Small-Signal Models of n-terminal devices
Small-signal models of MOSFET and BJT
Small-Signal Principle

Review from Last Lecture

Region around Q-Point

Q-point

$y = f(x)$

$Y_Q$

$x_Q$
Small-Signal Principle

Model of the nonlinear device at the Q-point

\[ i_{ss} = \frac{\partial I}{\partial V} \bigg|_{V=V_q} \]

\[ v_{ss} \]

\[ i_{ss} = \frac{\partial I}{\partial V} \bigg|_{V=V_q} \]

\[ v_{ss} \]

\[ i_{ss} = i \]

\[ v_{ss} = v \]

\[ y = \frac{\partial I}{\partial V} \bigg|_{V=V_q} \]

\[ i = y v \]
The small-signal model of this 2-terminal electrical network is a resistor of value $1/y$. One small-signal parameter characterizes this one-port but it is dependent on Q-point.
Review from Last Lecture

Solution for the example was based upon solving the nonlinear circuit for $V_{OUT}$ and then linearizing the solution by doing a Taylor’s series expansion.

- Solution of nonlinear equations very involved with two or more nonlinear devices.
- Taylor’s series linearization can get very tedious if multiple nonlinear devices are present.

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<th>Standard Approach to small-signal analysis of nonlinear networks</th>
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<td>1. Linearize nonlinear devices</td>
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<td>3. Replace all devices with small-signal equivalent</td>
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Review from Last Lecture

**Standard Approach to small-signal analysis of nonlinear networks**

1. **Nonlinear Network**
2. **dc Equivalent Network**
3. **Q-point**
4. **Values for small-signal parameters**
5. **Small-signal equivalent network**
6. **Small-signal output**
7. **Total output**
   (good approximation)
Review from Last Lecture

Linearized nonlinear devices
**Review from Last Lecture**

**Dc and small-signal equivalent elements**

<table>
<thead>
<tr>
<th>Element</th>
<th>ss equivalent</th>
<th>dc equivalent</th>
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<tr>
<td>dc Voltage Source</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
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<tr>
<td>ac Voltage Source</td>
<td>$V_{AC}$</td>
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<td>dc Current Source</td>
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<tr>
<td>ac Current Source</td>
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<tr>
<td>Resistor</td>
<td>$R$</td>
<td>$R$</td>
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### Dc and small-signal equivalent elements

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<thead>
<tr>
<th>Element</th>
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<tbody>
<tr>
<td>Capacitors</td>
<td>C (Large)</td>
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<tr>
<td></td>
<td>C (Small)</td>
<td></td>
</tr>
<tr>
<td>Inductors</td>
<td>L (Large)</td>
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<tr>
<td></td>
<td>L (Small)</td>
<td></td>
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<tr>
<td>Diodes</td>
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<td>MOS transistors</td>
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</table>
**Review from Last Lecture**

Dc and small-signal equivalent elements

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<tr>
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<tr>
<td><img src="image1.png" alt="Diagram of Bipolar Transistors" /></td>
<td><img src="image2.png" alt="Diagram of Bipolar Transistor ss equivalent" /></td>
<td><img src="image3.png" alt="Diagram of Bipolar Transistor dc equivalent" /></td>
</tr>
<tr>
<td><img src="image4.png" alt="Diagram of Dependent Sources" /></td>
<td><img src="image5.png" alt="Diagram of Dependent Source ss equivalent" /></td>
<td><img src="image6.png" alt="Diagram of Dependent Source dc equivalent" /></td>
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</tbody>
</table>

Simplified

Simplified

Simplified
How is the small-signal equivalent circuit obtained from the nonlinear circuit?

