EE 230
Lecture 23
Nonlinear Op Amp Applications
– Waveform Generators
Quiz 17

An oscillator based upon a comparator with hysteresis is shown. If \( V_{STAH} = 12\text{V} \) and \( V_{SATL} = -12\text{V} \), determine the peak value of

![Comparator with Hysteresis Diagram]
And the number is ?

1  3  8

5  3  4

2  ?  6

9  2  7
Quiz 17

An oscillator based upon a comparator with hysteresis is shown. If $V_{\text{STAH}}=12\text{V}$ and $V_{\text{SATL}}=-12\text{V}$, determine the peak value of $V_{\text{OUT1}}$.

Solution: The peak value of the $V_{\text{OUT1}}$ waveform is determined by the boundaries of the Hysteresis window.

$$V_{\text{OUT1MAX}} = V_{\text{SATH}} \frac{R_1}{R_1 + R_2} = 12\text{V} \cdot \frac{2\text{K}}{12\text{K}} = 2\text{V}$$
Modifications of Comparator with Hysteresis

\[ \theta = \frac{R_1}{R_1 + R_2} \]

Note this is the basic inverting amplifier with op amp terminals interchanged

Many other ways to control position and size of hysteresis window
Comparison of basic noninverting amplifier structures

- Serves as an amplifier **directly**
- Stable
- No hysteresis loop

- Not useful as an amplifier **directly**
- Unstable
- Serves as comparator with hysteresis

If ideal op amps both have gain

\[ A_{FB} = 1 + \frac{R_2}{R_1} \]
Review from Last Lecture

Waveform Generator

\[ \theta = \frac{R_1}{R_1 + R_2} \]

this process repeats itself

the rise time and the fall times are identical

the period of the nearly triangular waveform is thus \( 2t_1 \)

\[ T = 2t_1 = -2RC \ln \left( \frac{V_{SATL}(\theta-1)}{\theta V_{SATH} - V_{SATL}} \right) \]

If \( V_{SATL} = -V_{SATH} \), this simplifies to

\[ f = \frac{1}{T} = \frac{1}{2RC} \ln \left( \frac{\theta V_{SATH} - V_{SATL}}{V_{SATL}(\theta-1)} \right) \]

\[ f = \frac{1}{2RC} \ln \left( \frac{1}{1+\theta} \right) \]
Review from Last Lecture

\[ \theta = \frac{R_1}{R_1 + R_2} \]

for \( V_{\text{SATL}} = -V_{\text{SATH}} \)

\[ f = \frac{1}{2RC} \ln \left( \frac{1 + \theta}{1 - \theta} \right) \]

Square and distorted triangular output waveforms
Slope of square wave is determined by SR of Op Amp
Waveform Generator with Linear Triangle Waveform

Goal: Determine how this circuit operates, the output waveforms, and the frequency of the output
Waveform Generator with Linear Triangle Waveform

Let's first check stability

Since stability is determined by the poles of a linear network, must first assume devices are operating linearly
Waveform Generator with Linear Triangle Waveform

**Let's first check stability**

Since stability is determined by the poles of a linear network, must first assume devices are operating linearly.

What is the linear model of this comparator?

Linear region is area where slope is negative

Recall, in this region,

\[ V_{\text{OUT}} = -K_0 V_{\text{IN}} \]
Waveform Generator with Linear Triangle Waveform

Let's first check stability

Since stability is determined by the poles of a linear network, must first assume devices are operating linearly

(Recall do not need to provide excitation to find poles but details will be discussed later)
Waveform Generator with Linear Triangle Waveform

Let's first check stability

\[ V_{\text{IN}}(sC+G) = V_1sC + V_{\text{OUT}}G \]
\[ V_{\text{OUT}} = -K_0V_1 \]

\[
T(s) = \frac{V_{\text{OUT}}}{V_{\text{IN}}} = \frac{-K_0\left(s + \frac{1}{RC}\right)}{s - \frac{K_0}{RC}}
\]

Single pole at \( s = \frac{K_0}{RC} \)

\[ \therefore \text{The system is unstable!} \]
Waveform Generator with Linear Triangle Waveform

Since the comparator will be in one of two states, the current in the resistor will be constant when $V_{OUT2} = V_{SATH}$ and will be constant when $V_{OUT2} = V_{SATL}$.

