EE 230
Lecture 22

Nonlinear Op Amp Applications
– Waveform Generators
Quiz 16

Plot the transfer characteristics of the following circuit. Assume the op amp has $V_{SATH}=12V$ and $V_{SATL}=-12V$. 

![Circuit Diagram]

- $V_{IN}$
- $V_{OUT}$
- $R_2=20K$
- $R_1=4K$
And the number is ?

1 3 8

5 6 7

2 9

4 6 7
Quiz 16

Plot the transfer characteristics of the following circuit. Assume the op amp has $V_{SATH}=12V$ and $V_{SATL}=-12V$.

Solution

Define $\theta = \frac{R_1}{R_1+R_2}$
Review from Last Lecture

Before trashing the “bad” circuit which we saw was unstable, let’s see if it has any useful properties!

If ideal op amps both have gain

\[ A_{FB} = 1 + \frac{R_2}{R_1} \]

Usually the good circuit

Usually the bad circuit

This circuit is unstable!
Comparator with Hysteresis

\[ \theta = \frac{R_1}{R_1 + R_2} \]

Review from Last Lecture
Comparator with Hysteresis

\[ \theta = \frac{R_1}{R_1 + R_2} \]

Hysteresis Region

\[ V_{\text{SATL}} \]
\[ V_{\text{SATH}} \]
\[ \frac{V_{\text{SATL}}}{1 + \frac{R_2}{R_1}} \]
\[ \frac{V_{\text{SATH}}}{1 + \frac{R_2}{R_1}} \]
Modifications of Comparator with Hysteresis

Note this is the basic inverting amplifier with op amp terminals interchanged

Many other ways to control position and size of hysteresis window

Review from Last Lecture
Usually the good circuit

If ideal op amps both have gain

\[ A_{FB} = 1 + \frac{R_2}{R_1} \]

This circuit is unstable!

Will now analyze the “usually good” circuit using nonlinear analysis methods
Consider this circuit

\[ V_{\text{OUT}} = \begin{cases} V_{\text{SATL}} & \text{Region 1} \\ A_0 V_{\text{IN}} & \text{Region 2} \\ V_{\text{SATH}} & \text{Region 3} \end{cases} \]

Assume in Region 1 (must verify)

\[ V_{\text{OUT}} = V_{\text{SATH}} \]

Valid for

\[ V_{\text{IN}} - \theta V_{\text{SATH}} > \frac{V_{\text{SATH}}}{A_0} \]

\[ \theta V_{\text{SATH}} < V_{\text{IN}} \]
Consider this circuit

\[ V_{IN} \rightarrow \text{VOUT} \]

Assume in Region 3 (must verify)

\[ V_{OUT} = V_{SATL} \]

Valid for

\[ V_{IN} - \theta V_{OUT} < \frac{V_{SATL}}{A_0} \]

\[ \theta V_{SATL} > V_{IN} \]

Note this is now single-valued for the range considered so far
Consider this circuit

\[ V_{\text{IN}} \quad V_{\text{OUT}} \]

\[ R_1 \quad R_2 \]

\[ \theta = \frac{R_1}{R_1 + R_2} \]

Assume in Region 2 (must verify)

\[ V_{\text{DIFF}} = V_{\text{IN}} - \theta V_{\text{OUT}} \]

\[ V_{\text{OUT}} = A_0 V_{\text{DIFF}} \]

Valid for

\[
\frac{V_{\text{SATL}}}{A_0} < V_{\text{IN}} - \theta V_{\text{OUT}} < \frac{V_{\text{SATH}}}{A_0}
\]

(must use exact value for \( V_{\text{OUT}} \) to avoid degenerate conditions)

\[ \theta V_{\text{SATL}} < V_{\text{IN}} < \theta V_{\text{SATH}} \]

\[ V_{\text{OUT}} = \begin{cases} 
V_{\text{SATH}} & \text{Region 1} \\
A_0 V_{\text{IN}} & \text{Region 2} \\
V_{\text{SATL}} & \text{Region 3}
\end{cases} \]

Note this circuit does not have a hysteresis loop

Simply serves as a noninverting amplifier that saturates at extreme inputs
Comparison of basic noninverting amplifier structures

- Serves as an amplifier **directly**
- Stable
- No hysteresis loop

If ideal op amps both have gain

\[ A_{FB} = 1 + \frac{R_2}{R_1} \]

- Not useful as an amplifier **directly**
- Unstable
- Serves as comparator with hysteresis
Comparison of basic noninverting amplifier structures

If ideal op amps both have gain

$$A_{FB} = 1 + \frac{R_2}{R_1}$$

Usually a good amplifier

Usually not a good amplifier

Give examples where the circuit on the left serves as an amplifier and that on the right does not
Waveform Generator

\[ \theta = \frac{R_1}{R_1 + R_2} \]

**Lets first check stability**

Will add an excitation that does not change the “dead” network to obtain a transfer function

