Basic Feedback Configurations
Generalized Feedback Schemes

- Integrators
- Differentiators
- First-order active filters
- Second-order active filters
Input and Output Impedances with Feedback

\[ \beta = \frac{R_2}{R_1 + R_2} \]

Exact analysis: Consider amplifier as a two-port and use open/short analysis method

Will find \( R_{\text{INF}}, R_{\text{OF}}, A_v \) almost identical to previous calculations

Will see a small \( A_{\text{VR}} \) present but it plays almost no role since \( R_{\text{INF}} \) is so large (effectively unilateral)

\[ A_{\text{VF}} \approx \frac{1}{\beta} \]

\[ R_{\text{OF}} = \frac{R_0}{1 + \beta A_v} \]

\[ R_{\text{INF}} = R_{\text{IN}} \left(1 + A_v \beta \right) \]

\[ A_{\text{VR}} \approx \beta \]
Buffer Amplifier

One of the most widely used Op Amp circuits

Provides a signal to a load that is not affected by a source impedance

This provides for decoupling between stages in many circuits

Special case of basic noninverting amplifier with $R_1=\infty$ and $R_2=0$

$$\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_2}{R_1}$$
Input impedance of $R_1$ is unacceptable in many (but not all) applications.

This is not a voltage feedback amplifier (it is a feedback amplifier) of the type (note $R_{IN}$ is not high!)

Feedback concepts could be used to analyze this circuit but lots of detail required.
Summing Amplifier

- Output is a weighted sum of the input voltages
- Any number of inputs can be used
- Gains from all inputs can be adjusted together with $R_F$
- Gain for input $V_i$ can be adjusted independently with $R_i$ for $1 \leq i \leq k$
- All weights are negative
- Input impedance on each input is $R_i$
Generalized Inverting Amplifier

s-domain representation

\[ T(s) = -\frac{Z_F(s)}{Z_1(s)} \]

\( Z_1 \) and \( Z_F \) can be any s-domain circuits

If \( Z_1 = R, \ Z_F = 1/sC \), obtain

\[ T(s) = -\frac{1}{sRC} \]

What is this circuit?
Generalized Inverting Amplifier

What is this circuit?

Consider the differential equation

\[ y = K \int_{0}^{t} x(\tau) d\tau \]

Taking the Laplace Transform, obtain

\[ Y(s) = K \frac{X(s)}{s} \]

\[ T(s) = \frac{Y(s)}{X(s)} = \frac{K}{s} \]

K is the frequency where |T(j\omega)|=1 and is termed the Integrator Unity Gain Frequency

Thus, this circuit is an inverting integrator with a unity gain frequency of \( K = (RC)^{-1} \)
Inverting Integrator

\[ T(s) = -\frac{1}{sRC} \]

\[ T(j\omega) = -\frac{1}{j\omega RC} \]

\[ |T(j\omega)| = \frac{1}{\omega RC} \]

\[ \angle T(j\omega) = 90^\circ \]

Unity gain frequency is \( \omega_0 = \frac{1}{RC} \)
Integrators are widely used!

\[ R_{\text{IN}} = R \quad \text{and} \quad R_{\text{OUT}} = 0 \]

The integrator function itself is ill-conditioned and integrators are seldom used open-loop.

The ideal integrator has a pole at \( s = 0 \) which is not in the LHP.

If the input has any dc component present, since superposition applies, the output would diverge to ±\( \infty \) as time increases.

The offset voltage (discussed later) will also cause an integrator output to diverge.

\[ T(s) = -\frac{1}{sRC} \]

\[ V_{\text{OUT}} = -\frac{1}{RC} \int_0^t V_{\text{IN}}(\tau) d\tau + V_{\text{IN}}(0) \]
Inverting Integrator

\[ V_{OUT} = -\frac{1}{RC} \int_0^t V_{IN}(\tau) \, d\tau + V_{IN}(0) \]

What is the output of an ideal integrator if the input is an ideal square wave?

Amplitude of output dependent upon RC product.
Inverting Integrator

\[ V_{\text{OUT}} = -\frac{1}{RC} \int_{0}^{t} V_{\text{IN}}(\tau) \, d\tau + V_{\text{IN}}(0) \]

What is the output of an ideal integrator if the input is an ideal sine wave?

