EE 330
Lecture 26

- Small Signal Analysis
- Small Signal Models for MOSFET and BJT

Exam Schedule

Exam 2 will be given on Friday March 11

Exam 3 will be given on Friday April 15

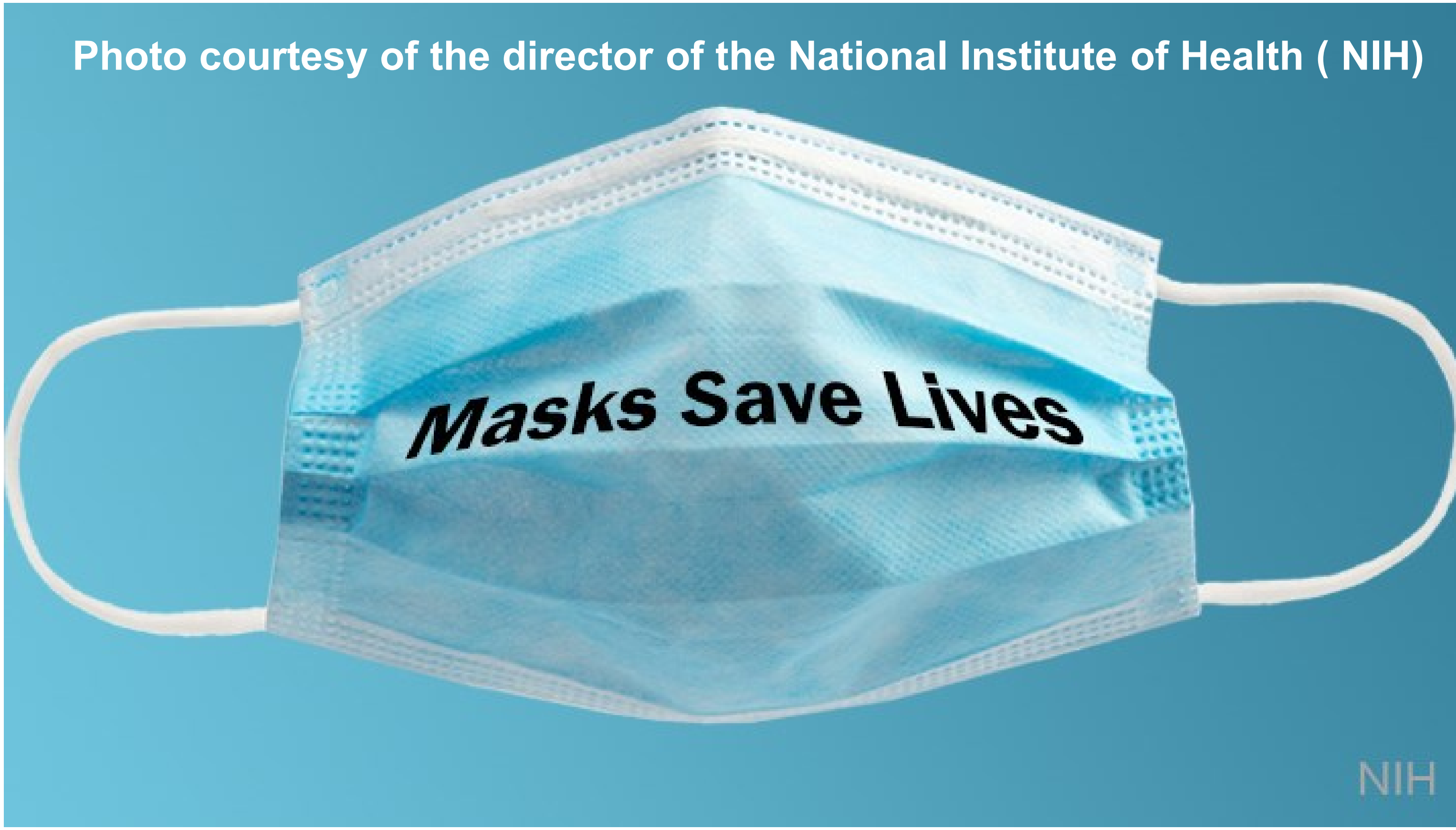


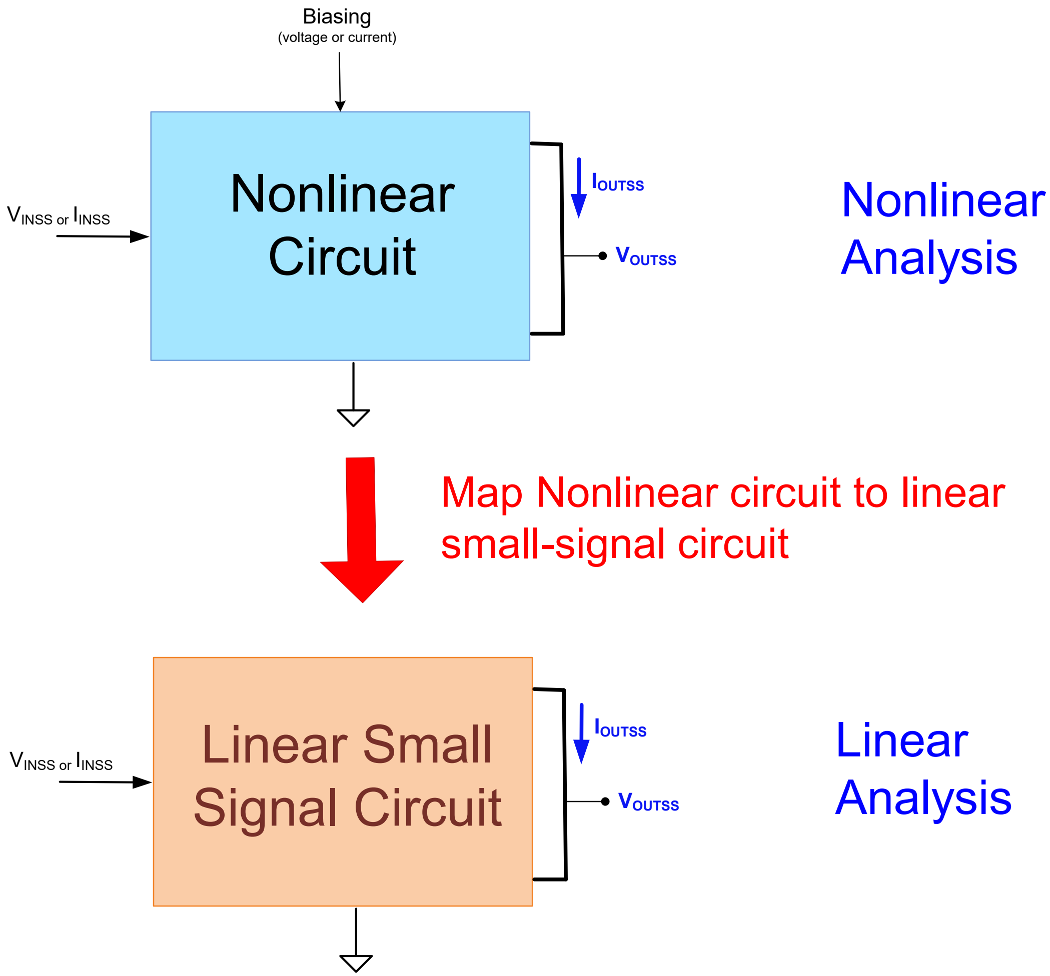
Photo courtesy of the director of the National Institute of Health (NIH)

As a courtesy to fellow classmates, TAs, and the instructor

Wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status

Review from Last Lecture

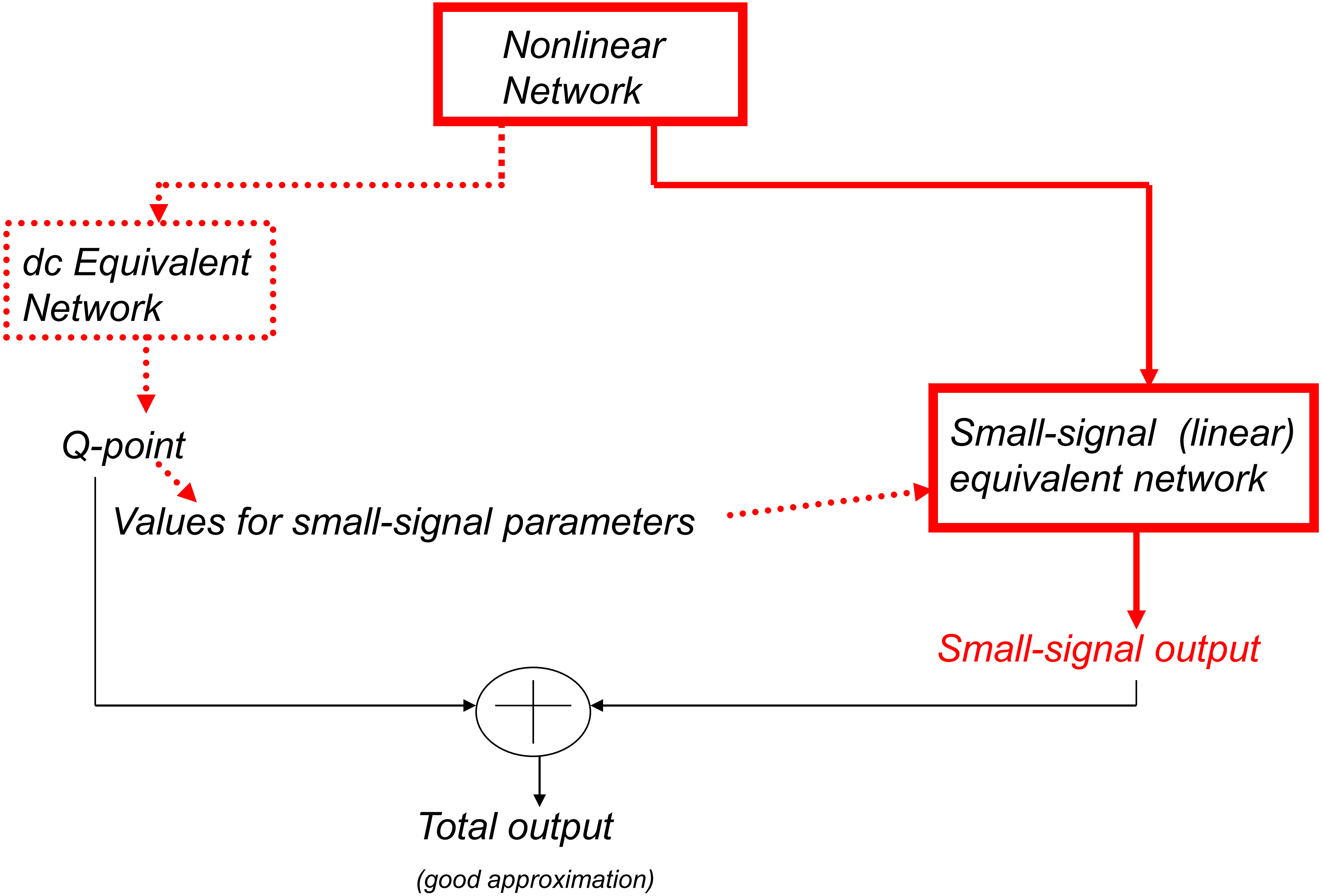
Small-Signal Analysis



- Will commit next several lectures to developing this approach
- Analysis will be MUCH simpler, faster, and provide significantly more insight
- Applicable to many fields of engineering

