Bipolar Device Modeling

Bipolar Process
Basic Devices and Device Models

- Resistor
- Diode
- Capacitor
- MOSFET
- BJT
Bipolar Junction Transistors

Operation and Modeling
With proper doping and device sizing these form Bipolar Transistors

- Bipolar Devices Show Basic Symmetry
- Electrical Properties not Symmetric
- Designation of C and E critical

Review from Last Lecture
Review from Last Lecture

Bipolar Operation

Consider npn transistor

So, what will happen?

Some will recombine with holes and contribute to base current and some will be attracted across BC junction and contribute to collector.

Size and thickness of base region and relative doping levels will play key role in percent of minority carriers injected into base contributing to collector current.
Bipolar Operation

Consider npn transistor

Under forward BE bias and reverse BC bias current flows into base region.

Efficiency at which minority carriers injected into base region and contribute to collector current is termed $\alpha$.

$\alpha$ is always less than 1 but for a good transistor, it is very close to 1.

For good transistors, $0.99 < \alpha < 0.999$.

Making the base region very thin makes $\alpha$ large.
In contrast to MOS devices where current flow in channel is by majority carriers, current flow in the critical base region of bipolar transistors is by minority carriers.
Bipolar Operation

\[ I_C = \beta I_B \]

*\( \beta \) is typically very large

Bipolar transistor can be thought of a current amplifier with a large current gain.

In contrast, MOS transistor is inherently a transconductance amplifier.

Current flow in base is governed by the diode equation:

\[ I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]

Collector current thus varies exponentially with \( V_{BE} \):

\[ I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]
Bipolar Operation

\[ I_C = \beta I_B \]

\( \beta \) is typically very large

Collector current thus varies exponentially with \( V_{BE} \)

\[ I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]

This exponential relationship (in contrast to the square-law relationship for the MOSFET) provides a very large gain for the BJT and this property is very useful for many applications!!
Bipolar Models

Simple dc Model

following convention, pick $I_C$ and $I_B$ as dependent variables and $V_{BE}$ and $V_{CE}$ as independent variables
Summary:

\[
I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}}
\]

\[
I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}}
\]

\[
V_t = \frac{kT}{q}
\]

This has the properties we are looking for but the variables we used in introducing these relationships are not standard.

It can be shown that \(\tilde{I}_S\) is proportional to the emitter area \(A_E\).

Define \(\tilde{I}_S = \beta^{-1} J_S A_E\) and substitute this into the above equations.
Simple dc model

\[ I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]
\[ I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]
\[ V_t = \frac{kT}{q} \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]
\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \]
\[ V_t = \frac{kT}{q} \]

\( J_S \) is termed the saturation current density

Process Parameters: \( J_S, \beta \)

Design Parameters: \( A_E \)

Environmental parameters and physical constants: \( k, T, q \)

At room temperature, \( V_t \) is around 26mV
\( J_S \) very small – around .25fA/u^2
Transfer Characteristics

\[ J_S = 0.25 \text{fA/u}^2 \]
\[ A_E = 400 \text{u}^2 \]

\( V_{BE} \) close to 0.6V for a two decade change in \( I_C \) around 1mA
Transfer Characteristics

$J_S = 0.25\text{fA}/u^2$

$A_E = 400u^2$

$V_{BE}$ close to 0.6V for a four decade change in $I_C$ around 1mA
Simple dc model

Output Characteristics

\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \]
Simple dc model

Better Model of Output Characteristics

\[ I_C \]

\[ V_{CE} \]

\[ V_{BE} \text{ or } I_B \]
Simple dc model

Typical Output Characteristics

Forward Active region of BJT is analogous to Saturation region of MOSFET

Saturation region of BJT is analogous to Triode region of MOSFET
Projections of these tangential lines all intercept the \(-V_{CE}\) axis at the same place and this is termed the Early voltage, \(V_{AF}\) (actually \(-V_{AF}\) is intercept)

Typical values of \(V_{AF}\) are in the 100V range
Simple dc model

Improved Model

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]
\[ I_C = J_S e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right) \]

Valid only in Forward Active Region
Simple dc model

Improved Model

\[ \begin{align*}
V_t &= \frac{kT}{q} \\
I_E &= -\frac{J_SA_E}{\alpha_F} \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) + J_SA_E \left( e^{\frac{V_{BC}}{V_t}} - 1 \right) \\
I_C &= J_SA_E \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) - \frac{J_SA_E}{\alpha_R} \left( e^{\frac{V_{BC}}{V_t}} - 1 \right)
\end{align*} \]

