Quiz 16

Obtain an expression for and plot the transfer characteristics of the following circuit. Assume $R_1=2\,\text{K}$, $R_2=8\,\text{K}$, $R=10\,\text{K}$, $V_{\text{DD}}+15\,\text{V}$, $V_{\text{SS}}=-15\,\text{V}$
And the number is ?
And the number is 3?
Quiz 16

Solution:

Obtain an expression for and plot the transfer characteristics of the following circuit. Assume $R_1=2\,\text{K}$, $R_2=8\,\text{K}$, $R=10\,\text{K}$, $V_{\text{DD}}=15\,\text{V}$, $V_{\text{SS}}=-15\,\text{V}$

\[ V_{\text{HYH}} = \theta V_{\text{SATL}} + (1-\theta) V_R \]

\[ V_{\text{HYL}} = \theta V_{\text{SATL}} + (1-\theta) V_R \]

\[ \theta = \frac{R_1}{R_1+R_2} \]

\[ V_{\text{OUT}} = \frac{V_1 + V_2}{2} \]
Obtain an expression for and plot the transfer characteristics of the following circuit. Assume $R_1=2K$, $R_2=8K$, $R=10K$, $V_{DD}=15V$, $V_{SS}=-15V$

\[
\theta = \frac{R_1}{R_1+R_2} = 0.2
\]

\[
V_{H\text{HH}} = \theta V_{S\text{ATH}} + (1-\theta) V_R = 3V + 4V = 7V
\]

\[
V_{H\text{HL}} = \theta V_{S\text{ATH}} + (1-\theta) V_R = 3V - 4V = -1V
\]

\[
V_{H\text{LL}} = \theta V_{S\text{ATL}} + (1-\theta) V_R = -3V + 4V = 1V
\]

\[
V_{H\text{LL}} = \theta V_{S\text{ATL}} + (1-\theta) V_R = -3V - 4V = -7V
\]
Obtain an expression for and plot the transfer characteristics of the following circuit. Assume $R_1=2\,\text{K}$, $R_2=8\,\text{K}$, $R=10\,\text{K}$, $V_{\text{DD}}=15\,\text{V}$, $V_{\text{SS}}=-15\,\text{V}$

\[
V_{\text{HYL}} = \theta V_{\text{SATL}} + (1-\theta) V_R = 3\,\text{V}-4\,\text{V} = -1\,\text{V} \\
V_{\text{HYH}} = \theta V_{\text{SATH}} + (1-\theta) V_R = 3\,\text{V}+4\,\text{V} = 7\,\text{V} \\
V_{\text{HLY}} = \theta V_{\text{SATL}} + (1-\theta) V_R = -3\,\text{V}+4\,\text{V} = 1\,\text{V} \\
V_{\text{HLY}} = \theta V_{\text{SATL}} + (1-\theta) V_R = -3\,\text{V}+4\,\text{V} = 1\,\text{V}
\]
Quiz 16

Solution:

Obtain an expression for and plot the transfer characteristics of the following circuit. Assume $R_1=2K$, $R_2=8K$, $R=10K$, $V_{DD}=15V$, $V_{SS}=-15V$

$$V_{HYH} = \theta V_{SATH} + (1-\theta) V_R = 3V - 4V = -1V$$  
$$V_{HYL} = \theta V_{SATL} + (1-\theta) V_R = -3V - 4V = -7V$$

$$V_{HYH} = \theta V_{SATH} + (1-\theta) V_R = 3V + 4V = 7V$$  
$$V_{HYL} = \theta V_{SATL} + (1-\theta) V_R = -3V + 4V = 1V$$
Shift can be to left or right depending upon sign of $V_R$.
Inversion of Hysteresis Loop

Noninverting Comparator with Hysteresis

\[ V_{\text{SATL}} \approx V_{SS} \]
\[ V_{\text{SATH}} \approx V_{DD} \]

\[ \theta = \frac{R_1}{R_1 + R_2} \]

\[ V_{HYL} = \frac{\theta}{\theta - 1} V_{\text{SATL}} \]
\[ V_{HYH} = \frac{V_{\text{SATH}}}{\theta} \]

If \( V_{\text{SATH}} = V_{DD} \), \( V_{\text{STAL}} = V_{SS} = -V_{DD} \)

\[ V_{HYH} = \frac{\theta}{1 - \theta} V_{DD} \]
\[ V_{HYL} = \frac{-\theta}{1 - \theta} V_{DD} \]
Shifted Inverted Hysteresis Loop

\[ R_\theta = \frac{R_1}{R_1 + R_2} \]

\[ V_{SATH} \approx V_{DD} \quad V_{SATL} \approx V_{SS} \]

\[ V_{HYH} = \frac{V_R}{1-\theta} + \frac{\theta}{\theta-1} V_{SATL} \]

\[ V_{HYL} = \frac{V_R}{1-\theta} + \frac{\theta}{\theta-1} V_{SATH} \]

\[ V_{CENT} = \frac{V_R}{1-\theta} + \frac{\theta}{\theta-1} \left( \frac{V_{SATH} - V_{SATL}}{2} \right) \]

