## EE 330 Class Seating

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EE 330
Lecture 22

- Small Signal Analysis
- Small Signal Analysis of BJT Amplifier
Review from Last Lecture

Comparison of Gains for MOSFET and BJT Circuits

**BJT**

\[ A_{VB} = - \frac{I_{CQ} R_1}{V_t} \]

If \( I_{DQ} R = I_{CQ} R_1 = 2V \), \( V_{SS} + V_T = -1V \), \( V_t = 25mV \)

\[ A_{VB} = - \frac{2V R_1}{25mV} = -80 \]

**MOSFET**

\[ A_{VM} = \frac{2I_{DQ} R}{V_{SS} + V_T} \]

\[ A_{VM} = \frac{4V R}{-1V} = -4 \]

Observe \( A_{VB} \gg A_{VM} \)

Due to exponential-law rather than square-law model
Operation with Small-Signal Inputs

- Analysis procedure for these simple circuits was very tedious
- This approach will be unmanageable for even modestly more complicated circuits
- **Faster analysis method is needed!**
Small-Signal Analysis

- Will commit next several lectures to developing this approach
- Analysis will be MUCH simpler, faster, and provide significantly more insight
- Applicable to many fields of engineering
Small-Signal Analysis

- Simple dc Model
  - Square-Law Model
- Small Signal
  - Frequency Dependent Small Signal
- Better Analytical dc Model
- Sophisticated Model for Computer Simulations
  - BSIM Model
  - Square-Law Model (with extensions for λ,γ effects)
  - Short-Channel α-law Model
- Simpler dc Model
  - Switch-Level Models
    - Ideal switches
    - \( R_{SW} \) and \( C_{GS} \)
Operation with Small-Signal Inputs

Why was this analysis so tedious?

Because of the nonlinearity in the device models

What was the key technique in the analysis that was used to obtain a simple expression for the output (and that related linearly to the input)?

\[
V_{OUT} = V_{CC} - J_S A_E R_1 e^{\frac{-V_{EE}}{V_t}} e^{\frac{V_M \sin(\omega t)}{V_t}}
\]

\[
V_{OUT} \approx \left[ V_{CC} - I_{CQ} R_1 \right] - \left( \frac{I_{CQ} R_1}{V_t} \right) V_M \sin(\omega t)
\]

Linearization of the nonlinear output expression at the operating point
Operation with Small-Signal Inputs

\[ I_{CQ} = J_S A_E e^{-\frac{V_{EE}}{V_t}} \]

\[ V_{OUT} \approx \left[ V_{CC} - I_{CQ} R_1 \right] - \left( \frac{I_{CQ} R_1}{V_t} \right) V_M \sin(\omega t) \]

Quiescent Output

Small-signal analysis strategy

1. Obtain Quiescent Output (Q-point)
2. Linearize circuit at Q-point instead of linearize the nonlinear solution
3. Analyze linear “small-signal” circuit
4. Add quiescent and small-signal outputs to obtain good approximation to actual output
Small-Signal Principle

Nonlinear function
\( y = f(x) \)

Q-point

\( X_Q \)

\( Y_Q \)
Small-Signal Principle

Region around Q-Point

Q-point

y = f(x)

y

x

Y_Q

X_Q
Small-Signal Principle

Relationship is nearly linear in a small enough region around Q-point
Region of linearity is often quite large
Linear relationship may be different for different Q-points
Relationship is nearly linear in a small enough region around Q-point
Region of linearity is often quite large
Linear relationship may be different for different Q-points
Small-Signal Principle

Device Behaves Linearly in Neighborhood of Q-Point
Can be characterized in terms of a small-signal coordinate system

\[ y = f(x) \]
### Small-Signal Principle

For a nonlinear function $y = f(x)$, the small-signal principle can be expressed as:

$$y - y_Q = \left. \frac{\partial f}{\partial x} \right|_{x=x_Q} (x - x_Q)$$

Or in matrix form:

$$y = \left[ \left. \frac{\partial f}{\partial x} \right|_{x=x_Q} \right] x + \left[ y_Q - x_Q \left. \frac{\partial f}{\partial x} \right|_{x=x_Q} \right]$$

