EE 230
Lecture 34

Small Signal Models
Small Signal Analysis
MOS Transistor Applications
(Digital Circuits)

- Termed CMOS Logic
- Widely used in industry today (millions of transistors in many ICs using this logic)
- Almost never used as discrete devices
Review from Last Time:

Bipolar Transistor

npn

pnp

B

C

E

B

C

E

p-type silicon

n-type silicon
Bipolar and MOS Region Comparisons

Review from Last Time:

MOSFET

- **Cutoff**
- **Saturation**
- **Triode**

BJT

- **Cutoff**
- **Forward Active**
- **Saturation**
Bipolar Transistor

Multi-Region Model

\[ I_C = \beta I_B \left(1 + \frac{V_{CE}}{V_{AF}}\right) \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ V_t = \frac{kT}{q} \]

Review from Last Time:

\[ V_{BE} > 0.4V \]
\[ V_{BC} < 0 \]

Forward Active

\[ V_{BE} = 0.7V \]
\[ V_{CE} = 0.2V \]

Saturation

\[ I_C < \beta I_B \]

Cutoff

\[ I_C = I_B = 0 \]
\[ V_{BE} < 0 \]
\[ V_{BC} < 0 \]
Small-Signal Principle

Relationship is nearly linear in a small enough region around Q-point
Region of linearity is often quite large
Linear relationship may be different for different Q-points
Mathematically, small signal model is simply Taylor’s series expansion at the Q-point truncated after first-order terms.
Small-Signal Principle

\[ y_{ss} = \frac{\partial f}{\partial x} \bigg|_{x=x_{ss}} \]

\[ i_{ss} = \frac{\partial I}{\partial V} \bigg|_{V=V_{ss}} \]
Small-Signal Principle

\[ i_{SS} = \frac{\partial I}{\partial V} \bigg|_{V=V_Q} \]

\[ V_{SS} = V \]

\[ y = \frac{\partial I}{\partial V} \bigg|_{V=V_Q} \]

\[ i = y \cdot V \]
The small-signal model of this 2-terminal electrical network is a resistor of value $1/y$. One small-signal parameter characterizes this one-port but it is dependent on Q-point.
Small-Signal Principle

Goal with small signal model is to predict performance of circuit or device in the vicinity of an operating point.

Operating point is often termed Q-point.

Will be extended to functions of two and three variables.
Small-signal Operation of Nonlinear Circuits

- Small-signal principles

Example Circuit

- Small-Signal Models

- Small-Signal Analysis of Nonlinear Circuits
Small signal analysis example

By selecting appropriate value of $V_{SS}$, $M_1$ will operate in the saturation region.

Assume $M_1$ operating in saturation region.

\[ V_{IN} = V_M \sin \omega t \]

\[ V_M \text{ is small} \]

\[ V_{IN} = V_M \sin \omega t \]

\[ V_{OUT} = V_{DD} - I_D R \]

\[ I_D = \frac{\mu C_{ox} W}{2L} (V_{IN} - V_{SS} - V_T)^2 \]

\[ I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{SS} + V_T)^2 \]

\[ V_{OUT} = V_{DD} - \frac{\mu C_{ox} W}{2L} (V_{IN} - V_{SS} - V_T)^2 R \]

\[ V_{OUT} = V_{DD} - \frac{\mu C_{ox} W}{2L} (V_M \sin \omega t - [V_{SS} + V_T]^2 R \]
Small signal analysis example

\[ V_{IN} = V_M \sin \omega t \]

\( V_M \) is small

\[ V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L} (V_M \sin \omega t - [V_{SS} + V_T])^2 R \]

Recall that if \( x \) is small, \((1+x)^2 \approx 1 + 2x\)

\[ V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L} [V_{SS} + V_T]^2 \left( 1 - \frac{V_M \sin \omega t}{[V_{SS} + V_T]} \right)^2 R \]

\[ V_{OUT} = \left\{ V_{DD} - \frac{\mu C_{OX} W}{2L} [V_{SS} + V_T]^2 R \right\} + \frac{\mu C_{OX} W}{2L} [V_{SS} + V_T]^2 \left( \frac{2V_M \sin \omega t}{[V_{SS} + V_T]} \right) R \]

\[ V_{OUT} = \left\{ V_{DD} - \frac{\mu C_{OX} W}{2L} [V_{SS} + V_T]^2 R \right\} + \left\{ \frac{\mu C_{OX} W}{L} [V_{SS} + V_T] R \right\} V_M \sin \omega t \]
Small signal analysis example

\[ V_{IN} = V_M \sin \omega t \]

