EE 230
Lecture 21
Nonlinear Circuits
Nonlinear Op Amp Applications
Quiz 14

Obtain an expression for and plot the dc transfer characteristics of the following circuit. Assume the op amp is biased with ±15V power supplies and the 50K potentiometer is set with the wiper at θ=0.25.
And the number is ?

1 3 8
5 4
2 6
9 7
And the number is 6?
Quiz 14

Obtain an expression for and plot the dc transfer characteristics of the following circuit. Assume the op amp is biased with ±15V power supplies and the 50K potentiometer is set with the wiper at \( \theta = 0.25 \).

Solution:

This circuit is a noninverting comparator with a trip point set at \( V_{\text{TRIP}} = 0.25 \times 6V = 1.5V \).
Quiz 14

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Solution:
This circuit is a noninverting comparator with a trip point set at $V_{TRIP}=0.25*6V = 1.5V$

$$V_{OUT} = \begin{cases} 
V_{SATH} = 15V & V_{IN} > 1.5V \\
V_{SATL} = -15V & V_{IN} < 1.5V 
\end{cases}$$
Quiz 14

Solution:

\[ V_{OUT} = \begin{cases} V_{SATH} = 15V & V_{IN} > 1.5V \\ V_{SATL} = -15V & V_{IN} < 1.5V \end{cases} \]
Methods of Analysis of Nonlinear Circuits

Will consider three different analysis requirements and techniques for some particularly common classes of nonlinear circuits

1. Circuits with continuously differential devices

   Interested in obtaining transfer characteristics of these circuits or outputs for given input signals

2. Circuits with piecewise continuous devices

   interested in obtaining transfer characteristics of these circuits or outputs for a given input signals

3. Circuits with small-signal inputs that vary around some operating point

   Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Other types of nonlinearities may exist and other types of analysis may be required but we will not attempt to categorize these scenarios in this course
3. Circuits with small-signal inputs that vary around some operating point

Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point.

Analysis Strategy:

Determine the operating point (using method 1 or 2 discussed above after all small signal independent inputs are set to 0)

Develop small signal (linear) model for all devices in the region of interest (around the operating point or “Q-point”)

Create small signal equivalent circuit by replacing all devices with small-signal equivalent

Solve the resultant small-signal (linear) circuit

Can use KCL, DVL, and other linear analysis tools such as superposition, voltage and current divider equations, Thevenin and Norton equivalence

Determine boundary of region where small signal analysis is valid
Op Amp Is Almost Never Used as an Open Loop High Gain Amplifier!!

But what will happen if an engineer attempts to use this circuit as an amplifier?

It only costs 25¢, lets for it!
Op Amp Is Almost Never Used as an Open Loop High Gain Amplifier!!

\[ \text{Review from Last Time:} \]

\[ V_{\text{OUT}} \approx \begin{cases} V_{\text{SATL}} & V_{\text{IN}} < 0 \\ V_{\text{SATL}} & V_{\text{IN}} > 0 \end{cases} \]

\[ \Delta V_{\text{sat}} \approx \frac{V_{\text{sat}} - V_{\text{sat}}} {A \text{ vs}} \text{ is very small} \]
Op Amp Is Almost Never Used as an Open Loop High Gain Amplifier !!

\[ V_{OUT} \begin{cases} V_{SATH} & V_{IN} < 0 \\ V_{SATL} & V_{IN} > 0 \end{cases} \]

This circuit serves as a comparator !

This circuit serves as a 1-bit analog to digital converter (ADC)
Consider:

\[ \text{Unstable Circuit} \]

Define \( \Theta = \frac{R_1}{R_1 + R_2} \)

How does this circuit perform?
Comparator with Hysteresis
Comparator with Hysteresis
Comparator with Hysteresis

If unstable region is entered from the left

\[ V_{\text{OUT}} \]

\[ V_{\text{IN}} \]

\[ V_{\text{SATL}} \]

\[ V_{\text{SATH}} \]

Unstable Region

\[ V_{\text{SATL}} = \frac{V_{\text{SATH}}}{1 + \frac{R_2}{R_1}} \]
Comparator with Hysteresis