Where $Q$-point is $(x_Q, y_Q)$ or $(x_Q, y_Q)$.
Small-Signal Principle

Changing coordinate systems:

\[ y_{SS} = y - y_Q \]
\[ x_{SS} = x - x_Q \]

\[ y - y_Q = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} (x - x_Q) \]
\[ y_{ss} = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} x_{ss} \]
Small-Signal Principle

Small-Signal Model: \[ y_{ss} = \frac{\partial f}{\partial x} \bigg|_{x=x_0} x_{ss} \]

- Linearized model for the nonlinear function \( y=f(x) \)
- Valid in the region of the Q-point
- Will show the small signal model is simply Taylor's series expansion of \( f(x) \) at the Q-point truncated after first-order terms
Small-Signal Principle

**Observe:**

\[ y - y_Q = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} (x - x_Q) \rightarrow y_{ss} = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} x_{ss} \]

\[ y_Q = f(x_Q) \]

\[ y = f(x_Q) + \frac{\partial f}{\partial x} \bigg|_{x=x_Q} (x - x_Q) \]

Recall Taylor's Series Expansion of nonlinear function \( f \) at expansion point \( x_0 \):

\[ y = f(x_0) + \sum_{k=1}^{\infty} \left( \frac{1}{k!} \frac{df}{dx} \bigg|_{x=x_0} (x-x_0)^k \right) \]

Truncating after first-order terms (and defining “o” as “Q”):

\[ y \approx f(x_Q) + \frac{\partial f}{\partial x} \bigg|_{x=x_Q} (x - x_Q) \rightarrow y_{ss} = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} x_{ss} \]

Mathematically, **linearized model** is simply Taylor's series expansion of the nonlinear function \( f \) at the Q-point truncated after first-order terms with notation \( x_Q = x_0 \).
Small-Signal Principle

\[ y = f(x_Q) + \frac{\partial f}{\partial x} \bigg|_{x=x_Q} x_{SS} \]

Quiescent Output

ss Gain

How can a \textbf{circuit} be linearized at an operating point as an alternative to linearizing a nonlinear function at an operating point?

Consider arbitrary nonlinear one-port network
Arbitrary Nonlinear One-Port

\[ I = f(V) \]

\[ i_{ss} = \left. \frac{\partial I}{\partial V} \right|_{V=V_Q} \]

\[ v_{ss} = \left. I \right|_{I=I_Q} \]

\[ y = \left. \frac{\partial I}{\partial V} \right|_{V=V_Q} \]

Linear model of the nonlinear device at the Q-point
**Arbitrary Nonlinear One-Port**

The small-signal model of this 2-terminal electrical network is a resistor of value $1/y$ or a conductor of value $y$.

**One small-signal parameter** characterizes this one-port but it is dependent on $Q$-point.

This applies to **ANY** nonlinear one-port that is differentiable at a $Q$-point (e.g. a diode).

---

**Linear small-signal model:**

\[ i = y \ v \]

\[ y = \left. \frac{\partial I}{\partial V} \right|_{V=V_Q} \]
Small-Signal Principle

Goal with small signal model is to predict performance of circuit or device in the vicinity of an operating point (Q-point)

Will be extended to functions of two and three variables (e.g. BJTs and MOSFETs)
Solution for the example of the previous lecture was based upon solving the nonlinear circuit for $V_{\text{OUT}}$ and then linearizing the solution by doing a Taylor’s series expansion.

- Solution of nonlinear equations very involved with two or more nonlinear devices.

- Taylor’s series linearization can get very tedious if multiple nonlinear devices are present.