Amplitude of output dependent upon RC product
Inverting Integrator

Ill-conditioned nature of open-loop integrator

\[ V_{\text{OUT}} = -\frac{1}{RC} \int_0^t V_{\text{IN}}(\tau) d\tau + V_{\text{IN}}(0) \]

If \( V_{\text{IN}}(0) = 0 \),

\[ V_{\text{OUT}} = -\frac{1}{RC} \int_0^t V_{\text{DC}} d\tau \]

\[ V_{\text{OUT}} = -\frac{V_{\text{DC}}}{RC} \int_0^t 1 d\tau \]

\[ V_{\text{OUT}} = -\frac{V_{\text{DC}}}{RC} \left( \tau \bigg|_0^t \right) = -\frac{V_{\text{DC}}}{RC} t \]

For any values of \( V_{\text{DC}}, R, \) and \( C \), the output will diverge to \pm \infty
Inverting Integrator

Ill-conditioned nature of open-loop integrator

\[ V_{\text{OUT}} = -\frac{1}{RC} \int_{0}^{t} V_{\text{IN}}(\tau) d\tau + V_{\text{IN}}(0) \]

Any periodic input that in which the average value is not EXACTLY 0 will have a dc component

\[ V_{\text{IN}}(t) = A_0 + \sum_{k=1}^{\infty} \sin(k\omega t + \theta_k) \]

This dc input will cause the output to diverge!
Noninverting Integrator

Obtained from inverting integrator by preceding or following with inverter

Requires more components

Also widely used

Same issues affect noninverting integrator

\[ V_{OUT} = \frac{1}{RC} \int_0^t V_{IN}(\tau) d\tau + V_{IN}(0) \]
Add a large resistor to slowly drain charge off of $C$ and prevent divergence.

Allows integrator to be used “open-loop”

Changes the dc gain from $-\infty$ to $-\frac{R_F}{R}$

But the lossy integrator is no longer a perfect integrator.
What if $R_F$ is not so large?

\[
T(s) = -\frac{Z_F(s)}{Z_1(s)}
\]
What if $R_F$ is not so large?

\[ T(s) = -\frac{Z_F(s)}{Z_1(s)} \]

\[
T(s) = -\left(\frac{R_2}{R_1}\right)\frac{1}{1+sCR_2}
\]
First-order lowpass filter with a dc gain of $R_2/R_1$

\[ T(s) = -\left(\frac{R_2}{R_1}\right) \frac{1}{1+sCR_2} \]

$R_2$ controls the pole (and also the dc gain)

$R_1$ controls the dc gain (and not the pole)
Summing Integrator

\[ T(s) = -\frac{1}{sRC} \]

\[ V_{OUT} = -\frac{1}{sRC} V_{IN} \]

By superposition

\[ V_{OUT} = -\frac{1}{sR_1C} V_1 - \frac{1}{sR_2C} V_2 - \cdots - \frac{1}{sR_kC} V_k \]

- All inverting functions
- Can have any number of inputs
- Weights independently controlled by resistor values
- Weights all changed by C

\[ V_{OUT} = -\sum_{i=1}^{k} \frac{1}{sR_iC} V_i \]
I've got a better noninverting integrator!
I've got a better noninverting integrator!

But is this a useful circuit?

It has a noninverting transfer function

But it is not a noninverting integrator!
This is a first-order high-pass amplifier (or filter) but the gain at dc goes to $\infty$ so applications probably limited.

But is this a useful circuit?
Two-capacitor noninverting integrator

\[ V_1(sC+G) = V_{OUT}sC \]
\[ V_1(sC+G) = V_{IN}G \]

\[ \frac{V_{OUT}}{V_{IN}} = + \frac{1}{sRC} \]

Requires matched resistors and matched capacitors

Actually uses a concept called “pole-zero cancellation”

Generally less practical than the cascade with an inverter
Generalized Inverting Amplifier

s-domain representation

\[ T(s) = -\frac{Z_F(s)}{Z_1(s)} \]

If \( Z_1 = 1/sC, \ Z_F = R \), obtain \( T(s) = -sRC \)

What is this circuit?
Generalized Inverting Amplifier

What is this circuit?

