EE 230
Lecture 43

Data Converters
Amplitude Quantization

Unwanted signals in the output of a system are called **noise**.

**Distortion**
- Smooth nonlinearities
- Frequency attenuation
- Large Abrupt Nonlinearities

**Signals coming from other sources**
- Movement of carriers in devices
- Interference from electrical coupling
- Interference from radiating sources

**Undesired outputs inherent in the data conversion process itself**
Characterization of Quantization Noise

Sinusoidal excitation

- Consider an ADC (clocked)

Theorem: If $n(t)$ is a random process, then

$$V_{\text{RMS}} \approx \sqrt{\sigma^2 + \mu^2}$$

provided that the RMS value is measured over a large interval where the parameters $\sigma$ and $\mu$ are the standard deviation and the mean of $\langle n(kT) \rangle$

This theorem can thus be represented as

$$V_{\text{RMS}} \approx \sqrt{\frac{1}{T_L \sum_{t_i}^{t_i+T_L} n^2(t) dt} \approx \sqrt{\sigma^2 + \mu^2}}$$

where $T$ is the sampling interval and $T_L$ is a large interval
Characterization of Quantization Noise

Saw tooth excitation

\[ X_{Q-RMS} \approx \frac{X_{LSB}}{\sqrt{12}} \]

\[ SNR = 2^n \]
\[ SNR_{dB} = 6.02n \]

Sinusoidal excitation

\[ SNR = 1.225 \cdot 2^n \]
\[ SNR_{dB} = 6.02n + 1.76 \]

Although derived for an ADC, same expressions apply for DAC
SNR for saw tooth and for triangle excitations are the same
SNR for sinusoidal excitation larger than that for saw tooth by 1.76dB
SNR will decrease if input is not full-scale
Equivalent Number of Bits (ENOB) often given relative to quantization noise \( SNR_{dB} \)
Remember – quantization noise is inherent in an ideal data converter!
Review from Last Time:

**Equivalent Number of Bits (ENOB)**

\[
\text{ENOB} = \frac{\text{SNR} - 1.76}{6.02}
\]

\[
\text{ENOB} = \frac{\text{SNDR} - 1.76}{6.02}
\]

These definitions of ENOB are based upon noise or noise and distortion.

Some other definitions of ENOB are used as well – e.g. if one is only interested in distortion, an ENOB based upon distortion can be defined.

ENOB is useful for determining whether the number of bits really being specified is really useful.
Engineering Issues for Using Data Converters

1. Inherent with Data Conversion Process
   • Amplitude Quantization
   • Time Quantization
     (Present even with Ideal Data Converters)

2. Nonideal Components
   • Uneven steps
   • Offsets
   • Gain errors
   • Response Time
   • Noise
     (Present to some degree in all physical Data Converters)

How do these issues ultimately impact performance?
Nonideal Transfer Characteristics

Uneven Steps

![Diagram showing nonideal transfer characteristics of DAC with uneven steps compared to an ideal transfer characteristic line.](image)
Nonideal Transfer Characteristics

Uneven Steps

Actual transfer characteristics can vary considerably from one device to another.
Nonideal Transfer Characteristics

Uneven Steps

This is termed a nonlinearity in the data converter

Linearity metrics (specifications) include INL, DNL, THD and SFDR
Characterization of Nonlinearities

End points are the outputs at the two extreme Boolean inputs

End Point

Nonideal Transfer Characteristics of DAC

End Point

$X_{OUT}$

$X_{IN}$
Characterization of Nonlinearities

End point fit line
Characterization of Nonlinearities

Integral Nonlinearity (INL)

Measure of worst-case deviation from linear

Define the INL at any input code $k$ by:

$$INL_k = X_{OUT}(k) - X_{FIT}(k)$$
Integral Nonlinearity (INL)

Define the INL by:

\[
\text{INL} = \max_{1 \leq k \leq N} \left\{ |\text{INL}_k| \right\}
\]
Integral Nonlinearity (INL)

\[ \text{INL} = \max_{1 \leq k \leq N} \{ |\text{INL}_k| \} \]

Often expressed in LSB:

\[ \text{INL}_{\text{LSB}} = \frac{\text{INL}}{X_{\text{LSBF}}} \]

where

\[ X_{\text{LSBF}} = \frac{X_{\text{OUT}(N)} - X_{\text{OUT}(1)}}{N-1} \]
Integral Nonlinearity (INL)

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but

\[ X_{\text{LSBF}} = \frac{X_{\text{OUT}}(N) - X_{\text{OUT}}(1)}{N-1} \cong \frac{X_{\text{REF}}}{2^n} \]

Linearity metrics:
- INL
- DNL
- THD
- SFDR
Characterization of Nonlinearities

Differential Nonlinearity (DNL)

