Basic Feedback Configurations
Second-Order Filters
Difference Amplifiers
Impedance Converters
Quiz 10

a) Determine the transfer function $T(s) = \frac{V_{OUT}(s)}{V_{IN}(s)}$ for the circuit shown.

b) Is the circuit stable?

Assume the op amps are ideal and all resistors are $1\Omega$ and all capacitors are $1F$.
And the number is ?
Quiz 10

a) Determine the transfer function $T(s) = \frac{V_{OUT}(s)}{V_{IN}(s)}$ for the circuit shown.

b) Is the circuit stable?

Solution:
Quiz 10

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a) Determine the transfer function $T(s) = \frac{V_{OUT}(s)}{V_{IN}(s)}$ for the circuit shown.

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Quiz 10

Solution:

a) Determine the transfer function \( T(s) = \frac{V_{\text{OUT}}(s)}{V_{\text{IN}}(s)} \) for the circuit shown.

b) Is the circuit stable?

\[
\frac{V_{\text{OUT}}}{s^2} = -\frac{V_{\text{IN}}}{s} + \frac{V_{\text{OUT}}}{s}
\]
Quiz 10

Solution:

a) Determine the transfer function $T(s) = \frac{V_{OUT}(s)}{V_{IN}(s)}$ for the circuit shown.

b) Is the circuit stable?

\[
V_{OUT} = -\frac{V_{IN}}{s} - \frac{V_{OUT}}{s} - \frac{V_{OUT}}{s^2}
\]
Quiz 10

Solution:

a) Determine the transfer function $T(s) = V_{OUT}(s)/V_{IN}(s)$ for the circuit shown.

$$V_{OUT} = \frac{V_{IN}}{s} - \frac{V_{OUT}}{s} - \frac{V_{OUT}}{s^2}$$

$$s^2V_{OUT} - sV_{OUT} + V_{OUT} = sV_{IN}$$

$$V_{OUT} \left( s^2 - s + 1 \right) = sV_{IN}$$

$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{s}{s^2 - s + 1}$$
Quiz 10

Solution:

a) Determine the transfer function \( T(s) = \frac{V_{OUT}}{V_{IN}} \) for the circuit shown.

\[
T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{s}{s^2 - s + 1}
\]

Poles at

\[
s = \frac{1+j\sqrt{3}}{2}, \quad s = \frac{1-j\sqrt{3}}{2}
\]
a) Determine the transfer function $T(s) = \frac{V_{OUT}(s)}{V_{IN}(s)}$ for the circuit shown.

\[
T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{s}{s^2 - s + 1}
\]

b) Is the circuit stable?

Poles at

\[
s = \frac{1+j\sqrt{3}}{2}
\]

RHP

\[
s = \frac{1-j\sqrt{3}}{2}
\]

RHP
a) Determine the transfer function \( T(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} \) for the circuit shown

\[
T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{s}{s^2 - s + 1}
\]

Poles at

\[
s = \frac{1+j\sqrt{3}}{2}
\]

\[
s = \frac{1-j\sqrt{3}}{2}
\]

Since there is one RHP pole, the circuit is unstable!
Inverting Integrator

\[ T(s) = -\frac{1}{sRC} \]

\[ T(j\omega) = -\frac{1}{j\omega RC} \]

\[ |T(j\omega)| = \frac{1}{\omega RC} \]

\[ \angle T(j\omega) = 90^\circ \]

Unity gain frequency is \( \omega_0 = \frac{1}{RC} \)
Noninverting Integrator

Obtained from inverting integrator by preceding or following with inverter

Requires more components

Also widely used

Same issues affect noninverting integrator
Review from Last Time

First-order lowpass filter with a dc gain of $R_2/R_1$

$$T(s) = -\left(\frac{R_2}{R_1}\right)\frac{1}{1+sCR_2}$$

$R_2$ controls the pole (and also the dc gain)

$R_1$ controls the dc gain (and not the pole)
This is a first-order high-pass amplifier (or filter)

\[
V_{\text{OUT}} = \frac{sRC}{1+sRC}
\]

3dB band edge at \( \omega = \frac{1}{RC} \)
Inverting Differentiator

Differentiator gain ideally goes to \( \infty \) at high frequencies

Differentiator not widely used

Differentiator relentlessly amplifies noise

Stability problems with implementation (not discussed here)

