EE 230
Lecture 28

Wien-Bridge Oscillator Stabilization
Nonlinear Amplifiers
Characteristic Equation Oscillation Criteria (CEOC)

If the characteristic equation $D(s)$ has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node.

Barkhausen Oscillation Criteria

A feedback amplifier will have sustained oscillation if $A\beta = -1$.

Differences:

1. Barkhausen requires a specific feedback amplifier architecture.
2. Sustained oscillation says nothing about wave shape.

Challenge:

It is impossible to place the poles of any network exactly on the imaginary axis.

Sinusoidal Oscillator Design Approach:

Place on pair of cc poles slightly in RHP and have no other RHP poles.

With this approach, will observe minor distortion of output waveforms.
Review from Last Time:

**Sinusoidal Oscillator Design Strategy**

Build networks with exactly one pair of complex conjugate roots slightly in the RHP and use nonlinearities in the amplifier part of the network to limit the amplitude of the output (i.e., $p = \alpha \pm j\beta$ $\alpha$ is very small but positive)

Nonlinearity will cause a small amount of distortion

Frequency of oscillation will be very close to but deviate slightly from $\beta$

Must be far enough in the RHP so the process and temperature variations do not cause movement back into LHP because if that happened, oscillation would cease!
Review from Last Time:

Know Barkhausen Criteria to satisfy interviewers questions but use CEOC to design sinusoidal oscillators
Amplitude Limiting in Noninverting Amplifier Structure

Observe:

- Amplifier gain changes from $3 + \varepsilon$ for $V_1 < V_{IN} < V_2$ to 0 for $V_{IN} < V_1$ or $V_{IN} > V_2$
- $V_{SATH}$ and $V_{SATL}$ strongly dependent upon op amp bias voltages $V_{DD}$ and $V_{SS}$
- This nonlinearity in the real amplifier will limit the output signal amplitude
- Can cause rather significant distortion
Can we do this?

Obtain slope >3 for \(-V_1 < V_{\text{IN}} < V_1\) and slope <3 for \(V_1 < V_{\text{IN}} < V_2\) and for \(-V_2 < V_{\text{IN}} < -V_1\)

Limit \(V_{\text{IN}}\) to interval \(-V_2 < V_{\text{IN}} < V_2\)

- If possible, hard nonlinearity associated with amplifier saturation will not be excited
- Dramatic reduction in distortion is anticipated
Consider:

NLD Transfer Characteristics

\[ V_D = V_{xx} \quad \text{for} \quad I_D > 0 \]

\[ I_D = 0 \quad \text{for} \quad V_D < V_{xx} \]

(will assume \( V_{xx} > 0 \))

**Analysis of Amplifier Circuit:**

**Case 1:** \( I_{D1} = 0 \) and \( I_{D2} = 0 \)

**Solution:**

\[ V_{OUT} = \left[ 1 + \left( \frac{R_2 + R_3}{R_1} \right) \right] V_{IN} \]

Must determine where this part of the solution is valid
Valid for $V_{D1} < V_{XX}$ and $V_{D2} < V_{XX}$
but
thus, valid for $V_{D1} = -V_{R3}$ and $V_{D2} = V_{R3}$
$V_{R3} > -V_{XX}$ and $V_{R3} < V_{XX}$

but
$V_{R3} = \frac{R_3}{R_2 + R_3} (V_o - V_{in})$

$V_{R3} = \frac{R_3}{R_2 + R_3} \left[ 1 + \frac{R_2 + R_3}{R_1} - 1 \right] V_{in}$

$V_{R3} = \frac{R_3}{R_2 + R_3} \left( \frac{R_2 + R_3}{R_1} \right) V_{in}$

$V_{R3} = \frac{R_3}{R_1} V_{in}$

∴ valid for

$\frac{R_3}{R_1} V_{in} < V_{XX}$ and $\frac{R_3}{R_1} V_{in} > -V_{XX}$

$-\frac{R_1}{R_3} V_{XX} < V_{in} < \frac{R_1}{R_3} V_{XX}$
Graph of solution for Case 1

\[ V_{OUT} = 1 + \frac{(R_2 + R_3)}{R_1} \]

\[ -\frac{R_1}{R_3} V_{xx} \]
Case 2: \( NLD_2 \) is in the conducting state \( (V_{D2} = V_{XX}) \)

\( NLD_1 \) is in the nonconducting state \((I_{D1} = 0)\)
Solution for Case 2 Continued

Applying superposition we obtain

\[ V_{\text{OUT}} = V_{\text{IN}} \left( 1 + \frac{R_2}{R_1} \right) + V_{\text{xx}} \]

This solution is valid for \( V_{\text{D1}} < 0 \) and \( I_{\text{D2}} > 0 \)

