Amplification with Transistors
Review from last time

Consider the following MOSFET and BJT Circuits

Assume BJT operating in FA region, MOSFET operating in Saturation
Assume same quiescent output voltage and same resistor $R_1$
One of the most widely used amplifier architectures
Small signal analysis example

\[ V_{IN} = V_M \sin \omega t \]

\( V_M \) is small

\[ V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L} V_M \sin \omega t - V_{SS} + V_T \sqrt{\frac{2}{R}} \]

\[ V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L} V_{SS} + V_T \sqrt{\frac{2}{1 - \frac{V_M \sin \omega t}{V_{SS} + V_T}}} R \]

Recall that if \( x \) is small

\[ 1 + x^2 \approx 1 + 2x \]

\[ V_{OUT} \approx V_{DD} - \frac{\mu C_{OX} W}{2L} V_{SS} + V_T \sqrt{\frac{2}{1 - \frac{2V_M \sin \omega t}{V_{SS} + V_T}}} R \]

\[ V_{OUT} \approx \left\{ V_{DD} - \frac{\mu C_{OX} W}{2L} V_{SS} + V_T R \right\} + \frac{\mu C_{OX} W}{2L} V_{SS} + V_T \left\{ \frac{2V_M \sin \omega t}{V_{SS} + V_T} \right\} R \]
Small signal analysis example

\[ V_{\text{OUT}} \approx V_{\text{OUTQ}} + A_v V_M \sin \omega t \]

\[ A_v = \frac{\mu C_{ox} W}{L} \left( v_{ss} + V_T \right) R \]

Review from last time
Small signal analysis example

\[ A_v = \frac{2I_{DQ} R}{V_{SS} + V_T} \]

Observe the small signal voltage gain is twice the Quiescent voltage across R divided by \( V_{SS} + V_T \)

- This analysis which required linearization of a nonlinear output voltage is quite tedious.
- This approach becomes unwieldy for even slightly more complicated circuits
- A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements
Small signal analysis example

\[
V_{\text{OUT}} \approx V_{\text{OUTQ}} + A_V V_M \sin \omega t
\]

However, there are invariably small errors in this analysis

\[
V_{\text{OUT}} = V_{\text{OUTQ}} + A_V V_M \sin \omega t + \epsilon t
\]

To see the effects of the approximations consider again

\[
V_{\text{OUT}} = V_{\text{DD}} - \frac{\mu C_{\text{Ox}} W}{2L} \left( V_M \sin \omega t - V_{SS} + V_T \right)^2 R
\]

\[
V_{\text{OUT}} = V_{\text{DD}} - \frac{\mu C_{\text{Ox}} R W}{2L} \left( V_M^2 \sin \omega t^2 - 2 V_{SS} + V_T \right) V_M \sin \omega t + V_{SS} + V_T^2
\]

\[
V_{\text{OUT}} = V_{\text{DD}} - \frac{\mu C_{\text{Ox}} R W}{2L} \left( V_M^2 \left[ \frac{1 - \cos 2\omega t}{2} \right] - 2 V_{SS} + V_T \right) V_M \sin \omega t + V_{SS} + V_T^2
\]

\[
V_{\text{OUT}} = \left\{ V_{\text{DD}} - \frac{\mu C_{\text{Ox}} R W}{2L} \left( \frac{V_M^2}{2} + V_{SS} + V_T^2 \right) \right\} + \left\{ \frac{\mu C_{\text{Ox}} W}{L} V_{SS} + V_T R \right\} V_M \sin \omega t + \left\{ \frac{\mu C_{\text{Ox}} R W}{4L} V_M^2 \cos 2\omega t \right\}
\]

Note presence of second harmonic distortion term!
Nonlinear distortion term

\[ V_{\text{out}} \approx V_{\text{outQ}} + A_V V_M \sin \omega t \]

\[ V_{\text{out}} = V_{\text{outQ}} + A_V V_M \sin \omega t + \varepsilon t \]

\[ V_{\text{out}} = \left\{ V_{\text{dd}} - \frac{\mu_{\text{ox}} R W}{2L} \left( \frac{V_M^2}{2} + V_{\text{ss}} + V_T^2 \right) \right\} + \left\{ \frac{\mu_{\text{ox}} W}{L} V_{\text{ss}} + V_T R \right\} V_M \sin \omega t + \left\{ \frac{\mu_{\text{ox}} R W}{4L} V_M^2 \right\} \cos 2\omega t \]

\[ V_{\text{out}} = \left\{ V_{\text{dd}} - \frac{\mu_{\text{ox}} R W}{2L} \left( \frac{V_M^2}{2} + V_{\text{ss}} + V_T^2 \right) \right\} + \left\{ A_V \left( \frac{\mu_{\text{ox}} W}{L} V_{\text{ss}} + V_T R \right) V_M \sin \omega t \right\} + \left\{ A_2 \left( \frac{\mu_{\text{ox}} R W}{4L} V_M^2 \right) \cos 2\omega t \right\} \]

\[ V_{\text{outQ}} = \left\{ V_{\text{dd}} - \frac{\mu_{\text{ox}} R W}{2L} \left( \frac{V_M^2}{2} + V_{\text{ss}} + V_T^2 \right) \right\} \]

\[ A_V = \frac{\mu_{\text{ox}} W}{L} V_{\text{ss}} + V_T R \]

\[ A_2 = \frac{\mu_{\text{ox}} R W}{4L} V_M \]

\[ V_{\text{out}} = V_{\text{outQ}} + A_V V_M \sin \omega t + A_2 V_M \cos 2\omega t \]
Small signal analysis example

