EE 330
Lecture 18

Bipolar Device Modeling

Bipolar Process
Basic Devices and Device Models

- Resistor
- Diode
- Capacitor
- MOSFET
- BJT
Bipolar Junction Transistors

Operation and Modeling
With proper doping and device sizing these form Bipolar Transistors

- Bipolar Devices Show Basic Symmetry
- Electrical Properties not Symmetric
- Designation of C and E critical
Consider npn transistor

So, what will happen?

Some will recombine with holes and contribute to base current and some will be attracted across BC junction and contribute to collector.

Size and thickness of base region and relative doping levels will play key role in percent of minority carriers injected into base contributing to collector current.
Under forward BE bias and reverse BC bias current flows into base region. Efficiency at which minority carriers injected into base region and contribute to collector current is termed $\alpha$. $\alpha$ is always less than 1 but for a good transistor, it is very close to 1. For good transistors $0.99 < \alpha < 0.999$. Making the base region very thin makes $\alpha$ large.
Bipolar Operation

Consider npn transistor

In contrast to MOS devices where current flow in channel is by majority carriers, current flow in the critical base region of bipolar transistors is by minority carriers.
Bipolar Operation

$\beta = \frac{I_C}{I_B}$ is typically very large

Bipolar transistor can be thought of a current amplifier with a large current gain

In contrast, MOS transistor is inherently a transconductance amplifier

Current flow in base is governed by the diode equation

$\begin{align*}
I_B &= \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \\
I_C &= \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}}
\end{align*}$

Collector current thus varies exponentially with $V_{BE}$
Simple dc model

Summary:

\[ I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]
\[ I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]
\[ V_t = \frac{kT}{q} \]

This has the properties we are looking for but the variables we used in introducing these relationships are not standard.

It can be shown that \( \tilde{I}_S \) is proportional to the emitter area \( A_E \).

Define \( \tilde{I}_S = \beta^{-1} J_S A_E \) and substitute this into the above equations.

Review from Last Lecture
Review from Last Lecture

Simple dc model

\[ I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]
\[ I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]
\[ V_t = \frac{kT}{q} \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]
\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \]
\[ V_t = \frac{kT}{q} \]

\[ J_S \] is termed the saturation current density

Process Parameters: \( J_S, \beta \)

Design Parameters: \( A_E \)

Environmental parameters and physical constants: \( k, T, q \)

At room temperature, \( V_t \) is around 26mV
\( J_S \) very small – around .25fA/u^2
Review from Last Lecture

Simple dc model

Typical Output Characteristics

Forward Active region of BJT is analogous to Saturation region of MOSFET
Saturation region of BJT is analogous to Triode region of MOSFET
Projections of these tangential lines all intercept the $-V_{CE}$ axis at the same place and this is termed the Early voltage, $V_{AF}$ (actually $-V_{AF}$ is intercept).

Typical values of $V_{AF}$ are in the 100V range.
Simple dc model

Improved Model

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ I_C = J_S e^{\frac{V_{BE}}{V_t}} \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

Valid only in Forward Active Region
Simple dc model

Improved Model

\[ V_t = \frac{kT}{q} \]

\[ I_E = -J_S A_E \frac{e^{V_{BE}/V_t} - 1}{\alpha_F} + J_S A_E \left( e^{V_{BC}/V_t} - 1 \right) \]

\[ I_C = J_S A_E \left( e^{V_{BE}/V_t} - 1 \right) - J_S A_E \left( e^{V_{BC}/V_t} - 1 \right) \frac{e^{V_{BC}/V_t}}{\alpha_R} \]

*Valid in All regions of operation*  
*Not mathematically easy to work with*  
*Note dependent variables changes*  
*Termed Ebers-Moll model*  
*Reduces to previous model in FA region*
Simple dc model

Ebers-Moll model

\[ I_E = -\frac{J_S A_E}{\alpha_F} \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) + J_S A_E \left( e^{\frac{V_{BC}}{V_t}} - 1 \right) \]

\[ V_t = \frac{kT}{q} \]

\[ I_C = J_S A_E \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) - \frac{J_S A_E}{\alpha_R} \left( e^{\frac{V_{BC}}{V_t}} - 1 \right) \]