Valid in all regions of operation

\( V_{AF} \) effects can be added

Not mathematically easy to work with

Note dependent variables changes

Termed Ebers-Moll model

Reduces to previous model in FA region
Simple dc model

Ebers-Moll model

\[
I_E = -\frac{J_SA_E}{\alpha_F} \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) + J_SA_E \left( e^{\frac{V_{BC}}{V_t}} - 1 \right) \\
V_t = \frac{kT}{q} \\
I_C = J_SA_E \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) - \frac{J_SA_E}{\alpha_R} \left( e^{\frac{V_{BC}}{V_t}} - 1 \right)
\]

Process Parameters: \{J_S, \alpha_F, \alpha_R\}

Design Parameters: \{A_E\}

\(\alpha_F\) is the parameter \(\alpha\) discussed earlier
\(\alpha_R\) is termed the “reverse \(\alpha\)”

\[
\beta_F = \frac{\alpha_F}{1-\alpha_F} \quad \beta_R = \frac{\alpha_R}{1-\alpha_R}
\]

Typical values for process parameters:

\(J_S \sim 10^{-16}A/\mu^2 \quad \beta_F \sim 100, \quad \beta_R \sim 0.4\)
Simple dc model

Ebers-Moll model

\[ I_E = - \frac{J_S A_E}{\alpha_F} \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) + J_S A_E \left( e^{\frac{V_{BC}}{V_t}} - 1 \right) \]

\[ V_t = \frac{kT}{q} \]

\[ I_C = J_S A_E \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) - \frac{J_S A_E}{\alpha_R} \left( e^{\frac{V_{BC}}{V_t}} - 1 \right) \]

With typical values for process parameters in forward active region (\( V_{BE} \sim 0.6V, V_{BC} \sim -3V \)), with \( V_t = 26mV \) and if \( A_E = 100\mu^2 \):

\( J_S \sim 10^{-16}A/\mu^2 \)

\( \beta_F \sim 100, \quad \beta_R \sim 0.4 \)

\[ I_C = J_S A_E \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) - \frac{J_S A_E}{\alpha_R} \left( e^{\frac{V_{BC}}{V_t}} - 1 \right) \]

\[ I_C = 10^{-14} \left( 1.05 \times 10^{10} - 1 \right) - \frac{10^{-14}}{.28} \left( 7.7 \times 10^{-51} - 1 \right) \]

Completely dominant!

Makes no sense to keep anything other than \( I_C = J_S A_E \left( e^{\frac{V_{BE}}{V_t}} \right) \) in forward active
Simple dc model

Ebes-Moll model

\[
I_E = -\frac{J_S A_E}{\alpha_F} \left( e^{v_{BE}/v_t} - 1 \right) + J_S A_E \left( e^{v_{BC}/v_t} - 1 \right)
\]

\[
V_t = \frac{kT}{q}
\]

\[
I_C = J_S A_E \left( e^{v_{BE}/v_t} - 1 \right) - \frac{J_S A_E}{\alpha_R} \left( e^{v_{BC}/v_t} - 1 \right)
\]

Alternate equivalent expressions for dependent variables \{I_C, I_B\} defined earlier for Ebers-Moll equations in terms of independent variables \{V_{BE}, V_{CE}\} after dropping the “-1” terms

\[
I_C = J_S A_E e^{v_{BE}/v_t} \left( 1 - \left[ \frac{1+\beta_R}{\beta_R} \right] e^{-v_{CE}/v_t} \right)
\]

\[
I_B = J_S A_E e^{v_{BE}/v_t} \left( \frac{1}{\beta_F} - \frac{1}{\beta_R} e^{-v_{CE}/v_t} \right)
\]

No more useful than previous equation but in form consistent with notation introduced earlier.
Simple dc model

Simplified Multi-Region Model

Ebers-Moll Model

Simplified Multi-Region Model

- Observe $V_{CE}$ around 0.2V when saturated
- $V_{BE}$ around 0.6V when saturated
- In most applications, exact $V_{CE}$ and $V_{BE}$ voltage in saturation not critical

