EE 330
Lecture 19

Bipolar Device Modeling
Bipolar Devices Show Basic Symmetry

Electrical Properties not Symmetric

Designation of C and E critical

With proper doping and device sizing these form Bipolar Transistors
So, what will happen?

Some will recombine with holes and contribute to base current and some will be attracted across BC junction and contribute to collector.

Size and thickness of base region and relative doping levels will play key role in percent of minority carriers injected into base contributing to collector current.
Bipolar Operation

Consider npn transistor – Forward Active Operation

\[ I_C + I_B = -I_E \]
\[ I_C = -\alpha I_E \]

\[ I_C = \frac{\alpha}{1-\alpha} I_B \]

\[ \beta = \frac{\alpha}{1-\alpha} \]
\[ I_C = \beta I_B \]

β is typically very large
often 50<β<999
Bipolar Operation

Consider npn transistor – Forward Active Operation

\[ I_C = \beta I_B \]

\( \beta \) is typically very large

Bipolar transistor can be thought of a current amplifier with a large current gain.

In contrast, MOS transistor is inherently a transconductance amplifier.

Current flow in base is governed by the diode equation:

\[ I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]

Collector current thus varies exponentially with \( V_{BE} \):

\[ I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]
Bipolar Operation

Consider npn transistor – Forward Active Operation

\[ I_C = \beta I_B \]

\( \beta \) is typically very large

Collector current thus varies exponentially with \( V_{BE} \)

\[ I_C = \beta I_S e^{\frac{V_{BE}}{V_t}} \]

This exponential relationship (in contrast to the square-law relationship for the MOSFET) provides a very large gain for the BJT and this property is very useful for many applications!!
Simple dc model

n-p-n transistor – Forward Active Operation

\[ I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]
\[ I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]
\[ V_t = \frac{kT}{q} \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]
\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \]
\[ V_t = \frac{kT}{q} \]

\( J_S \) is termed the saturation current density

Process Parameters: \( J_S, \beta \)

Design Parameters: \( A_E \)

Environmental parameters and physical constants: \( k, T, q \)

At room temperature, \( V_t \) is around 26mV

\( J_S \) very small – around .25fA/u^2
Simple dc model

npp transistor – Forward Active Operation

Output Characteristics

\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \]
Simple dc model

Better Model of Output Characteristics
Simple dc model

Typical Output Characteristics

Forward Active region of BJT is analogous to Saturation region of MOSFET
Saturation region of BJT is analogous to Triode region of MOSFET
Simple dc model

Output Characteristics in Forward Active Region

\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \]
Better Model of Output Characteristics

\[ I_C \]

\[ V_{BE} \text{ or } I_B \]

\[ V_{CE} \]
BJT and MOSFET Comparison

- Same general characteristics
- Spacings a bit different (Exponential vs square law)
- Slope steeper for small $V_{CE}$ compared to $V_{DS}$
Simple dc model

Typical Output Characteristics

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Simple dc model

Output Characteristics in Forward Active Region

\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \]
Better Model of Output Characteristics

With scaled $V_{CE}$ axis, transition in saturation very steep
BJT and MOSFET Comparison

• Same general characteristics
• Spacings a bit different (Exponential vs square law)
• Slope steeper for small $V_{CE}$ compared to $V_{DS}$
Simple dc model

Typical Output Characteristics

Projections of these tangential lines all intercept the \(-V_{CE}\) axis at the same place and this is termed the Early voltage, \(V_{AF}\) (actually \(-V_{AF}\) is intercept).

Typical values of \(V_{AF}\) are in the 100V range.
Simple BJT dc model

Typical Output Characteristics

$I_C$ vs $V_{CE}$ or $V_{BE}$ or $I_B$
Simple BJT dc model

Typical Output Characteristics

- Saturation
- Forward Active
- Cutoff

$I_C$ vs. $V_{CE}$

$V_{BE}$ or $I_B$
Simple BJT dc model

Typical Output Characteristics

Forward Active region of BJT is analogous to Saturation region of MOSFET
Saturation region of BJT is analogous to Triode region of MOSFET

Forward Active region of BJT is analogous to Saturation region of MOSFET
Saturation region of BJT is analogous to Triode region of MOSFET
Simple dc model