Analysis strategy: Guess state of the $V_{OUT2}$, solve circuit, and show where valid when $V_{OUT2} = V_{SATH}$, $I_R$ will be positive and $V_{OUT1}$ will be decreasing linearly when $V_{OUT2} = V_{SATL}$, $I_R$ will be positive and $V_{OUT1}$ will be increasing linearly.
Waveform Generator with Linear Triangle Waveform

Observe $T = t_3 - t_1 = (t_2 - t_1) + (t_3 - t_2)$

$V_{SATH} \approx V_{DD}$
$V_{SATL} \approx V_{SS}$
Waveform Generator with Linear Triangle Waveform

\[
V_{SATL} \approx V_{SS} \\
V_{SATH} \approx V_{DD}
\]

![Waveform Diagram](image)
Waveform Generator with Linear Triangle Waveform

Guess $V_{\text{OUT2}} = V_{\text{SATH}}$ will obtain $t_2 - t_1$

$$V_{\text{OUT1}} = -\frac{1}{RC} \int_{t_1}^{t} V_{\text{SATH}} \, dt + V_{\text{OUT1}}(t_1)$$

$$V_{\text{OUT1}}(t_1) = V_{\text{HYH}}$$

valid for $t_1 < t < t_2$
Waveform Generator with Linear Triangle Waveform

Guess $V_{OUT2} = V_{SATH}$ valid for $t_1 < t < t_2$

$$V_{OUT1} = -\frac{1}{RC} \int_{t_1}^{t} V_{SATH} \, dt + V_{OUT1}(t_1) \quad V_{OUT1}(t_1) = V_{HYH}$$

at $t=t_2$, $V_{OUT1}$ will become $V_{SATL}$

Substituting into integral expression for $V_{OUT1}$ we obtain

$$V_{HYL} = -\frac{1}{RC} \int_{t_1}^{t_2} V_{SATH} \, dt + V_{HYH}$$
Waveform Generator with Linear Triangle Waveform

Guess $V_{OUT2} = V_{SATH}$ valid for $t_1 < t < t_2$

\[
V_{HYL} = -\frac{1}{RC} \int_{t_1}^{t_2} V_{SATH} \, d\tau + V_{HYH}
\]

\[
V_{HYL} = -\frac{1}{RC} V_{SATH} \left[ t \bigg|_{t_1}^{t_2} \right] + V_{HYH}
\]

\[
V_{HYL} = -\frac{1}{RC} V_{SATH} \left( t_2 - t_1 \right) + V_{HYH}
\]

$V_{SATH} \approx V_{DD}$

$V_{SATL} \approx V_{SS}$
Waveform Generator with Linear Triangle Waveform

Guess $V_{OUT2}=V_{SATH}$ valid for $t_1 < t < t_2$

\[ V_{HYL} = -\frac{1}{RC} V_{SATH} (t_2 - t_1) + V_{HYH} \]

\[ t_2 - t_1 = RC \frac{(V_{HYH} - V_{HYL})}{V_{SATH}} \]
Waveform Generator with Linear Triangle Waveform