If \( V_{SATH} = V_{DD} \), \( V_{STAL} = V_{SS} = -V_{DD} \)

\[ V_{HYH} = \frac{V_R}{1-\theta} + \frac{\theta}{1-\theta} V_{DD} \]

\[ V_{HYL} = \frac{V_R}{1-\theta} + \frac{-\theta}{1-\theta} V_{DD} \]
Waveform Generator with Linear Triangle Waveform

Goal: Determine how this circuit operates, the output waveforms, and the frequency of the output
Waveform Generator with Linear Triangle Waveform

Since the comparator will be in one of two states, the current in the resistor will be constant when $V_{OUT2}=V_{SATH}$ and will be constant when $V_{OUT2}=V_{SATL}$

Analysis strategy: Guess state of the $V_{OUT2}$, solve circuit, and show where valid when $V_{OUT2}=V_{SATH}$, $I_R$ will be positive and $V_{OUT1}$ will be decreasing linearly when $V_{OUT2}=V_{SATH}$, $I_R$ will be positive and $V_{OUT1}$ will be increasing linearly
Waveform Generator with Linear Triangle Waveform

\[ V_{\text{SATH}} \approx V_{\text{DD}} \]
\[ V_{\text{SATL}} \approx V_{\text{SS}} \]

Observe \( T = t_3 - t_1 = (t_2 - t_1) + (t_3 - t_2) \)
Waveform Generator with Linear Triangle Waveform

V_{SATL} \approx V_{SS}
V_{SATH} \equiv V_{DD}

\begin{align*}
V_{OUT1} & \quad \text{Inverting Integrator} \\
V_{OUT2} & \quad \text{Noninverting Comparator with Hysteresis}
\end{align*}
Waveform Generator with Linear Triangle Waveform

Guess $V_{OUT2} = V_{SATH}$ will obtain $t_2 - t_1$

$$V_{OUT1} = -\frac{1}{RC} \int_{t_1}^{t} V_{SATH} \, dt + V_{OUT1}(t_1)$$

$$V_{OUT1}(t_1) = V_{HYH}$$

valid for $t_1 < t < t_2$
Waveform Generator with Linear Triangle Waveform

Guess $V_{OUT2} = V_{SATH}$ valid for $t_1 < t < t_2$

$V_{OUT1} = -\frac{1}{RC} \int_{t_1}^{t} V_{SATH} \, dt + V_{OUT1}(t_1) \quad V_{OUT1}(t_1) = V_{HYH}$

at $t=t_2$, $V_{OUT1}$ will become $V_{SATL}$

Substituting into integral expression for $V_{OUT1}$ we obtain

$V_{HYL} = -\frac{1}{RC} \int_{t_1}^{t} V_{SATH} \, dt + V_{HYH}$
Waveform Generator with Linear Triangle Waveform

Guess \( V_{OUT2} = V_{SATH} \) valid for \( t_1 < t < t_2 \)

\[
V_{HYL} = \frac{1}{RC} \int_{t_1}^{t} V_{SATH} d\tau + V_{HYH}
\]

\[
V_{HYL} = \frac{1}{RC} V_{SATH} \int_{t_1}^{t_2} 1 d\tau + V_{HYH}
\]

\[
V_{HYL} = \frac{1}{RC} V_{SATH} \left( \tau \bigg|_{t_1}^{t_2} \right) + V_{HYH}
\]

\[
V_{HYL} = \frac{1}{RC} V_{SATH} \left( t_2 - t_1 \right) + V_{HYH}
\]

\( V_{SATH} \approx V_{DD} \)

\( V_{SATL} \approx V_{SS} \)
Waveform Generator with Linear Triangle Waveform

Guess $V_{OUT2} = V_{SATH}$

valid for $t_1 < t < t_2$

$$V_{HYL} = -\frac{1}{RC} V_{SATH}(t_2 - t_1) + V_{HYH}$$

$$t_2 - t_1 = RC \frac{(V_{HYH} - V_{HYL})}{V_{SATH}}$$

$V_{SATH} \approx V_{DD}$

$V_{SATL} \approx V_{SS}$
Waveform Generator with Linear Triangle Waveform

Guess \( V_{\text{OUT2}} = V_{\text{SATL}} \) will obtain \( t_3 - t_2 \) valid for \( t_2 < t < t_3 \)