Consider the differential equation

\[ y = K \frac{dx(t)}{dt} \]

Taking the Laplace Transform, obtain

\[ Y(s) = KsX(s) \]

\[ T(s) = \frac{Y(s)}{X(s)} = Ks \]

\( K^{-1} \) is the frequency where \( |T(j\omega)| = 1 \)

Thus, this circuit is an **inverting** differentiator with a unity gain frequency of \( K^{-1} = (RC)^{-1} \)
Inverting Differentiator

Differentiator gain ideally goes to $\infty$ at high frequencies

Differentiator not widely used

Differentiator relentlessly amplifies noise

Stability problems with implementation (not discussed here)

Placing a resistor in series with C will result in a lossy differentiator that has some applications
First-order High-pass Filter

\[ T(s) = -\frac{R_2}{R_1 + \frac{1}{sC}} = -\frac{sR_2C}{1 + R_1Cs} \]

\[ |T(j\omega)| = \frac{\omega R_2 C}{\sqrt{1 + (\omega R_1 C)^2}} \]
Applications of integrators to solving differential equations

Standard Integral form of a differential equation

\[ X_{OUT} = b_1 \int X_{OUT} + b_2 \int \int X_{OUT} + b_3 \int \int \int X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \int \int X_{IN} + \ldots \]

Standard differential form of a differential equation

\[ X_{OUT} = \alpha_1 X'_{OUT} + \alpha_2 X''_{OUT} + \alpha_3 X'''_{OUT} + \ldots + \beta_1 X'_{IN} + \beta_2 X''_{IN} + \beta_3 X'''_{IN} + \ldots \]

Initial conditions not shown

Can express any system in either differential or integral form
Applications of integrators to solving differential equations

Consider the standard integral form

\[ X_{OUT} = b_1 \int X_{OUT} + b_2 \int \int X_{OUT} + b_3 \int \int \int X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \int \int X_{IN} + \ldots \]

This circuit is comprised of summers and integrators
Can solve an arbitrary linear differential equation
This concept was used in Analog Computers in the past
Applications of integrators to solving differential equations

Consider the standard integral form

\[ X_{OUT} = b_1 \int X_{OUT} + b_2 \int \int X_{OUT} + b_3 \int \int \int X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \int \int X_{IN} + \ldots \]

Take the Laplace transform of this equation

\[ \mathcal{X}_{OUT} = b_1 \frac{1}{s} \mathcal{X}_{OUT} + b_2 \frac{1}{s^2} \mathcal{X}_{OUT} + b_3 \frac{1}{s^3} \mathcal{X}_{OUT} + \ldots + b_n \frac{1}{s^n} + a_0 \mathcal{X}_{IN} + a_1 \frac{1}{s} \mathcal{X}_{IN} + a_2 \frac{1}{s^2} \mathcal{X}_{IN} + a_3 \frac{1}{s^3} \mathcal{X}_{IN} + \ldots + a_m \frac{1}{s^m} \]

Multiply by \( s^n \) and assume \( m=n \) (some of the coefficients can be 0)

\[ s^n \mathcal{X}_{OUT} = b_1 s^{n-1} \mathcal{X}_{OUT} + b_2 s^{n-2} \mathcal{X}_{OUT} + b_3 s^{n-3} \mathcal{X}_{OUT} + \ldots + b_n s^n \mathcal{X}_{IN} + a_0 s^n \mathcal{X}_{IN} + a_1 s^{n-1} \mathcal{X}_{IN} + a_2 s^{n-2} \mathcal{X}_{IN} + a_3 s^{n-3} \mathcal{X}_{IN} + \ldots + a_n \]

\[ \mathcal{X}_{OUT}(s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n) = \mathcal{X}_{IN}(a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n) \]

\[ T(s) = \frac{\mathcal{X}_{OUT}}{\mathcal{X}_{IN}} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n}{s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n} \]
Applications of integrators to solving differential equations