If unstable region is entered from the right

\[ V_{OUT} \quad V_{SATL} \quad \frac{V_{SATH}}{1 + \frac{R_2}{R_1}} \]

\[ V_{IN} \quad V_{SATL} \quad \frac{V_{SATH}}{1 + \frac{R_2}{R_1}} \]
Comparator with Hysteresis

If unstable region is entered from the left or right

\[ V_{\text{OUT}} \]

\[ V_{\text{VIN}} \]

\[ V_{\text{SATL}} \]

\[ V_{\text{SATL}} \]

\[ V_{\text{SATH}} \]

\[ \frac{V_{\text{SATH}}}{1 + \frac{R_2}{R_1}} \]
Comparator with Hysteresis

\[ V_{\text{IN}} \rightarrow V_{\text{OUT}} \]

\[ R_1 \]

\[ R_2 \]

\[ V_{\text{SATL}} \]

\[ V_{\text{SATH}} \]

Hysteresis Region

\[ \frac{V_{\text{SATL}}}{1 + \frac{R_2}{R_1}} \]

\[ \frac{V_{\text{SATH}}}{1 + \frac{R_2}{R_1}} \]
Comparator with Hysteresis

$$V_{\text{OUT}} = \frac{V_{\text{SATL}}}{1 + \frac{R_2}{R_1}}$$

$$\text{Width of Hysteresis Region} = \frac{V_{\text{SATH}} - V_{\text{SATL}}}{1 + \frac{R_2}{R_1}}$$

Hysteresis Region
Comparator with Hysteresis

\[ V_{\text{out}} = \begin{cases} V_{\text{DD}} & \text{if } V_{\text{in}} < \Theta V_{\text{DD}} \\ V_{\text{SS}} & \text{if } V_{\text{in}} > \Theta V_{\text{SS}} \end{cases} \]

- Hysteresis Loop is Formed
- Bistable if \( V_{\text{in}} \) is in hysteresis window
- Often \( \Theta \) is very small
- Widely used in control applications (provides "dead zone")
- Serves as a memory circuit if $V_{in} = 0$

- Output depends upon how hysteresis region is entered

- Output is indeterminate w/o information about how hysteresis region is entered
Waveform Generator

$$\Theta = \frac{R_1}{R_1 + R_2}$$

\[\begin{align*}
\omega \log, & \text{ assume } t_i = 0 \\
V_x &= F + (1 - F) e^{-t/RC} \\
F &= V_{SS}, \quad I = \Theta V_{PD} \\
\therefore V_x &= V_{SS} + (\Theta V_{PD} - V_{SS}) e^{-t/RC} \\
\therefore \Theta V_{SS} &= V_{SS} + (\Theta V_{PD} - V_{SS}) e^{-\frac{T}{2RC}} \\
\frac{V_{SS} (\Theta - 1)}{\Theta V_{PD} - V_{SS}} &= e^{-\frac{T}{2RC}}
\end{align*}\]
\[- \frac{T}{2RC} = \ln \left( \frac{\frac{V_{SS} [\theta - \frac{\Theta}{\Theta}]}{V_{DD} - V_{SS}}}{\Theta} \right) \]

\[T = -2RC \ln \left( \frac{\frac{V_{SS} [\theta - \frac{\Theta}{\Theta}]}{V_{DD} - V_{SS}}}{\Theta} \right)\]

\[f = \frac{1}{T} = \frac{1}{2RC} \cdot \ln \left( \frac{\frac{V_{SS} [\theta - \frac{\Theta}{\Theta}]}{V_{DD} - V_{SS}}}{\Theta} \right)\]

If \( V_{SS} = -V_{DD} \),

\[f = \frac{1}{2RC} \cdot \ln \left( \frac{\frac{V_{DD} [1 - \Theta]}{V_{DD} [1 + \Theta]}}{\Theta} \right)\]

\[f = \frac{1}{2RC} \cdot \frac{1}{\ln \left( \frac{1 - \Theta}{1 + \Theta} \right)}\]
How can these characterizing equations be solved to obtain $V_0(t)$?