EE 230
Lecture 36
Data Converters
Small Signal Model of BJT

3-terminal device

Forward Active Model:

\[ I_C = J_S A E \frac{V_{BE}}{V_t} \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

\[ I_B = \frac{J_S A E}{\beta} \frac{V_{BE}}{V_t} \]

Usually operated in Forward Active Region when small-signal model is needed
Small Signal Model of BJT

\[ i_B = g_{\pi} V_{BE} \]
\[ i_C = g_m V_{BE} + g_o V_{CE} \]

\[ g_{\pi} = \frac{I_{CQ}}{\beta V_t} \]
\[ g_m = \frac{I_{CQ}}{V_{t}} \]
\[ g_o = \frac{I_{CQ}}{V_{AF}} \]

Observe

\[ g_m V_{BE} = g_m \left[ \frac{i_b}{g_{\pi}} \right] = \left[ \frac{g_m}{g_{\pi}} \right] i_b \]
\[ \frac{g_m}{g_{\pi}} = \frac{V_t}{I_{CQ}} = \beta \]

\[ \therefore \quad g_m V_{BE} = \beta i_b \]

Alternate equivalent small-signal models of the BJT
Alternative Approach to small-signal analysis of nonlinear networks

1. Linearize nonlinear devices
   *(have small-signal model for key devices!)*

2. Replace all devices with small-signal equivalent

3. Solve linear small-signal network
Data Converters

Standard Symbols:

Analog to Digital Converter

Digital to Analog Converter
Data Converters

Other Symbols:

Analog to Digital Converter

Digital to Analog Converter
Data Converters

Analog variables: Voltage, Current, time, charge, occasionally other physical variables

Digital variables: Usually represented in binary form but other forms occasionally used (e.g. gray, Thermometer code)
Data Converters

Applications: Dominantly the interfaces between the continuous-time Continuous-amplitude physical environment and a digital system such as a computer, microprocessor, microcontroller, or finite state machine
Data Converters

\[ X_{IN} \rightarrow_{ DAC } X_{OUT} \]

\[ B: [b_1 b_2 \ldots b_n] \]  

\( X_{REF} \)

\( X_{OUT} \)

\( X_{IN} \)

000 001 010 011 100 101 110 111

An ideal DAC

(Some specific shifted versions of this DAC would also be termed an ideal DAC)
An ideal ADC

(Some specific shifted versions of this ADC would also be termed an ideal ADC)
Data Converters

Terminology:

B: \([b_1 b_2 \ldots b_n]\)

- \(b_1\): Most Significant Bit (MSB)
- \(b_n\): Least Significant Bit (LSB)

Resolution: Defines number of distinct levels for DAC or Boolean outputs for ADC. If there are \(N\) distinct levels, resolution generally defined as \(n = \log_2 N\) thus, \(N = 2^n\)

\(X_{\text{REF}}\): specifies the full-scale range of the data converter. Input range for ADC or output range for DAC is usually

\[
X_{\text{REF}} \left( \frac{2^n - 1}{2^n} \right)^{n \text{ large}} \approx X_{\text{REF}}
\]
Data Converters

Terminology:

**LSB (or \(X_{\text{LSB}}\))**: Analog change (in input to ADC or output of DAC) corresponding to one LSB digital change

\[ X_{\text{LSB}} = \frac{X_{\text{REF}}}{2^n} \]

**Transition Points (for ADC)**: values of \(X_{\text{IN}}\) where output changes by 1 LSB (an n-bit ADC has N-1 transition points partitioning input into N distinct intervals)

**Decimal Equivalent**: Decimal equivalent of \(B: [b_1, b_2, \ldots, b_n]\)

\[
D(B) = \left( \frac{b_1}{2} + \frac{b_2}{4} + \ldots + \frac{b_n}{2^n} \right) \quad \Rightarrow \quad D(B) = \sum_{k=1}^{n} \frac{b_k}{2^k}
\]
### Number of levels for different resolution

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Data Converters

B: \([b_1 \ b_2 \ldots \ b_n]\)  

An ideal DAC

\[ X_{\text{OUT}} = X_{\text{REF}} D(X_{\text{IN}}) = X_{\text{REF}} \sum_{k=1}^{n} \frac{b_k}{2^k} \]
Data Converters

$X_{IN} \rightarrow \text{ADC} \rightarrow X_{OUT}$

$X_{OUT}$

$X_{IN}$

$X_{REF}$

$X_{OUT} = B$

$X_{REF}D(B) < X_{IN} < X_{REF}D(B) + X_{LSB}$
Example

Determine $V_{\text{LSB}}$ for a 16-bit ADC if $X_{\text{REF}}$ is a voltage of 1V.

$$X_{\text{LSB}} = \frac{1\text{V}}{2^{16}} = 15.25\mu\text{V}$$

Observe $X_{\text{LSB}}$ is very small and for a 16-bit ADC, must resolve an input signal to $\pm X_{\text{LSB}}/2 = \pm 7.5\mu\text{V}$
Example

Determine the number of bits of resolution, \( n \), required in an ADC if it is to be used in a DMM that has accuracy corresponding to \( m \) decimal digits.

Resolution of an \( m \)-digit DMM is \( \frac{V_{\text{REF}}}{10^m} \)

Thus equating the resolution of an ADC represented in binary form to that of the DMM, we obtain the expression

\[
\frac{V_{\text{REF}}}{2^n} = \frac{V_{\text{REF}}}{10^m}
\]

It thus follows that

\[
m = n \log_{10} 2
\]

Solving for \( n \), we obtain

\[
n = \frac{m}{\log_{10} 2}
\]

If \( m = 6 \), \( n = 20 \)

If \( V_{\text{REF}} = 1 \text{V} \), \( V_{\text{LSB}} = 0.95 \mu \text{V} \)

If \( V_{\text{REF}} = 1 \text{V} \), \( V_{\text{LSB}} = 112 \text{nV} \)

Very high resolution is required in applications such as this!
Data Converters

Discrete implementations of data converters are seldom used
  – Not cost effective
  – Too large
  – Vary difficult to maintain acceptable accuracies of components

Integrated data converters usually have voltage or current as input or output variables
  – If conversion of other physical units is required, a transducer precedes or follows a voltage or current data converter