EE 330
Lecture 24

Amplification with Transistor Circuits
Amplification with Transistors

Review from Last Lecture

From Wikipedia:

Generally, an **amplifier** or simply **amp**, is any **device** that changes, usually increases, the amplitude of a **signal**. The "signal" is usually voltage or current.

- It is difficult to increase the voltage or current very much with passive RC circuits
- Voltage and current levels can be increased a lot with transformers but not practical in integrated circuits
- Power levels can not be increased with passive elements (R, L, C, and Transformers)
- Often an amplifier is defined to be a circuit that can increase power levels
- Transistors can be used to increase not only signal levels but power levels to a load
- In transistor circuits, power that is delivered in the signal path is supplied by a biasing network
Applications of Devices as Amplifiers

Review from Last Lecture

Typical Regions of Operation by Circuit Function

MOS
- Triode
- Cutoff

Bipolar
- Saturation
- Cutoff

Logic Circuits

Linear Circuits
- Saturation
- Forward Active
Review from Last Lecture

Consider the following MOSFET and BJT Circuits

- MOS and BJT Architectures often Identical
- Circuit are Highly Nonlinear
- Nonlinear Analysis Methods Must be used to analyze these and almost any other nonlinear circuit
Methods of Analysis of Nonlinear Circuits

Will consider three different analysis requirements and techniques for some particularly common classes of nonlinear circuits

1. **Circuits with continuously differential devices**

   Interested in obtaining transfer characteristics of these circuits or outputs for given input signals

2. **Circuits with piecewise continuous devices**

   Interested in obtaining transfer characteristics of these circuits or outputs for a given input signals

3. **Circuits with small-signal inputs that vary around some operating point**

   Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Other types of nonlinearities may exist and other types of analysis may be required but we will not attempt to categorize these scenarios in this course
Small signal operation of nonlinear circuits

$$V_{IN} = V_M \sin \omega t$$

$V_M$ is small

Practical methods of analyzing and designing circuits that operate with small signal inputs are really important

Two key questions:

How small must the input signals be to obtain locally-linear operation of a nonlinear circuit?

How can these locally-linear (alt small signal) circuits be analyzed and designed?
Small signal operation of nonlinear circuits

\[ V_{IN} = V_M \sin \omega t \]

\( V_M \) is small

Example of circuit that is widely used in locally-linear mode of operation

Two methods of analyzing locally-linear circuits will be considered, one of these is by far the most practical
Small signal operation of nonlinear circuits

$V_{IN} = V_M \sin \omega t$

$V_M$ is small

Two methods of analyzing locally-linear circuits for small-signal excitations will be considered, one of these is by far the most practical

1. Analysis using nonlinear models
2. Small signal analysis using locally-linearized models
Small signal analysis using nonlinear models

By selecting appropriate value of $V_{SS}$, $M_1$ will operate in the saturation region.

Assume $M_1$ operating in saturation region.

$V_{IN} = V_M \sin \omega t$

$V_M$ is small

$V_{OUT} = V_{DD} - I_D R$

$I_D = \frac{\mu C_{ox} W}{2L} (V_{IN} - V_{SS} - V_T)^2$

$I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{SS} + V_T)^2$

$V_{OUT} = V_{DD} - \frac{\mu C_{ox} W}{2L} (V_{IN} - V_{SS} - V_T)^2 R$

$V_{OUT} = V_{DD} - \frac{\mu C_{ox} W}{2L} \left(V_M \sin \omega t - [V_{SS} + V_T]^2\right) R$
Small signal analysis example

\[ V_{IN} = V_M \sin \omega t \]

\( V_M \) is small

\[ V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L} \left( V_M \sin \omega t - \left[ V_{SS} + V_T \right]^2 \right) R \]

\[ V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L} \left( V_{SS} + V_T \right)^2 \left( 1 - \frac{V_M \sin \omega t}{V_{SS} + V_T} \right)^2 R \]

Recall that if \( x \) is small

\[ (1+x)^2 \approx 1 + 2x \]

\[ V_{OUT} \approx V_{DD} - \frac{\mu C_{OX} W}{2L} \left[ V_{SS} + V_T \right]^2 \left( 1 - \frac{2V_M \sin \omega t}{V_{SS} + V_T} \right) R \]

\[ V_{OUT} \approx \left\{ V_{DD} - \frac{\mu C_{OX} W}{2L} \left[ V_{SS} + V_T \right]^2 R \right\} + \left\{ \frac{\mu C_{OX} W}{2L} \left[ V_{SS} + V_T \right] R \right\} V_M \sin \omega t \]
Small signal analysis example

By selecting appropriate value of $V_{SS}$, $M_1$ will operate in the saturation region