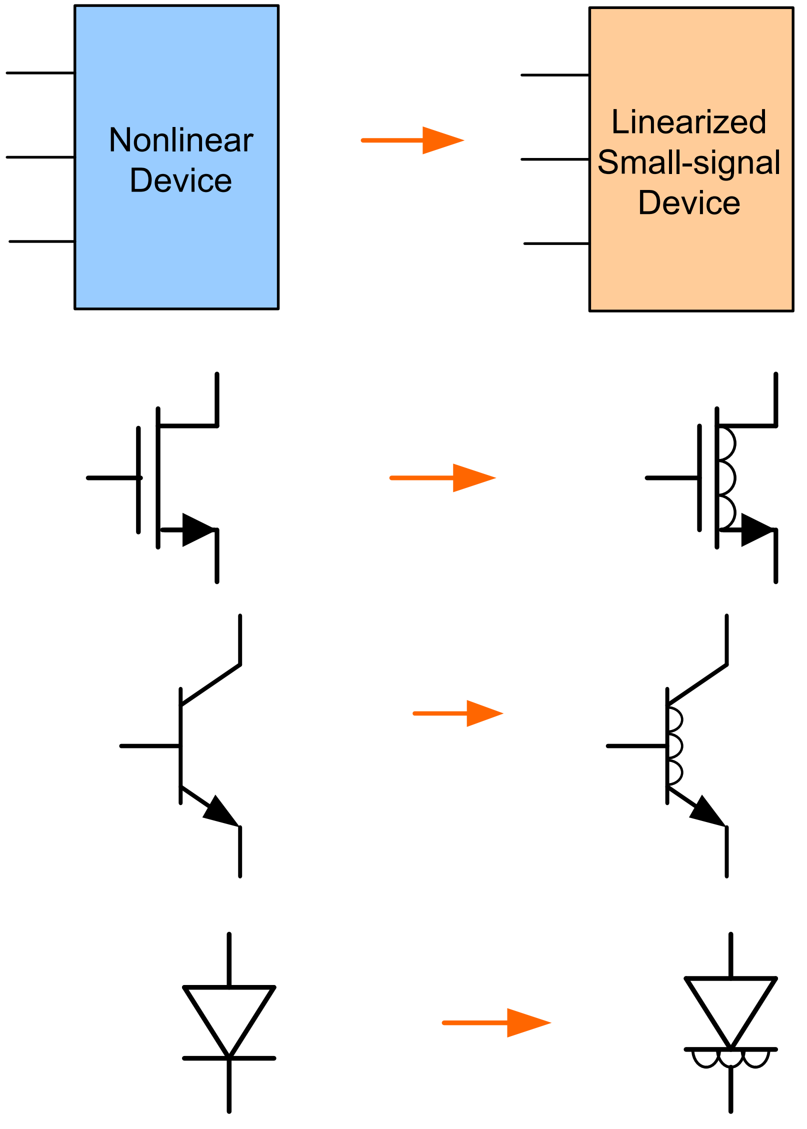
Review from Last Lecture

"Alternative" Approach to small-signal analysis of nonlinear networks



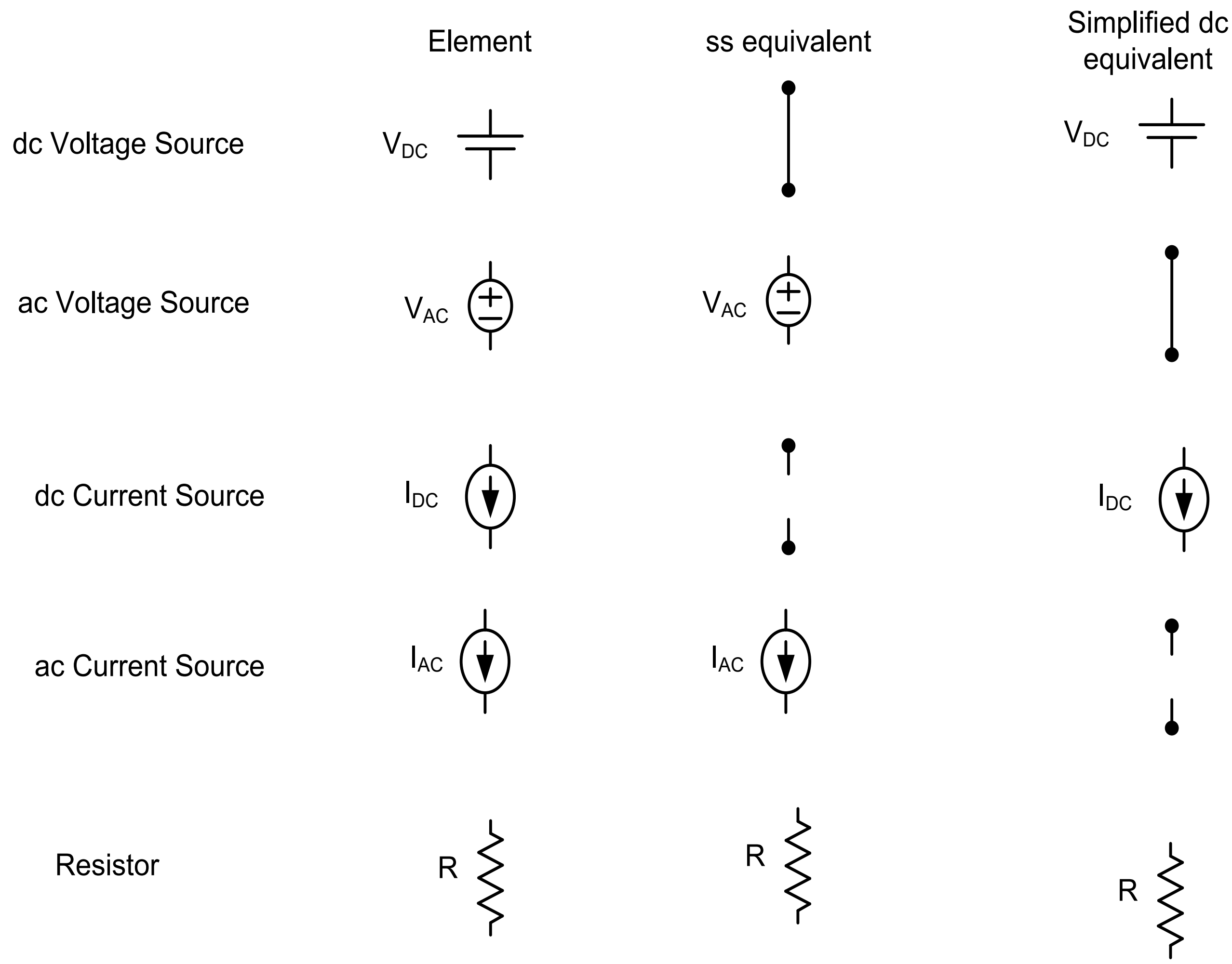
Review from Last Lecture