The roots of the denominator are the poles of the circuit

Actually can check stability without adding input but will not go into details at this time
Let's first check stability

\[ V_{\text{OUT2}} = \frac{GB}{s} (\theta V_{\text{OUT2}} - V_{\text{OUT1}}) \]

\[ V_{\text{OUT1}} (sC + G) = G V_{\text{OUT2}} + sCV_{\text{IN}} \]

\[
T(s) = \frac{V_{\text{OUT2}}}{V_{\text{IN}}} = \frac{sC}{s^2 \left( \frac{C}{GB} \right) + s \left( \frac{G}{GB} - \theta C \right) + G(1-\theta)}
\]

\[
T(s) = \frac{sC}{s^2 \left( \frac{C}{GB} \right) + s(-\theta C) + G(1-\theta)}
\]

It follows that there is a pole on the positive real axis!

Thus the circuit is unstable and will operate nonlinearly!

Can find the RHP pole of THIS circuit even if op amp is ideal!
Waveform Generator

\[ V_{\text{OUT1}} = R \cdot \frac{V_{\text{OUT2}}}{R + R} \]

Can also check stability for this with ideal op amp

\[ V_{\text{OUT1}} = \theta V_{\text{OUT2}} \]
\[ V_{\text{OUT1}}(sC+G) = GV_{\text{OUT2}} + sCV_{\text{IN}} \]

\[ T(s) = \frac{sC}{s(-\theta C) + G(1-\theta)} \]

Pole at \( p = \frac{G(1-\theta)}{\theta C} \)

Observe the pole is on the positive real axis!

Thus the circuit is unstable and will operate nonlinearly!
Waveform Generator

\[ \theta = \frac{R_1}{R_1 + R_2} \]

Assume \( V_{OUT2} \) is in either the high or low state at any time \( t \)

w.l.o.g. assume that \( V_{OUT1} = 0 \) at \( t=0 \) and \( V_{OUT2} = V_{SATH} \)

\[ V_{OUT1} = F + (1-F) e^{-\frac{t}{RC}} \]

\[ F = V_{SATH}, \quad I = 0 \]

\[ V_{OUT1} = V_{SATH} \left( 1 - e^{-\frac{t}{RC}} \right) \]

while in this state, \( V_{OUT1} \) is increasing, however, this is valid only for \( V_{OUT1} < \theta V_{SATH} \)

As soon as \( V_{OUT1} = \theta V_{SATH} \), comparator output will switch to \( V_{SATL} \).
For convenience reset the time axis as shown now, for $t>0$ have

$$V_{OUT1} = F + (1-F)e^{-\frac{t}{RC}}$$

$F=V_{SATL}$, $I=\theta V_{SATH}$

$$V_{OUT1} = V_{SATL} \left( 1-e^{-\frac{t}{RC}} \right) + \theta V_{SATH} e^{-\frac{t}{RC}}$$

while in this state, $V_{OUT1}$ is decreasing, however, this is valid only for $V_{OUT1}>\theta V_{SATL}$

As soon as $V_{OUT1}=\theta V_{SATL}$, comparator output will switch to $V_{SATH}$

If we define $t_1$ to be the time where this switch occurs, it follows that

$$\theta V_{SATL} = V_{SATL} \left( 1-e^{-\frac{t_1}{RC}} \right) + \theta V_{SATH} e^{-\frac{t_1}{RC}}$$

Solving for $t_1$, obtain

$$t_1 = -RC \ln \left( \frac{V_{SATL} (\theta-1)}{\theta V_{SATH} - V_{SATL}} \right)$$
Waveform Generator

![Comparator with Hysteresis](126x225 to 182x233)

\[
\theta = \frac{R_1}{R_1 + R_2}
\]

this process repeats itself

the rise time and the fall times are identical

the period of the nearly triangular waveform is thus \(2t_1\)

\[
T = 2t_1 = -2RC \ln \left( \frac{V_{SATL}(\theta-1)}{\theta V_{SATH} - V_{SATL}} \right)
\]

If \(V_{SATL} = -V_{SATH}\), this simplifies to

\[
f = \frac{1}{2RC} \ln \left( \frac{1}{\frac{\theta V_{SATH} - V_{SATL}}{V_{SATL}(\theta-1)}} \right)
\]

\[
f = \frac{1}{2RC} \ln \left( \frac{1}{\frac{1+\theta}{1-\theta}} \right)
\]
Waveform Generator

\[ \theta = \frac{R_1}{R_1 + R_2} \]

for \( V_{SATL} = -V_{SATH} \)

\[ T = 2RC\ln\left(\frac{1+\theta}{1-\theta}\right) \]
Square and distorted triangular output waveforms
Slope of square wave is determined by SR of Op Amp

for $V_{SATL} = -V_{SATH}$

$$f = \frac{1}{2RC} \ln \left( \frac{1 + \theta}{1 - \theta} \right)$$

$$\theta = \frac{R_1}{R_1 + R_2}$$
End of Lecture 22