By selecting appropriate value of \( V_{SS} \), \( M_1 \) will operate in the saturation region

Assume \( M_1 \) operating in saturation region

\[
V_{OUT} = \left( V_{DD} - \frac{\mu C_{ox} W}{2L} \left[ V_{ss} + V_T \right]^2 R \right) + \left( \frac{\mu C_{ox} W}{L} \left[ V_{ss} + V_T \right] R \right) V_M \sin \omega t
\]
Small signal analysis example

Assume $M_1$ operating in saturation region

\[ V_{\text{OUT}} = \left( V_{\text{DD}} - \frac{\mu C_{\text{ox}} W}{2L} \left[V_{\text{SS}} + V_T\right]^2 R \right) + \left( \frac{\mu C_{\text{ox}} W}{L} \left[V_{\text{SS}} + V_T\right] R \right) V_M \sin \omega t \]

Quiescent Output

ss Voltage Gain

\[ A_v = \frac{\mu C_{\text{ox}} W}{L} \left[V_{\text{SS}} + V_T\right] R \]

But – this expression gives little insight into how large the gain is!
Small signal analysis example

\[ V_{IN} = V_M \sin \omega t \]

\[ V_{OUT} = \left\{ V_{DD} - \frac{\mu C_{ox} W}{2L} \left[ V_{ss} + V_T \right]^2 R \right\} + \left\{ \frac{\mu C_{ox} W}{L} \left[ V_{ss} + V_T \right] R \right\} V_M \sin \omega t \]

\[ A_v = \frac{\mu C_{ox} W}{L} \left[ V_{ss} + V_T \right] R \]

But recall:

\[ I_{DQ} = \frac{\mu C_{ox} W}{2L} \left( V_{ss} + V_T \right)^2 \]

Thus, substituting from the expression for \( I_{DQ} \) we obtain

\[ A_v = \frac{2I_{DQ} R}{\left[ V_{ss} + V_T \right]} \]

Note this is negative since \( V_{ss} + V_T < 0 \)
Small signal analysis example

\[ V_{IN} = V_M \sin \omega t \]

\[ A_v = \frac{2I_{DQ} R}{[V_{SS} + V_T]} \]

Observe the small signal voltage gain is twice the Quiescent voltage across R divided by \( V_{SS} + V_T \)

- This analysis which required linearization of a nonlinear output voltage is quite tedious.
- This approach becomes unwieldy for even slightly more complicated circuits
- A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements
Small-signal Operation of Nonlinear Circuits

• Small-signal principles

• Example Circuit

Small-Signal Models

• Small-Signal Analysis of Nonlinear Circuits
Solution for the example was based upon solving the nonlinear circuit for $V_{OUT}$ and then linearizing the solution by doing a Taylor’s series expansion

- Solution of nonlinear equations very involved with two or more nonlinear devices
- Taylor’s series linearization can get very tedious if multiple nonlinear devices are present

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<th>Standard Approach to small-signal analysis of nonlinear networks</th>
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<td>1. Solve nonlinear network</td>
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<td>2. Linearize solution</td>
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<th>Alternative Approach to small-signal analysis of nonlinear networks</th>
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<td>1. Linearize nonlinear devices</td>
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<td>2. Replace all devices with small-signal equivalent</td>
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<td>3. Solve linear small-signal network</td>
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Alternative Approach to small-signal analysis of nonlinear networks

1. Linearize nonlinear devices

2. Replace all devices with small-signal equivalent

3. Solve linear small-signal network

- Must only develop linearized model once for any nonlinear device
e.g. once for a MOSFET, once for a JFET, and once for a BJT

Linearized model for nonlinear device termed “small-signal model”

derrivation of small-signal model for most nonlinear devices is less complicated than solving even one simple nonlinear circuit

- Solution of linear network much easier than solution of nonlinear network
Standard Approach to small-signal analysis of nonlinear networks

1. Linearize nonlinear devices

2. Replace all devices with small-signal equivalent

3. Solve linear small-signal network
Standard Approach to analysis of nonlinear networks

Nonlinear Network

dc Equivalent Network

Q-point

Values for small-signal parameters

Small-signal equivalent network

Small-signal output

Total output
  (good approximation)
Standard Approach to small-signal analysis of nonlinear networks

