EE 230
Lecture 8

Amplifiers
Quiz 7

A nonideal transresistance amplifier is shown. Represent this same amplifier as a nonideal voltage amplifier.
And the number is ?
Quiz 7

A nonideal transresistance amplifier is shown. Represent this same amplifier as a nonideal voltage amplifier.

Solution: The nonideal circuits are identical so

Alternately,

\[ I_1 = \frac{V_1}{10\Omega} \]
Amplifiers are generally not ideal
(but can be nearly ideal)

Gain can vary with frequency

\[ X_{\text{OUT}} = K X_{\text{IN}} \quad \quad X_{\text{OUT}} = T(s) X_{\text{IN}} \]

Region of Constant Gain
Amplifiers are generally not ideal
(but can be nearly ideal)

Amplifier will display some nonlinearity at extreme inputs (in transfer characteristics)

\[ X_{\text{OUT}} = K X_{\text{IN}} \quad \text{versus} \quad X_{\text{OUT}} = f(X_{\text{IN}}) \]

The region of linear gain is often quite large for good amplifiers
Amplifiers are generally not ideal
(but can be nearly ideal)

The input and output impedances may not be ideal

Example: Consider a Voltage Amplifier with a gain of 10
Review from Last Time

Amplifiers are generally not ideal (but can be nearly ideal)

Amplifier will display some nonlinearity throughout (in transfer characteristics)

\[ X_{\text{OUT}} = K X_{\text{IN}} \quad \text{and} \quad X_{\text{OUT}} = f(X_{\text{IN}}) \]

Nonlinear distortion components will be present throughout region of operation.
Amplifiers

Equivalence of functional form of all four basic types

When nonideal, the functional form of all four basic amplifiers are identical and they differ only in the value of the model parameters.
Half-power Frequency and Amplifier Bandwidth

The half-power frequency $\omega_H$ is the frequency where the output power drops to $\frac{1}{2}$ of the peak output power.

Claim: The half-power frequency is the frequency where the magnitude of the voltage gain drops to $\frac{A_0}{\sqrt{2}}$ where $A_0$ is the maximum gain.

Proof:
Half-power Frequency and Amplifier Bandwidth

Definition: The amplifier bandwidth is the width of the “passband” of the amplifier.

For a first-order lowpass amplifier,
\[ BW = \omega_H \]

For a wide passband with first-order high and low frequency performance,
\[ BW = \omega_{HH} - \omega_{HL} \]
Half-power Frequency and Amplifier Bandwidth

First-Order Lowpass Amplifier

Claim: If an amplifier has a first-order lowpass response, then the half-power frequency (in rad/sec) is the magnitude of the pole

Proof:

\[
A(s) = \frac{\pm A_0 p}{s-p}
\]

\[
A(j\omega) = \frac{\pm A_0 p}{j\omega - p}
\]

\[
\frac{A_0}{\sqrt{2}} = |A(j\omega_H)|
\]

\[
\frac{A_0}{\sqrt{2}} = \frac{|A_0 p|}{\sqrt{\omega_H^2 + p^2}}
\]

\[
2p^2 = \omega_H^2 + p^2
\]

\[
p^2 = \omega_H^2
\]

\[
\omega_H = |p|
\]

∴ BW = |p|
Frequency Response of Amplifiers

First-Order Lowpass Amplifier

Gain Bandwidth Product

Consider now the logarithmic frequency and gain axis

\[ A(s) = \frac{\pm A_0}{s - \frac{1}{p}} \]

Definition: Gain Bandwidth Product

\[ GB = A_0 BW \]

Thus, for the first-order lowpass amplifier

\[ GB = A_0 |p| \]

The concept of gain-bandwidth product will come up on several occasions when discussing amplifier performance.
Frequency Response of Amplifiers

Roll-off rate of first-order amplifiers in the stop band

First-Order Lowpass Amplifier

Consider now the logarithmic frequency and gain axis

First-order rolloff
Linear frequency axis

First-order rolloff
Logarithmic frequency and gain axis
Frequency Response of Amplifiers

First-Order Lowpass Amplifier

Roll-off rate of first-order amplifiers in the stop band

Consider now the logarithmic frequency and gain axis

\[
A(s) = \frac{\pm A_0}{s - 1} \\
BW = |p|
\]

In one decade, gain in dB decreases by

\[
\Delta A_{dB} = 20\log_{10}\left(\frac{GB}{\omega_x}\right) - 20\log_{10}\left(\frac{GB}{10\omega_x}\right)
\]

Note this is independent of \(\omega_x\)

Therefore, the slope (roll-off) is \(-20\text{dB/decade}\)

Can be shown this slope is also \(-6.02\text{dB/octave}\)

\[
A(j\omega) = \frac{\pm A_0}{j\omega - 1} \\
\omega \left| A(j\omega) \right| = \frac{A_0}{\sqrt{\frac{\omega^2}{p^2} + 1}} \\
\left| A(j\omega) \right| = \frac{A_0}{\sqrt{\frac{\omega^2}{\omega^2 + 1}}}
\]

