Homework 12
Note many problems are design problems and have $\infty$ solutions, the solution here is one example and I have given reasoning for each decision.

Problem 1)
\[ I_{out} = \frac{40}{2} \cdot \frac{20}{5} \cdot \frac{2}{2} \cdot 50 \mu A \]
\[ I_{out} = 100 \mu A \]

Problem 2)
One design option is this,

![Diagram](image)

a) With this we have to choose the sizes of $M_1$ and $M_5$ to determine the 100$\mu$A sinking current and $M_1, M_2, M_3,$ and $M_4$ to create the 20mA sourcing current
As we can choose whatever sizes we want, I am going to use $W_1 = L_1 = 1\mu$, so $\frac{W_1}{L_1} = 1$.
From there I can set $M_5$, I want $\frac{W_5}{L_5} = \frac{100\mu A}{5mA} = 0.02$.
So I will use $W_5 = 1\mu$ and $L_5 = 50\mu$.
We then want to convert 5mA to 20mA, or times 4. This can be easily done in a single mirror, so I will use, $M_2 = L_2 = 1\mu$
$M_3 = L_3 = 1\mu$
Then I just need $M_4/L_4 = 4$, so $M_4 = 4\mu, L_4 = 1\mu$

b) $I_1 = \frac{\mu p C_{ox} W_4}{2L_4} (|V_{GS}| - |V_T|)^2 = 20mA$
$33 \cdot 10^{-6} \cdot 4 \cdot \frac{(|V_{GS}| - |V_T|)^2}{2 \cdot 1} = 20mA$
$|V_{GS}| - |V_T| = \sqrt{\frac{2 \cdot 1 \cdot 20mA}{33 \cdot 10^{-6} \cdot 4} = 17.4V}$
Problem 3)
This design problem has a few parts that make it a bit difficult. First it needs an input impedance of 100k-200kΩ. This seems easy enough, but our basic amplifier structures have $\infty$ except for the Common Gate design. I am not going to use that design because of a problem with getting the proper $R_{in}$, so instead I will use two CSwRS amplifiers and an extra resister, because determining the gain of CSwRS amplifiers is the easiest.

![Amplifier Circuit Diagram]

The gain of this is fairly easy to calculate, as the second stage is $A_{V2} = -\frac{R_4}{R_5}$ and the first stage is $A_{V2} = -\frac{(R_2 \parallel R_{in2})}{R_3}$, and $R_{in2} = \infty \rightarrow A_{V2} = -\frac{R_2}{R_3}$

$A_V = A_{V1} \times A_{V2} = \frac{R_2 R_4}{R_3 R_5}$

We can set,

$R_2 = 10k\Omega$

$R_3 = 1k\Omega$

$R_4 = 2k\Omega$

$R_5 = 1k\Omega$

$A_V = \frac{10 \times 2}{1 \times 1} = 20$

Finally we need to create the proper input impedance. The amplifier structure as an $R_{in1} = \infty$, but it is in parallel with $R_1$

$R_{in} = R_{in1} \parallel R_1 = R_1$ so we set $R_1$ between 100K and 200K,

$R_1 = 150k\Omega$

Problem 4)

$M = \frac{W_2 + 2\Delta w}{W_1 + 2\Delta w} \times \frac{L_1}{L_2}$

$M = \frac{12 - 0.2}{4 - 0.2} \times \frac{4}{4}$

$M = 3.1$
Problem 5)

a)  
\[ V_{01} = I_3 R_2 = \frac{A_3}{A_2} \beta l_1 R_2 \]
\[ V_{01} = \left( \frac{V_{DD} - 0.6}{R_1} \right) \beta \frac{A_3}{A_2} * R_2 \]
\[ V_{02} = I_6 R_3 = \frac{w_6}{w_5} * \frac{A_4}{A_1} * \beta l_1 R_3 \]
\[ V_{02} = \left( \frac{V_{DD} - 0.6}{R_1} \right) * A_4 \frac{W_6 L_5}{W_5 L_6} R_3 \]

b)  
\[ \left( \frac{10 - 0.6}{60k} \right) * 100 * \frac{25}{100} * R_2 = 3V \]
\[ R_2 = 255.3\Omega \]
\[ \left( \frac{10 - 0.6}{60k} \right) * 100 * \frac{300}{100} * \frac{16 * 1}{10 * 4} * R_3 = 6V \]
\[ R_3 = 319.1\Omega \]