Nonlinear Network

dc Equivalent Network

Q-point

Values for small-signal parameters

Small-signal equivalent network

Small-signal output

Total output

(good approximation)
Linearized nonlinear devices
Example:

Nonlinear network

Linearized small-signal network
Dc and small-signal equivalent elements

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<tr>
<th>Element</th>
<th>ss equivalent</th>
<th>dc equivalent</th>
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<tr>
<td>dc Voltage Source</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
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<tr>
<td>ac Voltage Source</td>
<td>$V_{AC}$</td>
<td>$V_{AC}$</td>
</tr>
<tr>
<td>dc Current Source</td>
<td>$I_{DC}$</td>
<td>$I_{DC}$</td>
</tr>
<tr>
<td>ac Current Source</td>
<td>$I_{AC}$</td>
<td>$I_{AC}$</td>
</tr>
<tr>
<td>Resistor</td>
<td>$R$</td>
<td>$R$</td>
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</table>
Dc and small-signal equivalent elements

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<tr>
<th>Element</th>
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<tr>
<td>Capacitors</td>
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<tr>
<td>Large Capacitor</td>
<td>C</td>
<td></td>
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<tr>
<td>Small Capacitor</td>
<td>C</td>
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<tr>
<td>Inductors</td>
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<tr>
<td>Large Inductor</td>
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<tr>
<td>Small Inductor</td>
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<tr>
<td>MOS Transistors</td>
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<td>Simplified</td>
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Dc and small-signal equivalent elements

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<tr>
<th>Element</th>
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<tr>
<td>Bipolar Transistors</td>
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How is the small-signal equivalent circuit obtained from the nonlinear circuit?

*What is the small-signal equivalent of the MOSFET and BJT?*
Small-Signal Model

Consider 4-terminal network

\[ I_1 = f_1(V_1, V_2, V_3) \]
\[ I_2 = f_2(V_1, V_2, V_3) \]
\[ I_3 = f_3(V_1, V_2, V_3) \]

Define

\[ i_1 = I_1 - I_{1Q} \]
\[ i_2 = I_2 - I_{2Q} \]
\[ i_3 = I_3 - I_{3Q} \]

\[ v_1 = V_1 - V_{1Q} \]
\[ v_2 = V_2 - V_{2Q} \]
\[ v_3 = V_3 - V_{3Q} \]

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents.
Small-Signal Model

Consider 4-terminal network

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents

For small signals, this relationship should be linear

Can be thought of as a change in coordinate systems from the large signal coordinate system to the small-signal coordinate system
Recall for a function of one variable

\[ y = f(x) \]

Taylor’s Series Expansion about the point \( x_0 \)

\[ y = f(x) = f(x) \bigg|_{x=x_0} + \frac{\partial f}{\partial x} \bigg|_{x=x_0} (x - x_0) + \frac{\partial^2 f}{\partial x^2} \bigg|_{x=x_0} \frac{1}{2!} (x - x_0)^2 + ... \]

If \( x-x_0 \) is small

\[ y \approx f(x) \bigg|_{x=x_0} + \frac{\partial f}{\partial x} \bigg|_{x=x_0} (x - x_0) \]

\[ y \approx y_0 + \frac{\partial f}{\partial x} \bigg|_{x=x_0} (x - x_0) \]
Recall for a function of one variable

\[ y = f(x) \]

If \( x-x_0 \) is small

\[ y \approx y_0 + \left. \frac{\partial f}{\partial x} \right|_{x=x_0} (x-x_0) \]

\[ y - y_0 = \left. \frac{\partial f}{\partial x} \right|_{x=x_0} (x-x_0) \]

If we define the small signal variables as

\[ y = y - y_0 \]

\[ x = x - x_0 \]
Recall for a function of one variable

\[ y = f(x) \]

If \( x-x_0 \) is small

\[ y - y_0 = \left. \frac{\partial f}{\partial x} \right|_{x=x_0} (x - x_0) \]

If we define the small signal variables as

\[ y = y - y_0 \]

\[ \chi = x - x_0 \]

Then

\[ y = \left. \frac{\partial f}{\partial x} \right|_{x=x_0} \chi \]