At high frequencies, gain given by
Half-power Frequency and Amplifier Bandwidth

Wide-band bandpass with first-order band edges

This type of response can be approximated by the expression

\[ A(s) = \frac{-A_0 s}{(s-p_L) \left( \frac{s}{p_H} - 1 \right)} \]

\[ A(s) \approx \left\{ \begin{align*}
A_0 \\
\frac{-A_0 s}{p_L} \\
\frac{-A_0 p_H}{s} \\
\frac{s}{p_H}
\end{align*} \right. \]

\[
\begin{align*}
\omega &< \omega_{HL} \\
\omega_{HL} &< \omega < \omega_{HH} \\
\omega &> \omega_{HH}
\end{align*}
\]
Half-power Frequency and Amplifier Bandwidth

Wide-band bandpass with first-order band edges

Around the low-frequency transition

\[
A(s) = \frac{A_0 s}{s-p_L} \quad A(j\omega) = \frac{j\omega A_0}{j\omega-p_L} \quad |A(j\omega_{HL})| = \frac{A_0}{\sqrt{2}} = \frac{\omega_{HL} A_0}{\sqrt{\omega_{HL}^2 + p_L^2}} \quad \omega_{HL} = |p_L|
\]

Around the high-frequency transition

\[
A(s) = \frac{-A_0}{\left(\frac{s}{p_H} - 1\right)} \quad \text{but we found previously that} \quad \omega_{HH} = |p_H|
\]

Thus, the bandwidth is given by

\[
\text{BW} = \omega_{HH} - \omega_{HL} = |p_H| - |p_L| = -p_H + p_L
\]
Frequency Response of Amplifiers

Roll-off rate of first-order amplifiers in the stop band

Wide-band bandpass with first-order band edges

Consider now the logarithmic frequency and gain axis

Wide-band bandpass with first-order band edges

Logarithmic frequency axis and gain axis
Frequency Response of Amplifiers

Roll-off rate of first-order amplifiers in the stop band

Wide-band bandpass with first-order band edges

Consider now the logarithmic frequency and gain axis

\[ |A(j\omega)| \]

-20dB/decade

20dB/decade
Increasing Power in a Signal with Amplifiers

The power delivered to the load is

\[ P_{\text{LOAD}} = \frac{V_{\text{OUT}}^2}{R_L} \]

\[ P_{\text{LOAD}} = V_{\text{IN}}^2 \frac{R_L}{(R_L + R_S)^2} \]

For a given power source \((V_S \text{ and } R_S)\), it can be shown that the power delivered to the load will be maximized when \(R_L = R_S\)

For a given power source \((V_S \text{ and } R_S)\), it can be shown that the power transfer efficiency is an increasing function of \(R_L\) and approaches the upper bound of 100% as \(R_L\) approaches \(\infty\)
Increasing Power in a Signal with Amplifiers

When $R_L$ selected for maximum power delivery to load,

$$P_{LOAD,MAX} = V_{IN}^2 \frac{R_L}{(R_L + R_S)^2} = V_{IN}^2 \frac{R_S}{(R_S + R_S)^2} = \frac{V_{IN}^2}{4R_S}$$

And, in this case, the same power is dissipated internal to the source in $R_S$

Thus, the source is 50% efficient at delivering power from $V_{IN}$ to the load at maximum power transfer.

Matching the load impedance to the source impedance for the purpose of delivering maximum power to the load is often termed “impedance matching”

Highly undesirable to force the load impedance to match $R_S$ for maximum power transfer!
Increasing Power in a Signal with Amplifiers

\[ P_{\text{LOAD,MAX}} = \frac{V_{\text{IN}}^2}{4R_S} \]

With an ideal amplifier

\[ P_{\text{LOAD}} = V_{\text{IN}}^2 \frac{A_V^2}{R_L} \quad P_{V_{\text{IN}}} = 0 \]

Power to load can be MUCH LARGER than power delivered by source!

Power can be increased to an arbitrary level by making \( A_V \) large!

No requirements for any impedance matching between \( R_L \) and \( R_S \)!

Concept of impedance matching irrelevant when amplifiers are used

Where does the power delivered to the load come from?

dc power supply (not shown)
Increasing Power in a Signal with Amplifiers

Can a transformer be used to boost the voltage to the load and thus increase the power transferred to the load?

\[
\begin{align*}
P_{\text{LOAD}} &= \frac{V_{\text{OUT}}^2}{R_L} = \frac{r^2V_1^2}{R_L} \\
V_1I_1 &= \frac{r^2V_1^2}{R_L} \\
V_1 &= \frac{R_L}{R_S + \frac{R_L}{r^2}}V_{\text{IN}} \\
\end{align*}
\]

\[
\begin{align*}
P_{\text{OUT}} &= \frac{V_{\text{OUT}}^2}{R_S + R_L + \frac{r^2}{r^2}} \\
P_{\text{IN}} &= \frac{V_{\text{IN}}^2}{R_S} \\
P_{\text{LOAD}} &= \frac{R_L}{R_S + \frac{R_L}{r^2}} \\
P_{\text{LOAD}} &= \frac{R_L}{R_S + \frac{R_L}{r^2}} \\
\end{align*}
\]
Increasing Power in a Signal with Amplifiers

Can a transformer be used to boost the voltage to the load and thus increase the power transferred to the load?

\[
P_{\text{LOAD,MAX}} = \frac{V_{\text{IN}}^2}{4R_S}
\]

This is always less than 1

Although a transformer can boost the voltage, it can never deliver more power to the load than is supplied by the source.
Increasing Power in a Signal with Amplifiers

Amplifier circuits can increase the average power delivered to the load but the average power delivered to the load with any passive circuit will always be less than the power supplied by the excitation.