EE 330
Lecture 22

Amplification with Transistors
Review from last time

Enhancement and Depletion Devices

- Enhancement Mode n-channel devices
  \[ V_T > 0 \]
- Enhancement Mode p-channel devices
  \[ V_T < 0 \]
- Depletion Mode n-channel devices
  \[ V_T < 0 \]
- Depletion Mode p-channel devices
  \[ V_T > 0 \]
Review from last time

The JFET

In saturation (pinch-off) region under reverse bias and large $V_{DS}$ (channel pinches off)
The Schottky Diode

- Metal-Semiconductor Junction
- One contact is ohmic, other is rectifying
- Not available in all processes
- Relatively inexpensive adder in some processes
- Lower cut-in voltage than pn junction diode
- High speed
Review from last time

Note: Not to vertical Scale
Area Comparison between BJT and MOSFET

- BJT Area = 3600 $\lambda^2$
- n-channel MOSFET Area = 168 $\lambda^2$
- Area Ratio = 21:1

Review from last time
Quiz 16

What processing step in a bipolar process is the dominant contributor to the large size inherent in the bipolar transistors in the BJT discussed in class?
And the number is ....
And the number is ....
Quiz 16

What processing step in a bipolar process is the dominant contributor to the large size inherent in the bipolar transistors in the BJT discussed in class?

Solution:

Isolation diffusion since it must diffuse completely through the thick epitaxial layer.
Amplification with Transistors

From Wikipedia:

Generally, an **amplifier** or simply **amp**, is any **device** that changes, usually increases, the amplitude of a **signal**. The "signal" is usually voltage or current.

- It is difficult to increase the voltage or current very much with passive RC circuits
- Voltage and current levels can be increased a lot with transformers but not practical in integrated circuits
- Power levels can not be increased with passive elements (R, L, C, and Transformers)
- Often an amplifier is defined to be a circuit that can increase power levels
- Transistors can be used to increase not only signal levels but power levels to a load
- In transistor circuits, power that is delivered in the signal path is supplied by a biasing network
Amplification with Transistors
Applications of Devices as Amplifiers

Typical Regions of Operation by Circuit Function

- Logic Circuits
  - MOS: Triode and Cutoff
  - Bipolar: Saturation and Cutoff
- Linear Circuits
  - MOS: Saturation
  - Bipolar: Forward Active
Consider the following MOSFET and BJT Circuits

**BJT**

- Assume BJT operating in FA region, MOSFET operating in Saturation
- Assume same quiescent output voltage and same resistor $R_1$
- One of the most widely used amplifier architectures
Consider the following MOSFET and BJT Circuits

**BJT**

- $V_{CC}$
- $R_1$
- $V_{OUT}$
- $Q_1$
- $V_{EE}$
- $V_{IN(t)}$  

**MOSFET**

- $V_{DD}$
- $R_1$
- $V_{OUT}$
- $M_1$
- $V_{SS}$
- $V_{IN(t)}$  

- MOS and BJT Architectures often Identical
- Circuit are Highly Nonlinear
- Nonlinear Analysis Methods Must be used to analyze these and almost any other nonlinear circuit
Methods of Analysis of Nonlinear Circuits

KCL and KVL apply to both linear and nonlinear circuits

Superposition, voltage divider and current divider equations, Thevenin and Norton equivalence apply only to linear circuits!

Some other analysis techniques that have been developed may apply only to linear circuits as well
Methods of Analysis of Nonlinear Circuits

Will consider three different analysis requirements and techniques for some particularly common classes of nonlinear circuits

1. Circuits with continuously differential devices

   Interested in obtaining transfer characteristics of these circuits or outputs for given input signals

2. Circuits with piecewise continuous devices

   Interested in obtaining transfer characteristics of these circuits or outputs for a given input signals

3. Circuits with small-signal inputs that vary around some operating point

   Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Other types of nonlinearities may exist and other types of analysis may be required but we will not attempt to categorize these scenarios in this course
1. Nonlinear circuits with continuously differential devices

Analysis Strategy:

Use KVL and KCL for analysis

Represent nonlinear models for devices either mathematically or graphically

Solve the resultant set of equations for the variables of interest
2. Circuits with piecewise continuous devices

e.g. \( f(x) = \begin{cases} f_1(x) & x < x_1 \text{ region 1} \\ f_2(x) & x > x_1 \text{ region 2} \end{cases} \)

Analysis Strategy:

Guess region of operation

Solve resultant circuit using the previous method

Verify region of operation is valid

Repeat the previous 3 steps as often as necessary until region of operation is verified

It helps to guess right the first time but a wrong guess will not result in an incorrect solution because a wrong guess cannot be verified
3. Circuits with small-signal inputs that vary around some operating point

Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point.

