EE 330
Lecture 23

• Small Signal Analysis
  – SS Models for BJT
  – Alternate two-port models
  – Graphical small-signal analysis
Exam Schedule

Exam 1        Monday Feb 23
Exam 2        Wednesday March 25
Exam 3        Monday April 20
Review from Last Lecture

Comparison of Gains for MOSFET and BJT Circuits

**BJT**

\[ A_{VB} = - \frac{I_{CQ}}{V_t} R_1 \]

**MOSFET**

\[ A_{VM} = \frac{2I_{DQ} R}{V_{SS} + V_T} \]
Small Signal Model of MOSFET

by convention, \( y_{21} = g_m, y_{22} = g_0 \)

\[
\therefore \quad y_{21} \equiv g_m = \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_T)
\]

\[
y_{22} = g_0 \equiv \lambda I_{DQ}
\]

Note: \( g_o \) vanishes when \( \lambda = 0 \)
Consider again:

**Small signal analysis example**

For \( \lambda = 0 \), \( g_o = \lambda I_{DQ} = 0 \)

\[
A_v = \frac{V_{OUT}}{V_{IN}} = -\frac{g_m}{g_o + 1/R}
\]

Same expression as derived before

More accurate gain can be obtained if \( \lambda \) effects are included and does not significantly increase complexity of small signal analysis
Review from Last Lecture

**Small Signal Model of BJT**

An equivalent circuit

\[
\begin{align*}
  i_B &= y_{11} V_{BE} + y_{12} V_{CE} \\
  i_C &= y_{21} V_{BE} + y_{22} V_{CE} \\
  i_B &= g_\pi V_{BE} \\
  i_C &= g_m V_{BE} + g_O V_{CE}
\end{align*}
\]
Consider again:

**Small signal analysis example**

\[ V_{IN} = V_M \sin(\omega t) \]

Recall the derivation was very tedious and time consuming!

\[ A_{VB} = -\frac{I_{CQ} R}{V_t} \]

Derived for \( V_{AF} = 0 \)
Neglect $V_{AF}$ effects (i.e. $V_{AF}=\infty$) to be consistent with earlier analysis

$$g_o = \frac{I_{CQ}}{V_{AF}} \Bigg|_{V_{AF}=\infty} = 0$$

$$V_{OUT} = -g_m R V_{BE}$$

$$V_{IN} = V_{BE}$$

$$A_V = \frac{V_{OUT}}{V_{IN}} = -g_m R$$

$$g_m = \frac{I_{CQ}}{V_t}$$

$$A_V = -\frac{I_{CQ}R}{V_t}$$

Note this is identical to what was obtained with the direct nonlinear analysis
Small Signal BJT Model – alternate representation

Observe:

\[ g_{m} u_{be} = i_{b} \]

\[ g_{m} u_{be} = i_{b} \frac{g_{m}}{g_{\pi}} \]

\[ g_{m} = \frac{I_{Q}}{V_{t}} \]

\[ g_{\pi} = \frac{I_{Q}}{\beta V_{t}} \]

\[ g_{o} \approx \frac{I_{Q}}{V_{AF}} \]

\[ g_{m} u_{be} = \beta i_{b} \]

Can replace the voltage dependent current source with a current dependent current source
Alternate equivalent small signal model

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]
Small-Signal Model Representations

(3-terminal network – also relevant with 4-terminal networks)

Have developed small-signal models for the MOSFET and BJT

Models have been based upon arbitrary assumption that \( v_1, v_2 \) are independent variables

Have already seen some alternatives for parameter definitions in these models

Alternative representations are sometimes used
Small-Signal Model Representations

Bi-Linear Relationship between \( i_1, i_2, v_1, v_2 \)

The good, the bad, and the unnecessary!!

what we have developed:

The hybrid parameters:
Small-Signal Model Representations

The z-parameters

\[ v_1 = z_{11}i_1 + z_{12}i_2 \]
\[ v_2 = z_{21}i_1 + z_{22}i_2 \]

The ABCD parameters:

\[ v_1 = Av_2 - Bi_2 \]
\[ i_1 = Cv_2 - Di_2 \]
Small-Signal Model Representations

The S-parameters

The T parameters:
Small-Signal Model Representations

Bi-Linear Relationship between $i_1, i_2, v_1, v_2$

The good, the bad, and the **unnecessary** !!