Measure of worst-case resolving capabilities

Define the DNL at any input code $k$ by:

$$\text{DNL}_k = X_{\text{OUT}}(k) - X_{\text{OUT}}(k-1) - X_{\text{LSBF}} \approx X_{\text{OUT}}(k) - X_{\text{OUT}}(k-1) - X_{\text{LSBF}}$$
Differential Nonlinearity (DNL)

Linearity metrics:
- INL
- DNL
- THD
- SFDR

\[
DNL_k \approx X_{OUT}(k) - X_{OUT}(k-1) - X_{LSB}
\]

\[
DNL = \max_{1 \leq k \leq N} |DNL_k|
\]

Often expressed in LSB

\[
DNL_{LSB} = \frac{DNL}{X_{LSBF}}
\]

\[
DNL_{LSB} \approx 2^n \frac{DNL}{X_{REF}}
\]
Characterization of Nonlinearities

Linearity metrics:
- INL
- DNL
- THD
- SFDR

Linearity Metrics for ADC and DAC are Analogous to Each Other
Integral Nonlinearity (INL)

Linearity metrics:
- INL
- DNL
- THD
- SFDR
Integral Nonlinearity (INL)

Nonideal Transfer Characteristics of ADC

Linearity metrics:
- INL
- DNL
- THD
- SFDR

$X_{OUT}$

Code $k$

$X_{TRAN}(k)$

$X_{IN}$

$X_{REF}$

End Point

End Point Fit Line
Integral Nonlinearity (INL)

\[ \text{INL}_k = X_{\text{TRAN}(k)} - X_{\text{FIT}(k)} \]

\[ X_{\text{FIT}(k)} = X_{\text{TRAN}(1)} + \left( \frac{k-1}{N-2} \right) \frac{X_{\text{TRAN}(N-1)} - X_{\text{TRAN}(1)}}{N-2} \]
Integral Nonlinearity (INL)

\[ \text{INL}_k = X_{\text{TRAN}}(k) - X_{\text{FIT}}(k) \]

\[ \text{INL} = \max_{1 \leq k \leq N} \left\{ \left| \text{INL}_k \right| \right\} \]

\[ \text{INL}_{\text{LSB}} = \frac{\text{INL}}{X_{\text{LSBF}}} \]

\[ \text{INL}_{\text{LSB}} \approx 2^n \frac{\text{INL}}{X_{\text{REF}}} \]
Differential Nonlinearity (DNL)

\[ \text{DNL}_k \approx X_{\text{TRANS}(k)} - X_{\text{TRANS}(k-1)} - X_{\text{LSB}} \]

\[ \text{DNL} = \max_{1 \leq k \leq N} \{|\text{DNL}_k|\} \]

Linearity metrics:
- INL
- DNL
- THD
- SFDR

Nonideal Transfer Characteristics of ADC

Presentation of code transitions and linearity metrics.
Equivalent Number of Bits - ENOB (based upon linearity)

Generally expect INL to be less than $\frac{1}{2}$ LSB

If INL larger than $\frac{1}{2}$ LSB, effective resolution is less than specified resolution
Equivalent Number of Bits - ENOB (based upon linearity)

If \( \nu \) is the INL in LSB

\[
\text{ENOB} = n - 1 - \frac{\log_{10} \nu}{\log_{10} 2}
\]

<table>
<thead>
<tr>
<th>( \nu )</th>
<th>( \text{res} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>( n )</td>
</tr>
<tr>
<td>1</td>
<td>( n-1 )</td>
</tr>
<tr>
<td>2</td>
<td>( n-2 )</td>
</tr>
<tr>
<td>4</td>
<td>( n-3 )</td>
</tr>
<tr>
<td>8</td>
<td>( n-4 )</td>
</tr>
<tr>
<td>16</td>
<td>( n-5 )</td>
</tr>
</tbody>
</table>
Spectral Characterization

Linearity metrics:
- INL
- DNL
- THD
- SFDR

If nonlinearities present, $X_{\text{OUT}}$ given by

$$X_{\text{OUT}} = A_0 + A_1 \sin(\omega t + \theta_1) + \sum_{k=2}^{\infty} A_k \sin(k \omega t + \theta + \gamma_k)$$
Spectral Characterization

\[ X_{\text{IN}} = X_M \sin(\omega t + \theta) \]

\[ X_{\text{OUT}} = A_0 + A_1 \sin(\omega t + \theta + \gamma_1) + \sum_{k=2}^{\infty} A_k \sin(k\omega t + \theta + \gamma_k) \]

\[ A_k, \quad k>1 \text{ are all spectral distortion components} \]