What is the small-signal equivalent of the MOSFET, BJT, and diode?
Small-Signal Diode Model

A Small Signal Equivalent Circuit

Thus, for the diode

\[ R_d = \left( \frac{\partial I_D}{\partial V_D} \right)_Q^{-1} \]
Review from Last Lecture

Small-Signal Diode Model

For the diode

\[ I_D = I_S e^{\frac{V_D}{V_t}} \]

\[ R_d = \left( \frac{\partial I_D}{\partial V_D} \right)^{-1}_Q \]

\[ \frac{\partial I_D}{\partial V_t} |_Q = \left[ I_S e^{\frac{V_D}{V_t}} \left( \frac{1}{V_t} \right) \right]_Q \cdot \frac{I_D}{V_t} \]

\[ R_d = \frac{V_t}{I_D Q} \]
Small signal model is that which represents the relationship between the small signal voltages and the small signal currents.

Consider 4-terminal network

\[ I_1 = f_1(V_1, V_2, V_3) \]
\[ I_2 = f_2(V_1, V_2, V_3) \]
\[ I_3 = f_3(V_1, V_2, V_3) \]

Define

\[ i_1 = I_1 - I_{1Q} \]
\[ i_2 = I_2 - I_{2Q} \]
\[ i_3 = I_3 - I_{3Q} \]

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents.
Small-Signal Model

Consider 4-terminal network

\[ i_1 = g_1(v_1, v_2, v_3) \]
\[ i_2 = g_2(v_1, v_2, v_3) \]
\[ i_3 = g_3(v_1, v_2, v_3) \]

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents.

For small signals, this relationship should be linear.

Can be thought of as a change in coordinate systems from the large signal coordinate system to the small-signal coordinate system.
Recall for a function of one variable

\[ y = f(x) \]

Taylor’s Series Expansion about the point \( x_0 \)

\[
y = f(x) = f(x)\bigg|_{x=x_0} + \left. \frac{\partial f}{\partial x} \right|_{x=x_0} (x - x_0) + \left. \frac{\partial^2 f}{\partial x^2} \right|_{x=x_0} \frac{1}{2!} (x - x_0)^2 + ... \\
\]

If \( x-x_0 \) is small

\[
y \approx f(x)\bigg|_{x=x_0} + \left. \frac{\partial f}{\partial x} \right|_{x=x_0} (x - x_0) \\
\]

\[
y \approx y_0 + \left. \frac{\partial f}{\partial x} \right|_{x=x_0} (x - x_0) \\
\]
Recall for a function of one variable

\[ y = f(x) \]

If \( x - x_0 \) is small

\[ y \approx y_0 + \left. \frac{\partial f}{\partial x} \right|_{x=x_0} (x - x_0) \]

\[ y - y_0 = \left. \frac{\partial f}{\partial x} \right|_{x=x_0} (x - x_0) \]

If we define the small signal variables as

\[ \mathbf{y} = y - y_0 \]

\[ \mathbf{x} = x - x_0 \]
Recall for a function of one variable

\[ y = f(x) \]

If \( x - x_0 \) is small

\[ y - y_0 = \frac{\partial f}{\partial x} \bigg|_{x=x_0} (x - x_0) \]

If we define the small signal variables as

\[ y = y - y_0 \]

\[ x = x - x_0 \]

Then

\[ y = \frac{\partial f}{\partial x} \bigg|_{x=x_0} x \]

This relationship is linear!
Consider now a function of $n$ variables

$$y = f(x_1, \ldots x_n) = f(\bar{x})$$

If we define the small signal variables as

$$\bar{X}_0 = \{x_{10}, x_{20}, \ldots x_{n0}\}$$

The multivariate Taylor's series expansion around the point $\bar{X}_0$ is given by

$$y = f(\bar{x}) = f(x)\bigg|_{x = \bar{x}_0} + \sum_{k=1}^{n} \left( \frac{\partial f}{\partial x_k} \bigg|_{x = \bar{x}_0} (x_k - x_{k0}) \right)$$

$$+ \sum_{k=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 f}{\partial x_j \partial x_k} \bigg|_{x = \bar{x}_0} \frac{1}{2!} (x_j - x_{j0})(x_k - x_{k0}) + \ldots \text{(H.O.T.)}$$

