Amplification with Transistor Circuits
Small Signal Modelling
Amplification with Transistors

From Wikipedia:

Generally, an **amplifier** or simply **amp**, is any **device** that changes, usually increases, the amplitude of a **signal**. The "signal" is usually voltage or current.

- It is difficult to increase the voltage or current very much with passive RC circuits.
- Voltage and current levels can be increased a lot with transformers but not practical in integrated circuits.
- Power levels can not be increased with passive elements (R, L, C, and Transformers).
- Often an amplifier is defined to be a circuit that can increase power levels.
- Transistors can be used to increase not only signal levels but power levels to a load.
- In transistor circuits, power that is delivered in the signal path is supplied by a biasing network.

Review from Last Lecture
Consider the following MOSFET and BJT Circuits

Assume BJT operating in FA region, MOSFET operating in Saturation
Assume same quiescent output voltage and same resistor $R_1$
One of the most widely used amplifier architectures
Consider the following MOSFET and BJT Circuits

**BJT**

- $V_{CC}$
- $R_1$
- $Q_1$
- $V_{OUT}$
- $V_{EE}$

**MOSFET**

- $V_{DD}$
- $R_1$
- $M_1$
- $V_{OUT}$
- $V_{SS}$

- MOS and BJT Architectures often Identical
- Circuit are Highly Nonlinear
- Nonlinear Analysis Methods Must be used to analyze these and almost any other nonlinear circuit
KCL and KVL apply to both linear and nonlinear circuits

Superposition, voltage divider and current divider equations, Thevenin and Norton equivalence apply only to linear circuits!

Some other analysis techniques that have been developed may apply only to linear circuits as well
Methods of Analysis of Nonlinear Circuits

Will consider three different analysis requirements and techniques for some particularly common classes of nonlinear circuits

1. Circuits with continuously differential devices

   Interested in obtaining transfer characteristics of these circuits or outputs for given input signals

2. Circuits with piecewise continuous devices

   Interested in obtaining transfer characteristics of these circuits or outputs for a given input signals

3. Circuits with small-signal inputs that vary around some operating point

   Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Other types of nonlinearities may exist and other types of analysis may be required but we will not attempt to categorize these scenarios in this course
Analysis Strategy:

Use KVL and KCL for analysis

Represent nonlinear models for devices either mathematically or graphically

Solve the resultant set of equations for the variables of interest
2. Circuits with piecewise continuous devices

\[ f(x) = \begin{cases} f_1(x) & x < x_i \text{ region 1} \\ f_2(x) & x > x_i \text{ region 2} \end{cases} \]

Analysis Strategy:

Guess region of operation

Solve resultant circuit using the previous method

Verify region of operation is valid

Repeat the previous 3 steps as often as necessary until region of operation is verified

It helps to guess right the first time but a wrong guess will not result in an incorrect solution because a wrong guess cannot be verified.

Review from Last Lecture
3. Circuits with small-signal inputs that vary around some operating point

Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point.

Analysis Strategy:

Use methods from previous class of nonlinear circuits.

More Practical Analysis Strategy:

Determine the operating point (using method 1 or 2 discussed above after all small signal independent inputs are set to 0).

Develop small signal (linear) model for all devices in the region of interest (around the operating point or “Q-point”).

Create small signal equivalent circuit by replacing all devices with small-signal equivalent.

Solve the resultant small-signal (linear) circuit. Can use KCL, DVL, and other linear analysis tools such as superposition, voltage and current divider equations, Thevenin and Norton equivalence.

Determine boundary of region where small signal analysis is valid.
If \( V_M \) is sufficiently small, then any nonlinear circuit operating at a region where there are no abrupt nonlinearities will have a nearly sinusoidal output and the variance of the magnitude of this output with \( V_M \) will be nearly linear (could be viewed as “locally linear”)

This is termed the “small signal” operation of the nonlinear circuit

When operating with “small signals”, the nonlinear circuit performs linearly with respect to these small signals thus other properties of linear networks such as superposition apply provided the sum of all superimposed signals remains sufficiently small

Other types of “small signals”, e.g. square waves, triangular waves, or even arbitrary waveforms often are used as inputs as well but the performance of the nonlinear network also behaves linearly for these inputs

Many useful electronic systems require the processing of these small signals

Practical methods of analyzing and designing circuits that operate with small signal inputs are really important
Small signal operation of nonlinear circuits

$V_{\text{IN}} = V_M \sin \omega t$

$V_M$ is small

Practical methods of analyzing and designing circuits that operate with small signal inputs are really important

Two key questions:

How small must the input signals be to obtain locally-linear operation of a nonlinear circuit?