Nonlinear distortion term

\[ V_{\text{OUT}} = \tilde{V}_{\text{OUTQ}} + A_v V_m \sin \omega t + A_2 V_m \cos 2\omega t \]

\[ \tilde{V}_{\text{OUTQ}} = \left\{ V_{\text{DD}} - \frac{\mu C_{\text{OX}} W}{2L} \left( \frac{V_m^2}{2} + V_{\text{SS}} + V_T^2 \right) \right\} \]

\[ A_v = \frac{\mu C_{\text{OX}} W}{L} \cdot V_{\text{SS}} + V_T \cdot R \]

\[ A_2 = \frac{\mu C_{\text{OX}} R W}{4L} \cdot V_m \]

Total Harmonic Distortion:

Recall, if \( x(t) = \sum_{k=0}^{\infty} b_k \sin k\omega T \) then

\[ \text{THD} = \sqrt{\sum_{k=2}^{\infty} b_k^2} / b_1 \]

Thus, for this amplifier, as long as \( M_1 \) stays in the saturation region

\[ \text{THD} = \frac{A_2 V_m}{A_v V_m} = \frac{A_2}{A_v} = \frac{\frac{\mu C_{\text{OX}} W}{4L} RV_m}{R \left| V_{\text{SS}} + V_T \right|} = \frac{V_m}{4 \left| V_{\text{SS}} + V_T \right|} \]

Distortion will be small for \( V_m \ll |V_{\text{SS}} + V_T| \)

Distortion will be much worse (larger and more harmonic terms) if \( M_1 \) leaves saturation region.
Small signal analysis using nonlinear models

By selecting appropriate value of $V_{SS}$, $M_1$ will operate in the saturation region

Assume $M_1$ operating in saturation region

$V_{IN} = V_M \sin \omega t$

$V_M$ is small

$I_C = J_s A_e \frac{V_{IN} \cdot V_{EE}}{V_t}$

$I_{CQ} = J_s A_e e^{-\frac{V_{EE}}{V_t}}$

$V_{OUT} = V_{CC} - J_s A_e R e^{\frac{V_{IN} \cdot V_{EE}}{V_t}}$

$V_{OUT} = V_{CC} - J_s A_e R e^{\frac{V_M \sin \omega t \cdot V_{EE}}{V_t}}$

$V_{OUT} = V_{CC} - J_s A_e R e^{\frac{V_{IN} \cdot V_{EE}}{V_t}}$
Small signal analysis using nonlinear models

\[ V_{\text{OUT}} = V_{\text{CC}} - J A R e \]

\[ V_{\text{OUT}} = V_{\text{CC}} - J A R e e \ln \left( \frac{-V_{\text{EE}}}{V_t} \right) \]

*Recall that if \( x \) is small \( e^x \approx 1 + x \) (truncated Taylor’s series)*

\[ V_{\text{IN}} = V_M \sin \omega t \]

\[ V_M \text{ is small} \]

\[ V_{\text{OUT}} \approx \left[ V_{\text{CC}} - J A R e \ln \left( \frac{-V_{\text{EE}}}{V_t} \right) \right] - \left[ J A R e \ln \left( \frac{-V_{\text{EE}}}{V_t} \right) \right] \]
Small signal analysis using nonlinear models

\[ V_{\text{OUT}} \approx V_{CC} - J S A R_e \left( \frac{-V_{EE}}{v_t} \right) - J S A R_e \left( \frac{-V_{EE}}{v_t} \right) \frac{V_m \sin \omega t}{v_t} \]

\[ I_{CQ} = J_S A_E e^{\frac{V_{EE}}{v_t}} \]

\[ V_{IN} = V_M \sin \omega t \]

\[ V_M \text{ is small} \]

\[ V_{\text{OUT}} \approx \left[ V_{CC} - I_{CQ} R_1 \right] - \left( \frac{I_{CQ} R_1}{V_t} \right) V_m \sin \omega t \]

Quiescent Output

ss Voltage Gain
Comparison of Gains for MOSFET and BJT Circuits

**BJT**

\[ A_{vb} = - \frac{I_{CQ} R_1}{V_t} \]

If \( I_{DQ} R = I_{CQ} R_1 = 2V \), \( V_{SS} + V_T = -1V \), \( V_t = 25mV \)

\[ A_{vb} = - \frac{I_{CQ} R_1}{V_t} = - \frac{2V}{25mV} = -80 \]

**MOSFET**

\[ A_{vm} = \frac{2I_{DQ} R}{V_{SS} + V_T} \]

\[ A_{vm} = \frac{2I_{DQ} R}{V_{SS} + V_T} = \frac{4V}{-1V} = -4 \]

Observe \( A_{vb} \gg A_{vm} \)

Due to exponential-law rather than square-law model
Operation with Small-Signal Inputs

- Analysis procedure for these simple circuits was very tedious
- This approach will be unmanageable for even modestly more complicated circuits
- Faster analysis method is needed!
Operation with Small-Signal Inputs

Why was this analysis so tedious?