Truncating after first-order terms, we obtain the approximation

$$y - y_0 \approx \sum_{k=1}^{n} \left( \frac{\partial f}{\partial x_k} \bigg|_{x = \bar{x}_0} (x_k - x_{k0}) \right)$$

where $y_0 = f(x)\bigg|_{x = \bar{x}_0}$
Multivariate Taylors Series Expansion

\[ y = f(x_1, \ldots x_n) = f(\bar{x}) \]

Linearized approximation

\[ y - y_0 \approx \sum_{k=1}^{n} \left( \frac{\partial f}{\partial x_k} \bigg|_{\bar{x}=x_0} (x_k - x_{k0}) \right) \]

This can be expressed as

\[ y_{ss} \approx \sum_{k=1}^{n} a_k x_{ss} \]

where

\[ y_{ss} = y - y_0 \]

\[ x_{kss} = x_k - x_{k0} \]

\[ a_k = \frac{\partial f}{\partial x_k} \bigg|_{\bar{x}=x_0} \]
In the more general form\(^1\), the multivariate Taylor’s series expansion can be expressed as

\[
\begin{align*}
f(x_1, \ldots, x_n) &= \alpha_0 + \sum_{m=1}^{\infty} \left( \sum_{k_1, \ldots, k_n} \alpha_{k_1, \ldots, k_n; m} \left( x_1 - x_{1,0} \right)^{k_1} \cdots \left( x_n - x_{n,0} \right)^{k_n} \right) \quad (7) \\
\alpha_0 &= f(x_{10}, \ldots, x_{n0}) \\
\alpha_{k_1, \ldots, k_n; m} &= \frac{1}{k_1! \cdots k_n!} \left. \frac{\partial^m f}{\partial x_1^{k_1} \cdots \partial x_n^{k_n}} \right|_{x_{10}, \ldots, x_{n0}} \quad (8)
\end{align*}
\]

\(^1\) http://www.chem.mtu.edu/~tbco/cm416/taylor.html
Consider 4-terminal network

\[ I_1 = f_1(V_1, V_2, V_3) \]
\[ I_2 = f_2(V_1, V_2, V_3) \]
\[ I_3 = f_3(V_1, V_2, V_3) \]

Nonlinear network characterized by 3 functions each functions of 3 variables
Consider now 3 functions each functions of 3 variables

\[ I_1 = f_1(V_1, V_2, V_3) \]
\[ I_2 = f_2(V_1, V_2, V_3) \]
\[ I_3 = f_3(V_1, V_2, V_3) \]

Define

\[ \vec{V}_Q = \begin{bmatrix} V_{1Q} \\ V_{2Q} \\ V_{3Q} \end{bmatrix} \]

In what follows, we will use \( \vec{V}_Q \) as an expansion point in a Taylor’s series expansion.
Consider now 3 functions each functions of 3 variables

\[
\begin{align*}
l_1 &= f_1(V_1, V_2, V_3) \\
l_2 &= f_2(V_1, V_2, V_3) \\
l_3 &= f_3(V_1, V_2, V_3)
\end{align*}
\]

Define

\[
\vec{V}_Q = \begin{bmatrix} V_{1Q} \\ V_{2Q} \\ V_{3Q} \end{bmatrix}
\]

Consider first the function \( I_1 \)

The multivariate Taylor Series expansion of \( I_1 \), around the operating point \( \vec{V}_Q \), when truncated after first-order terms, can be expressed as:

\[
l_1 = f_1(V_1, V_2, V_3) \approx f_1(V_{1Q}, V_{2Q}, V_{3Q}) + \\
\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1} \bigg|_{V = \vec{V}_Q} (V_1 - V_{1Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2} \bigg|_{V = \vec{V}_Q} (V_2 - V_{2Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3} \bigg|_{V = \vec{V}_Q} (V_3 - V_{3Q})
\]

or equivalently as:

