Purpose: The purpose of this laboratory experiment is to investigate the use of small signal concepts for designing and analyzing nonlinear circuits that operate linearly in a restricted region. A second purpose of this laboratory will be to explore methods of characterizing the performance of three-terminal or four-terminal nonlinear devices.

Equipment:
Computer with SPICE software
HP E3631A or equivalent power supply
HP 33120A or equivalent signal generator
HP 34401A or equivalent multimeter
HP 54602B or equivalent oscilloscope

Parts:
CD4007 MOSFET array
2N440 or equivalent NPN BJT
Assortment of resistors and capacitors

Reference Material:
Data sheet for the CD4007 MOSFET array
Data sheet for the 2N4400 NPN BJT

Background:
Essentially all circuits that incorporate amplifiers and many other analog circuits that are treated as linear circuits are actually constructed from sub circuits that contain one or more highly nonlinear devices such as MOS transistors, bipolar transistors, or possibly diodes. In spite of the highly nonlinear characteristics of the constituent subcomponents, the linearity of the resultant circuits is often sufficiently linear that they can be treated as linear circuits or components. The dependent sources discussed in EE 201 and the amplifiers discussed earlier in this course are two classes of circuits that have these properties. In this experiment, the linearization of nonlinear devices will be investigated and the widely used small-signal modeling techniques will be explored.

Small-signal Models
An arbitrary 3-terminal nonlinear device is shown in Fig. 1. If $I_1$ and $I_2$ are treated as the dependent variables, the nonlinear relationship between the port voltages and the port currents can be expressed by two equations of the form...
\[
\begin{align*}
\begin{cases}
I_1 &= f_1(V_1, V_2) \\
I_2 &= f_2(V_1, V_2)
\end{cases}
\end{align*}
\]  \quad (1)

The functions \(f_1\) and \(f_2\) are dependent upon the properties of the specific nonlinear device that is of interest. A BJT is a 3-terminal device and a MOSFET, though actually a 4-terminal device, is often modeled as a 3-terminal device and actually becomes a 3-terminal device when the bulk terminal is connected to the source.

![Fig. 1: Arbitrary 3-terminal Nonlinear Device](image)

It was shown in class that a small-signal linear model of any such device can be developed and is given by the equation

\[
\begin{align*}
\begin{cases}
i_1 &= y_{11}V_1 + y_{12}V_2 \\
i_2 &= y_{21}V_1 + y_{22}V_2
\end{cases}
\end{align*}
\]  \quad (2)

where the currents and voltages in these equations are the small signal currents and voltages and where the four parameters \(y_{11}, y_{12}, y_{21},\) and \(y_{22}\) are given by the expression

\[
y_{ij} = \frac{\partial f_i(V_1, V_2)}{\partial V_j} \bigg|_{V_1 = V_1^0, V_2 = V_2^0}
\]  \quad (3)

A small-signal equivalent circuit that captures exactly the same relationship given in (2) is shown in Fig. 2.

![Fig. 2: Small-signal Model of Nonlinear Device](image)
**Small-signal MOSFET Models**

If the nonlinear device is an n-channel MOSFET, with the source connected to the bulk, it is often modeled by the square-law equations

\[ I_D = 0 \]

\[ I_s = \begin{cases} 0 & V_{gs} \leq V_T \\ \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_T - \frac{V_{ds}}{2} \right) V_{ds} & V_{gs} \geq V_T, \ V_{ds} < V_{gs} - V_T \\ \mu C_{ox} \frac{W}{2L} (V_{gs} - V_T)^2 (1 + \lambda V_{ds}) & V_{gs} \geq V_T, \ V_{ds} \geq V_{gs} - V_T \end{cases} \]

Graphically, the drain current predicted by these equations is as shown in Fig. 3.

![Fig. 3 Output characteristics of n-channel MOSFET](image)

When operated in the saturation region, the functions \( f_1 \) and \( f_2 \) that model this nonlinear device become

\[ I_D = 0 \]

\[ I_D = \frac{\mu C_{ox} W}{2L} \left( V_{gs} - V_T \right)^2 (1 + \lambda V_{ds}) \]

where the parameters \( \mu \), \( C_{ox} \), \( W \), \( L \) and \( \lambda \) characterize the device.

The small-signal model for this device, when operating in the saturation region, is as shown in Fig. 4.
The small signal parameters $g_m$ and $g_o$ are given by the expressions

$$g_m = \mu C_{ox} \frac{W}{L} (V_{GS} - V_T)$$

$$g_o = \lambda I_D$$

In quite a few applications, the parameter $\lambda$ is sufficiently small that the parameter $g_o$ can be assumed to be zero.

**Application of MOSFET in Voltage Controlled Amplifiers**

Although the MOSFET is usually operated in the saturation region when used in analog circuits, there are some applications of the MOSFET in the triode region as well. If the MOSFET is operating in the triode region with a small value of $V_{DS}$, the $I_D-V_{DS}$ characteristics are nearly linear and the device can be used as a voltage controlled resistor. One application of the MOSFET as a voltage variable resistor is in the voltage controlled amplifier of Fig. 5.
Part 1 I-V Characteristics of MOSFET

Measure the $I_D-V_{DS}$ characteristics of an n-channel MOS transistor. The characteristics should look similar to that shown in Fig. 3. What is the $V_T$ of the transistor? What is $\frac{\mu C_{ox}}{L}$? Use one of the n-channel MOS transistors in the CD 4007 for these measurements. How do your measured results compare with what is predicted from the datasheet?

Part 2 Analyze a MOSFET Amplifier

The circuit shown below can be used as a voltage amplifier. If $R_L=10K$ and $R_2=100K$, determine the value of $R_1$ needed to establish a quiescent output voltage at 5V if $V_{DD}=8V$. Assume $C_C$ is large (you may wish to use an electrolytic capacitor for $C_C$ but be sure to note the correct polarity) For $M_1$ use a device in the CD4007 MOSFET array. Develop a small-signal model for $M_1$ (you may neglect $\lambda$ effects) at the operating point specified. When operating at this Q-point, predict the small signal voltage gain based upon a small-signal analysis and compare with measured results. How large of output signals can be obtained without seeing excessive distortion of the output?

![Figure 6 Amplifier Circuit.](image)

Part 3 Voltage Controlled Amplifier

Design and test a voltage controlled amplifier that has a gain that can be adjusted between 20 and 50 by adjusting a dc control voltage. Compare measured and theoretical results.