Because of the nonlinearity in the device models

What was the key technique in the analysis that was used to obtain a simple expression for the output?

\[ V_{\text{OUT}} = V_{\text{CC}} - J_{S} A_{E} R_{1} e^{\frac{-V_{\text{EE}}}{V_{t}}} \frac{V_{M} \sin \omega t}{V_{t}} \]

\[ V_{\text{OUT}} \equiv \left[ V_{\text{CC}} - I_{CQ} R_{1} \right] - \left( \frac{I_{CQ} R_{1}}{V_{t}} \right) V_{M} \sin \omega t \]

Linearization of the nonlinear output expression at the operating point
Operation with Small-Signal Inputs

\[ I_{CQ} = J_S A_E e^{v_t} \]

\[ V_{OUT} \approx \left[ V_{CC} - I_{CQ} R_1 \right] - \left( \frac{I_{CQ} R_1}{V_t} \right) V_m \sin \omega t \]

Quiescent Output

ss Voltage Gain

Small-signal analysis strategy

1. Obtain Quiescent Output (Q-point)
2. Linearize circuit at Q-point instead of linearize the nonlinear solution
3. Analyze linear “small-signal” circuit
4. Add quiescent and small-signal outputs to obtain good approximation to actual output
Small-Signal Principle

Nonlinear function

\[ y = f(x) \]

\[ X_Q \]

\[ Y_Q \]

Q-point
Small-Signal Principle

Region around Q-Point

Q-point

y = f(x)

Y_Q

x_Q

y

x
Small-Signal Principle

Relationship is nearly linear in a small enough region around Q-point
Region of linearity is often quite large
Linear relationship may be different for different Q-points
Small-Signal Principle

Relationship is nearly linear in a small enough region around Q-point. Region of linearity is often quite large. Linear relationship may be different for different Q-points.
Device Behaves Linearly in Neighborhood of Q-Point
Can be characterized in terms of a small-signal coordinate system
Small-Signal Principle

Linear Model at Q-point

\[ y = f(x) \]

\[ y - y_Q = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} (x - x_Q) \]
Small-Signal Principle

Linear Model at Q-point

\[ \frac{y - y_Q}{x - x_Q} = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} \]

\[ y - y_Q = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} \]

y = f(x)

Q-point

y_Q

x = x_Q

X_Q

Y_Q

x_Q

X_QINT
Small-Signal Principle

\[ y = mx + b \]

\( y = f(x) \)

\( y - y_Q = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} (x - x_Q) \)

\[ y = \left[ \frac{\partial f}{\partial x} \right]_{x=x_Q} x + \left[ y_Q - x_Q \frac{\partial f}{\partial x} \right]_{x=x_Q} \]

\[ m = \left[ \frac{\partial f}{\partial x} \right]_{x=x_Q} \]

\[ b = \left[ y_Q - x_Q \frac{\partial f}{\partial x} \right]_{x=x_Q} \]
Small-Signal Principle

Changing coordinate systems:

\[ y_{SS} = y - y_Q \]
\[ x_{SS} = x - x_Q \]

\[ y - y_Q = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} \]
\[ x - x_Q \]

\[ y_{ss} = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} \]
\[ x_{ss} \]
Small-Signal Model:

\[ y_{ss} = \left. \frac{\partial f}{\partial x} \right|_{x=x_Q} x_{ss} \]

- Linearized model for the nonlinear function \( y=f(x) \)
- Valid in the region of the Q-point
- Will show the small signal model is simply Taylor’s series expansion at the Q-point truncated after first-order terms
Small-Signal Principle

Observe:

\[ y - y_Q = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} (x - x_Q) \]

\[ y = y_Q + \frac{\partial f}{\partial x} \bigg|_{x=x_Q} (x - x_Q) \]

Recall Taylor's Series Expansion of nonlinear function \( f \) at expansion point \( x_0 \)

\[ y = f(x) \bigg|_{x=x_0} \sum_{k=1}^{\infty} \left( \frac{1}{k!} \frac{\partial^k f}{\partial x^k} \right|_{x=x_0} (x-x_0)^k \]

Small-Signal Model:

\[ y = f(x) \bigg|_{x=x_Q} + \frac{\partial f}{\partial x} \bigg|_{x=x_Q} (x - x_Q) \]

\[ y_{ss} = \frac{\partial f}{\partial x} \bigg|_{x=x_Q} (x_{ss} - x_Q) \]

Mathematically, linearized model is simply Taylor's series expansion at the Q-point truncated after first-order terms with notation \( x_Q = x_0 \)
End of Lecture 23