\[
l_1 - I_{1Q} = \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1} \bigg|_{V = \vec{V}_Q} (V_1 - V_{1Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2} \bigg|_{V = \vec{V}_Q} (V_2 - V_{2Q}) + \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3} \bigg|_{V = \vec{V}_Q} (V_3 - V_{3Q})
\]
repeating from previous slide:

\[ I_1 - I_{1Q} = \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1} \right|_{V=\bar{V}_Q} (V_1 - V_{1Q}) + \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2} \right|_{V=\bar{V}_Q} (V_2 - V_{2Q}) + \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3} \right|_{V=\bar{V}_Q} (V_3 - V_{3Q}) \]

Make the following definitions

\[ y_{11} = \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1} \right|_{V=\bar{V}_Q} \]
\[ y_{12} = \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2} \right|_{V=\bar{V}_Q} \]
\[ y_{13} = \left. \frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3} \right|_{V=\bar{V}_Q} \]

It thus follows that

\[ i_1 = y_{11} v_1 + y_{12} v_2 + y_{13} v_3 \]

This is a linear relationship between the small signal electrical variables
Consider now 3 functions, each a function of 3 variables

\[ i_1 = y_{11} v_1 + y_{12} v_2 + y_{13} v_3 \]

Extending this approach to the two nonlinear functions \( I_2 \) and \( I_3 \)

\[ \begin{align*}
  i_1 & = y_{11} v_1 + y_{12} v_2 + y_{13} v_3 \\
  i_2 & = y_{21} v_1 + y_{22} v_2 + y_{23} v_3 \\
  i_3 & = y_{31} v_1 + y_{32} v_2 + y_{33} v_3
\end{align*} \]

where

\[ y_{ij} = \frac{\partial f_i(v_1, v_2, v_3)}{\partial v_j} \bigg|_{v = \bar{v}_q} \]

This is a small-signal model of a 4-terminal network and it is linear
9 small-signal parameters characterize the linear 4-terminal network
Small-signal model parameters dependent upon Q-point!
A small-signal equivalent circuit of a 4-terminal nonlinear network

\[ y_{ij} = \left. \frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j} \right|_{V=\bar{V}_Q} \]

Equivalent circuit is not unique
Equivalent circuit is a three-port network
4-terminal small-signal network summary

Small signal model:

\[ i_1 = y_{11} v_1 + y_{12} v_2 + y_{13} v_3 \]
\[ i_2 = y_{21} v_1 + y_{22} v_2 + y_{23} v_3 \]
\[ i_3 = y_{31} v_1 + y_{32} v_2 + y_{33} v_3 \]

\[ y_{ij} = \frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j} \bigg|_{v=v_Q} \]
Consider 3-terminal network

Small-Signal Model

\[ \begin{aligned}
    i_1 &= g_1(v_1, v_2, v_3) \\
    i_2 &= g_2(v_1, v_2, v_3) \\
    i_3 &= g_3(v_1, v_2, v_3)
\end{aligned} \]

\[ \begin{aligned}
    i_1 &= y_{11}v_1 + y_{12}v_2 + y_{13}v_3 \\
    i_2 &= y_{21}v_1 + y_{22}v_2 + y_{23}v_3 \\
    i_3 &= y_{31}v_1 + y_{32}v_2 + y_{33}v_3
\end{aligned} \]

\[ y_{ij} = \left. \frac{\partial f_i(v_1, v_2, v_3)}{\partial v_j} \right|_{\bar{v} = v_0} \]
Consider 3-terminal network

**Small-Signal Model**

\[
\begin{align*}
I_1 &= f_1(V_1, V_2) \\
I_2 &= f_2(V_1, V_2)
\end{align*}
\]

Define

\[
\begin{align*}
i_1 &= I_1 - I_{1Q} \\
i_2 &= I_2 - I_{2Q}
\end{align*}
\]

\[
\begin{align*}
u_1 &= V_1 - V_{1Q} \\
u_2 &= V_2 - V_{2Q}
\end{align*}
\]

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents.
Consider 3-terminal network

Small-Signal Model

\[ i_1 = y_{11} v_1 + y_{12} v_2 \]
\[ i_2 = y_{21} v_1 + y_{22} v_2 \]

4 small-signal parameters characterize this 3-terminal (two-port) linear network.