Consider the standard integral form

\[ X_{OUT} = b_1 \int X_{OUT} + b_2 \int \int X_{OUT} + b_3 \int \int \int X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \int \int X_{IN} + \ldots \]

\[ T(s) = \frac{X_{OUT}}{X_{IN}} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n}{s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n} \]

This can be written in more standard form

\[ T(s) = \frac{\alpha_n s^n + \alpha_{n-1} s^{n-1} + \ldots + \alpha_1 s + \alpha_0}{s^n + \beta_{n-1} s^{n-1} + \ldots + \beta_1 s + \beta_0} \]
Applications of integrators to filter design

$T(s) = \frac{\alpha_n s^n + \alpha_{m-1} s^{n-1} + \ldots + \alpha_1 s + \alpha_0}{s^n + \beta_{n-1} s^{n-1} + \ldots + \beta_1 s + \beta_0}$

Can design (synthesize) any $T(s)$ with just integrators and summers!

Integrators are not used “open loop” so loss is not added

Although this approach to filter design works, often more practical methods are used
End of Lecture 12
Applications of integrators to filter design

This is a two-integrator-loop filter

\[
X_{OUT1} = \left( -\frac{l_{01}}{s} \right) (X_{IN} + X_{OUT2} + \alpha X_{OUT1}) \]

\[
X_{OUT2} = \left( \frac{l_{02}}{s} \right) X_{OUT1} \]

These are 2-nd order filters

If \( l_{01} = l_{02} = l_0 \), these transfer functions reduce to

\[
T_1(s) = \frac{-l_0 s}{s^2 + \alpha l_0 s + l_0^2} \]

\[
T_2(s) = \frac{-l_0^2}{s^2 + \alpha l_0 s + l_0^2} \]
Applications of integrators to filter design

Consider $T_1(j\omega)$

$$T_1(j\omega) = \frac{-j\omega l_0}{(l_0^2 - \omega^2) + j\omega\alpha l_0}$$

$$|T_1(j\omega)| = \frac{\omega l_0}{\sqrt{(l_0^2 - \omega^2)^2 + (\omega\alpha l_0)^2}}$$

This is the standard 2nd order bandpass transfer function

Now lets determine the BW and $\omega_p$
Applications of integrators to filter design

Determine the BW and $\omega_p$

$$T_1(s) = \frac{-I_0s}{s^2 + \alpha I_0 s + I_0^2}$$

$$|T_1(j\omega)| = \frac{\omega I_0}{\sqrt{(I_0^2-\omega^2)^2 + (\omega \alpha I_0)^2}}$$

To determine $\omega_p$, must set

$$\frac{d |T_1(j\omega)|}{d\omega} = 0$$

This will occur also when

$$\frac{d |T_1(j\omega)|^2}{d\omega^2} = 0$$

and the latter is easier to work with

$$|T_1(j\omega)|^2 = \frac{\omega^2 I_0^2}{(I_0^2-\omega^2)^2 + (\omega \alpha I_0)^2}$$

$$\frac{d |T_1(j\omega)|^2}{d\omega^2} = \frac{\left((I_0^2-\omega^2)^2 + (\omega \alpha I_0)^2\right) I_0^2 - \omega^2 I_0^2 \left(-2(I_0^2-\omega^2) + (\alpha I_0)^2\right)}{\left[(I_0^2-\omega^2)^2 + (\omega \alpha I_0)^2\right]^2} = 0$$
Applications of integrators to filter design

The 2\textsuperscript{nd} order Bandpass Filter

Determine the BW and $\omega_p$

\[ T_1(s) = \frac{-l_0s}{s^2 + \alpha l_0s + l_0^2} \]

\[ |T_1(j\omega)| = \frac{\omega l_0}{\sqrt{(l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2}} \]

\[ \frac{d|T_1(j\omega)|^2}{d\omega^2} = \frac{\left((l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2\right)l_0^2 - \omega^2 l_0^2 (-2(l_0^2 - \omega^2) + (\alpha l_0)^2)}{\left[(l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2\right]^2} = 0 \]

It suffices to set the numerator to 0

\[ \left((l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2\right)l_0^2 = \omega^2 l_0^2 (-2(l_0^2 - \omega^2) + (\alpha l_0)^2) \]