How can these locally-linear (alt small signal) circuits be analyzed and designed?
Consider the following MOSFET and BJT Circuits

One of the most widely used amplifier architectures
Small signal operation of nonlinear circuits

\[ V_{IN} = V_M \sin \omega t \]

\( V_M \) is small

Example of circuit that is widely used in locally-linear mode of operation

Two methods of analyzing locally-linear circuits will be considered, one of these is by far the most practical.
Small signal operation of nonlinear circuits

\[ V_{IN} = V_M \sin \omega t \]

\( V_M \) is small

Two methods of analyzing locally-linear circuits for small-signal excitations will be considered, one of these is by far the most practical

1. Analysis using nonlinear models
2. Small signal analysis using locally-linearized models
Small signal analysis using nonlinear models

By selecting appropriate value of $V_{SS}$, $M_1$ will operate in the saturation region

Assume $M_1$ operating in saturation region

$V_{IN} = V_M \sin \omega t$

$V_M$ is small

$V_{OUT} = V_{DD} - I_D R$

$I_D = \frac{\mu C_{OX} W}{2L} (V_{IN} - V_{SS} - V_T)^2$

$I_{DQ} = \frac{\mu C_{OX} W}{2L} (V_{SS} + V_T)^2$

$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L} (V_{IN} - V_{SS} - V_T)^2 R$

$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L} (V_M \sin \omega t - [V_{SS} + V_T])^2 R$
Small signal analysis example

\[ V_{IN} = V_M \sin \omega t \]

\[ V_M \text{ is small} \]

\[ V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L} \left( V_M \sin \omega t - \left[ V_{SS} + V_T \right] \right)^2 R \]

\[ V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L} \left[ V_{SS} + V_T \right]^2 \left( 1 - \frac{V_M \sin \omega t}{\left[ V_{SS} + V_T \right]} \right)^2 R \]

Recall that if \( x \) is small \( (1+x)^2 \approx 1+2x \)

\[ V_{OUT} \approx V_{DD} - \frac{\mu C_{OX} W}{2L} \left[ V_{SS} + V_T \right]^2 \left( 1 - \frac{2V_M \sin \omega t}{\left[ V_{SS} + V_T \right]} \right) R \]

\[ V_{OUT} \approx \left\{ V_{DD} - \frac{\mu C_{OX} W}{2L} \left[ V_{SS} + V_T \right]^2 R \right\} + \left\{ \frac{\mu C_{OX} W}{2L} \left[ V_{SS} + V_T \right]^2 \left( 1 - \frac{2V_M \sin \omega t}{\left[ V_{SS} + V_T \right]} \right) R \right\} V_M \sin \omega t \]
Small signal analysis example

By selecting appropriate value of $V_{SS}$, $M_1$ will operate in the saturation region

Assume $M_1$ operating in saturation region

$$V_{OUT} \approx \left\{ V_{DD} - \frac{\mu C_{ox} W}{2L} \left[ V_{SS} + V_T \right]^2 R \right\} + \left\{ \frac{\mu C_{ox} W}{L} \left[ V_{SS} + V_T \right] R \right\} V_M \sin \omega t$$
Small signal analysis example

Assume $M_1$ operating in saturation region

\[ V_{\text{OUT}} = V_{DD} - \frac{\mu C_{\text{ox}} W}{2L} \left[ V_{SS} + V_T \right]^2 R \] + \left\{ \frac{\mu C_{\text{ox}} W}{L} \left[ V_{SS} + V_T \right] R \right\} V_M \sin \omega t \]

**Quiescent Output**

**ss Voltage Gain**

\[ A_v = \frac{\mu C_{\text{ox}} W}{L} \left[ V_{SS} + V_T \right] R \]

\[ V_{\text{OUTQ}} = \left\{ V_{DD} - \frac{\mu C_{\text{ox}} W}{2L} \left[ V_{SS} + V_T \right]^2 R \right\} \]

\[ V_{OUT} \approx V_{\text{OUTQ}} + A_v V_M \sin \omega t \]