**Standard Approach to small-signal analysis of nonlinear networks**

1. Solve nonlinear network
2. Linearize solution

**Alternative Approach to small-signal analysis of nonlinear networks**

1. Linearize nonlinear devices (all)
2. Obtain small-signal model from linearized device models
3. Replace all devices with small-signal equivalent
4. Solve linear small-signal network
Alternative Approach to small-signal analysis of nonlinear networks

1. Linearize nonlinear devices

2. Obtain small-signal model from linearized device models

3. Replace all devices with small-signal equivalent

4. Solve linear small-signal network

• Must only develop linearized model once for any nonlinear device (steps 1. and 2.)
  e.g. once for a MOSFET, once for a JFET, and once for a BJT

Linearized model for nonlinear device termed “small-signal model”

derivation of small-signal model for most nonlinear devices is less complicated than solving even one simple nonlinear circuit

• Solution of linear network much easier than solution of nonlinear network
Alternative Approach to small-signal analysis of nonlinear networks

1. Linearize nonlinear devices

2. Obtain small-signal model from linearized device models

3. Replace all devices with small-signal equivalent

4. Solve linear small-signal network

The “Alternative” approach is used almost exclusively for the small-signal analysis of nonlinear networks
“Alternative” Approach to small-signal analysis of nonlinear networks

Nonlinear Network

dc Equivalent Network

Q-point
Values for small-signal parameters

Small-signal (linear) equivalent network

Small-signal output

Total output
(good approximation)
This terminology will be used in THIS course to emphasize difference between nonlinear model and linearized small signal model.
Example:

It will be shown that the nonlinear circuit has the linearized small-signal network given.

*Nonlinear network*

*Linearized small-signal network*
Linearized Circuit Elements

Must obtain the linearized circuit element for ALL linear and nonlinear circuit elements

(Will also give models that are usually used for Q-point calculations : Simplified dc models)
Small-signal and simplified dc equivalent elements

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<th>Simplified dc equivalent</th>
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<td>dc Voltage Source</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
</tr>
<tr>
<td>ac Voltage Source</td>
<td>$V_{AC}$</td>
<td>$V_{AC}$</td>
</tr>
<tr>
<td>dc Current Source</td>
<td>$I_{DC}$</td>
<td>$I_{DC}$</td>
</tr>
<tr>
<td>ac Current Source</td>
<td>$I_{AC}$</td>
<td>$I_{AC}$</td>
</tr>
<tr>
<td>Resistor</td>
<td>$R$</td>
<td>$R$</td>
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Small-signal and simplified dc equivalent elements

**Capacitors**
- Large: $C$
- Small: $C$

**Inductors**
- Large: $L$
- Small: $L$

**Diodes**

**MOS transistors**
(MOSFET (enhancement or depletion), JFET)
Small-signal and simplified dc equivalent elements

Element | ss equivalent | Simplified dc equivalent
---|---|---
Bipolar Transistors
Dependent Sources (Linear)

\[ V_O = A_V I_{IN} \]
\[ I_O = A_I I_{IN} \]
\[ V_O = R_{T} I_{IN} \]
\[ I_O = G_T V_{IN} \]
Example: Obtain the small-signal equivalent circuit

\[ \text{V}_{\text{INSS}} \quad \text{C} \quad \text{R}_1 \quad \text{V}_{\text{OUT}} \]

C is large
Example: Obtain the small-signal equivalent circuit
Example: Obtain the small-signal equivalent circuit

\[ V_{INSS} \]

\[ V_{DD} \]

\[ V_{OUT} \]

\[ V_{SS} \]

\[ R_{1} \]

\[ R_{2} \]

\[ R_{3} \]

\[ R_{4} \]

\[ R_{5} \]

\[ R_{6} \]

\[ R_{7} \]

\[ R_{L} \]

\[ C_{1} \]

\[ C_{2} \]

\[ C_{3} \]

\[ C_{4} \]

\[ \text{C}_{1}, \text{C}_{2}, \text{C}_{3} \text{ large} \]

\[ \text{C}_{4} \text{ small} \]
How is the small-signal equivalent circuit obtained from the nonlinear circuit?