Problem 6)

a)  
\[ A_{VCC} = - \frac{g_{m1}}{g_{01}} \]

b)  
\[ A_{VCC} = - \frac{1}{2} \left( \frac{g_{m1}}{g_{01}} * \frac{g_{m2}}{g_{02}} \right) \]

c)  
\[ A_{VCC} = - \left[ \frac{g_{m1}}{g_{01}} \frac{g_{m2}}{g_{02}} \right] \]

Problem 7

A)  
\[ A_{V3} = - \frac{g_{m5}}{-g_{05} + g_{m6} + g_{06}} \approx - \frac{g_{m5}}{g_{m6}} \]
\[ A_{V2} = - \frac{g_{m4}}{g_{m3}} ; \quad A_{V1} = - \frac{g_{m1}}{g_{m2}} \]
\[ A_V = A_{V1} A_{V2} A_{V3} = - \frac{g_{m1} g_{m4} g_{m5}}{g_{m2} g_{m3} g_{m6}} \]
B)

\[ l_{m1} = l_{m4} = l_{m5} = 500\mu A \]

\[ g_m = \frac{2I_{CQ}}{V_{GSQ} - V_T} \]

\[ V_{GS2} = V_{TP} + \frac{l_{M1}}{\mu C_{OX} \left( \frac{W_2}{2L_2} \right)} = 2.5V \]

\[ V_{GS3} = V_{TP} + \frac{l_{M4}}{\mu C_{OX} \left( \frac{W_3}{2L_3} \right)} = 2.5V \]

\[ V_{GS6} = V_{TP} + \frac{l_{M5}}{\mu C_{OX} \left( \frac{W_6}{2L_6} \right)} = 4V \]

\[ g_{m1} = \frac{2 \times 500\mu}{5 - 0.5} = 1000\mu A \]

\[ g_{m2} = g_{m3} = g_{m4} = g_{m5} = \frac{2 \times 500\mu}{2.5 - 0.5} = \frac{500\mu A}{V} \]

\[ g_{m6} = \frac{2 \times 500\mu}{4 - 0.5} = \frac{1000\mu A}{3.5 V} \]

\[ A_V = -\frac{g_{m1} g_{m4} g_{m5}}{g_{m2} g_{m3} g_{m6}} = -\frac{g_{m1}}{g_{m6}} \]

\[ A_V = -\frac{3.5}{4.5} = -0.777 \]
Problem 8)

\[
\frac{V_{out}}{V_A} = -\frac{R_2}{R_3}
\]

\[
\frac{V_A}{V_{in}} = g_{m2}R_1 \left( \frac{g_{m1}}{g_{m1}g_{m2}} \right)
\]

\[
A_V = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_A} \ast \frac{V_A}{V_{in}} = -\frac{R_2}{R_3} g_{m2}R_1 \ast \left( \frac{g_{m1}}{g_{m1} + g_{m2}} \right)
\]

Problem 9)

\[
A_V = -\frac{R_2}{R_3} g_{m2}R_1 \ast \left( \frac{g_{m1}}{g_{m1} + g_{m2}} \right)
\]

\[
g_{m1} = \frac{2I_{CQ1}}{V_{GSS1} - V_{TN}} ; I_{CQ1} = 200\mu + I_{CQ2}
\]

\[
I_{CQ2} = J_s A_E_2 \exp \left( \frac{V_{BE}}{V_t} \right) = 10^{-14} \ast 100 \ast \exp \left( \frac{0.6}{0.0259} \right) = 0.0115A
\]

\[
I_{CQ1} = 200\mu + 11.5m = 11.7mA
\]

\[
g_{m2} = \frac{I_{CQ2}}{V_t} = \frac{0.0115}{0.0259} = 0.444A
\]

\[
g_{m3} = \frac{2(0.0117)}{(V_{GSS1} - V_{TN})} = \sqrt{2\mu_n C_{ox} \left( \frac{w_1}{L_1} \right) I_{CQ1}} = 3.06 \ast 10^{-3}
\]

\[
A_V = -\frac{25000}{12500} \ast 0.444 \ast 3.06 \ast 10^{-3}
\]

\[
A_V = -48.6
\]
Problem 10)

\[ A_v = (-g_{m1}R_1) * g_{m3} \left( \frac{1}{g_{m2}} || R_{in4} \right) * \left( -\frac{R_3 || R_6}{R_2} \right) ; \quad R_{in4} = r_{\pi4} + \beta R_2 \]