Analysis Strategy:

Use methods from previous class of nonlinear circuits.

More Practical Analysis Strategy:

Determine the operating point (using method 1 or 2 discussed above after all small signal independent inputs are set to 0).

Develop small signal (linear) model for all devices in the region of interest (around the operating point or “Q-point”).

Create small signal equivalent circuit by replacing all devices with small-signal equivalent.

Solve the resultant small-signal (linear) circuit.

Can use KCL, DVL, and other linear analysis tools such as superposition, voltage and current divider equations, Thevenin and Norton equivalence.

Determine boundary of region where small signal analysis is valid.
Small signal analysis using nonlinear models

By selecting appropriate value of $V_{SS}$, $M_1$ will operate in the saturation region

Assume $M_1$ operating in saturation region

$$V_{IN} = V_M \sin \omega t$$

$V_M$ is small

$$I_D = \frac{\mu C_{ox} W}{2L} (V_{IN} - V_{SS} - V_T)^2$$

$$I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{SS} + V_T)^2$$

$$V_{OUT} = V_{DD} - \frac{\mu C_{ox} W}{2L} (V_{IN} - V_{SS} - V_T)^2 R$$

$$V_{OUT} = V_{DD} - \frac{\mu C_{ox} W}{2L} (V_M \sin \omega t - [V_{SS} + V_T]^2) R$$
Small signal analysis example

\[ V_{IN} = V_M \sin \omega t \]

\(V_M\) is small

\[
V_{OUT} = V_{DD} - \frac{\mu C_{ox} W}{2L} \left( V_M \sin \omega t - \left[ V_{SS} + V_T \right] \right)^2 R
\]

Recall that if \(x\) is small \((1+x)^2 \approx 1+2x\)

\[
V_{OUT} = V_{DD} - \frac{\mu C_{ox} W}{2L} \left[ V_{SS} + V_T \right]^2 \left( 1 - \frac{V_M \sin \omega t}{V_{SS} + V_T} \right)^2 R
\]

\[
V_{OUT} = \left\{ V_{DD} - \frac{\mu C_{ox} W}{2L} \left[ V_{SS} + V_T \right]^2 R \right\} + \frac{\mu C_{ox} W}{2L} \left[ V_{SS} + V_T \right]^2 \left( 1 - \frac{2V_M \sin \omega t}{V_{SS} + V_T} \right) R
\]

\[
V_{OUT} = \left\{ V_{DD} - \frac{\mu C_{ox} W}{2L} \left[ V_{SS} + V_T \right]^2 R \right\} + \left\{ \frac{\mu C_{ox} W}{L} \left[ V_{SS} + V_T \right] R \right\} V_M \sin \omega t
\]
Small signal analysis example

By selecting appropriate value of $V_{SS}$, $M_1$ will operate in the saturation region

Assume $M_1$ operating in saturation region

$$V_{OUT} = \left( V_{DD} - \frac{\mu C_{ox} W}{2L} \left( V_{SS} + V_T \right)^2 R \right) + \left( \frac{\mu C_{ox} W}{L} \left( V_{SS} + V_T \right) R \right) V_M \sin \omega t$$
Small signal analysis example

Assume $M_1$ operating in saturation region

$$V_{\text{IN}} = V_M \sin \omega t$$

$$V_{\text{OUT}} = \left( V_{\text{DD}} - \frac{\mu C_{\text{ox}} W}{2L} [V_{\text{ss}} + V_T]^2 R \right) + \left( \frac{\mu C_{\text{ox}} W}{L} [V_{\text{ss}} + V_T] R \right) V_M \sin \omega t$$

- Quiescent Output
- ss Voltage Gain

$$A_v = \frac{\mu C_{\text{ox}} W}{L} [V_{\text{ss}} + V_T] R$$

But – this expression gives little insight into how large the gain is!
And the analysis for even this very simple circuit was messy!
Small signal analysis example

\[ V_{\text{OUT}} = \left\{ \frac{V_{\text{DD}} - \frac{\mu C_{\text{OX}} W}{2L} \left[ V_{\text{SS}} + V_T \right]^2}{R} \right\} + \left\{ \frac{\mu C_{\text{OX}} W}{L} \left[ V_{\text{SS}} + V_T \right] R \right\} V_m \sin \omega t \]

- **Quiescent Output**
- **ss Voltage Gain**

\[ A_v = \frac{\mu C_{\text{OX}} W}{L} \left[ V_{\text{SS}} + V_T \right] R \]
Small signal analysis example

\[ V_{\text{IN}} = V_M \sin \omega t \]

\[ V_{\text{OUT}} = \left\{ V_{\text{DD}} - \frac{\mu C_{\text{ox}} W}{2L} \left[ V_{\text{SS}} + V_T \right]^2 R \right\} + \left\{ \frac{\mu C_{\text{ox}} W}{L} \left[ V_{\text{SS}} + V_T \right] R \right\} V_M \sin \omega t \]