Placing a resistor in series with C will result in a lossy differentiator that has some applications
**First-order High-pass Filter**

\[ T(s) = -\frac{R_2}{R_1 + \frac{1}{sC}} = -\frac{sR_2C}{1 + R_1Cs} \]

\[ |T(j\omega)| = \frac{\omega R_2 C}{\sqrt{1 + (\omega R_1 C)^2}} \]

**Review from Last Time**
Review from Last Time

Applications of integrators to solving differential equations

Consider the standard integral form

$$X_{OUT} = b_1 \int X_{OUT} + b_2 \int \int X_{OUT} + b_3 \int \int \int X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \int \int X_{IN} + \ldots$$

This circuit is comprised of summers and integrators.
Can solve an arbitrary linear differential equation.
This concept was used in Analog Computers in the past.
Applications of integrators to filter design

Can design (synthesize) any $T(s)$ with just integrators and summers!

Integrators are not used “open loop” so loss is not added.

Although this approach to filter design works, often more practical methods are used.

Review from Last Time
Applications of integrators to filter design

This is a two-integrator-loop filter

\[ X_{\text{OUT1}} = \left( -\frac{l_{01}}{s} \right) (X_{\text{IN}} + X_{\text{OUT2}} + \alpha X_{\text{OUT1}}) \]

\[ X_{\text{OUT2}} = \left( \frac{l_{02}}{s} \right) X_{\text{OUT1}} \]

These are 2nd order filters

If \( l_{01} = l_{02} = l_0 \), these transfer functions reduce to

\[ T_1(s) = \frac{-l_0s}{s^2 + \alpha l_0s + l_0^2} \]

\[ T_2(s) = \frac{-l_0^2}{s^2 + \alpha l_0s + l_0^2} \]
Applications of integrators to filter design

\[ T_1(s) = \frac{-l_0 s}{s^2 + \alpha l_0 s + l_0^2} \]
\[ T_2(s) = \frac{-l_0^2}{s^2 + \alpha l_0 s + l_0^2} \]

Consider \( T_1(j\omega) \)

\[ T_1(j\omega) = \frac{-j\omega l_0}{(l_0^2 - \omega^2) + j\omega \alpha l_0} \]
\[ |T_1(j\omega)| = \frac{\omega l_0}{\sqrt{(l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2}} \]

This is the standard 2\textsuperscript{nd} order bandpass transfer function

Now lets determine the BW and \( \omega_p \)
Applications of integrators to filter design

Determine the BW and $\omega_p$

$$T_1(s) = \frac{-l_0 s}{s^2 + \alpha l_0 s + l_0^2}$$

$$|T_1(j\omega)| = \frac{\omega l_0}{\sqrt{(l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2}}$$

To determine $\omega_p$, must set

$$\frac{d|T_1(j\omega)|}{d\omega} = 0$$

This will occur also when

$$\frac{d|T_1(j\omega)|^2}{d\omega^2} = 0$$

and the latter is easier to work with

$$|T_1(j\omega)|^2 = \frac{\omega^2 l_0}{(l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2}$$

$$\frac{d|T_1(j\omega)|^2}{d\omega^2} = \frac{\left((l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2\right)l_0^2 - \omega^2 l_0^2 \left(-2(l_0^2 - \omega^2) + (\alpha l_0)^2\right)}{\left[(l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2\right]^2} = 0$$
Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and $\omega_p$

$$T_1(s) = \frac{-l_0 s}{s^2 + \alpha l_0 s + l_0^2}$$

$$|T_1(j\omega)| = \frac{\omega l_0}{\sqrt{(l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2}}$$

$$\frac{d|T_1(j\omega)|^2}{d\omega^2} = \frac{\left((l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2\right)l_0^2 - \omega^2 l_0^2 (-2(l_0^2 - \omega^2) + (\alpha l_0)^2)}{\left[(l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2\right]^2} = 0$$

It suffices to set the numerator to 0

$$\left((l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2\right)l_0^2 = \omega^2 l_0^2 (-2(l_0^2 - \omega^2) + (\alpha l_0)^2)$$