Note the ss voltage gain is negative since $V_{SS} + V_T < 0!$
Small signal analysis example

Assume $M_1$ operating in saturation region

$$V_{OUT} \approx V_{OUTQ} + A_v V_M \sin \omega t$$

$$A_v = \frac{\mu C_{ox} W}{L} \left[ V_{SS} + V_T \right] R$$

$$V_{OUTQ} = \left\{ V_{DD} - \frac{\mu C_{ox} W}{2L} \left[ V_{SS} + V_T \right]^2 R \right\}$$

But – this expression gives little insight into how large the gain is!
And the analysis for even this very simple circuit was messy!
Small signal analysis example

\[ V_{\text{OUT}} \approx V_{\text{OUTQ}} + A_v V_M \sin \omega t \]
Small signal analysis example

\[ V_{\text{OUT}} \approx V_{\text{OUTQ}} + A_v V_M \sin \omega t \]

\[ A_v = \frac{\mu C_{ox} W}{L} \left[ V_{ss} + V_T \right] R \]

\[ V_{\text{IN}} = V_M \sin \omega t \]

\[ V_{\text{IN}} = V_{\text{M}} \sin \omega t \]

\[ V_{\text{OUT}} = V_{\text{OUTQ}} + A_v V_M \sin \omega t \]

\[ A_v = \frac{\mu C_{ox} W}{L} \left[ V_{ss} + V_T \right] R \]
Small signal analysis example

\[ V_{\text{OUT}} \approx V_{\text{OUTQ}} + A_v V_M \sin \omega t \]

\[ A_v = \frac{\mu C_{\text{ox}} W}{L} [V_{\text{ss}} + V_T] R \]
Small signal analysis example

\[ V_{\text{OUT}} \approx V_{\text{OUTQ}} + A_v V_M \sin \omega t \]

\[ A_v = \frac{\mu C_{\text{ox}} W}{L} \left[ V_{\text{SS}} + V_T \right] R \]

Serious Distortion occurs if signal is too large or Q-point non-optimal.
Here “clipping” occurs for high \( V_{\text{OUT}} \)
Small signal analysis example

\[ V_{\text{OUT}} \approx V_{\text{OUTQ}} + A_V V_M \sin \omega t \]

\[ A_V = \frac{\mu C_{\text{ox}} W}{L} \left[ V_{\text{ss}} + V_T \right] R \]

Serious Distortion occurs if signal is too large or Q-point non-optimal

Here “clipping” occurs for low \( V_{\text{OUT}} \)
Small signal analysis example

\( V_{IN} = V_M \sin \omega t \)

\[ V_{OUT} \approx V_{OUTQ} + A_V V_M \sin \omega t \]

\[ A_V = \frac{\mu C_{ox} W}{L} \left[ V_{SS} + V_T \right] R \]

But recall:

Thus, substituting from the expression for \( I_{DQ} \) we obtain

\[ I_{DQ} = \frac{\mu C_{ox} W}{2L} \left( V_{SS} + V_T \right)^2 \]

\[ A_V = \frac{2 I_{DQ} R}{\left[ V_{SS} + V_T \right]} \]

Note this is negative since \( V_{SS} + V_T < 0 \)
Small signal analysis example

\[ A_v = \frac{2I_{DQ} R}{V_{SS} + V_T} \]

Observe the small signal voltage gain is twice the Quiescent voltage across \( R \) divided by \( V_{SS} + V_T \)

Can make \(|A_v|\) large by making \(|V_{SS} + V_T|\) small

- This analysis which required linearization of a nonlinear output voltage is quite tedious.
- This approach becomes unwieldy for even slightly more complicated circuits
- A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements
Small signal analysis example
(Consider what was neglected in the previous analysis)

\[ V_{\text{OUT}} \approx V_{\text{OUTQ}} + AV_{\text{M}} \sin \omega t \]

However, there are invariably small errors in this analysis

\[ V_{\text{OUT}} = V_{\text{OUTQ}} + AV_{\text{M}} \sin \omega t + \epsilon(t) \]