$V_{BE}=0.7V$

$V_{CE}=0.2V$

Saturation
Simple dc model

Simplified Multi-Region Model

\[
I_C = J_S A_e \frac{v_{BE}}{v_t} \left( 1 - \left[ \frac{1 + \beta_R}{\beta_R} \right] e^{-\frac{v_{CE}}{v_t}} \right) \left( 1 + \frac{v_{CE}}{V_{AF}} \right)
\]

\[
I_B = J_S A_e \frac{v_{BE}}{v_t} \left( \frac{1}{\beta_F} - \frac{1}{\beta_R} e^{-\frac{v_{CE}}{v_t}} \right)
\]
Simple dc model

Simplified Multi-Region Model

\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right) \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ V_t = \frac{kT}{q} \]

Forward Active

\[ V_{BE} = 0.7V \]

Saturation

\[ V_{CE} = 0.2V \]

Cutoff

\[ I_C = I_B = 0 \]
Simple dc model

Simplified Multi-Region Model

\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ V_t = \frac{kT}{q} \]

- \( V_{BE} > 0.4V \) \( \Rightarrow \) Forward Active \( V_{BC} < 0 \)

- \( V_{BE} = 0.7V \)
- \( V_{CE} = 0.2V \) \( \Rightarrow \) Saturation \( I_C < \beta I_B \)

- \( I_C = I_B = 0 \) \( \Rightarrow \) Cutoff \( V_{BE} < 0 \)
- \( V_{BC} < 0 \)

A small portion of the operating region is missed with this model but seldom operate in the missing region.
Simple dc model

Equivalent Simplified Multi-Region Model

\[ I_C = \beta I_B \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ V_t = \frac{kT}{q} \]

- \( V_{BE} > 0.4V \) \quad Forward Active
- \( V_{BC} < 0 \)

\( V_{BE}=0.7V \) \quad \( I_C < \beta I_B \) \quad Saturation

\( V_{CE}=0.2V \)

\( I_C=I_B=0 \) \quad \( V_{BE}<0 \) \quad Cutoff

\( V_{BC}<0 \)

A small portion of the operating region is missed with this model but seldom operate in the missing region.
Simplified dc model

Forward Active

Adequate when it makes little difference whether $V_{BE}=0.6V$ or $V_{BE}=0.7V$
Simplified dc model

Forward Active

Mathematically

\[ V_{BE} = 0.6V \]
\[ I_C = \beta I_B \]

Or, if want to show slope in \( I_C - V_{CE} \) characteristics

\[ V_{BE} = 0.6V \]
\[ I_C = \beta I_B (1 + V_{CE}/V_{AF}) \]
Simplified dc model

Equivalent Simplified Multi-Region Model

\[ I_C = \beta I_B \]

\[ V_{BE} = 0.6V \]

\[ V_t = \frac{kT}{q} \]

- \( V_{BE} > 0.4V \) and \( V_{BC} < 0 \) \( \rightarrow \) Forward Active

- \( V_{BE} = 0.7V \) and \( V_{CE} = 0.2V \) \( \rightarrow \) Saturation

- \( I_C = I_B = 0 \) \( \rightarrow \) Cutoff

A small portion of the operating region is missed with this model but seldom operate in the missing region
Conditions for Regions of Operation in Multi-Region Model

\[ V_{BE} > 0.4V \]  
\[ V_{BC} < 0 \]  
Forward Active

\[ I_C < \beta I_B \]  
Saturation

\[ V_{BE} < 0 \]  
\[ V_{BC} < 0 \]  
Cutoff

Note: One condition is on dependent variables!

Observe that in saturation, \( I_C < \beta I_B \)

Can’t condition on independent variables in saturation because they are fixed in the model.
Regions of Operation in Independent Parameter Domain used in multi-region models

Seldom operate in regions excluded in this picture
Excessive Power Dissipation if either junction has large forward bias
Safe regions of operation

Simplified FW Saturation Model

Melt Down!!

Forward Active

Cutoff

Reverse Active

Saturation

V_{BE}

V_{BC}

0.4V

0.4V
Actually cutoff, forward active, and reverse active regions can be extended modestly as shown and multi-region models still are quite good.
Sufficient regions of operation for most applications

- Forward Active
- Cutoff
- Reverse Active
  - Simplified FW Saturation Model
  - Saturation

Voltage regions:
- $V_{BE}$
- $V_{BC}$
End of Lecture 18