Solving, we obtain

$$\omega_p = l_0$$

Substituting back into the magnitude expression, we obtain

$$|T_1(j\omega_p)| = \frac{l_0 l_0}{\sqrt{(l_0^2 - l_0^2) + (l_0 \alpha)^2}} = \frac{1}{\alpha}$$

Although the analysis is somewhat tedious, the results are clean
Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and $\omega_p$

$$T_1(s) = \frac{-l_0s}{s^2 + \omega l_0 s + l_0^2}$$

$$|T_1(j\omega)| = \frac{\omega l_0}{\sqrt{(l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2}}$$

To obtain $\omega_L$ and $\omega_H$, must solve

$$|T_1(j\omega)| = \frac{1}{\sqrt{2\alpha}}$$

This becomes

$$\frac{1}{2\alpha^2} = \frac{(l_0^2 - \omega^2)^2 + (\omega \alpha l_0)^2}{[l_0^2 - \omega^2 + (\omega \alpha l_0)^2]^2}$$

The expressions for $\omega_L$ and $\omega_H$ can be easily obtained but are somewhat messy, but from these expressions, we obtain the simple expressions

$$BW = \omega_H - \omega_L = \alpha l_0$$

$$\sqrt{\omega_H \omega_L} = l_0$$
Applications of integrators to filter design

The 2\textsuperscript{nd} order Bandpass Filter

Determine the BW and $\omega_p$

$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$

$$|T_1(j\omega)| = \frac{\omega I_0}{\sqrt{(I_0^2 - \omega^2)^2 + (\omega \alpha I_0)^2}}$$

$\omega_p = I_0$

$\text{BW} = \alpha I_0$

$$|T_1(j\omega_p)| = \frac{1}{\alpha}$$
Applications of integrators to filter design

The 2\textsuperscript{nd} order Bandpass Filter

Determine the BW and $\omega_p$

$$T_1(s) = \frac{-l_0s}{s^2 + \alpha l_0 s + l_0^2}$$

$$BW = \alpha l_0 \sqrt{\omega_H \omega_L} = l_0$$

Often express the standard 2\textsuperscript{nd} order bandpass transfer function as

$$T_1(s) = \frac{-l_0s}{s^2 + BW s + l_0^2}$$
Applications of integrators to filter design

The 2nd order Bandpass Filter

These results can be generalized

\[ T_{BP}(s) = \frac{Hs}{s^2 + as + b} \]

\[ BW = a \]

\[ \omega_p = \sqrt{b} \]

\[ K = \frac{|H|}{a} \]
Applications of integrators to filter design

The 2nd order Bandpass Filter

Determine the BW and $\omega_p$

\[ T_1(s) = \frac{-l_0 s}{s^2 + \alpha l_0 s + l_0^2} \]

\[ T_1(s) = \frac{-l_0 s}{s^2 + BW s + l_0^2} \]

Can readily be implemented with a summing inverting integrator and a noninverting integrator
Applications of integrators to filter design

The 2\textsuperscript{nd} order Bandpass Filter

Determine the BW and $\omega_P$

$T_1(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$

$T_1(s) = \frac{-I_0 s}{s^2 + BW s + I_0^2}$

- Widely used 2\textsuperscript{nd} order Bandpass Filter
- BW can be adjusted with $R_Q$
- Peak gain changes with $R_Q$
- Note no loss is added to the integrators

$\omega_P = I_0$

$BW = \alpha I_0$
Applications of integrators to filter design

The 2nd order Bandpass Filter

Design Strategy

Assume BW and $\omega_P$ are specified

$$T_{BP}(s) = \frac{-I_0 s}{s^2 + \alpha I_0 s + I_0^2}$$

1. Pick C (use some practical or convenient value)
2. Solve expression $\omega_P = \frac{1}{RC}$ to obtain R
3. Solve expression $BW = \frac{\alpha}{RC}$ to obtain $\alpha$ and thus $R_Q$
Applications of integrators to filter design

The 2nd order Lowpass Filter

\[ T_2(s) = \frac{-I_0^2}{s^2 + \alpha I_0 s + I_0^2} \]

\[ X_{\text{OUT}} \]

\[ I_{01} = I_{02} = I_0 \]

Exact expressions for BW and \( \omega_p \) are very complicated but \( \omega_p \approx I_0 \)

- Widely used 2nd order Lowpass Filter
- BW can be adjusted with \( R_Q \) but expression not so simple
- Peak gain changes with \( R_Q \)
- Note no loss is added to the integrators