Improved Model

\[ I_C = J_S A_E \exp\left(\frac{V_{BE}}{V_t}\right) \left(1 + \frac{V_{CE}}{V_{AF}}\right) \]

\[ I_B = \frac{J_S A_E}{\beta} \exp\left(\frac{V_{BE}}{V_t}\right) \]

Valid only in Forward Active Region
Improved dc model

\[ V_t = \frac{kT}{q} \]

\[
I_E = -J_S A_E \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) + J_S A_E \left( e^{\frac{V_{BC}}{V_t}} - 1 \right)
\]

\[
I_C = J_S A_E \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) - J_S A_E \left( \frac{V_{BC}}{V_t} - 1 \right)
\]

- Valid in All regions of operation
- \( V_{AF} \) effects can be added
- Not mathematically easy to work with
- Note dependent variables changes
- Termed Ebers-Moll model
- Reduces to previous model in FA region
Improved dc model

- Valid in all regions of operation
- $V_{AF}$ effects can be added
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$V_t = \frac{kT}{q}$

$I_E = -J_S A_E \left( \frac{V_{BE}}{V_t} - 1 \right) + J_S A_E \left( e^{\frac{V_{BC}}{V_t}} - 1 \right)$

$I_C = J_S A_E \left( e^{\frac{V_{BE}}{V_t}} - 1 \right) - J_S A_E \alpha_R \left( e^{\frac{V_{BC}}{V_t}} - 1 \right)$
Simplified Multi-Region Model

- Observe $V_{CE}$ around 0.2V when saturated
- $V_{BE}$ around 0.6V when saturated
- In most applications, exact $V_{CE}$ and $V_{BE}$ voltage in saturation not critical

$V_{BE} = 0.7V$
$V_{CE} = 0.2V$

Saturation
Simplified Multi-Region Model

\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right) \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ V_t = \frac{kT}{q} \]

Forward Active

Saturation

Cutoff

\[ V_{BE} = 0.7V \]
\[ V_{CE} = 0.2V \]

\[ I_C = I_B = 0 \]
Simplified Multi-Region Model

\[ I_C = J_S A_E \left( \frac{V_{BE}}{V_t} \right) \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

\[ I_B = \frac{J_S A_E}{\beta} \left( \frac{V_{BE}}{V_t} \right) \]

\[ V_t = \frac{kT}{q} \]

- **Forward Active**
  \[ V_{BE} > 0.4V \]
  \[ V_{BC} < 0 \]

- **Saturation**
  \[ V_{BE} = 0.7V \]
  \[ V_{CE} = 0.2V \]
  \[ I_C < \beta I_B \]

- **Cutoff**
  \[ I_C = I_B = 0 \]
  \[ V_{BE} < 0 \]
  \[ V_{BC} < 0 \]

A small portion of the operating region is missed with this model but seldom operate in the missing region.
Simplified Multi-Region Model

Alternate equivalent model

\[ I_C = \beta I_B \left(1 + \frac{V_{CE}}{V_{AF}} \right) \]
\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]
\[ V_t = \frac{kT}{q} \]

\[ V_{BE} > 0.4V \quad V_{BC} < 0 \]
Forward Active

\[ V_{BE} = 0.7V \quad I_C < \beta I_B \]
Saturation

\[ V_{BE} = 0 \quad I_C = I_B = 0 \]
Cutoff

A small portion of the operating region is missed with this model but seldom operate in the missing region
Adequate when it makes little difference whether $V_{BE}=0.6V$ or $V_{BE}=0.7V$
Simplified Multi-Region dc Model

Forward Active

Mathematically

\[ V_{BE} = 0.6V \]
\[ I_C = \beta I_B \]

Or, if want to show slope in \( I_C - V_{CE} \) characteristics

\[ V_{BE} = 0.6V \]
\[ I_C = \beta I_B (1 + V_{CE}/V_{AF}) \]

\[ R_{DS} = \frac{V_{AF}}{I_{CQ}} \]
\[ R_{DS} \text{ highly nonlinear} \]
A small portion of the operating region is missed with this model but seldom operate in the missing region.
Conditions for Regions of Operation in Multi-Region Model

\[ V_{BE} > 0.4V \quad \text{Forward Active} \]
\[ V_{BC} < 0 \]

\[ I_C < \beta I_B \quad \text{Saturation} \]

\[ V_{BE} < 0 \]
\[ V_{BC} < 0 \quad \text{Cutoff} \]

Note: One condition is on dependent variables!