Linearized nonlinear devices



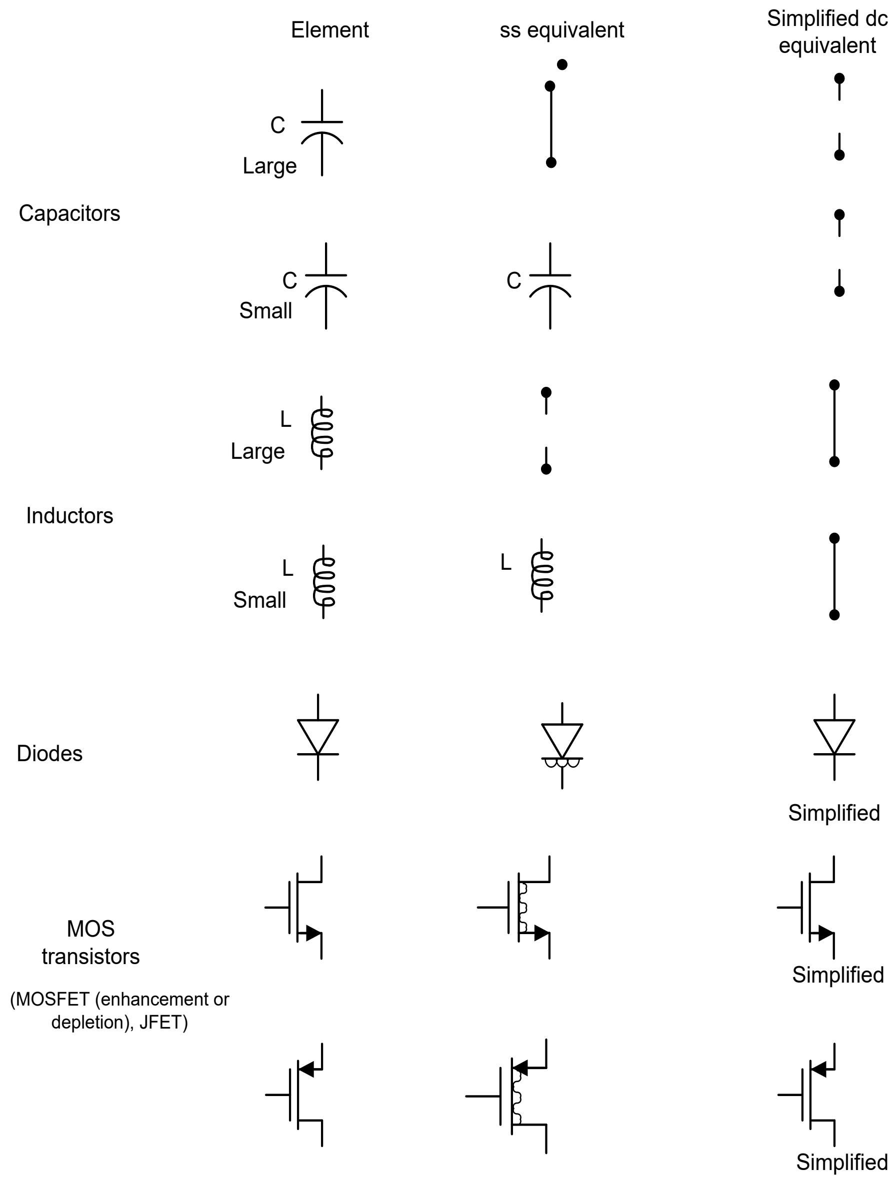
This terminology will be used in THIS course to emphasize difference between nonlinear model and linearized small signal model