Design procedure to realize a given 2nd order lowpass function is straightforward
Another 2\textsuperscript{nd}-order Bandpass Filter

\[ V_1(sC_1+sC_2+G_2+G_3) = V_{\text{OUT}}sC_2 + V_{\text{IN}}G_3 \]

\[ V_1sC_1 + V_{\text{OUT}}G_1 = 0 \]

\[ T(s) = \frac{s}{s^2 + \frac{1}{R_1C_1} + \frac{1}{R_2/R_3R_1C_1}} \]

If the capacitors are matched and equal to \( C \)

\[ T(s) = -\frac{s}{R_3C} \left( s^2 + \frac{2}{R_1C} + \frac{1}{(R_2/R_3)R_1C^2} \right) \]

Since this is of the general form of a 2\textsuperscript{nd} order BP transfer function, obtain

\[ \omega_p = \frac{1}{\sqrt{R_1(R_2//R_3)C}} \]

\[ \text{BW} = \frac{2}{R_1C} \quad K = \frac{R_1}{2R_3} \]
Another 2\textsuperscript{nd}-order Bandpass Filter

Design Strategy

Assume BW, \( \omega_p \), and \( K \) are specified

\[
T(s) = -\frac{s}{s^2 + s\left(\frac{2}{R_1C} + \frac{1}{(R_2//R_3)R_1C^2}\right)}
\]

\[
BW = \frac{2}{R_1C}, \quad \omega_p = \frac{1}{\sqrt{R_1(R_2//R_3)}C}
\]

\[
K = \frac{R_1}{2R_3}
\]

1. Pick \( C \) to some practical or convenient value
2. Solve expression \( BW = \frac{2}{R_1C} \) to obtain \( R_1 \)
3. Solve expression \( K = \frac{R_1}{2R_3} \) to obtain \( \alpha \) and thus \( R_3 \)
4. Solve expression \( \omega_p = \frac{1}{\sqrt{R_1(R_2//R_3)}C} \) to obtain \( R_2 \)
Another 2\textsuperscript{nd}-order Bandpass Filter

Termed the “STAR” biquad by inventors at Bell Labs

\[
\begin{align*}
V_1(sC + sC + G_2 + G_3) &= V_{\text{OUT}}sC + V_{\text{IN}}G_3 + V_{\text{OUT}}sC \\
\frac{V_{\text{OUT}}}{H}(sC + G_1) &= V_1sC + V_{\text{OUT}}G_1
\end{align*}
\]

\[
T(s) = \frac{s}{s^2 + \frac{2}{R_1C} - \frac{1}{(R_2//R_3)(H-1)} + \frac{1}{(R_2//R_3)R_1C^2}}
\]

\[
\omega_p = \frac{1}{\sqrt{R_1(R_2//R_3)C}}
\]

\[
\text{BW} = \frac{2}{R_1C} - \frac{1}{(R_2//R_3)(H-1)}
\]

\[
K = \frac{1 \left(\frac{H}{H-1}\right)}{\left(\frac{2}{R_1} - \frac{1}{(R_2//R_3)(H-1)}\right)}
\]

For the appropriate selection of component values, this is one of the best 2\textsuperscript{nd} order bandpass filters that has been published.
STAR 2\textsuperscript{nd}-order Bandpass Filter

\[ T(s) = -\frac{s}{R_3 C \left( \frac{H}{H-1} \right)} \frac{s^2 + s}{\frac{2}{R_1 C} - \frac{1}{(R_2//R_3)(H-1)} + \frac{1}{(R_2//R_3)R_1C^2}} \]

Implementation:

But the filter doesn’t work!
STAR 2nd-order Bandpass Filter

\[ T(s) = -\frac{s}{R_3C} \left( \frac{H}{H-1} \right) \]

\[ s^2 + s \left( \frac{2}{R_1C} - \frac{1}{(R_2/R_3)(H-1))} \right) + \frac{1}{(R_2/R_3)R_1C^2} \]

Implementation:

If op amp ideal, \( \frac{V_{OUT}}{V_{IN}} = H \)

Works fine!

Will discuss why this happens later!

Reduces to previous bandpass filter at H gets large

Note that the “H” amplifier has feedback to positive terminal