Observe that in saturation, \( I_C < \beta I_B \)

Can’t condition on independent variables in saturation because they are fixed in the model.
Regions of Operation in Independent Parameter Domain used in multi-region models

Seldom operate in regions excluded in this picture
Excessive Power Dissipation if either junction has large forward bias

- Forward Active
- Cutoff
- Reverse Active
- Saturation
- Melt Down!!
Safe regions of operation

- Forward Active
- Cutoff
- Reverse Active
- Saturation
- Simplified Forward Saturation

V_{BE} = 0.4V
V_{BC} = 0.4V

Melt Down!!
Actually cutoff, forward active, and reverse active regions can be extended modestly as shown and multi-region models still are quite good.
Sufficient regions of operation for most applications

- Forward Active
- Cutoff
- Reverse Active
- Simplified Forward Saturation
- Saturation

$V_{BC}$
$V_{BE}$

$0.4V$
Example: Determine $I_C$ and $V_{OUT}$

\[ J_s = 10^{16} \text{A/} \mu^2 \]
\[ \beta = 100 \]
Example: Determine $I_C$ and $V_{OUT}$

Solution:

1. Guess Forward Active Region
2. Solve Circuit with Guess
   
   $J_s = 10^{-16} \text{A/}/\mu^2$
   $\beta = 100$

   \[
   I_B = \frac{(12 - 0.6)}{500K} \]
   \[
   I_C = \beta I_B = 100 \left( \frac{12 - 0.6}{500K} \right) = 2.28mA
   \]
   \[
   V_{OUT} = 12 - I_C \cdot 4K = 2.88V
   \]

3. Verify FA Region

   $V_{BE} = 0.6V > 0.4V$$\quad V_{BE} > 0.4V$ and $V_{BC} < 0$

   Verify Passes so solution is valid

   $I_C = 2.28mA$
   $V_{OUT} = 2.88V$
Example: Determine $I_C$ and $V_{OUT}$.

- $J_s = 10^{-16} \text{A/}\mu^2$
- $\beta = 100$
Example: Determine $I_C$ and $V_{OUT}$.

Solution:

1. Guess Forward Active Region
2. Solve Circuit with Guess

$$J_s = 10^{-16} \text{A/} \mu \text{m}^2$$
$$\beta = 100$$

$$I_B = \frac{(12 - 0.6)}{50K}$$
$$I_C = \beta I_B = 100 \frac{(12 - 0.6)}{50K} = 22.8mA$$
$$V_{OUT} = 12 - I_C \cdot 4K = -79.2V$$

3. Verify FA Region $V_{BE} > 0.4V$ and $V_{BC} < 0$

Verify Fails so solution is not valid
Example: Determine $I_C$ and $V_{OUT}$

Solution:

4. Guess Saturation
5. Solve Circuit with Guess

$J_s = 10^{-16} \text{A}/ \mu^2$

$\beta = 100$

$I_B = \frac{(12 - 0.6)}{50K} = 228 \mu A$

$I_C = \frac{(12 - 0.2)}{4K} = 2.95 mA$

$V_{OUT} = 0.2V$

6. Verify SAT Region

$\beta I_B = 100 \times 228 \mu A = 22.8 mA$

$I_C = 2.95 mA$

$I_C = 2.95 mA < \beta I_B = 22.8 mA$

Verify Passes so solution is valid

$I_C = 2.95 mA \quad V_{OUT} = 0.2 V$
Example: Determine $I_C$ and $V_{OUT}$. Assume $C$ is large and $V_{IN}$ is very small.
Example: Determine $I_C$ and $V_{OUT}$. Assume $C$ is large and $V_{IN}$ is very small.

Solution:

Assume $V_{IN}=0$, then no current flows through $C$ so circuit is identical to circuit of previous example so

$$I_C = 2.28mA \quad V_{OUT} = 2.88V$$

Note: Since $V_{IN}$ is coupled directly to base, will amplify $V_{IN}$ if it is a small time varying signal and gain will be very large.
End of Lecture 18