This relationship is linear!
Consider 4-terminal network

Nonlinear network characterized by 3 functions each functions of 3 variables

\[ I_1 = f_1(V_1, V_2, V_3) \]
\[ I_2 = f_2(V_1, V_2, V_3) \]
\[ I_3 = f_3(V_1, V_2, V_3) \]
Consider now 3 functions each functions of 3 variables

\[
\begin{align*}
I_1 &= f_1(V_1, V_2, V_3) \\
I_2 &= f_2(V_1, V_2, V_3) \\
I_3 &= f_3(V_1, V_2, V_3)
\end{align*}
\]

Define

\[
\tilde{V}_Q = \begin{bmatrix} V_{1Q} \\
V_{2Q} \\
V_{3Q} \end{bmatrix}
\]
Consider now 3 functions each functions of 3 variables

\[
\begin{align*}
I_1 &= f_1(V_1, V_2, V_3) \\
I_2 &= f_2(V_1, V_2, V_3) \\
I_3 &= f_3(V_1, V_2, V_3)
\end{align*}
\]

Define

\[
\vec{V}_Q = \begin{bmatrix} V_{1Q} \\ V_{2Q} \\ V_{3Q} \end{bmatrix}
\]

\[
I_1 = f_1(V_1, V_2, V_3) \approx f_1(V_{1Q}, V_{2Q}, V_{3Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1} \bigg|_{V = V_a} (V_1 - V_{1Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2} \bigg|_{V = V_a} (V_2 - V_{2Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3} \bigg|_{V = V_a} (V_3 - V_{3Q})
\]

\[
I_1 - I_{1Q} = \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1} \bigg|_{V = V_a} (V_1 - V_{1Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2} \bigg|_{V = V_a} (V_2 - V_{2Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3} \bigg|_{V = V_a} (V_3 - V_{3Q})
\]
Consider now 3 functions each function of 3 variables

\[
I_1 - I_{1Q} = \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1} \right|_{\tilde{V} = \tilde{v}_Q} (V_1 - V_{1Q}) + \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2} \right|_{\tilde{V} = \tilde{v}_Q} (V_2 - V_{2Q}) + \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3} \right|_{\tilde{V} = \tilde{v}_Q} (V_3 - V_{3Q})
\]

\[
y_{11} = \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1} \right|_{\tilde{V} = \tilde{v}_Q}
\]

\[
y_{12} = \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2} \right|_{\tilde{V} = \tilde{v}_Q}
\]

\[
y_{13} = \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3} \right|_{\tilde{V} = \tilde{v}_Q}
\]

\[
i_1 = I_1 - I_{1Q}
\]

\[
i_2 = I_2 - I_{2Q}
\]

\[
i_3 = I_3 - I_{3Q}
\]

\[
u_1 = V_1 - V_{1Q}
\]

\[
u_2 = V_2 - V_{2Q}
\]

\[
u_3 = V_3 - V_{3Q}
\]
Consider now 3 functions each functions of 3 variables

\[ I_1 - I_{1q} = \left. \frac{\partial f_1(v_1, v_2, v_3)}{\partial v_1} \right|_{\bar{v} = \bar{v}_a} (v_1 - v_{1q}) + \left. \frac{\partial f_1(v_1, v_2, v_3)}{\partial v_2} \right|_{\bar{v} = \bar{v}_a} (v_2 - v_{2q}) + \left. \frac{\partial f_1(v_1, v_2, v_3)}{\partial v_3} \right|_{\bar{v} = \bar{v}_a} (v_3 - v_{3q}) \]

\[ i_1 = y_{11} v_1 + y_{12} v_2 + y_{13} v_3 \]

This is now a linear relationship between the small signal electrical variables
Consider now 3 functions each functions of 3 variables

\[ i_1 = y_{11}v_1 + y_{12}v_2 + y_{13}v_3 \]

Let's now extend this to \( I_2 \) and \( I_3 \)

Define

\[ y_{ij} = \frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j} \bigg|_{V = V_0} \]

\[ i_1 = y_{11}v_1 + y_{12}v_2 + y_{13}v_3 \]

\[ i_2 = y_{21}v_1 + y_{22}v_2 + y_{23}v_3 \]

\[ i_3 = y_{31}v_1 + y_{32}v_2 + y_{33}v_3 \]

This is a small-signal model of a 4-terminal network and it is linear
9 small-signal parameters characterize the linear 4-terminal network
Small-signal model parameters dependent upon Q-point!
A small-signal equivalent circuit of a 4-terminal nonlinear network

\[ y_{ij} = \left. \frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j} \right|_{V=\bar{V}_Q} \]