Small-signal and simplified dc equivalent elements



Review from Last Lecture

Small-signal and simplified dc equivalent elements



Review from Last Lecture

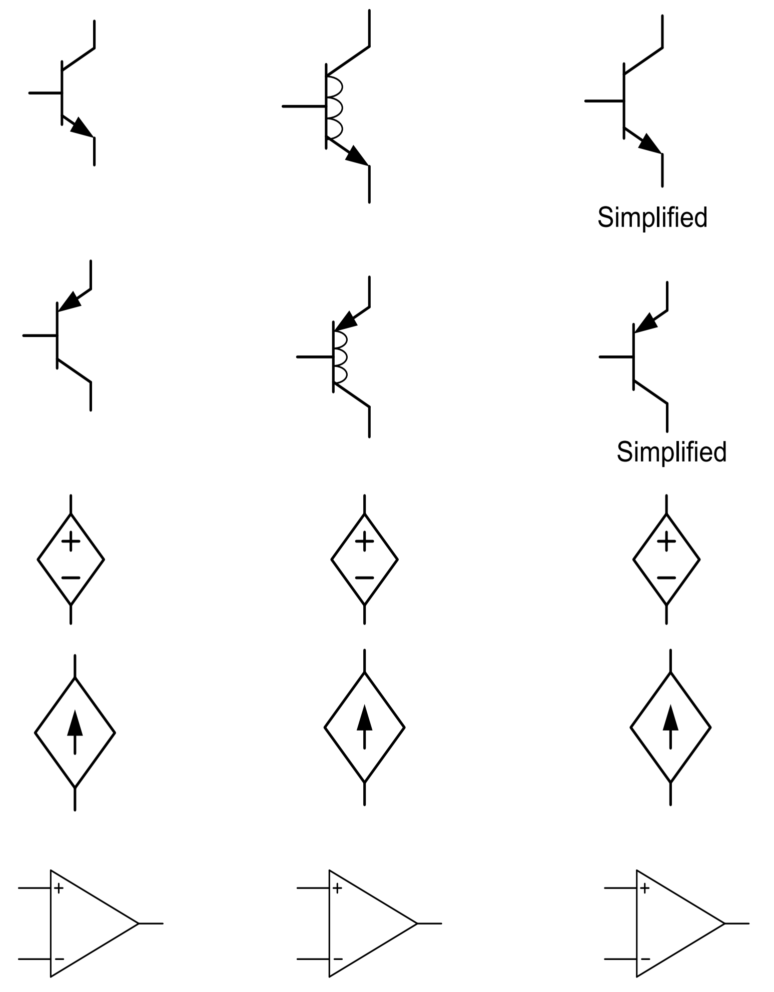
Small-signal and simplified dc equivalent elements

Element | ss equivalent | Simplified dc equivalent

Bipolar Transistors

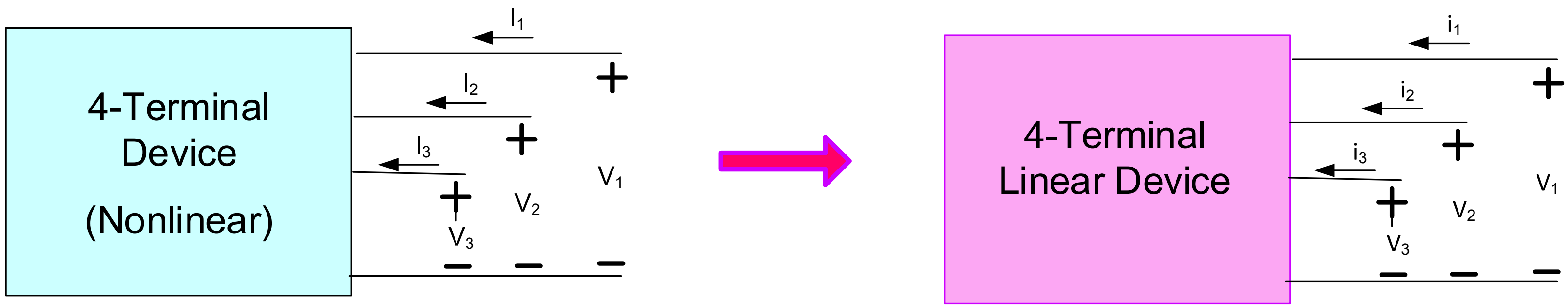
Dependent Sources (Linear)

$V_O=A_V V_{IN}$ $I_O=A_I I_{IN}$
$V_O=R_T I_{IN}$ $I_O=G_T V_{IN}$



Review from Last Lecture

Small-Signal Model of 4-Terminal Network



$$\left.\begin{array}{l} \mathbf{I_1 = f_1(V_1, V_2, V_3)} \\ \mathbf{I_2 = f_2(V_1, V_2, V_3)} \\ \mathbf{I_3 = f_3(V_1, V_2, V_3)} \end{array}\right\} \qquad \left.\begin{array}{l} i_1 = g_1(v_1, v_2, v_3) \\ i_2 = g_2(v_1, v_2, v_3) \\ i_3 = g_3(v_1, v_2, v_3) \end{array}\right\}$$

Mapping is unique (with same models)