What is the small-signal equivalent of the MOSFET, BJT, and diode?
A Small Signal Equivalent Circuit

Thus, for the diode

\[ R_d = \left( \frac{\partial I_D}{\partial V_D} \right)_Q^{-1} \]
Small-Signal Diode Model

For the diode

\[ V_D = \frac{I_D}{V_t} \]

\[ I_D = I_s e^{\frac{V_D}{V_t}} \]

\[ R_d = \left( \frac{\partial I_D}{\partial V_D} \right)^{-1} \]

\[ I_{DQ} = I_s e^{\frac{V_D}{V_t}} \]

\[ R_d = \frac{V_t}{I_{DQ}} \]
Example of diode circuit where small-signal diode model is useful

Voltage Reference

Small-signal model of Voltage Reference (useful for compensation when parasitic Cs included)
Small-Signal Model of BJT and MOSFET

Consider 4-terminal network

\[
\begin{align*}
I_1 &= f_1(V_1, V_2, V_3) \\
I_2 &= f_2(V_1, V_2, V_3) \\
I_3 &= f_3(V_1, V_2, V_3)
\end{align*}
\]

Define

\[
\begin{align*}
i_1 &= I_1 - I_{1Q} \\
i_2 &= I_2 - I_{2Q} \\
i_3 &= I_3 - I_{3Q}
\end{align*}
\]

\[
\begin{align*}
u_1 &= V_1 - V_{1Q} \\
u_2 &= V_2 - V_{2Q} \\
u_3 &= V_3 - V_{3Q}
\end{align*}
\]

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents.
Small-Signal Model of 4-Terminal Network

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents.

For small signals, this relationship should be linear.

Can be thought of as a change in coordinate systems from the large signal coordinate system to the small-signal coordinate system.
Recall for a function of one variable

\[ y = f(x) \]

Taylor’s Series Expansion about the point \( x_0 \)

\[ y = f(x) = f(x)\bigg|_{x=x_0} + \frac{\partial f}{\partial x}\bigg|_{x=x_0} (x-x_0) + \frac{\partial^2 f}{\partial x^2}\bigg|_{x=x_0} \frac{1}{2!} (x-x_0)^2 + \ldots \]

If \( x-x_0 \) is small

\[ y \approx f(x)\bigg|_{x=x_0} + \frac{\partial f}{\partial x}\bigg|_{x=x_0} (x-x_0) \]

\[ y \approx y_0 + \frac{\partial f}{\partial x}\bigg|_{x=x_0} (x-x_0) \]
Recall for a function of one variable

\[ y = f(x) \]

If \( x - x_0 \) is small

\[ y \approx y_0 + \left. \frac{\partial f}{\partial x} \right|_{x=x_0} (x - x_0) \]

\[ y - y_0 = \left. \frac{\partial f}{\partial x} \right|_{x=x_0} (x - x_0) \]

If we define the small signal variables as

\[ \mathbf{y} = y - y_0 \]

\[ \mathbf{x} = x - x_0 \]
Recall for a function of one variable

\[ y = f(x) \]

If \( x - x_0 \) is small

\[ y - y_0 = \frac{\partial f}{\partial x} \bigg|_{x=x_0} (x - x_0) \]

If we define the small signal variables as

\[ y = y - y_0 \]

\[ \alpha = x - x_0 \]

Then

\[ y = \left. \frac{\partial f}{\partial x} \right|_{x=x_0} \alpha \quad \text{This relationship is linear!} \]
Consider now a function of \( n \) variables

\[
y = f (x_1, \ldots, x_n) = f (\bar{x})
\]

If we define the small signal variables as

\[
\bar{X}_0 = \{ x_{10}, x_{20}, \ldots, x_{n0} \}
\]