Assume $M_1$ operating in saturation region

$$V_{OUT} \approx \left\{ V_{DD} - \frac{\mu C_{ox} W}{2L} \left[ V_{SS} + V_T \right]^2 R \right\} + \left\{ \frac{\mu C_{ox} W}{L} \left[ V_{SS} + V_T \right] R \right\} V_M \sin \omega t$$
Small signal analysis example

Assume $M_1$ operating in saturation region

$$V_{IN} = V_M \sin \omega t$$

$$V_{OUT} \approx \left\{ \frac{V_{DD}}{2} - \frac{\mu C_{ox} W}{2L} \left[ V_{SS} + V_T \right]^2 R \right\} + \left\{ \frac{\mu C_{ox} W}{L} \left[ V_{SS} + V_T \right] R \right\} V_M \sin \omega t$$

**Quiescent Output**

**ss Voltage Gain**

$$A_v = \frac{\mu C_{ox} W}{L} \left[ V_{SS} + V_T \right] R$$

$$V_{OUTQ} = \left\{ V_{DD} - \frac{\mu C_{ox} W}{2L} \left[ V_{SS} + V_T \right]^2 R \right\}$$

$$V_{OUT} \approx V_{OUTQ} + A_v V_M \sin \omega t$$

Note the ss voltage gain is negative since $V_{SS} + V_T < 0$!
Small signal analysis example

Assume $M_1$ operating in saturation region

$$V_{OUT} \approx V_{OUTQ} + A_v V_M \sin \omega t$$

$$A_v = \frac{\mu C_{ox} W}{L} \left[V_{ss} + V_T\right] R$$

$$V_{OUTQ} = \left\{ V_{DD} - \frac{\mu C_{ox} W}{2L} \left[V_{ss} + V_T\right]^2 R \right\}$$

But – this expression gives little insight into how large the gain is!
And the analysis for even this very simple circuit was messy!
Small signal analysis example

\[ V_{\text{out}} \approx V_{\text{outq}} + A_v V_m \sin \omega t \]
Small signal analysis example

\[ V_{\text{OUT}} \approx V_{\text{OUTQ}} + A_v V_m \sin \omega t \]

\[ A_v = \frac{\mu C_{ox} W}{L} \left[ V_{ss} + V_T \right] R \]
Small signal analysis example

\[ V_{OUT} \approx V_{OUTQ} + A_v V_M \sin \omega t \]

\[ A_v = \frac{\mu C_{ox} W}{L} \left[ V_{SS} + V_T \right] R \]
Small signal analysis example

\[ V_{\text{OUT}} \approx V_{\text{OUTQ}} + A_V V_M \sin \omega t \]

Serious Distortion occurs if signal is too large or Q-point non-optimal
Here “clipping” occurs for high \( V_{\text{OUT}} \)

\[ A_v = \frac{\mu C_{ox} W}{L} \left[ V_{\text{ss}} + V_T \right] R \]
Small signal analysis example

\[ V_{OUT} = V_{OUTQ} + A_v V_M \sin \omega t \]

\[ A_v = \frac{\mu C_{ox} W}{L} [V_{ss} + V_T] R \]

Serious Distortion occurs if signal is too large or Q-point non-optimal
Here “clipping” occurs for low \( V_{OUT} \)
Small signal analysis example

\[ V_{\text{out}} \approx V_{\text{OUTQ}} + A_V V_M \sin \omega t \]

\[ A_V = \frac{\mu C_{ox} W}{L} \left[ V_{ss} + V_T \right] R \]

**But recall:**

*Thus, substituting from the expression for \( I_{DQ} \) we obtain*

\[ A_V = \frac{2I_{DQ} R}{\left[ V_{ss} + V_T \right]} \]

**Note this is negative since** \( V_{ss} + V_T < 0 \)
Small signal analysis example

\[ A_v = \frac{2I_{DQ} R}{V_{SS} + V_T} \]

Observe the small signal voltage gain is twice the Quiescent voltage across \( R \) divided by \( V_{SS} + V_T \)

Can make \( |A_v| \) large by making \( |V_{SS} + V_T| \) small

• This analysis which required linearization of a nonlinear output voltage is quite tedious.