Equivalent circuit is not unique
4-terminal small-signal network summary

\[
\begin{align*}
I_1 &= f_1(V_1, V_2, V_3) \\
I_2 &= f_2(V_1, V_2, V_3) \\
I_3 &= f_3(V_1, V_2, V_3)
\end{align*}
\]

Small signal model:

\[
\begin{align*}
I_1 &= y_{11} V_1 + y_{12} V_2 + y_{13} V_3 \\
I_2 &= y_{21} V_1 + y_{22} V_2 + y_{23} V_3 \\
I_3 &= y_{31} V_1 + y_{32} V_2 + y_{33} V_3
\end{align*}
\]

\[
y_{ij} = \left. \frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j} \right|_{V=V_Q}
\]
Consider 3-terminal network

Small-Signal Model

\[ i_1 = g_1(v_1, v_2, v_3) \]
\[ i_2 = g_2(v_1, v_2, v_3) \]
\[ i_3 = g_3(v_1, v_2, v_3) \]

\[ i_1 = y_{11} v_1 + y_{12} v_2 + y_{13} v_3 \]
\[ i_2 = y_{21} v_1 + y_{22} v_2 + y_{23} v_3 \]
\[ i_3 = y_{31} v_1 + y_{32} v_2 + y_{33} v_3 \]

\[ y_{ij} = \frac{\partial f_i(v_1, v_2, v_3)}{\partial v_j} \bigg|_{\bar{v}} = v_q \]
Consider 3-terminal network

**Small-Signal Model**

\[
\begin{align*}
I_1 &= f_1(V_1, V_2) \\
I_2 &= f_2(V_1, V_2)
\end{align*}
\]

Define
\[
\begin{align*}
i_1 &= I_1 - I_{1Q} \\
i_2 &= I_2 - I_{2Q}
\end{align*}
\]

\[
\begin{align*}
v_1 &= V_1 - V_{1Q} \\
v_2 &= V_2 - V_{2Q}
\end{align*}
\]

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents.
Consider 3-terminal network

Small-Signal Model

\[ i_1 = y_{11} v_1 + y_{12} v_2 \]
\[ i_2 = y_{21} v_1 + y_{22} v_2 \]

A Small Signal Equivalent Circuit

\[ y_{ij} = \left. \frac{\partial f_i(v_1, v_2)}{\partial v_j} \right|_{v=v_Q} \]
\[ v = \begin{pmatrix} v_{1Q} \\ v_{2Q} \end{pmatrix} \]

4 small-signal parameters characterize this 3-terminal (two-port) linear network
Small signal parameters dependent upon Q-point
3-terminal small-signal network summary

\[ I_1 = f_1(V_1, V_2) \]
\[ I_2 = f_2(V_1, V_2) \]

Small signal model:

\[ i_1 = y_{11} V_1 + y_{12} V_2 \]
\[ i_2 = y_{21} V_1 + y_{22} V_2 \]

\[ y_{ij} = \left. \frac{\partial f_i(V_1, V_2)}{\partial V_j} \right|_{\bar{V} = \bar{V}_Q} \]
Consider 2-terminal network

**Small-Signal Model**

\[
\begin{align*}
    i_1 &= g_1(v_1, v_2, v_3) \\
    i_2 &= g_2(v_1, v_2, v_3) \\
    i_3 &= g_3(v_1, v_2, v_3)
\end{align*}
\]

\[
\begin{align*}
    i_1 &= y_{11}v_1 + y_{12}v_2 + y_{13}v_3 \\
    i_2 &= y_{21}v_1 + y_{22}v_2 + y_{23}v_3 \\
    i_3 &= y_{31}v_1 + y_{32}v_2 + y_{33}v_3
\end{align*}
\]

\[y_{ij} = \left. \frac{\partial f_i(v_1, v_2, v_3)}{\partial v_j} \right|_{\bar{v}=v_Q}\]
Consider 2-terminal network

Small-Signal Model

\[ I_1 = f_1(V_1) \]

Define

\[ i_1 = I_1 - I_{1Q} \]

\[ v_1 = V_1 - V_{1Q} \]

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents.
Consider 2-terminal network

Small-Signal Model

\[ i_1 = y_{11} V_1 \]

\[ y_{11} = \left. \frac{\partial f_1(V_1)}{\partial V_1} \right|_{V=V_0} \]

A Small Signal Equivalent Circuit
MOSFET is actually a 4-terminal device but for many applications acceptable predictions of performance can be obtained by treating it as a 3-terminal device by neglecting the bulk terminal.