Problem 11)

For this problem we have BJTs to create a gain of 100 and input impedance greater than 400k. The load is 1k. The easiest amplifier to create is the CEwRE (Common Emitter with Emitter Resistor), but has the problem of creating a negative gain. So we need two stages of negative gain.

We have to work backwards for this circuit because the input impedance of Q2 (\(R_{in2}\)) will affect the gain of Q1. First I will set \(V_{dd} = 5V\)

\(R_4\) is already determined, as it is the load resistor, \(R_4 = 1k\Omega\)

We want a total gain of 100, and the gain of the above is \(-\frac{R_C}{R_E} = -\frac{R_4}{R_5}\). Therefore if we use \(R_5 = 500\Omega\) we get a gain of \(A_{v2} = -\frac{1000}{500} = -2\) and need a gain of -50 in the first stage.

The first stage has a gain of \(-\frac{R_2 || R_{in2}}{R_3} \cdot R_{in2} = \beta \left( \frac{V_t}{I_{CQ2}} + R_E \right) = 100 \left( \frac{0.0259}{I_{CQ2}} + 500\Omega \right)\).

\(I_{CQ} = \beta I_{B2}, I_{B2} = \frac{5V - 0.6 - V_{R5}}{R_2} = \frac{V_{R5}}{500}\) We want this small so that the resistance is large.

I am going to set \(R_2 = 500k\Omega\). This sets \(V_{R5} = 4.39mV\) and \(I_B = 8.79\mu A\). \(I_C = 100(i_B) = 879\mu A\)

\(R_{in2} = 100 * \left( \frac{0.0259}{0.000879 + 500} \right) = 52.9k\Omega\)

\(R_2 || R_{in2} = 47.9k\Omega\), so we want \(R_3 = \frac{47.9k}{30} = 1595\Omega\)

\(A_{v1} = -\frac{47900}{1595} \approx -30\)

\(A_v = A_{v1} * A_{v2} = -30 * -2 \rightarrow A_v = 60\frac{V}{V}\)

The input impedance of the system is \(100 * \left( \frac{0.0259}{I_{CQ1}} + 957 \right)\). \(I_{CQ1} = \beta I_{B1}\), we want this really small try \(R_1 = 7M\Omega\) → \(I_{B1} = 0.628\mu A\) → \(I_{C1} = 62.8\mu A\) → \(R_{in1} = 200.7k\Omega\)
Problem 12) A)

SS equivalent circuit:

\[
A_V = -(g_{m1} + g_{m2}) \times \left( 1 + \frac{1}{g_{01}} \right)
\]

\[
I_{DQ} = I_{CQ} = \lambda I_C \exp \left( \frac{V_{BE}}{V_t} \right) = 10.5\text{mA}
\]

\[
g_{m2} = \frac{2I_{CQ}}{V_{GS} - V_t} = \frac{2 \times 0.0105}{5V - 0.5V} = \frac{0.021}{4.5} = 0.004667
\]

\[
g_{m1} = \frac{I_{CQ}}{V_t} = \frac{0.0105}{0.0259} = 0.4054
\]

\[
g_{01} = \frac{I_{CQ}}{V_{AF}} = \frac{0.0105}{100} = 0.000105
\]

\[
g_{02} = \lambda I_{DQ} = 0.01 \times 0.0105 = 0.000105
\]

\[
\frac{1}{g_{02}} = \frac{1}{g_{01}} = \frac{1}{0.000105} = 9523.8
\]

\[
A_V = -(0.4054 + 0.00467) \times \left( 1 + \frac{1}{1000} + \frac{1}{9523} + \frac{1}{9523} \right)
\]

\[
A_V = -338.9\text{V}
\]

B)

\[
V_{out} = A_V V_{in}
\]

\[
V_{out} = 3.38\sin(1000t)
\]