• This approach becomes unwieldy for even slightly more complicated circuits

• A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements
Small signal analysis example

\[ V_{\text{out}} \approx V_{\text{outq}} + A_V V_M \sin \omega t \]

However, there are invariably small errors in this analysis

\[ V_{\text{out}} = V_{\text{outq}} + A_V V_M \sin \omega t + \varepsilon(t) \]

To see the effects of the approximations consider again

\[ V_{\text{out}} = V_{\text{dd}} - \frac{\mu C_{\text{ox}} W}{2L} \left( V_M \sin \omega t - \left[ V_{\text{ss}} + V_T \right]^2 \right) R \]

\[ V_{\text{out}} = V_{\text{dd}} - \frac{\mu C_{\text{ox}} W}{2L} \left( V_M^2 \sin(\omega t)^2 - 2[V_{\text{ss}} + V_T] V_M \sin \omega t + [V_{\text{ss}} + V_T]^2 \right) \]

\[ V_{\text{out}} = V_{\text{dd}} - \frac{\mu C_{\text{ox}} W}{2L} \left( V_M^2 \left[ \frac{1 - \cos 2\omega t}{2} \right] - 2[V_{\text{ss}} + V_T] V_M \sin \omega t + [V_{\text{ss}} + V_T]^2 \right) \]

\[ V_{\text{out}} = \left\{ V_{\text{dd}} - \frac{\mu C_{\text{ox}} W}{2L} \left( V_M^2 / 2 + [V_{\text{ss}} + V_T]^2 \right) \right\} + \left\{ \frac{\mu C_{\text{ox}} W}{L} \left[ V_{\text{ss}} + V_T \right] R \right\} V_M \sin \omega t \] + \left\{ \frac{\mu C_{\text{ox}} W R}{4L} V_M^2 \cos 2\omega t \right\} \]

Note presence of second harmonic distortion term!
Small signal analysis example

Nonlinear distortion term

\[ V_{\text{OUT}} \approx V_{\text{OUTQ}} + A_V V_M \sin \omega t \]

\[ V_{\text{OUT}} = V_{\text{OUTQ}} + A_V V_M \sin \omega t + \epsilon(t) \]

\[
V_{\text{OUT}} = \left\{ V_{\text{DD}} - \frac{\mu_{\text{Cox}} R W}{2L} \left( \frac{V_M^2}{2} + [V_{SS} + V_T]^2 \right) \right\} + \left\{ \frac{\mu_{\text{Cox}} W}{L} [V_{SS} + V_T] R \right\} V_M \sin \omega t + \left\{ \frac{\mu_{\text{Cox}} R W}{4L} V_M^2 \right\} \cos 2\omega t \]

\[
V_{\text{OUT}} = \tilde{V}_{\text{OUTQ}} \left\{ V_{\text{DD}} - \frac{\mu_{\text{Cox}} R W}{2L} \left( \frac{V_M^2}{2} + [V_{SS} + V_T]^2 \right) \right\} + \left\{ A_V \left( \frac{\mu_{\text{Cox}} W}{L} [V_{SS} + V_T] R \right) \right\} V_M \sin \omega t + \left\{ A_2 \left( \frac{\mu_{\text{Cox}} R W}{4L} V_M^2 \right) \right\} \cos 2\omega t \]

\[
\tilde{V}_{\text{OUTQ}} = \left\{ V_{\text{DD}} - \frac{\mu_{\text{Cox}} R W}{2L} \left( \frac{V_M^2}{2} + [V_{SS} + V_T]^2 \right) \right\} \\
A_V = \frac{\mu_{\text{Cox}} W}{L} [V_{SS} + V_T] R \\
A_2 = \frac{\mu_{\text{Cox}} R W}{4L} V_M \\
V_{\text{OUT}} = \tilde{V}_{\text{OUTQ}} + \left\{ A_V V_M \sin \omega t \right\} + \left\{ A_2 V_M \cos 2\omega t \right\} \]
Small signal analysis example

Nonlinear distortion term

\[ V_{OUT} = \tilde{V}_{OUTQ} + \{A_v V_m \sin \omega t\} + \{A_2 V_m \cos 2\omega t\} \]

\[ \tilde{V}_{OUTQ} = \left\{ V_{DD} - \frac{\mu C_{ox} R W}{2L} \left( \frac{V_m^2}{2} + [V_{SS} + V_T]^2 \right) \right\} \]

\[ A_v = \frac{\mu C_{ox} W}{L} [V_{SS} + V_T] R \]

\[ A_2 = \frac{\mu C_{ox} R W}{4L} V_m \]

Total Harmonic Distortion:

Recall, if \( x(t) = \sum_{k=0}^{\infty} b_k \sin (k\omega T) \) then

\[ \text{THD} = \frac{\sqrt{\sum_{k=2}^{\infty} b_k^2}}{b_1} \]

Thus, for this amplifier, as long as \( M_1 \) stays in the saturation region

\[ \text{THD} = \frac{A_2 V_m}{A_v V_m} = \frac{A_2}{A_v} = \frac{\mu C_{ox} W}{4L} \frac{R V_m}{R |V_{SS} + V_T|} = \frac{V_m}{4|V_{SS} + V_T|} \]

Distortion will be small for \( V_m < < |V_{SS} + V_T| \)

