Resolution	F	Т	Т	Т
Need Full Spectrum Test	T*	F	Т	Т
Only Needs ENOB and SNDR	Т	Т	Т	Т
Need Multi-tone Test	T*	F	Τ^	F
With Clipped Data	F	Т	F	Т

TABLE 6: Ability of four methods to provide accuratehigh performance test results based on Test Setup.

*: Provided the bin containing the frequency of interest is not in the primary lobe and each lobe is well separated.

^: Provided the tones are well separated and less number of tones are present in the signal.

Hence, Table 6 provides guidelines for selecting a method based on the available test setup to obtain accurate spectral results.

The FIRE method could be improved to accommodate for multi-tone testing with additional work. Although two conditions are simultaneously relaxed in FERARI method, there is still an obvious need for methods that can relax all the stringent conditions for accurate spectral testing.

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