- Equivalent circuits often given for each representation
- All provide identical characterization
- Easy to move from any one to another
Small-Signal Model Representations

Conversions between $S$, $Z$, $Y$, $h$, $ABCD$, and $T$ parameters which are valid for complex source and load impedances

Dean A. Frickey, Member, IEEE
What are the simplified dc equivalent models?
What are the simplified dc equivalent models?

\[ G \]  
\[ D \]  
\[ S \]  
\[ V_{GSQ} \]  
\[ \frac{\mu_{COX} W}{2L} (V_{GSQ} - V_{Th})^2 \]  
\[ \frac{\mu_{COX} W}{2L} (V_{GSQ} - V_{Tp})^2 \]  
\[ \beta I_{BQ} \]  
\[ I_{BQ} \]  
\[ 0.6V \]  
\[ 0.6V \]
Small-signal Operation of Nonlinear Circuits

• Small-signal principles

• Example Circuit

• Small-Signal Models

Small-Signal Analysis of Nonlinear Circuits
Recall:

Alternative Approach to small-signal analysis of nonlinear networks

1. Linearize nonlinear devices
   *(have small-signal model for key devices!)*

2. Replace all devices with small-signal equivalent

3. Solve linear small-signal network

Remember that the small-signal model is operating point dependent!

Thus need Q-point to obtain values for small signal parameters
Example:

Determine the small signal voltage gain $A_V = \frac{V_{\text{OUT}}}{V_{\text{IN}}}$. Assume $M_1$ and $M_2$ are operating in the saturation region and that $\lambda = 0$. 
Example: Determine the small signal voltage gain $A_V = \frac{V_{OUT}}{V_{IN}}$. Assume $M_1$ and $M_2$ are operating in the saturation region and that $\lambda = 0$. 

Small-signal circuit
Example: Determine the small signal voltage gain $A_V = \frac{v_{OUT}}{v_{IN}}$. Assume $M_1$ and $M_2$ are operating in the saturation region and that $\lambda = 0$.

Small-signal circuit

Small-signal MOSFET model for $\lambda = 0$
Example: Determine the small signal voltage gain $A_v = \frac{V_{OUT}}{V_{IN}}$. Assume $M_1$ and $M_2$ are operating in the saturation region and that $\lambda = 0$. 

![Small-signal circuit diagram]

Small-signal circuit
Example:

![Small-signal circuit](image)

**Analysis:**

**By KCL**

\[ g_{m1} V_{GS1} = g_{m2} V_{GS2} \]

but

\[ V_{GS1} = V_{IN} \]

\[ -V_{GS2} = V'_{OUT} \]

thus:

\[ A_v = \frac{V'_{OUT}}{V_{IN}} = -\frac{g_{m1}}{g_{m2}} \]
Example:

Small-signal circuit

Analysis:

\[ A_v = \frac{V_{OUT}}{V_{IN}} = -\frac{g_{m1}}{g_{m2}} \]

Recall:

\[ g_m = -\sqrt{2I_D \mu C_{ox}} \sqrt{\frac{W_1}{L_1}} \]

\[ A_v = -\sqrt{2I_D \mu C_{ox} \frac{W_1}{L_1}} = -\sqrt{\frac{W_1}{W_2}} \sqrt{\frac{L_2}{L_1}} \]
Example:

Small-signal circuit

Analysis:

\[ A_V = \frac{v_{OUT}}{v_{IN}} = -\frac{g_{m1}}{g_{m2}} \]

Recall:

\[ A_V = -\sqrt{\frac{W_1}{W_2}} \sqrt{\frac{L_2}{L_1}} \]

If \( L_1 = L_2 \), obtain

\[ A_V = -\sqrt{\frac{W_1}{W_2}} \sqrt{\frac{L_2}{L_1}} = -\sqrt{\frac{W_1}{W_2}} \]

The width and length ratios can be accurately set when designed in a standard CMOS process
Example:

Obtain the small signal model of the following circuit. Assume MOSFET is operating in the saturation region.
Example

Obtain the small signal model of the following circuit. Assume MOSFET is operating in the saturation region

Solution:

\[ V \left( g_m + g_0 \right) = I \]

\[ R_{EQ} = \frac{1}{g_m + g_0} \approx \frac{1}{g_m} \]
End of Lecture 23