Guess $V_{OUT2} = V_{SATL}$ will obtain $t_3 - t_2$

Following the same approach observe (valid for $t_2 < t < t_3$)

$$V_{OUT1} = -\frac{1}{RC} \int_{t_2}^{t} V_{SATL} \, dt + V_{OUT1}(t_2)$$

$$V_{OUT1}(t_2) = V_{HYL}$$

It thus follows that

$$V_{HYH} = -\frac{1}{RC} V_{SATL}(t_3 - t_2) + V_{HYL} \quad t_3 - t_2 = RC \left( \frac{V_{HYL} - V_{HYH}}{V_{SATL}} \right)$$
Waveform Generator with Linear Triangle Waveform

\[ T = (t_2 - t_1) + (t_3 - t_2) \]

\[ t_2 - t_1 = RC \left( \frac{V_{HYH} - V_{HYL}}{V_{SATH}} \right) \]

\[ t_3 - t_2 = RC \left( \frac{V_{HYL} - V_{HYH}}{V_{SATL}} \right) \]

\[ T = RC \left( V_{HYH} - V_{HYL} \right) \left( \frac{1}{V_{SATH}} - \frac{1}{V_{SATL}} \right) \]

\[ f = \frac{1}{t} = \frac{1}{RC \left( V_{HYH} - V_{HYL} \right) \left( V_{SATL} - V_{SATH} \right)} \]

\[ V_{SATH} \approx V_{DD} \]

\[ V_{SATL} \approx V_{SS} \]
Waveform Generator with Linear Triangle Waveform

If we use the noninverting comparator with hysteresis circuit developed previously and if

If $V_{SATH} = V_{DD}$, $V_{STAL} = V_{SS} = -V_{DD}$,

then

$$V_{HYH} = \frac{\theta}{1-\theta} V_{DD} \quad V_{HYL} = -\frac{\theta}{1-\theta} V_{DD}$$

$$f = \frac{1}{2RC} \frac{1}{\theta} \frac{V_{SATH} V_{SATL}}{(V_{HYH} - V_{HYL})(V_{STAL} - V_{SATH})}$$
Example:

Obtain an expression for and plot the transfer characteristics of the following circuit. Assume $R_1=2\,\text{K}$, $R_2=8\,\text{K}$, $R=10\,\text{K}$, $V_{\text{DD}}=15\,\text{V}$, $V_{\text{SS}}=-15\,\text{V}$
Example:

Solution:

Obtain an expression for and plot the transfer characteristics of the following circuit. Assume $R_1=2\,\text{K}, \ R_2=8\,\text{K}, \ R=10\,\text{K}, \ V_{DD}=15\,\text{V}, \ V_{SS}=-15\,\text{V}$

\[
V_{HYH} = \theta V_{SATH} + (1-\theta)V_R
\]

\[
V_{HYL} = \theta V_{SATL} + (1-\theta)V_R
\]

\[
\theta = \frac{R_1}{R_1+R_2}
\]

\[
V_{OUT} = \frac{V_1+V_2}{2}
\]
Example:

Solution:

Obtain an expression for and plot the transfer characteristics of the following circuit. Assume $R_1=2K$, $R_2=8K$, $R=10K$, $V_{DD}=15V$, $V_{SS}=-15V$

\[ V_{OUT} = \frac{V_1 + V_2}{2} \]

\[ \theta = \frac{R_1}{R_1 + R_2} = 0.2 \]

Upper Circuit

\[ V_{HH} = \theta V_{SATH} + (1-\theta) V_R = 3V + 4V = 7V \]
\[ V_{HL} = \theta V_{SATL} + (1-\theta) V_R = -3V + 4V = 1V \]

Lower Circuit

\[ V_{HH} = \theta V_{SATH} + (1-\theta) V_R = 3V - 4V = -1V \]
\[ V_{HL} = \theta V_{SATL} + (1-\theta) V_R = -3V - 4V = -7V \]
Example:

Solution:

\[ V_{\text{HYH}} = \theta V_{\text{SATH}} + (1-\theta) V_R = 3V + 4V = 7V \]
\[ V_{\text{HYL}} = \theta V_{\text{SATL}} + (1-\theta) V_R = -3V + 4V = 1V \]

\[ V_{\text{HYH}} = \theta V_{\text{SATH}} + (1-\theta) V_R = 3V - 4V = -1V \]
\[ V_{\text{HYL}} = \theta V_{\text{SATL}} + (1-\theta) V_R = -3V - 4V = -7V \]
Example:

Solution:

Assuming $V_{\text{SATL}} = -V_{\text{SATH}}$

$$V_{\text{OUT}} = \frac{V_1 + V_2}{2}$$
Poles of a Network

\[ T(s) = \frac{X_{\text{OUT}}(s)}{X_{\text{IN}}(s)} \]

\[ T(s) = \frac{N(s)}{D(s)} \]

T(s) can be expressed as

where N(s) and D(s) are polynomials in s

- D(s) is termed the characteristic equation or the characteristic polynomial of the network
- Roots of D(s) are the poles of the network
Poles are inherent and unique characteristics of any linear network.

Theorem: The poles of any transfer function of a linear system are independent of where the excitation is applied and where the response is taken provided the dead networks are the same.

Equivalently, the characteristic equation, $D(s)$, is characteristic of a network (or the corresponding dead network) and is independent of where the excitation is applied and where the response is taken.

Poles are inherent and unique characteristics of any linear network.
Poles of a Network

\[ T(s) = \frac{N(s)}{D(s)} \]
Poles of Networks – some examples

\[ T(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{1+RCs} \]

\[ D(s) = 1 + RCs \]
Poles of Networks – some examples

\[ T(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{RCs}{1+RCs} \]

\[ D(s) = 1 + RCs \]
Poles of Networks – some examples

$$T(s) = \frac{V_{out}}{I_{in}} = \frac{R}{1 + RC_s}$$

$$D(s) = 1 + RC_s$$
Poles of Networks – some examples

\[ T(s) = \frac{I_{\text{OUT}}}{I_{\text{IN}}} = \frac{1}{1 + RCs} \]

\[ D(s) = 1 + RCs \]
Poles of Networks – some examples

\[ T(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{2} \frac{2}{1+RCs} \]

\[ D(s) = 1 + RCs \]
Poles of Networks – some examples

\[ T(s) = \frac{V_{\text{OUT}}}{I_{\text{IN}}} = \frac{2R}{1 + 2RCs} \]

\[ D(s) = 1 + 2RCs \]

Note dead network has changed as has \( D(s) \) and thus the pole
Strategies for determining poles of networks with no excitations:

1) Apply any excitation that does not alter the dead network
   - Obtain transfer function \( T(s) = \frac{N(s)}{D(s)} \)
   - Poles are roots of \( D(s) \)
   - By theorem, they are unique (independent of where excitation is applied or response is taken)

2) Develop new strategy that does not require assigning excitation and reduces calculation requirements
Consider the dead network

\[ V_x (SC + \frac{1}{R}) = 0 \]

\[ V_x (RCS + 1) = 0 \]

Observe that even though no excitation was applied, the last equation is of the form

\[ V_x D(s) = 0 \]
This method for obtaining \( D(s) \) is not just a coincidence applicable to this example but rather can be applied to an arbitrary linear network as stated in the following Theorem.

**Theorem:** The characteristic polynomial \( D(s) \) of a system can be obtained by assigning an output variable to the "dead network" and using circuit analysis techniques to obtain an expression that involves only the output variable expressed in the form \( X_0 F(s) = 0 \). When expressed in this form, \( F(s) \) when written in polynomial form is the characteristic polynomial of the system, i.e. \( D(s) = F(s) \).
Example. Determine the characteristic polynomial for the following circuit

\[ R_2 \quad R_1 \quad C_2 \quad C_1 \]

Solution: Assign \( V_0 \) to one of the nodes

\[ V_0 \quad V_1 \]

\( R_2 \quad C_2 \quad R_1 \quad C_1 \)