The multivariate Taylor’s series expansion around the point \( \bar{X}_0 \) is given by

\[
y = f (\bar{x}) = f (x) \bigg|_{\bar{x} = x_0} + \sum_{k=1}^{n} \left( \frac{\partial f}{\partial x_k} \bigg|_{x = x_0} (x_k - x_{k0}) \right) \\
+ \sum_{k=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 f}{\partial x_j \partial x_k} \bigg|_{\bar{x} = x_0} \frac{1}{2!} (x_j - x_{j0}) (x_k - x_{k0}) + \ldots \text{(H.O.T.)}
\]

Truncating after first-order terms, we obtain the approximation

\[
y - y_0 = \sum_{k=1}^{n} \left( \frac{\partial f}{\partial x_k} \bigg|_{x = x_0} (x_k - x_{k0}) \right) \\
\text{where } y_0 = f (x) \bigg|_{\bar{x} = x_0}
\]
Multivariate Taylors Series Expansion

\[ y = f(x_1, \ldots, x_n) = f(\bar{x}) \]

Linearized approximation

\[ y - y_0 \approx \sum_{k=1}^{n} \left( \frac{\partial f}{\partial x_k} \bigg|_{x = \bar{x}_0} (x_k - x_{k0}) \right) \]

This can be expressed as

\[ y_{ss} \approx \sum_{k=1}^{n} a_k x_{ssk} \]

where

\[ y_{ss} = y - y_0 \]
\[ x_{ssk} = x_k - x_{k0} \]
\[ a_k = \frac{\partial f}{\partial x_k} \bigg|_{x = \bar{x}_0} \]

\[ y = \sum_{k=1}^{n} a_k \bar{x}_k \]
In the more general form\(^1\), the multivariate Taylor’s series expansion can be expressed as

\[
f(x_1, \ldots, x_n) = \alpha_o + \sum_{m=1}^{\infty} \sum_{\substack{k_1, \ldots, k_n \geq 0 \atop \sum k_j = m}} \alpha_{k_1, \ldots, k_n; m} (x_1 - x_{1,o})^{k_1} \cdots (x_n - x_{n,o})^{k_n}
\]

\[
\alpha_o = f(x_{1,o}, \ldots, x_{n,o})
\]

\[
\alpha_{k_1, \ldots, k_n; m} = \frac{1}{k_1! \cdots k_n!} \frac{\partial^m f}{\partial x_1^{k_1} \cdots \partial x_n^{k_n}} \bigg|_{x_1 = x_{1,o}, \ldots, x_n = x_{n,o}}
\]

\(^1\) http://www.chemistry.mtu.edu/~tbco/cm416/taylor.html
Consider 4-terminal network

Nonlinear network characterized by 3 functions each functions of 3 variables
Consider now 3 functions each function of 3 variables

\[
\begin{align*}
I_1 &= f_1(V_1, V_2, V_3) \\
I_2 &= f_2(V_1, V_2, V_3) \\
I_3 &= f_3(V_1, V_2, V_3)
\end{align*}
\]

Define

\[
\vec{V}_Q = \begin{bmatrix}
V_{1Q} \\
V_{2Q} \\
V_{3Q}
\end{bmatrix}
\]

In what follows, we will use \( \vec{V}_Q \) as an expansion point in a Taylor’s series expansion.
Consider now 3 functions each functions of 3 variables

\[
\begin{align*}
I_1 &= f_1(V_1, V_2, V_3) \\
I_2 &= f_2(V_1, V_2, V_3) \\
I_3 &= f_3(V_1, V_2, V_3)
\end{align*}
\]

Define

\[
\tilde{V}_Q = \begin{bmatrix} V_{1Q} \\ V_{2Q} \\ V_{3Q} \end{bmatrix}
\]

Consider first the function \(I_1\)