Small signal parameters dependent upon Q-point
3-terminal small-signal network summary

\[ I_1 = f_1(V_1, V_2) \]
\[ I_2 = f_2(V_1, V_2) \]

Small signal model:

\[ i_1 = y_{11} v_1 + y_{12} v_2 \]
\[ i_2 = y_{21} v_1 + y_{22} v_2 \]

\[ y_{ij} = \frac{\partial f_i(V_1, V_2)}{\partial v_j} \bigg|_{v=v_0} \]
Consider 2-terminal network

**Small-Signal Model**

\[
\begin{align*}
\mathbf{i}_1 &= y_{11} \mathbf{v}_1 + y_{12} \mathbf{v}_2 + y_{13} \mathbf{v}_3 \\
\mathbf{i}_2 &= y_{21} \mathbf{v}_1 + y_{22} \mathbf{v}_2 + y_{23} \mathbf{v}_3 \\
\mathbf{i}_3 &= y_{31} \mathbf{v}_1 + y_{32} \mathbf{v}_2 + y_{33} \mathbf{v}_3 \\
\end{align*}
\]

\[
\begin{align*}
\mathbf{i}_1 &= g_1(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3) \\
\mathbf{i}_2 &= g_2(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3) \\
\mathbf{i}_3 &= g_3(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3) \\
\end{align*}
\]

\[
y_{ij} = \left. \frac{\partial f_i(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)}{\partial \mathbf{v}_j} \right|_{\mathbf{v}=\mathbf{v}_q}
\]
Consider 2-terminal network

Small-Signal Model

\[ I_1 = f_1(V_1) \]

Define

\[ i_1 = I_1 - I_{1Q} \]
\[ v_1 = V_1 - V_{1Q} \]

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents.
Consider 2-terminal network

Small-Signal Model

\[ i_1 = y_{11} V_1 \]

\[ y_{11} = \left. \frac{\partial f_1 (V_1)}{\partial V_1} \right|_{V=V_0} \]

\[ \tilde{V} = V_{1Q} \]

A Small Signal Equivalent Circuit
Linearized nonlinear devices
How is the small-signal equivalent circuit obtained from the nonlinear circuit?

What is the small-signal equivalent of the MOSFET, BJT, and diode?
Small Signal Model of MOSFET

MOSFET is actually a 4-terminal device but for many applications acceptable predictions of performance can be obtained by treating it as a 3-terminal device by neglecting the bulk terminal.

In this course, we have been treating it as a 3-terminal device and in this lecture will develop the small-signal model by treating it as a 3-terminal device.

If considered as a 4-terminal device, the bulk voltage must also be considered and it introduces one additional term to the small signal model which is often either negligibly small or has no effect on circuit performance.
Small Signal Model of MOSFET

Large Signal Model

\[ I_G = 0 \]

3-terminal device

\[
I_D = \begin{cases} 
0 & V_{GS} \leq V_T \\
\mu C_{OX} \frac{W}{L} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T \quad V_{DS} < V_{GS} - V_T \\
\mu C_{OX} \frac{W}{2L} \left( V_{GS} - V_T \right)^2 \left( 1 + \lambda V_{DS} \right) & V_{GS} \geq V_T \quad V_{DS} \geq V_{GS} - V_T
\end{cases}
\]

MOSFET is usually operated in saturation region in linear applications where a small-signal model is needed so will develop the small-signal model in the saturation region.
Small Signal Model of MOSFET

\[ I_1 = f_1(V_1, V_2) \quad \leftrightarrow \quad I_G = 0 \]

\[ I_2 = f_2(V_1, V_2) \quad \leftrightarrow \quad I_D = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \]