Generally only first few terms are large enough to represent significant distortion

\[ THD = \frac{\sum_{k=2}^{\infty} A_k^2}{A_1^2} \]

\[ THD_{dB} = 10 \log_{10} \left( \frac{\sum_{k=2}^{\infty} A_k^2}{A_1^2} \right) \]

\[ SFDR = \frac{|A_1|}{\max_{1<k} \{|A_k|\}} \]

\[ SFDR_{dB} = 20 \log_{10} \left( \frac{|A_1|}{\max_{1<k} \{|A_k|\}} \right) \]
Spectral Characterization

\[ X_{\text{IN}} = X_M \sin(\omega t + \theta) \]

\[ X_{\text{OUT}} = A_0 + A_1 \sin(\omega t + \theta + \gamma_1) + \sum_{k=2}^{\infty} A_k \sin(k\omega t + \theta + \gamma_k) \]

Generally \( X_M \) is chosen nearly full-scale and input is offset by \( X_{\text{REF}}/2 \)

\[ X_{\text{IN}} = \frac{X_{\text{REF}}}{2} + \left( \frac{X_{\text{REF}}}{2} - \varepsilon \right) \sin(\omega t + \theta) \]

Direct measurement of \( A_k \) terms not feasible

\( A_k \) generally calculated from a large number of samples of \( X_{\text{OUT}}(t) \)
Spectral Characterization

\[ X_{IN} = X_M \sin(\omega t + \theta) \]

\[ X_{OUT} = A_0 + A_1 \sin(\omega t + \theta + \gamma_1) + \sum_{k=2}^{\infty} A_k \sin(k\omega t + \theta + \gamma_k) \]

Key theorem useful for spectral characterization

Theorem: If a periodic signal \( x(t) \) with period \( T = 1/f \) is band-limited to frequency \( hf \) and if the signal is sampled \( N \) times over an integral number of periods, \( N_p \), then

\[ |A_m| = \frac{2}{N} |X(mN_p + 1)| \quad \text{for} \quad 0 \leq m \leq h-1 \]

where \( \left< X(k) \right>_{k=1}^{N-1} \) is the DFT of the sampled sequence \( \left< x(kT_s) \right>_{k=1}^{N-1} \) where \( T_s \) is the sampling period.

\[ T_s = \frac{T \cdot N_p}{N} \]
Spectral Characterization

\[ X_{\text{IN}} = X_M \sin(\omega t + \theta) \]

\[ X_{\text{OUT}} = A_0 + A_1 \sin(\omega t + \theta + \gamma_1) + \sum_{k=2}^{\infty} A_k \sin(k\omega t + \theta + \gamma_k) \]

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- This theorem is usually not stated although widely used
- Often this theorem is misunderstood or misused
- If hypothesis not exactly satisfied, major problems with trying to use this theorem
Spectral Characterization

\[ X_{\text{IN}} = X_M \sin(\omega t + \theta) \]

\[ X_{\text{OUT}} = A_0 + A_1 \sin(\omega t + \theta + \gamma_1) + \sum_{k=2}^{\infty} A_k \sin(k\omega t + \theta + \gamma_k) \]

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Spectral Characterization

Key theorem useful for spectral characterization

Theorem: If a periodic signal $x(t)$ with period $T=1/f$ is band-limited to frequency $hf$ and if the signal is sampled $N$ times over an integral number of periods, $N_p$, then

$$|A_m| = \frac{2}{N} |X(mN_p + 1)|$$

for $0 \leq m \leq h-1$

where $\left<X(k)\right>_{k=1}^{N-1}$ is the DFT of the sampled sequence $\left<x(kT_S)\right>_{k=1}^{N-1}$ where $T_S$ is the sampling period.

- Usually $N_p$ is a prime number (e.g. 11, 21, 29, 31)
- If $N$ is a power of 2, the Fast Fourier Transform (FFT) is a computationally efficient method for calculating the DFT
- Often $N=4096, 65,536, \ldots$
- FFT is available in Matlab and as subroutines for C++
Spectral Characterization

Key theorem useful for spectral characterization

Theorem: If a periodic signal $x(t)$ with period $T=1/f$ is band-limited to frequency $hf$ and if the signal is sampled $N$ times over an integral number of periods, $N_P$, then

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$A_0, A_1, A_2, A_3, \ldots$ are the magnitudes of the DFT elements $X(0)$, $X(N_P+1)$, $X(2N_P+1)$, $X(3N_P+1)$, $\ldots$ respectively
Spectral Characterization

$T_{DFT \, \text{WINDOW}}$

$T_{\text{PERIOD}}$

$T_{\text{SIG}}$

$T_{\text{CLOCK}}$

Sampling Clock

$T_{DFT \, \text{CLOCK}}$

DFT Clock
Spectral Characterization

\[ |X(k)| \]

\[ A_0, A_1, A_2, A_3, A_4, \ldots \]

\[ N_p+1, 2N_p+1, 3N_p+1, 4N_p+1 \]