Distortion will be much worse (larger and more harmonic terms) if \( M_1 \) leaves saturation region.
Small signal analysis using nonlinear models

**By selecting appropriate value of** $V_{SS}$, $M_1$ **will operate in the saturation region**

Assume $M_1$ operating in saturation region

$$V_{OUT} = V_{CC} - I_C R_1$$

$$I_C = J_S A e^{\frac{V_{IN} - V_{EE}}{V_t}}$$

$$I_{CQ} = J_S A e^{\frac{-V_{EE}}{V_t}}$$

$$V_{OUT} = V_{CC} - J_S A e^{\frac{V_{IN} - V_{EE}}{V_t}}$$

$$V_{OUT} = V_{CC} - J_S A e^{\frac{V_{M \sin(\omega t)} - V_{EE}}{V_t}}$$

$V_{IN} = V_m \sin \omega t$

$V_m$ is small
Small signal analysis using nonlinear models

\[ V_{\text{OUT}} = V_{\text{CC}} - J_S A_R e^{-\frac{V_{\text{EE}}}{V_t}} \]

\[ V_{\text{OUT}} = V_{\text{CC}} - J_S A_R e^{\frac{V_M \sin(\omega t) - V_{\text{EE}}}{V_t}} e^{-\frac{V_{\text{EE}}}{V_t}} \]

Recall that if \( x \) is small

\[ e^x \approx 1 + x \]

(truncated Taylor's series)

\[ V_{\text{IN}} = V_M \sin(\omega t) \]

\[ V_M \text{ is small} \]

\[ V_{\text{OUT}} \approx \left[ V_{\text{CC}} - J_S A_R e^{-\frac{V_{\text{EE}}}{V_t}} \right] - \left[ J_S A_R e^{-\frac{V_{\text{EE}}}{V_t}} \frac{V_M \sin(\omega t)}{V_t} \right] \]
Small signal analysis using nonlinear models

\[ V_{OUT} \approx \left[ V_{CC} - J_s A_s R_1 e^{-\frac{V_{EE}}{V_t}} \right] - \left[ J_s A_s R_1 e^{-\frac{V_{EE}}{V_t}} \frac{V_M \sin(\omega t)}{V_t} \right] \]

\[ I_{CQ} = J_s A_s e^{\frac{-V_{EE}}{V_t}} \]

\[ V_{IN} = V_m \sin(\omega t) \]

\( V_m \) is small

**Quiescent Output**

**ss Voltage Gain**
Comparison of Gains for MOSFET and BJT Circuits

**BJT**

\[ A_{VB} = - \frac{I_{CQ} R_1}{V_t} \]

If \( I_{DQ} R = I_{CQ} R_1 = 2V, V_{SS} + V_T = -1V, V_t = 25mV \)

\[ A_{VB} = - \frac{I_{CQ} R_1}{V_t} = - \frac{2V}{25mV} = -80 \]

**MOSFET**

\[ A_{VM} = \frac{2I_{DQ} R}{V_{SS} + V_T} \]

\[ A_{VM} = \frac{2I_{DQ} R}{V_{SS} + V_T} = \frac{4V}{-1V} = -4 \]

Observe \( A_{VB} >> A_{VM} \)

Due to exponential-law rather than square-law model
Operation with Small-Signal Inputs

• Analysis procedure for these simple circuits was very tedious

• This approach will be unmanageable for even modestly more complicated circuits

• Faster analysis method is needed!
Operation with Small-Signal Inputs

Why was this analysis so tedious?

Because of the nonlinearity in the device models

What was the key technique in the analysis that was used to obtain a simple expression for the output?

\[ V_{\text{OUT}} = V_{CC} - J_s A_E R_1 e^{\frac{-V_{EE}}{V_l}} e^{\frac{V_m \sin(\omega t)}{V_l}} \]

\[ V_{\text{OUT}} \approx \left[ V_{CC} - I_{CQ} R_1 \right] - \left( \frac{I_{CQ} R_1}{V_l} \right) V_m \sin(\omega t) \]

Linearization of the nonlinear output expression at the operating point
Operation with Small-Signal Inputs

\[ I_{CQ} = J_S A_E e^{\frac{-V_{EE}}{V_t}} \]

\[ V_{OUT} \approx \left[ V_{CC} - I_{CQ} R_1 \right] - \left( \frac{I_{CQ} R_1}{V_t} \right) V_m \sin(\omega t) \]

Quiescent Output

ss Voltage Gain

Small-signal analysis strategy

1. Obtain Quiescent Output (Q-point)
2. Linearize circuit at Q-point instead of linearize the nonlinear solution
3. Analyze linear “small-signal” circuit
4. Add quiescent and small-signal outputs to obtain good approximation to actual output
End of Lecture 23