By KCL

\[
\begin{align*}
V_0 \left( G_2 + G_1 + SC_2 \right) &= V_1 G_1 \\
V_1 \left( G_1 + SC_1 \right) &= V_0 G_1
\end{align*}
\]
Eliminating $V_1$ between these two equations we obtain

$$V_0 \left( G_1 + G_2 + SC_2 \right) = G_1 \frac{V_0 \cdot G_1}{G_1 + SC_1}$$

or

$$V_0 \left( G_1^2 + G_1 G_2 + SC_2 G_1 + SC_1 G_2 + SC_1 G_1 + S^2 C_1 C_2 - G_1^2 \right) = 0$$

or

$$V_0 \left( S^2 C_1 C_2 + S \left( C_1 \left[ G_1 + G_2 \right] + C_2 G_1 \right) + G_1 G_2 \right) = 0$$

or

$$V_0 \left( S^2 + S \left( \frac{G_1 + G_2}{C_2} + \frac{G_1}{C_1} \right) + \frac{G_1 G_2}{C_1 C_2} \right) = 0$$

The characteristic polynomial for this circuit is

$$D(s) = S^2 + S \left( \frac{G_1 + G_2}{C_2} + \frac{G_1}{C_1} \right) + \frac{G_1 G_2}{C_1 C_2}$$
Consider again the waveform generator. A resistor \( R_x \) has been added but if \( R_x = \infty \) this becomes the waveform generator considered earlier.

\[
\Theta = \frac{R_1}{R_1 + R_2}
\]

Obtain the poles of this circuit. Assume OA is ideal, solving, we obtain:

\[
\begin{align*}
U_1 &= \Theta U_0 \\
U_1 &= \left( \frac{1}{R} + \frac{1}{R_x} + sC \right) = \frac{U_0}{R} \\
V_0 (S + \left[ \frac{1}{Rc} \frac{\Theta - 1}{\Theta} + \frac{1}{R_x C} \right])
\end{align*}
\]
\[ D(s) = s + \frac{1}{RC} \left( \frac{\Theta - 1}{\Theta} \right) + \frac{1}{R_x C} \]

If \( R_x = \infty \), this circuit has a single pole at

\[ P = \frac{1}{RC} \left( \frac{1-\Theta}{\Theta} \right) \]

Since \( 0 < \Theta < 1 \), this circuit has a single pole on the positive real axis and is unstable.
Determine the minimum value of $R_x$ that will maintain instability in the previous circuit.

$$D(s) = s + \frac{1}{RC} \left( \frac{\Theta - 1}{\Theta} \right) + \frac{1}{R_x C}$$

$\therefore$ pole at

$$P = \frac{1-\Theta}{\Theta} \left( \frac{1}{RC} \right) - \frac{1}{R_x C}$$

Setting this equal to 0 (when it leaves RHP) obtain

$$\frac{1}{C R_{x \text{min}}} = \frac{1-\Theta}{\Theta} \left( \frac{1}{RC} \right)$$

or

$$R_{x \text{min}} = R \left( \frac{\Theta}{1-\Theta} \right)$$
Where are the poles of the following waveform generator?

\[
\theta = \frac{R_1}{R_1+R_2}
\]

Inverting Integrator

Noninverting Comparator with Hysteresis

Linear region of operation (slope = \[-\frac{R_2}{R_1}\])

Transfer Characteristics of Comparator
Must use linear mode of operation to find poles of the circuit

\[ V_0 = -\frac{R_2}{R_1} V_1 \]

\[ V_1 = -\frac{1}{RC} V_0 \]

\[ V_0 (s - \frac{R_2}{R_1} \frac{1}{RC}) = 0 \]

\[ \therefore \text{single pole at} \quad p = \left( \frac{R_2}{R_1} \right) \frac{1}{RC} \]

Note pole in RHP on positive real axis
Stability and Waveform Generation

- Waveform generators provide an output with no excitation.

- Waveform circuits are circuits that, when operated in quiescent linear condition, have one or more poles in the right half-plane.

- Will now investigate the pole locations of waveform generators:
  - Conditions for oscillation
  - Triangle/Square/Sinusoidal Oscillations
End of Lecture 23