*Small-signal model:*\[ y_{ij} = \left. \frac{\partial f_i(V_1, V_2)}{\partial V_j} \right|_{V=V_Q} \]

\[ y_{11} = \left. \frac{\partial I_G}{\partial V_{GS}} \right|_{V=V_Q} \]

\[ y_{21} = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V=V_Q} \]

\[ y_{12} = \left. \frac{\partial I_G}{\partial V_{DS}} \right|_{V=V_Q} \]

\[ y_{22} = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V=V_Q} \]
Small Signal Model of MOSFET

\[ I_1 = f_1(V_1, V_2) \quad \Longleftrightarrow \quad I_G = 0 \]

\[ I_2 = f_2(V_1, V_2) \quad \Longleftrightarrow \quad I_D = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \]

Small-signal model:

\[ y_{11} = \frac{\partial I_G}{\partial V_{GS}} \bigg|_{V=V_0} = 0 \]
\[ y_{12} = \frac{\partial I_G}{\partial V_{DS}} \bigg|_{V=V_0} = 0 \]

\[ y_{21} = \frac{\partial I_D}{\partial V_{GS}} \bigg|_{V=V_0} = 2\mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^1 (1 + \lambda V_{DS}) \bigg|_{V=V_0} = \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_T) (1 + \lambda V_{DSQ}) \]

\[ y_{21} \approx \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_T) \]

\[ y_{22} = \frac{\partial I_D}{\partial V_{DS}} \bigg|_{V=V_0} = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2 \lambda \bigg|_{V=V_0} \approx \lambda I_{DQ} \]
Small Signal Model of MOSFET

\[ I_G = 0 \]

\[ I_D = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \]

\[ y_{12} = 0 \]

\[ y_{11} = 0 \]

\[ y_{21} \approx \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_T) \]

\[ y_{22} \approx \lambda I_{DQ} \]

\[ i_G = y_{11} v_{GS} + y_{12} v_{DS} \]

\[ i_D = y_{21} v_{GS} + y_{22} v_{DS} \]

An equivalent circuit
Small Signal Model of MOSFET

by convention, \( y_{21} = g_m \), \( y_{22} = g_0 \)

\[
\therefore \quad y_{21} \approx g_m = \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_t)
\]

\[
y_{22} = g_o \approx \lambda I_{DQ}
\]
Small Signal Model of MOSFET

\[ g_m = \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_T) \]

\[ g_o \approx \lambda I_{DQ} \]

Alternate equivalent expressions:

\[ I_{DQ} = \mu C_{ox} \frac{W}{2L} (V_{GSQ} - V_T)^2 \left(1 + \lambda V_{DSQ}\right) \approx \mu C_{ox} \frac{W}{2L} (V_{GSQ} - V_T)^2 \]

\[ g_m = \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_T) \]

\[ g_m = \sqrt{2 \mu C_{ox} \frac{W}{L}} \cdot \sqrt{I_{DQ}} \]

\[ g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} \]
Consider again:

Small signal analysis example

\[ V_{IN} = V_M \sin \omega t \]

\[ A_v = \frac{2I_{DQ} R}{\left[ V_{SS} + V_T \right]} \]

Derived for \( \lambda = 0 \)

\[ I_{DQ} = \mu C_{ox} \frac{W}{2L} \left( V_{GSQ} - V_T \right)^2 \]
Consider again:

Small signal analysis example

For $\lambda = 0$, $g_o = \lambda_{IDQ} = 0$

$$A_v = \frac{V_{OUT}}{V_{IN}} = -\frac{g_m}{g_o + 1/R}$$

but

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T}$$

thus

$$V_{GSQ} = -V_{SS}$$

And

$$A_v = \frac{2I_{DQ} R}{[V_{SS} + V_T]}$$
Consider again:

Small signal analysis example

For $\lambda = 0$, $g_O = \lambda_{IDQ} = 0$

$$A_v = \frac{V_{OUT}}{V_{IN}} = -\frac{g_m}{g_o + 1/R}$$

Same expression as derived before

More accurate gain can be obtained if $\lambda$ effects are included and does not significantly increase complexity of small signal analysis
Small Signal Model of BJT

3-terminal device

Forward Active Model:

\[ I_C = J_S A E \left( \frac{V_{BE}}{V_t} \right) \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

\[ I_B = \frac{J_S A E}{\beta} \left( \frac{V_{BE}}{V_t} \right) \]

Usually operated in Forward Active Region when small-signal model is needed
Small Signal Model of BJT

\[ I_1 = f_1 (V_1, V_2) \]  \[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ I_2 = f_2 (V_1, V_2) \]  \[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

Small-signal model:

\[ y_{ij} = \left. \frac{\partial f_i (V_1, V_2)}{\partial V_j} \right|_{V=V_Q} \]

\[ y_{11} = g_\pi = \left. \frac{\partial I_B}{\partial V_{BE}} \right|_{V=V_Q} \]

\[ y_{12} = \left. \frac{\partial I_B}{\partial V_{CE}} \right|_{V=V_Q} \]

\[ y_{21} = g_m = \left. \frac{\partial I_C}{\partial V_{BE}} \right|_{V=V_Q} \]

\[ y_{22} = g_o = \left. \frac{\partial I_C}{\partial V_{CE}} \right|_{V=V_Q} \]
Small Signal Model of BJT

\[ I_B = \frac{J_A}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ I_C = J_A e^{\frac{V_{BE}}{V_t}} \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

**Small-signal model:**

\[ g_m = \frac{\partial I_B}{\partial V_{BE}} \bigg|_{V = V_o} = \frac{1}{V} J_A e^{\frac{V_{BE}}{V_t}} \bigg|_{V = V_o} \approx \frac{I_{BO}}{V_t} \]

\[ y_{12} = \frac{\partial I_B}{\partial V_{CE}} \bigg|_{V = V_o} = 0 \]

\[ y_{21} = g_m = \frac{\partial I_C}{\partial V_{BE}} \bigg|_{V = V_o} = \frac{1}{V} J_A e^{\frac{V_{BE}}{V_t}} \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \bigg|_{V = V_o} = \frac{I_{CQ}}{V_t} \]

\[ y_{22} = g_o = \frac{\partial I_C}{\partial V_{CE}} \bigg|_{V = V_o} = \frac{J_A e^{\frac{V_t}{V_{AF}}}}{V_{AF}} \bigg|_{V = V_o} \approx \frac{I_{CQ}}{V_{AF}} \]
Small Signal Model of BJT

\[ i_B = y_{11} V_{BE} + y_{12} V_{CE} \]
\[ i_C = y_{21} V_{BE} + y_{22} V_{CE} \]

\[ i_B = g_\pi V_{BE} \]
\[ i_C = g_m V_{BE} + g_o V_{CE} \]

\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_o = \frac{I_{CQ}}{V_{AF}} \]

An equivalent circuit
Small Signal BJT Model

Observe:

\[ g_{\Pi} v_{be} = i_b \]
\[ g_m v_{be} = i_b \frac{g_m}{g_{\Pi}} \]
\[ \frac{g_m}{g_{\Pi}} \begin{bmatrix} \frac{I_Q}{V_t} \\ \frac{I_Q}{\beta V_t} \end{bmatrix} = \beta \]

Can replace the voltage dependent current source with a current dependent current source
Small Signal BJT Model

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

Alternate equivalent small signal model

\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]
What are the simplified dc equivalent models?
Active Device Model Summary

What are the simplified dc equivalent models?

dc equivalent

![Diagram of simplified dc equivalent models]
Small-signal Operation of Nonlinear Circuits

- Small-signal principles
- Example Circuit
- Small-Signal Models

Small-Signal Analysis of Nonlinear Circuits