In this course, we have been treating it as a 3-terminal device and in this lecture will develop the small-signal model by treating it as a 3-terminal device.
Small Signal Model of MOSFET

Large Signal Model

\[ I_G = 0 \]

3-terminal device

\[
I_D = \begin{cases}
0 & V_{GS} \leq V_T \\
\mu C_{OX} \frac{W}{L} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T, V_{DS} < V_{GS} - V_T \\
\mu C_{OX} \frac{W}{2L} \left( V_{GS} - V_T \right)^2 \left( 1 + \lambda V_{DS} \right) & V_{GS} \geq V_T, V_{DS} \geq V_{GS} - V_T
\end{cases}
\]

MOSFET is usually operated in saturation region in linear applications where a small-signal model is needed so will develop the small-signal model in the saturation region
Small Signal Model of MOSFET

\[ I_1 = f_1(V_1, V_2) \quad \leftrightarrow \quad I_G = 0 \]

\[ I_2 = f_2(V_1, V_2) \quad \leftrightarrow \quad I_D = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \]

Small-signal model:

\[ y_{ij} = \left. \frac{\partial f_i(V_1, V_2)}{\partial V_j} \right|_{V=V_Q} \]

\[ y_{11} = \left. \frac{\partial I_G}{\partial V_{GS}} \right|_{V=V_Q} \]

\[ y_{21} = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V=V_Q} \]

\[ y_{12} = \left. \frac{\partial I_G}{\partial V_{DS}} \right|_{V=V_Q} \]

\[ y_{22} = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V=V_Q} \]
Small Signal Model of MOSFET

\[ I_1 = f_1(V_1, V_2) \quad \dashv \quad I_G = 0 \]

\[ I_2 = f_2(V_1, V_2) \quad \dashv \quad I_D = \mu C_{ox} \frac{W}{2L} (V_{gs} - V_T)^2 (1 + \lambda V_{ds}) \]

**Small-signal model:**

\[ y_{11} = \left. \frac{\partial I_G}{\partial V_{gs}} \right|_{V=V_0} = 0 \]

\[ y_{12} = \left. \frac{\partial I_G}{\partial V_{ds}} \right|_{V=V_0} = 0 \]

\[ y_{21} = \left. \frac{\partial I_D}{\partial V_{gs}} \right|_{V=V_0} = 2\mu C_{ox} \frac{W}{2L} (V_{gs} - V_T)^2 (1 + \lambda V_{ds}) \]

\[ y_{21} \approx \mu C_{ox} \frac{W}{L} (V_{gs} - V_T) \]

\[ y_{22} = \left. \frac{\partial I_D}{\partial V_{ds}} \right|_{V=V_0} = \mu C_{ox} \frac{W}{2L} (V_{gs} - V_T)^2 \lambda \]

\[ y_{22} \approx \lambda I_{DO} \]
Small Signal Model of MOSFET

\[ I_G = 0 \]

\[ I_D = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \]

\[ y_{12} = 0 \]
\[ y_{11} = 0 \]
\[ y_{21} \approx \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_T) \]
\[ y_{22} \approx \lambda I_{DQ} \]

\[ i_G = y_{11} V_{GS} + y_{12} V_{DS} \]
\[ i_D = y_{21} V_{GS} + y_{22} V_{DS} \]
Small Signal Model of MOSFET

by convention, $y_{21} = g_m$, $y_{22} = g_0$

$\therefore y_{21} \approx g_m = \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_t)$

$y_{22} = g_o \approx \lambda I_{DQ}$
Small Signal Model of MOSFET

\[ g_m = \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_T) \]

\[ g_o \approx \lambda I_{DQ} \]

Alternate equivalent expressions:

\[ I_{DQ} = \mu C_{ox} \frac{W}{2L} (V_{GSQ} - V_T)^2 \left(1 + \lambda V_{DSQ}\right) \approx \mu C_{ox} \frac{W}{2L} (V_{GSQ} - V_T)^2 \]

\[ g_m = \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_T) \]

\[ g_m = \sqrt{2\mu C_{ox} \frac{W}{L} \cdot \sqrt{I_{DQ}}} \]

\[ g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} \]