Small Signal Model

$$i_1 = y_{11} v_1 + y_{12} v_2 + y_{13} v_3$$
$$i_2 = y_{21} v_1 + y_{22} v_2 + y_{23} v_3$$
$$i_3 = y_{31} v_1 + y_{32} v_2 + y_{33} v_3$$

where

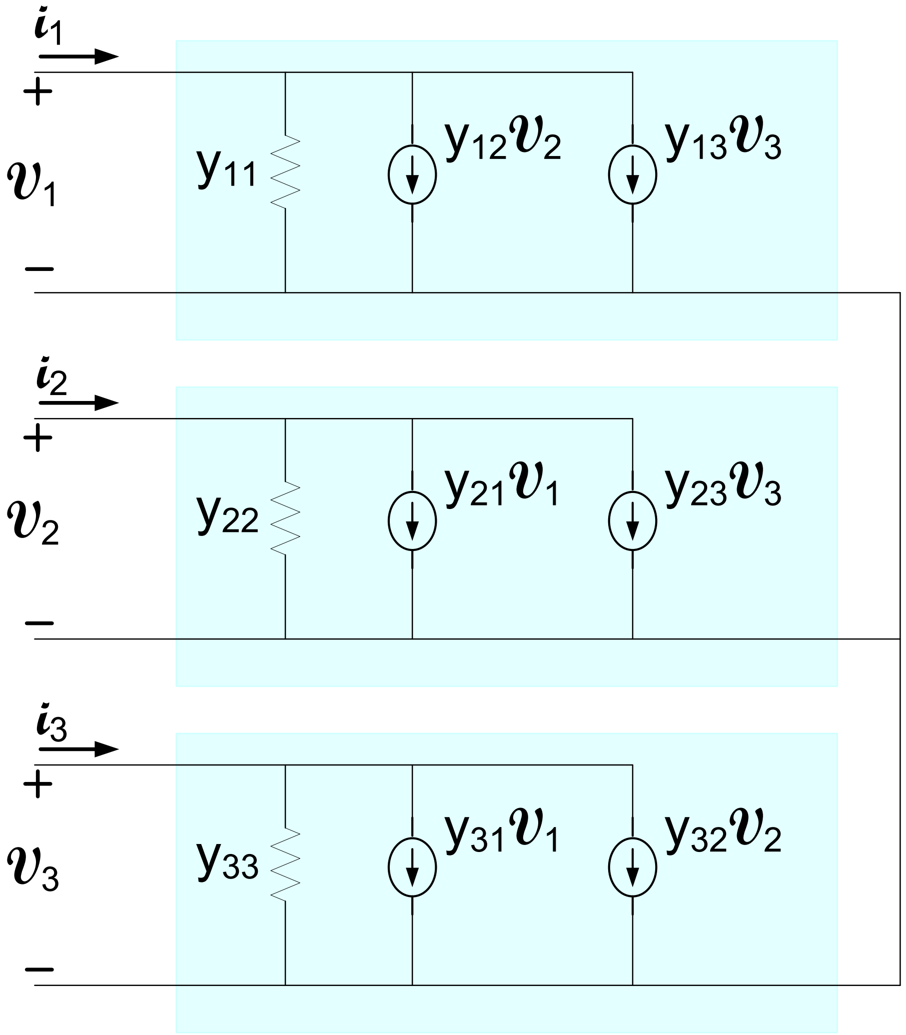
$$\mathbf{y}_{ij} = \left. \frac{\partial \mathbf{f}_i(\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3)}{\partial \mathbf{V}_j} \right|_{\vec{\mathbf{V}} = \vec{\mathbf{V}}_Q}$$

- This is a small-signal model of a 4-terminal network and it is linear
- 9 small-signal parameters characterize the linear 4-terminal network
- Small-signal model parameters dependent upon Q-point !
- Termed the y-parameter model or “admittance” –parameter model

Review from Last Lecture

A small-signal equivalent circuit of a 4-terminal nonlinear network

(equivalent circuit because has exactly the same port equations)



$$\mathbf{y_{ij}} = \left.\frac{\partial \mathbf{f_i}(\mathbf{V_1},\mathbf{V_2},\mathbf{V_3})}{\partial \mathbf{V_j}}\right|_{\vec{\mathbf{V}}=\vec{\mathbf{V}}_{\mathbf{Q}}}$$

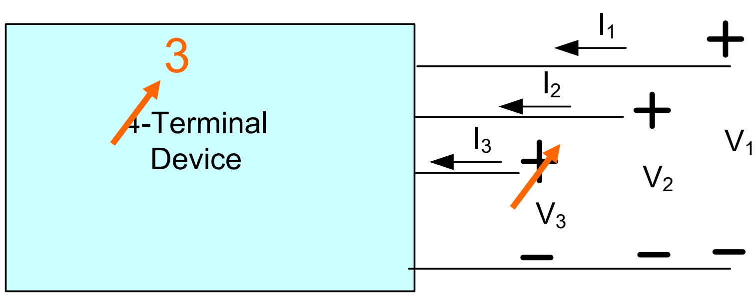
Equivalent circuit is not unique

Equivalent circuit is a three-port network

Review from Last Lecture

Consider 3-terminal network

Small-Signal Model



$$i_1 = g_1(v_1, v_2, v_3)$$
$$i_2 = g_2(v_1, v_2, v_3)$$
$$i_3 = g_3(v_1, v_2, v_3)$$

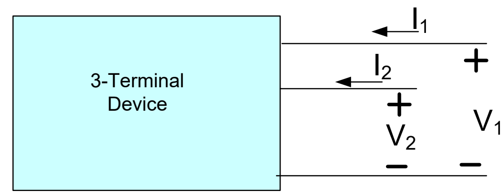
$$i_1 = y_{11}v_1 + y_{12}v_2 + y_{13}v_3$$
$$i_2 = y_{21}v_1 + y_{22}v_2 + y_{23}v_3$$
$$i_3 = y_{31}v_1 + y_{32}v_2 + y_{33}v_3$$

$$\mathbf{y_{ij}} = \left.\frac{\partial \mathbf{f_i}(\mathbf{V_1},\mathbf{V_2},\mathbf{V_3})}{\partial \mathbf{V_j}}\right|_{\vec{\mathbf{V}}=\vec{\mathbf{V}}_\mathbf{Q}}$$