To see the effects of the approximations consider again

\[ V_{\text{OUT}} = V_{\text{DD}} - \frac{\mu C_{\text{OX}} W}{2L} \left( V_{\text{M}} \sin \omega t - \left[ V_{\text{SS}} + V_{\text{T}} \right] \right)^2 R \]

\[ V_{\text{OUT}} = V_{\text{DD}} - \frac{\mu C_{\text{OX}} R W}{2L} \left( V_{\text{M}}^2 \sin^2(\omega t) - 2 \left[ V_{\text{SS}} + V_{\text{T}} \right] V_{\text{M}} \sin \omega t + \left[ V_{\text{SS}} + V_{\text{T}} \right]^2 \right) \]

\[ V_{\text{OUT}} = V_{\text{DD}} - \frac{\mu C_{\text{OX}} R W}{2L} \left( V_{\text{M}}^2 \left[ \frac{1 - \cos 2\omega t}{2} \right] - 2 \left[ V_{\text{SS}} + V_{\text{T}} \right] V_{\text{M}} \sin \omega t + \left[ V_{\text{SS}} + V_{\text{T}} \right]^2 \right) \]

\[ V_{\text{OUT}} = \left\{ V_{\text{DD}} - \frac{\mu C_{\text{OX}} R W}{2L} \left( \frac{V_{\text{M}}^2}{2} + \left[ V_{\text{SS}} + V_{\text{T}} \right]^2 \right) \right\} + \left\{ \frac{\mu C_{\text{OX}} W}{L} \left[ V_{\text{SS}} + V_{\text{T}} \right] R \right\} V_{\text{M}} \sin \omega t \]

\[ + \left\{ \frac{\mu C_{\text{OX}} R W}{4L} V_{\text{M}}^2 \right\} \cos 2\omega t \]

Note presence of second harmonic distortion term!
Small signal analysis example

Nonlinear distortion term

\[ V_{\text{OUT}} \approx V_{\text{OUTQ}} + A_V V_M \sin \omega t \]

\[ V_{\text{OUT}} = V_{\text{OUTQ}} + A_V V_M \sin \omega t + \varepsilon(t) \]

\[ V_{\text{OUT}} = \left\{ V_{\text{DD}} - \frac{\mu C_{\text{OX}} RW}{2L} \left( \frac{V_M^2}{2} + [V_{SS} + V_T]^2 \right) \right\} + \left\{ \left( \frac{\mu C_{\text{OX}} W}{L} \right) [V_{SS} + V_T] V_M \sin \omega t \right\} + \left\{ \left( \frac{\mu C_{\text{OX}} RW}{4L} V_M^2 \right) \cos 2\omega t \right\} \]

\[ \tilde{V}_{\text{OUTQ}} = \left\{ V_{\text{DD}} - \frac{\mu C_{\text{OX}} RW}{2L} \left( \frac{V_M^2}{2} + [V_{SS} + V_T]^2 \right) \right\} \]

\[ A_V = \frac{\mu C_{\text{OX}} W}{L} [V_{SS} + V_T] R \]

\[ A_2 = \frac{\mu C_{\text{OX}} RW}{4L} V_M \]

\[ V_{\text{OUT}} = \tilde{V}_{\text{OUTQ}} + \{ A_V V_M \sin \omega t \} + \{ A_2 V_M \cos 2\omega t \} \]
Small signal analysis example

Nonlinear distortion term

\[ V_{OUT} = \tilde{V}_{OUTQ} + \{ A_v V_m \sin \omega t \} + \{ A_2 V_m \cos 2\omega t \} \]

\[ \tilde{V}_{OUTQ} = \left\{ V_{DD} - \frac{\mu_{C_{OX}} W}{2L} \left( \frac{V_m^2}{2} + [V_{SS} + V_T]^2 \right) \right\} \]

\[ A_v = \frac{\mu_{C_{OX}} W}{L} [V_{SS} + V_T] R \]

\[ A_2 = \frac{\mu_{C_{OX}} RW}{4L} V_m \]

Total Harmonic Distortion:

Recall, if \[ x(t) = \sum_{k=0}^{\infty} b_k \sin(k\omega t + \phi_k) \] then

\[ THD = \sqrt{\sum_{k=2}^{\infty} b_k^2} \]