Solving, we obtain

\[ \omega_P = l_0 \]

Substituting back into the magnitude expression, we obtain

\[ |T_1(j\omega_P)| = \frac{l_0 l_0}{\sqrt{(l_0^2 - l_0^2)^2 + (l_0 \alpha)^2}} = \frac{1}{\alpha} \]

Although the analysis is somewhat tedious, the results are clean
Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and $\omega_p$

$$T_1(s) = \frac{-l_0 s}{s^2 + \alpha l_0 s + l_0^2}$$

$$|T_1(j\omega)| = \frac{\omega l_0}{\sqrt{(l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2}}$$

To obtain $\omega_L$ and $\omega_H$, must solve

$$|T_1(j\omega)| = \frac{1}{\sqrt{2\alpha}}$$

This becomes

$$\frac{1}{2\alpha^2} = \frac{\left[\left(l_0^2 - \omega^2\right)^2 + (\omega \alpha l_0)^2\right]l_0^2 - \omega^2 l_0^2 \left(-2\left(l_0^2 - \omega^2\right) + (\alpha l_0)^2\right)}{\left[(l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2\right]^2}$$

The expressions for $\omega_L$ and $\omega_H$ can be easily obtained but are somewhat messy, but from these expressions, we obtain the simple expressions

$$BW = \omega_H - \omega_L = \alpha l_0$$

$$\sqrt{\omega_H \omega_L} = l_0$$
Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and $\omega_p$

$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$

$$|T_1(j\omega)| = \frac{\omega I_0}{\sqrt{(I_0^2 - \omega^2)^2 + (\omega \alpha I_0)^2}}$$

$$\omega_p = I_0$$

$$|T_1(j\omega_p)| = \frac{1}{\alpha}$$
Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and $\omega_p$

$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$

$$BW = \alpha I_0 \sqrt{\omega_H \omega_L} = I_0$$

Often express the standard 2nd order bandpass transfer function as

$$T_1(s) = \frac{-I_0 s}{s^2 + BW s + I_0^2}$$
Applications of integrators to filter design

The 2nd order Bandpass Filter

These results can be generalized

\[ T_{BP}(s) = \frac{Hs}{s^2 + as + b} \]

\( BW = a \)

\( \omega_p = \sqrt{b} \)

\( K = \frac{|H|}{a} \)
Applications of integrators to filter design

The 2\textsuperscript{nd} order Bandpass Filter

Determine the BW and $\omega_p$

\[ T_1(s) = \frac{-l_0s}{s^2 + \alpha l_0 s + l_0^2} \]

\[ T_1(s) = \frac{-l_0s}{s^2 + BWs + l_0^2} \]

Can readily be implemented with a summing inverting integrator and a noninverting integrator
Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and $\omega_P$

$$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$

$$T_1(s) = \frac{-I_0 s}{s^2 + BW s + I_0^2}$$

- Widely used 2nd order Bandpass Filter
- BW can be adjusted with $R_Q$
- Peak gain changes with $R_Q$
- Note no loss is added to the integrators

$$l_0 = \frac{1}{RC} \quad \therefore \text{BW} = \frac{\alpha}{RC}$$

$$\omega_P = l_0$$

$$BW = \alpha l_0$$
Applications of integrators to filter design

The 2\textsuperscript{nd} order Bandpass Filter

Design Strategy
Assume BW and $\omega_p$ are specified

$$T_{BP}(s) = \frac{-I_0s}{s^2 + \alpha I_0 s + I_0^2}$$

1. Pick $C$ (use some practical or convenient value)
2. Solve expression $\omega_p = \frac{1}{RC}$ to obtain $R$
3. Solve expression $\text{BW} = \frac{\alpha}{RC}$ to obtain $\alpha$ and thus $R_Q$
Applications of integrators to filter design

The 2nd order Lowpass Filter

\[
T_2(s) = \frac{-l_0^2}{s^2 + \alpha l_0 s + l_0^2}
\]

\[X_{IN} \rightarrow \begin{array}{c}
- \frac{l_{01}}{s} \\
\alpha
\end{array} \rightarrow \frac{l_{02}}{s} \rightarrow X_{OUT2}
\]