But: \( V_{\text{D1}} = -V_{\text{xx}} \) and \( I_{\text{D2}} = \frac{V_{\text{OUT}} - V_{\text{IN}} - V_{\text{xx}}}{R_2} - \frac{V_{\text{xx}}}{R_3} \)

Substituting the validity conditions, we obtain

\(-V_{\text{xx}} < 0\) and \( \frac{V_{\text{OUT}} - V_{\text{IN}} - V_{\text{xx}}}{R_2} - \frac{V_{\text{xx}}}{R_3} > 0 \)

The first of these inequalities is valid provided \( V_{\text{xx}} > 0 \) and substituting the expression for \( V_{\text{OUT}} \) into the second, we obtain after simplification

\( V_{\text{IN}} > \frac{R_1}{R_3} V_{\text{xx}} \)
Solution for Case 2 Continued

\[ V_{xx} > 0 \quad V_{IN} > \frac{R_1}{R_3} V_{xx} \]

Assuming \( V_{xx} > 0 \), the region where Case 2 is valid is thus determined by the second inequality.
Case 3: NLD₂ is nonconducting \((I_D2 = 0)\)
NLD₁ is conducting \((V_D1 = V_{XX})\)

\[
V_{OUT} = \left(1 + \frac{R_2}{R_1}\right)V_{IN} - V_{XX}
\]
Solution for Case 3 continued:

This solution is valid for $V_{D2} < 0$ and $I_{D1} > 0$

But: $V_{D2} = -V_{XX}$ and

$$I_{D1} = \frac{V_{IN} - V_{OUT} - V_{XX}}{R_2} - \frac{V_{XX}}{R_3}$$

Substituting the validity conditions, we obtain

$$-V_{XX} < 0 \quad \text{and} \quad \frac{V_{IN} - V_{OUT} - V_{XX}}{R_2} - \frac{V_{XX}}{R_3} > 0$$

The first of these inequalities is valid provided $V_{XX} > 0$ and substituting the expression for $V_{OUT}$ into the second, we obtain after simplification

$$V_{IN} < -\frac{R_1}{R_3} V_{XX}$$

![Diagram](image)
Case 4: NLD₁ and NLD₂ both conducting (this case never happens and need not be considered since we already have a solution for all inputs)

Thus, if we neglect the saturation of the op amp, we can write an expression for the output as

\[
V_{\text{OUT}} = \begin{cases} 
1 + \frac{R_2}{R_1} & V_{\text{IN}} + V_{xx} \\
1 + \frac{R_2 + R_3}{R_1} & V_{\text{IN}} \\
1 + \frac{R_2}{R_1} & V_{\text{IN}} - V_{xx}
\end{cases}
\]

\[
V_{\text{IN}} > \frac{R_1}{R_3} V_{xx} \quad "2" \\
-\frac{R_1}{R_3} V_{xx} < V_{\text{IN}} < \frac{R_1}{R_3} V_{xx} \quad "1" \\
V_{\text{IN}} < -\frac{R_1}{R_3} V_{xx} \quad "3"
\]

This is shown graphically, along with the saturation of the op amp, on the following slide.
Overall Transfer Characteristics

\[
\begin{align*}
V_{\text{OUT}} &= -V_2 \frac{R_3}{R_x} V_{sx} \\
V_{\text{SATL}} &= 1 + \frac{R_2}{R_1} \\
V_{\text{SATL}} &= 1 + \frac{R_2}{R_1} \\
V_{\text{SATL}} &= 1 + \left(1 + \frac{R_2}{R_1} \right) \\
V_{\text{SATL}} &= 1 + \left(1 + \frac{(R_2 + R_1)}{R_1} \right)
\end{align*}
\]
Overall Transfer Characteristics

$$V_D = V_{XX} \text{ for } I_D > 0$$
$$I_D = 0 \text{ for } V_D < V_{XX}$$

If $V_{XX} = 0.6\text{V}$, this represents a good approximation to the transfer characteristics of a silicon diode. We thus can replace the NLD with a diode and obtain the amplitude stabilized Wien-Bridge oscillator.
Wein – Bridge Oscillator with Amplitude Stabilization

\[ R_2 < 2R_1 \]
\[ R_2 + R_3 > 2R_1 \]
\[ \omega_{osc} \approx \frac{1}{RC} \]
Wein – Bridge Oscillator
– an alternative view of same circuit using feedback concepts

\[ \beta(s) = \frac{R}{1+RCs} + \frac{1+RCs}{Cs} \]

\[ A_{FB} = 1 + \frac{R_2 + R_3}{R_1} \]

\[ D(s) = s^2 + s \left( \frac{3-A_{FB}}{RC} \right) + \frac{1}{(RC)^2} \]

\[ A_{FB} = 3 + \epsilon \]

\[ \omega_{OSC} \approx \frac{1}{RC} \]

\[ R_2 < 2R_1 \]

\[ R_2 + R_3 > 2R_1 \]