The multivariate Taylors Series expansion of \(I_1\), around the operating point \(\tilde{V}_Q\), when truncated after first-order terms, can be expressed as:

\[
I_1 = f_1(V_1, V_2, V_3) \approx f_1(V_{1Q}, V_{2Q}, V_{3Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1} (V_1 - V_{1Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2} (V_2 - V_{2Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3} (V_3 - V_{3Q})
\]

or equivalently as:

\[
I_1 - I_{1Q} = \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1} (V_1 - V_{1Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2} (V_2 - V_{2Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3} (V_3 - V_{3Q})
\]
repeating from previous slide:

\[ I_1 - I_{1Q} = \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1} \right|_{V = V_0} (V_1 - V_{1Q}) + \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2} \right|_{V = V_0} (V_2 - V_{2Q}) + \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3} \right|_{V = V_0} (V_3 - V_{3Q}) \]

Make the following definitions

\[ y_{11} = \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1} \right|_{V = \bar{V}_0} \]

\[ y_{12} = \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2} \right|_{V = \bar{V}_0} \]

\[ y_{13} = \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3} \right|_{V = \bar{V}_0} \]

It thus follows that

\[ \mathbf{i}_1 = y_{11} \mathbf{v}_1 + y_{12} \mathbf{v}_2 + y_{13} \mathbf{v}_3 \]

This is a linear relationship between the small signal electrical variables
Small Signal Model Development

Nonlinear Model

\[ I_1 = f_1(V_1, V_2, V_3) \]
\[ I_2 = f_2(V_1, V_2, V_3) \]
\[ I_3 = f_3(V_1, V_2, V_3) \]

Linear Model at \( \bar{V}_Q \)
(alt. small signal model)

\[ \mathbf{i}_1 = y_{11} \mathbf{v}_1 + y_{12} \mathbf{v}_2 + y_{13} \mathbf{v}_3 \]

Extending this approach to the two nonlinear functions \( I_2 \) and \( I_3 \)

\[ \mathbf{i}_2 = y_{21} \mathbf{v}_1 + y_{22} \mathbf{v}_2 + y_{23} \mathbf{v}_3 \]
\[ \mathbf{i}_3 = y_{31} \mathbf{v}_1 + y_{32} \mathbf{v}_2 + y_{33} \mathbf{v}_3 \]

where

\[ y_{ij} = \left. \frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j} \right|_{\mathbf{V}=\bar{V}_Q} \]
Small Signal Model Development

**Nonlinear Model**

\[ I_1 = f_1(V_1, V_2, V_3) \]
\[ I_2 = f_2(V_1, V_2, V_3) \]
\[ I_3 = f_3(V_1, V_2, V_3) \]

**Linear Model at \( \tilde{V}_Q \) (alt. small signal model)**

\[ i_1 = y_{11} v_1 + y_{12} v_2 + y_{13} v_3 \]
\[ i_2 = y_{21} v_1 + y_{22} v_2 + y_{23} v_3 \]
\[ i_3 = y_{31} v_1 + y_{32} v_2 + y_{33} v_3 \]

where

\[ y_{ij} = \left. \frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j} \right|_{v=\tilde{v}_Q} \]
Small Signal Model

\[
\begin{align*}
\mathbf{i}_1 &= y_{11} \mathbf{v}_1 + y_{12} \mathbf{v}_2 + y_{13} \mathbf{v}_3 \\
\mathbf{i}_2 &= y_{21} \mathbf{v}_1 + y_{22} \mathbf{v}_2 + y_{23} \mathbf{v}_3 \\
\mathbf{i}_3 &= y_{31} \mathbf{v}_1 + y_{32} \mathbf{v}_2 + y_{33} \mathbf{v}_3
\end{align*}
\]

where

\[y_{ij} = \left. \frac{\partial f_i(\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3)}{\partial \mathbf{V}_j} \right|_{\mathbf{V} = \mathbf{V}_Q}\]

- This is a small-signal model of a 4-terminal network and it is linear.
- 9 small-signal parameters characterize the linear 4-terminal network.
- Small-signal model parameters dependent upon Q-point!
End of Lecture 21