Exact expressions for BW and \( \omega_P \) are very complicated but \( \omega_P \approx l_0 \)

- Widely used 2nd order Lowpass Filter
- BW can be adjusted with \( R_Q \) but expression not so simple
- Peak gain changes with \( R_Q \)
- Note no loss is added to the integrators

Design procedure to realize a given 2nd order lowpass function is straightforward.
Another 2\textsuperscript{nd}-order Bandpass Filter

\[ V_1(sC_1 + sC_2 + G_2 + G_3) = V_{\text{OUT}}sC_2 + V_{\text{IN}}G_3 \]

\[ V_1sC_1 + V_{\text{OUT}}G_1 = 0 \]

If the capacitors are matched and equal to C

\[ T(s) = -\frac{sR_3C_2}{s^2 + s\left(\frac{2}{R_1C} + \frac{1}{(R_2//R_3)R_1C^2}\right) + \frac{1}{R_1C} + \frac{1}{R_2//R_3})} \]

Since this is of the general form of a 2\textsuperscript{nd} order BP transfer function, obtain

\[ \omega_p = \frac{1}{\sqrt{R_1(R_2//R_3)C}} \]

\[ BW = \frac{2}{R_1C} \quad K = \frac{R_1}{2R_3} \]
Another 2\textsuperscript{nd}-order Bandpass Filter

Design Strategy

Assume BW, \( \omega_p \), and \( K \) are specified

\[ T(s) = \frac{s}{s^2 + \left( \frac{2}{R_1 C} \right) + \frac{1}{(R_2 // R_3) R_1 C^2}} \]

\[ BW = \frac{2}{R_1 C} \quad \omega_p = \frac{1}{\sqrt{R_1 (R_2 // R_3) C}} \]

\[ K = \frac{R_1}{2R_3} \]

1. Pick \( C \) to some practical or convenient value
2. Solve expression \( BW = \frac{2}{R_1 C} \) to obtain \( R_1 \)
3. Solve expression \( K = \frac{R_1}{2R_3} \) to obtain \( \alpha \) and thus \( R_3 \)
4. Solve expression \( \omega_p = \frac{1}{\sqrt{R_1 (R_2 // R_3) C}} \) to obtain \( R_2 \)
Another 2\textsuperscript{nd}-order Bandpass Filter

Termed the “STAR” biquad by inventors at Bell Labs

\[ V_1(sC + sC + G_2 + G_3) = V_{OUT}sC + V_{IN}G_3 + V_{OUT}sC \]

\[ \frac{V_{OUT}}{H} (sC + G_1) = V_1sC + V_{OUT}G_1 \]

\[ \omega_p = \frac{1}{\sqrt{R_1(R_2/R_3)C}} \]

\[ BW = \frac{2}{R_1C} \frac{1}{(R_2/R_3)(H-1)} \]

\[ K = \frac{1}{R_3(H-1)} \left( \frac{1}{R_1} \frac{1}{(R_2/R_3)(H-1)} \right) \]

For the appropriate selection of component values, this is one of the best 2\textsuperscript{nd} order bandpass filters that has been published.
STAR 2\textsuperscript{nd}-order Bandpass Filter

\[ T(s) = \frac{s}{R_3C} \left( \frac{H}{H-1} \right) \]

Implementation:

But the filter doesn’t work !
STAR 2nd-order Bandpass Filter

\[ T(s) = - \frac{s}{R_3 C} \left( \frac{H}{H-1} \right) \]
\[ \frac{s^2 + s \left( \frac{2}{R_1 C} - \frac{1}{R_2 R_3 (H-1)} \right) + \frac{1}{(R_2/R_3)R_1 C^2}} {s^2 + s \left( \frac{2}{R_1 C} - \frac{1}{R_2 R_3 (H-1)} \right) + \frac{1}{(R_2/R_3)R_1 C^2}} \]

Implementation:

If op amp ideal, \( \frac{V_{OUT}}{V_{IN}} = H \)

Works fine! Will discuss why this happens later!

Reduces to previous bandpass filter at H gets large

Note that the “H” amplifier has feedback to positive terminal