Following the same approach observe

\[
V_{\text{OUT1}} = -\frac{1}{RC} \int_{t_2}^{t} V_{\text{SATL}} \, d\tau + V_{\text{OUT1}}(t_2)
\]

\[
V_{\text{OUT1}}(t_2) = V_{\text{HYL}}
\]

It thus follows that

\[
V_{\text{HYH}} = -\frac{1}{RC} V_{\text{SATL}}(t_3 - t_2) + V_{\text{HYL}} \quad t_3 - t_2 = RC \left( \frac{V_{\text{HYL}} - V_{\text{HYH}}}{V_{\text{SATL}}} \right)
\]

\( V_{\text{SATH}} \approx V_{\text{DD}} \)

\( V_{\text{SATL}} \approx V_{\text{SS}} \)
Waveform Generator with Linear Triangle Waveform

$$\begin{align*}
T &= (t_2-t_1) + (t_3-t_2) \\
t_2-t_1 &= RC \left( \frac{V_{\text{HYH}} - V_{\text{HYL}}}{V_{\text{SATL}}} \right) \\
t_3-t_2 &= RC \left( \frac{V_{\text{HYL}} - V_{\text{HYH}}}{V_{\text{SATL}}} \right) \\
T &= RC \left( V_{\text{HYH}} - V_{\text{HYL}} \right) \left( \frac{1}{V_{\text{SATL}}} - \frac{1}{V_{\text{SATL}}} \right) \\
f &= \frac{1}{t} = \frac{1}{RC \left( V_{\text{HYH}} - V_{\text{HYL}} \right) \left( V_{\text{SATL}} - V_{\text{SATL}} \right)}
\end{align*}$$

$$V_{\text{SATL}} \cong V_{\text{SS}}$$

$$V_{\text{SATH}} \cong V_{\text{DD}}$$
Waveform Generator with Linear Triangle Waveform

If we use the noninverting comparator with hysteresis circuit developed previously and if

\[ V_{SATL} = V_{DD}, \quad V_{STAL} = V_{SS} = -V_{DD} \]

\[ \theta = \frac{R_1}{R_1 + R_2} \]

then

\[ V_{HYH} = \frac{\theta}{1-\theta} V_{DD} \]

\[ V_{HYL} = \frac{-\theta}{1-\theta} V_{DD} \]

\[ f = \frac{1}{2RC} \frac{1}{1-\theta} \]
Stability and Waveform Generation

• Waveform generators provide an output with no excitation

• Waveform circuits are circuits that, when operated in quiescent linear condition, have one or more poles in the right half-plane

• Will now investigate the pole locations of waveform generators
  – Conditions for oscillation
  – Triangle/Square/Sinusoidal Oscillations
Poles of a Network

\[ T(s) = \frac{X_{\text{OUT}}(s)}{X_{\text{IN}}(s)} \]

\[ T(s) = \frac{N(s)}{D(s)} \]

where \( N(s) \) and \( D(s) \) are polynomials in \( s \)

- \( D(s) \) is termed the characteristic equation or the characteristic polynomial of the network
- Roots of \( D(s) \) are the poles of the network
Theorem: The poles of any transfer function of a linear system are independent of where the excitation is applied and where the response is taken provided the dead networks are the same.

Equivalently, the characteristic equation, \( D(s) \), is characteristic of a network (or the corresponding dead network) and is independent of where the excitation is applied and where the response is taken.
Poles of a Network

\[ T(s) = \frac{N(s)}{D(s)} \]
Poles of Networks – some examples

\[ T(s) = \frac{V_{out}}{V_{in}} = \frac{1}{1+RC_s} \]

\[ D(s) = 1 + RC_s \]
Poles of Networks – some examples

\[ T(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{\text{RCs}}{1+\text{RCs}} \]

\[ D(s) = 1 + \text{RCs} \]
Poles of Networks – some examples

\[ V_{OUT} = \frac{V_{OUT}}{I_{IN}} = \frac{R}{1 + RCs} \]

\[ T(s) = \frac{V_{OUT}}{I_{IN}} = \frac{R}{1 + RCs} \]

\[ D(s) = 1 + RCs \]
Poles of Networks – some examples

\[ D(s) = \frac{1}{1 + R \cdot C_s} \]

\[ T(s) = \frac{I_{out}}{I_{in}} = \frac{1}{1 + R \cdot C_s} \]

Dead Network
Poles of Networks – some examples

\[ T(s) = \frac{V_{\text{OUT}}}{V_{\text{IN}}} = \frac{1}{2} \frac{1}{1 + R C s} \]

\[ D(s) = 1 + R C s \]

Dead Network
Poles of Networks – some examples

\[ T(s) = \frac{V_{\text{OUT}}}{I_{\text{IN}}} = \frac{2R}{1+2RCs} \]

\[ D(s) = 1 + 2RCs \]

Note dead network has changed as has \( D(s) \) and thus the pole.