Quiescent Output

ss Voltage Gain

\[ A_v = \frac{\mu C_{\text{ox}} W}{L} \left[ V_{\text{SS}} + V_T \right] R \]
Small signal analysis example

\[ V_{\text{IN}} = V_M \sin \omega t \]

\[ V_{\text{OUT}} = \left\{ V_{\text{DD}} - \frac{\mu C_{\text{ox}} W}{2L} \left[ V_{\text{SS}} + V_T \right]^2 R \right\} + \left\{ \frac{\mu C_{\text{ox}} W}{L} \left[ V_{\text{SS}} + V_T \right] R \right\} V_M \sin \omega t \]

**Quiescent Output**

**ss Voltage Gain**

\[ A_v = \frac{\mu C_{\text{ox}} W}{L} \left[ V_{\text{SS}} + V_T \right] R \]
Small signal analysis example

\[ V_{\text{IN}} = V_M \sin \omega t \]

\[ V_{\text{out}} = \left\{ V_{\text{DD}} - \frac{\mu C_{\text{ox}} W}{2L} \left( V_{\text{SS}} + V_T \right)^2 R \right\} + \left\{ \frac{\mu C_{\text{ox}} W}{L} \left( V_{\text{SS}} + V_T \right) R \right\} V_M \sin \omega t \]

**Quiescent Output**

**ss Voltage Gain**

\[ A_v = \frac{\mu C_{\text{ox}} W}{L} \left[ V_{\text{SS}} + V_T \right] R \]

Serious Distortion occurs if signal is too large or Q-point non-optimal
Here “clipping” occurs for high \( V_{\text{OUT}} \)
Small signal analysis example

\[ V_{\text{IN}} = V_M \sin \omega t \]

\[ V_{\text{OUT}} = \left\{ V_{\text{DD}} - \frac{\mu C_{\text{ox}} W}{2L} \left[ V_{\text{SS}} + V_T \right]^2 R \right\} + \left\{ \frac{\mu C_{\text{ox}} W}{L} \left[ V_{\text{SS}} + V_T \right] R \right\} V_M \sin \omega t \]

\text{Quiescent Output}

\text{ss Voltage Gain}

Serious Distortion occurs if signal is too large or Q-point non-optimal

Here “clipping” occurs for low \( V_{\text{OUT}} \)
Small signal analysis example

\[ V_{IN} = V_M \sin \omega t \]

\[ V_{OUT} = \left( V_{DD} - \frac{\mu C_{ox} W}{2L} [V_{ss} + V_T]^2 R \right) + \left( \frac{\mu C_{ox} W}{L} [V_{ss} + V_T] R \right) V_M \sin \omega t \]

But recall:

\[ A_v = \frac{\mu C_{ox} W}{L} [V_{ss} + V_T] R \]

\[ I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{ss} + V_T)^2 \]

Thus, substituting from the expression for \( I_{DQ} \) we obtain

\[ A_v = \frac{2I_{DQ} R}{[V_{ss} + V_T]} \]

Note this is negative since \( V_{ss} + V_T < 0 \)
Small signal analysis example

\[ A_v = \frac{2I_{DQ} R}{V_{SS} + V_T} \]

Observe the small signal voltage gain is twice the Quiescent voltage across \( R \) divided by \( V_{SS} + V_T \)

- This analysis which required linearization of a nonlinear output voltage is quite tedious.

- This approach becomes unwieldy for even slightly more complicated circuits

- A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements
Small signal analysis using nonlinear models

By selecting appropriate value of $V_{SS}$, $M_1$ will operate in the saturation region

Assume $M_1$ operating in saturation region

$V_{IN} = V_M \sin \omega t$

$V_M$ is small

$I_C = J_S A_E e^{\frac{V_{IN} - V_{EE}}{V_t}}$

$I_{CQ} = J_S A_E e^{\frac{-V_{EE}}{V_t}}$

$V_{OUT} = V_{CC} - J_S A_E R_1 e^{\frac{V_{IN} - V_{EE}}{V_t}}$

$V_{OUT} = V_{CC} - J_S A_E R_1 e^{\frac{V_{M\sin(\omega t)} - V_{EE}}{V_t}}$
Small signal analysis using nonlinear models

\[
V_{OUT} = V_{CC} - J_{SE}ARe \frac{-V_{EE}}{V_t} \]

\[
V_{OUT} = V_{CC} - J_{SE}ARe \frac{-V_{EE}}{V_t} \frac{V_{Msin(\omega t)} - V_{EE}}{V_t} \]