Review from Last Lecture

Consider 3-terminal network

Small-Signal Model

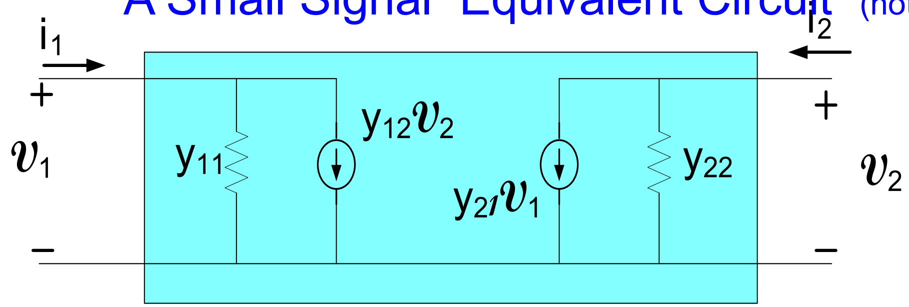


$$i_1 = y_{11} v_1 + y_{12} v_2$$
$$i_2 = y_{21} v_1 + y_{22} v_2$$

$$\mathbf{y_{ij}} = \left. \frac{\partial \mathbf{f_i}(\mathbf{V_1}, \mathbf{V_2})}{\partial \mathbf{V_j}} \right|_{\vec{\mathbf{V}} = \vec{\mathbf{V}}_\mathbf{Q}}$$

$$\vec{\mathbf{V}} = \begin{pmatrix} \mathbf{V_{1Q}} \\ \mathbf{V_{2Q}} \end{pmatrix}$$

A Small Signal Equivalent Circuit (not unique)

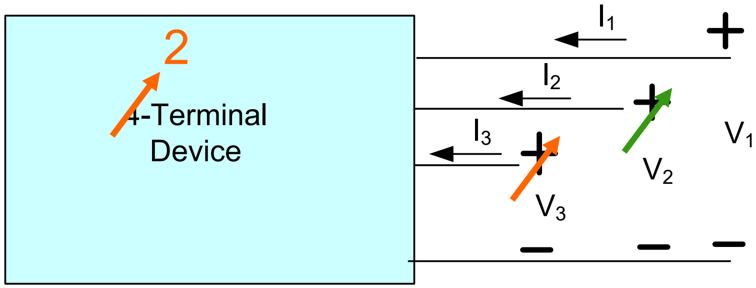


- *Small-signal model is a "two-port"*
- *4 small-signal parameters characterize this 3-terminal linear network*
- *Small signal parameters dependent upon Q-point*

Review from Last Lecture

Consider 2-terminal network

Small-Signal Model



$$i_1 = g_1(v_1, v_2, v_3)$$
$$i_2 = g_2(v_1, v_2, v_3)$$
$$i_3 = g_3(v_1, v_2, v_3)$$

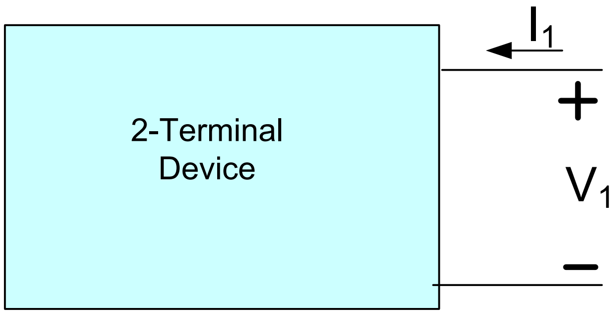
$$i_1 = y_{11}v_1 + y_{12}v_2 + y_{13}v_3$$
$$i_2 = y_{21}v_1 + y_{22}v_2 + y_{23}v_3$$
$$i_3 = y_{31}v_1 + y_{32}v_2 + y_{33}v_3$$

$$\mathbf{y}_{ij} = \left.\frac{\partial \mathbf{f}_i(\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3)}{\partial \mathbf{V}_j}\right|_{\vec{\mathbf{V}} = \vec{\mathbf{V}}_Q}$$

Consider 2-terminal network

Review from Last Lecture

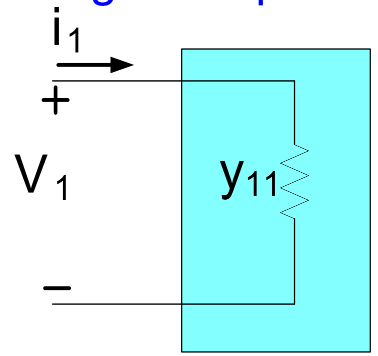
Small-Signal Model



$$i_1 = y_{11} v_1$$

$$y_{11} = \left. \frac{\partial f_1(V_1)}{\partial V_1} \right|_{\vec{V}=\vec{V}_Q} \qquad \vec{V} = V_{1Q}$$