Process Parameters: \{J_S, \alpha_F, \alpha_R\}

Design Parameters: \{A_E\}

\( \alpha_F \) is the parameter \( \alpha \) discussed earlier
\( \alpha_R \) is termed the “reverse \( \alpha \)

\[ \beta_F = \frac{\alpha_F}{1 - \alpha_F} \quad \beta_R = \frac{\alpha_R}{1 - \alpha_R} \]

Typical values for process parameters:

\( J_S \sim 10^{-16} A/\mu^2 \quad \beta_F \sim 100, \quad \beta_R \sim 0.4 \)
Simple dc model

Ebers-Moll model

\[ I_E = -\frac{J_S A_E}{\alpha_F} \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) + J_S A_E \left( e^{\frac{V_{BC}}{V_t}} - 1 \right) \]

\[ V_t = \frac{kT}{q} \]

\[ I_C = J_S A_E \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) - \frac{J_S A_E}{\alpha_R} \left( e^{\frac{V_{BC}}{V_t}} - 1 \right) \]

With typical values for process parameters in forward active region (\( V_{BE} \sim 0.6\, \text{V}, \ V_{BC} \sim 3\, \text{V} \)), with \( V_t = 26\, \text{mV} \) and if \( A_E = 100\mu^2 \):

\[ J_S \sim 10^{-16} \, \text{A/\mu}^2 \]

\[ \beta_F \sim 100, \quad \beta_R \sim 0.4 \]

\[ I_C = J_S A_E \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) - \frac{J_S A_E}{\alpha_R} \left( e^{\frac{V_{BC}}{V_t}} - 1 \right) \]

\[ I_C = 10^{-14} \left( 1.05 \times 10^{10} \right) - 10^{-14} \left( \frac{7.7 \times 10^{-51}}{.28} - 1 \right) \]

Makes no sense to keep anything other than \( I_C = J_S A_E \left( e^{\frac{V_{BE}}{V_t}} \right) \) in forward active
Simple dc model

Ebes-Moll model

\[
I_E = -J_S A_E \left( \frac{V_{BE}}{e^{V_t} - 1} \right) + J_S A_E \left( \frac{V_{BC}}{e^{V_t} - 1} \right)
\]

\[
V_t = \frac{kT}{q}
\]

\[
I_C = J_S A_E \left( e^{\frac{V_{BE}}{V_t} - 1} \right) - \frac{J_S A_E}{\alpha_R} \left( \frac{V_{BC}}{e^{V_t} - 1} \right)
\]

Alternate equivalent expressions for dependent variables \(\{I_C, I_B\}\) defined earlier for Ebers-Moll equations in terms of independent variables \(\{V_{BE}, V_{CE}\}\) after dropping the “-1” terms

\[
I_C = J_S A_E \left( \frac{V_{BE}}{V_t} \right) \left( 1 - \frac{1+\beta_R}{\beta_R} \right) e^{-\frac{V_{CE}}{V_t}}
\]

\[
I_B = J_S A_E \left( \frac{1}{\beta_F} - \frac{1}{\beta_R} \right) e^{-\frac{V_{CE}}{V_t}}
\]

No more useful than previous equation but in form consistent with notation Introduced earlier
Simple dc model

Simplified Multi-Region Model

- Observe $V_{CE}$ around 0.2V when saturated
- $V_{BE}$ around 0.6V when saturated
- In most applications, exact $V_{CE}$ and $V_{BE}$ voltage in saturation not critical

$V_{BE} = 0.7V$

$V_{CE} = 0.2V$
Simple dc model

Simplified Multi-Region Model

I_c = J_s A_e \frac{v_b}{v_t} \left[ 1 - \left( \frac{1}{\beta_F} - 1 \right) e^{-\frac{v_c}{v_t}} \right] \left( 1 + \frac{v_c}{v_{AF}} \right)

I_b = \frac{J_s A_e}{\beta} \left( \frac{v_b}{v_t} \right) \left( 1 + \frac{v_c}{v_{AF}} \right)

V_{BE} = 0.7V
V_{CE} = 0.2V
I_c = I_b = 0

Forward Active
Saturation
Cutoff
Simple dc model

Simplified Multi-Region Model

\[ I_C = J_S A_E \frac{V_{BE}}{V_t} \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

\[ I_B = \frac{J_S A_E}{\beta} \frac{V_{BE}}{V_t} \]

\[ V_t = \frac{kT}{q} \]