Recall that if \( x \) is small, \( e^x \approx 1 + x \) (truncated Taylor's series)

\[ V_{IN} = V_M \sin \omega t \]

\[ V_M \text{ is small} \]

\[
V_{OUT} \approx \left[ V_{CC} - J_{SE}ARe \frac{-V_{EE}}{V_t} \right] - \left[ J_{SE}ARe \frac{-V_{EE}}{V_t} \frac{V_{Msin(\omega t)}}{V_t} \right]
\]

\[
I_{CQ} = J_S A_E e^{\frac{-V_{EE}}{V_t}}
\]
Small signal analysis using nonlinear models

\[ I_{CQ} = J_S A_E e^{\frac{-V_{EE}}{V_t}} \]

\[ V_{OUT} \equiv V_{CC} - J_S A_E R_1 e^{\frac{-V_{EE}}{V_t}} - J_S A_E R_1 \frac{V_M \sin(\omega t)}{V_t} \]

\[ V_{IN} = V_M \sin(\omega t) \]

\[ V_M \text{ is small} \]

**Quiescent Output**

**ss Voltage Gain**
Comparison of Gains for MOSFET and BJT Circuits

**BJT**

\[ A_{VB} = - \frac{I_{CQ} R_1}{V_t} \]

If \( I_{DQ} R = I_{CQ} R_1 = 2V, V_{SS} + V_T = -1V, V_t = 25mV \)

\[ A_{VB} = - \frac{2V}{25mV} = -80 \]

**MOSFET**

\[ A_{VM} = \frac{2I_{DQ} R}{V_{SS} + V_T} \]

\[ A_{VM} = \frac{2I_{DQ} R}{V_{SS} + V_T} = \frac{4V}{-1V} = -4 \]

Observe \( A_{VB} >> A_{VM} \)

Due to exponential-law rather than square-law model
Operation with Small-Signal Inputs

• Analysis procedure for these simple circuits was very tedious
• This approach will be unmanageable for even modestly more complicated circuits
• Faster analysis method is needed!
Operation with Small-Signal Inputs

Why was this analysis so tedious?

Because of the nonlinearity in the device models

What was the key technique in the analysis that was used to obtain a simple expression for the output?

\[ V_{\text{OUT}} = V_{\text{CC}} - J_s A_E R_i \left( \frac{V_{\text{EE}}}{V_i} \right) \left( \frac{V_m \sin(\omega t)}{V_i} \right) \]

Linearization of the nonlinear output expression at the operating point
Operation with Small-Signal Inputs

\[ I_{CQ} = J_S A_E e^{-\frac{V_{EE}}{V_t}} \]

\[ V_{OUT} \approx \left[ V_{CC} - I_{CQ} R_1 \right] - \left( \frac{I_{CQ} R_1}{V_t} \right) V_M \sin(\omega t) \]

**Quiescent Output**

**ss Voltage Gain**

Small-signal analysis strategy

1. Obtain Quiescent Output (Q-point)
2. Linearize circuit at Q-point instead of linearize the nonlinear solution
3. Analyze linear “small-signal” circuit
4. Add quiescent and small-signal outputs to obtain good approximation to actual output
Small-Signal Principle

Nonlinear function
\[ y = f(x) \]

\( y \)-axis
\( x \)-axis

\( Y_Q \)
\( X_Q \)

Q-point
Small-Signal Principle

Region around Q-Point

y = f(x)

Q-point

y

x

y_Q

x_Q
Small-Signal Principle

Relationship is nearly linear in a small enough region around Q-point
Region of linearity is often quite large
Linear relationship may be different for different Q-points
Small-Signal Principle

Relationship is nearly linear in a small enough region around Q-point. Region of linearity is often quite large. Linear relationship may be different for different Q-points.
Small-Signal Principle

Device Behaves Linearly in Neighborhood of Q-Point
Can be characterized in terms of a small-signal coordinate system

\[ y = f(x) \]
Small-Signal Principle

Linear Model at Q-point

\[ \frac{y - y_Q}{x - x_Q} = \left. \frac{\partial f}{\partial x} \right|_{x=x_Q} \]

\[ y - y_Q = \left. \frac{\partial f}{\partial x} \right|_{x=x_Q} (x - x_Q) \]
Small-Signal Principle

\[ \frac{y - y_Q}{x - x_Q} = \left. \frac{\partial f}{\partial x} \right|_{x=x_Q} \]

\[ y - y_Q = \left. \frac{\partial f}{\partial x} \right|_{x=x_Q} (x - x_Q) \]