Thus, for this amplifier, as long as \( M_1 \) stays in the saturation region

\[ THD = \frac{A_2 V_M}{A_v V_M} = \frac{A_2}{A_v} = \frac{\mu_{C_{OX}} W}{4L} \frac{R V_m}{R|V_{SS} + V_T|} = \frac{V_m}{4|V_{SS} + V_T|} \]

Distortion will be small for \( V_M \ll |V_{SS} + V_T| \)

Distortion will be much worse (larger and more harmonic terms) if \( M_1 \) leaves saturation region.
Consider the following MOSFET and BJT Circuits

One of the most widely used amplifier architectures

- Analysis was very time consuming
- Issue of operation of circuit was obscured in the details of the analysis
Consider the following MOSFET and BJT Circuits

One of the most widely used amplifier architectures
Small signal analysis using nonlinear models

Assume $M_1$ operating in saturation region

By selecting appropriate value of $V_{SS}$, $M_1$ will operate in the saturation region

$I_C = J_S A_E e^{\frac{V_{IN}-V_{EE}}{V_t}}$

$I_{CQ} = J_S A_E e^{\frac{-V_{EE}}{V_t}}$

$V_{OUT} = V_{CC} - J_S A_E R e^{\frac{V_{IN}-V_{EE}}{V_t}}$

$V_{OUT} = V_{CC} - J_S A_E R e^{\frac{V_{M} \sin(\omega t)-V_{EE}}{V_t}}$

$V_{IN} = V_M \sin \omega t$

$V_M$ is small

$V_{IN}(t)$

$V_{CC}$

$R_1$

$V_{OUT}$

$Q_1$

$V_{EE}$

$V_{IN}$

$t$
Small signal analysis using nonlinear models

\[ V_{\text{OUT}} = V_{\text{CC}} - J S A R e^{\left(-\frac{V_{\text{EE}}}{V_t}\right)} \]

\[ V_{\text{OUT}} = V_{\text{CC}} - J S A R e^{\left(-\frac{V_{\text{EE}}}{V_t}\right)} \]

Recall that if \( x \) is small \( e^x \approx 1 + x \) (truncated Taylor’s series)

\[ V_{\text{IN}} = V_M \sin \omega t \]

\[ V_M \text{ is small} \]

\[ V_{\text{OUT}} \approx \left[ V_{\text{CC}} - J S A R e^{\left(-\frac{V_{\text{EE}}}{V_t}\right)} \right] - \left[ J S A R e^{\left(-\frac{V_{\text{EE}}}{V_t}\right)} \frac{V_M \sin(\omega t)}{V_t} \right] \]

\[ I_{\text{CQ}} = J_S A E e^{\left(-\frac{V_{\text{EE}}}{V_t}\right)} \]

\[ \frac{V_{\text{M}} \sin(\omega t) - V_{\text{EE}}}{V_t} \]
Small signal analysis using nonlinear models

\[ V_{OUT} \approx \left[ V_{CC} - J S A E \frac{-V_{EE}}{V_t} R_1 \right] - \left[ J S A E \frac{-V_{EE}}{V_t} \right] \frac{V_M \sin(\omega t)}{V_t} \]

\[ I_{CQ} = J S A E \frac{-V_{EE}}{V_t} \]

\[ V_{IN} = V_M \sin \omega t \]

\[ V_M \text{ is small} \]

Quiescent Output

ss Voltage Gain
Comparison of Gains for MOSFET and BJT Circuits

**BJT**

\[ A_{VB} = - \frac{I_{CQ} R_1}{V_t} \]

If \( I_{DQ} R = I_{CQ} R_1 = 2V, V_{SS} + V_T = -1V, V_t = 25mV \)

\[ A_{VB} = - \frac{2V R_1}{25mV} = -80 \]

**MOSFET**

\[ A_{VM} = \frac{2I_{DQ} R}{V_{SS} + V_T} \]

\[ A_{VM} = \frac{2I_{DQ} R}{4V - 1V} = -4 \]

Observe \( A_{VB} \gg A_{VM} \)

Due to exponential-law rather than square-law model
Operation with Small-Signal Inputs

- Analysis procedure for these simple circuits was very tedious
- This approach will be unmanageable for even modestly more complicated circuits
- **Faster analysis method is needed!**
End of Lecture 24