Forward Active

Saturation

\[ V_{BE} = 0.7V \]
\[ V_{CE} = 0.2V \]

Cutoff

\[ I_C = I_B = 0 \]
Simple dc model

Simplified Multi-Region Model

\[ I_C = J_S A_e \left( \frac{V_{BE}}{V_t} \right) \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]  
\[ I_B = \frac{J_S A_e}{\beta} \left( \frac{V_{BE}}{V_t} \right) \]

- **Forward Active**
  - \( V_{BE} > 0.4V \)
  - \( V_{BC} < 0 \)

- **Saturation**
  - \( V_{BE} = 0.7V \)
  - \( V_{CE} = 0.2V \)
  - \( I_C < \beta I_B \)

- **Cutoff**
  - \( I_C = I_B = 0 \)
  - \( V_{BE} < 0 \)
  - \( V_{BC} < 0 \)

A small portion of the operating region is missed with this model but seldom operate in the missing region.
Simple dc model

Equivalent Simplified Multi-Region Model

\[ I_C = \beta I_B \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ V_t = \frac{kT}{q} \]

\[ V_{BE} = 0.7V \]

\[ V_{CE} = 0.2V \]

\[ I_C = I_B = 0 \]

\[ V_{BE} < 0 \]

\[ V_{BC} < 0 \]

A small portion of the operating region is missed with this model but seldom operate in the missing region.
Simplified dc model

Forward Active

Adequate when it makes little difference whether $V_{BE} = 0.6V$ or $V_{BE} = 0.7V$
Simplified dc model

Forward Active

Mathematically

\[ V_{BE} = 0.6V \]
\[ I_C = \beta I_B \]

Or, if want to show slope in \( I_C-V_{CE} \) characteristics

\[ V_{BE} = 0.6V \]
\[ I_C = \beta I_B (1 + V_{CE}/V_{AF}) \]
Simplified dc model

Equivalent Simplified Multi-Region Model

\[ I_C = \beta I_B \]

\[ V_{BE} = 0.6V \]

\[ V_t = \frac{kT}{q} \]

- \[ V_{BE} > 0.4V \] --- Forward Active
- \[ V_{BC} < 0 \]

- \[ V_{BE} = 0.7V \]
- \[ V_{CE} = 0.2V \] --- Saturation
- \[ I_C < \beta I_B \]

- \[ I_C = I_B = 0 \] --- Cutoff

\[ V_{BE} < 0 \]
\[ V_{BC} < 0 \]

A small portion of the operating region is missed with this model but seldom operate in the missing region.
Conditions for Regions of Operation in Multi-Region Model

Note: One condition is on dependent variables!

\[ V_{BE} > 0.4V \quad \text{Forward Active} \]
\[ V_{BC} < 0 \]
\[ I_C < \beta I_B \quad \text{Saturation} \]
\[ V_{BE} < 0 \]
\[ V_{BC} < 0 \quad \text{Cutoff} \]

Observe that in saturation, \( I_C < \beta I_B \)

Can’t condition on independent variables in saturation because they are fixed in the model.
Regions of Operation in Independent Parameter Domain used in multi-region models

Seldom operate in regions excluded in this picture
Excessive Power Dissipation if either junction has large forward bias
Safe regions of operation

- Forward Active
- Cutoff
- Reverse Active
- Simplified FW Saturation Model
- Melt Down!!
Actually cutoff, forward active, and reverse active regions can be extended modestly as shown and multi-region models still are quite good.
Sufficient regions of operation for most applications
Example: Determine $I_C$ and $V_{OUT}$

$J_s = 10^{-16} \text{A/}\mu^2$

$\beta = 100$
Example: Determine $I_C$ and $V_{OUT}$, assume $C$ is large and $V_{IN}$ is very small.

$$V_{OUT}$$

$$A_\text{e}=100\mu^2$$

$$J_s=10^{-16}\text{A/}\mu^2$$

$$\beta=100$$
Example: Determine $I_C$ and $V_{\text{OUT}}$. Assume $C$ is large and $V_{\text{IN}}$ is very small.
End of Lecture 18