A Small Signal Equivalent Circuit

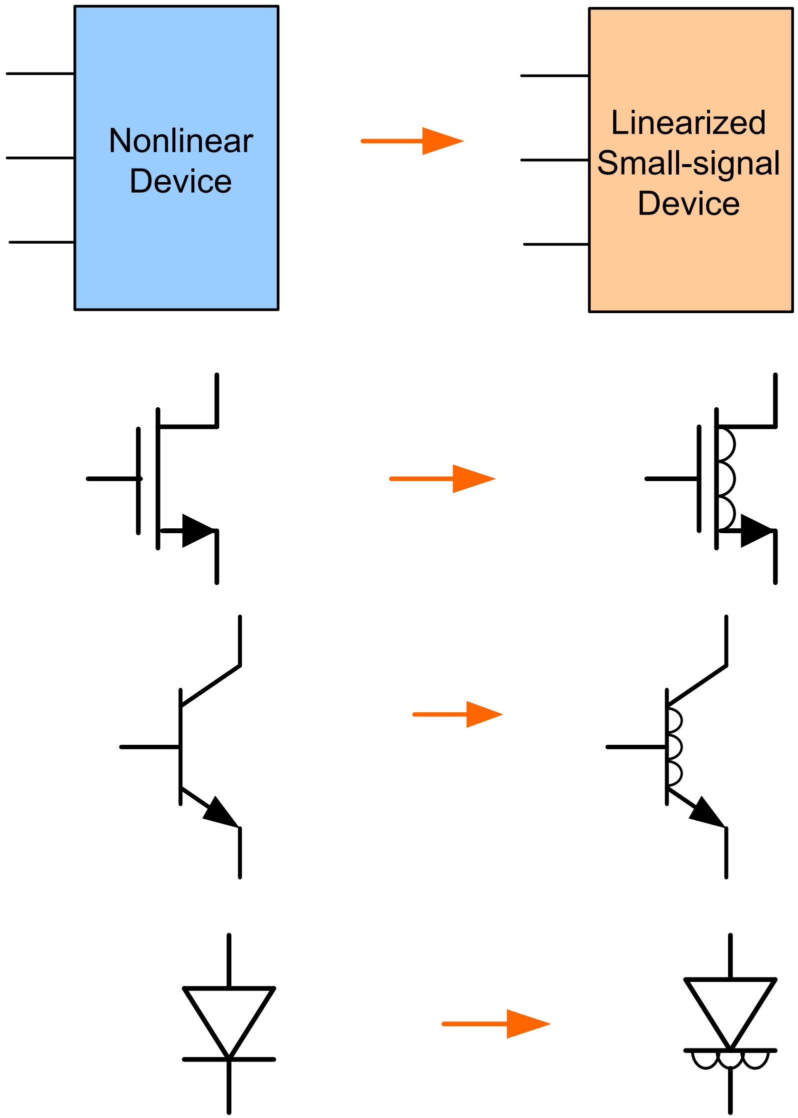


Small-signal model is a one-port

This was actually developed earlier !

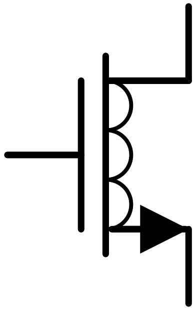
Review from Last Lecture

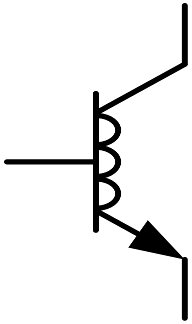
Linearized nonlinear devices

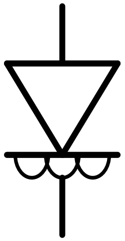


How is the small-signal equivalent circuit obtained from the nonlinear circuit?

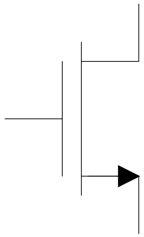
What is the small-signal equivalent of the MOSFET, BJT, and diode ?



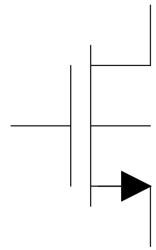




Small Signal Model of MOSFET



3-terminal device



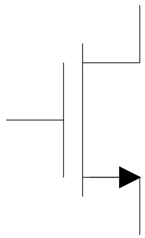
4-terminal device

MOSFET is actually a 4-terminal device but for many applications acceptable predictions of performance can be obtained by treating it as a 3-terminal device by neglecting the bulk terminal

In this course, we have been treating it as a 3-terminal device and in <u>this</u> lecture will develop the small-signal model by treating it as a 3-terminal device

When treated as a 4-terminal device, the bulk voltage introduces one additional term to the small signal model which is often either negligibly small or has no effect on circuit performance (will develop 4-terminal ss model later)

Small Signal Model of MOSFET

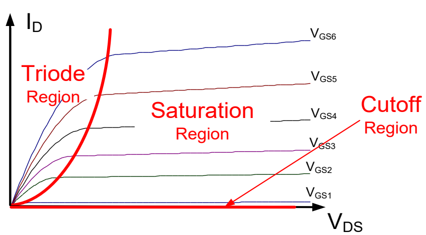


Large Signal Model

$$I_G = 0$$

3-terminal device

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$



MOSFET is usually operated in saturation region in linear applications where a small-signal model is needed so will develop the small-signal model in the saturation region

Small Signal Model of MOSFET

$$I_1 = f_1(V_1, V_2) \quad \Longleftrightarrow \quad I_G = 0$$

$$I_2 = f_2(V_1, V_2) \quad \Longleftrightarrow \quad I_D = \mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$$I_G = f_1(V_{GS}, V_{DS})$$

$$I_D = f_2(V_{GS}, V_{DS})$$

Small-signal model:

$$y_{ij} = \left. \frac{\partial f_i(V_1, V_2)}{\partial V_j} \right|_{\vec{V} = \vec{V}_Q}$$

$$y_{11} = \left. \frac{\partial I_G}{\partial V_{GS}} \right|_{\vec{V} = \vec{V}_Q} \qquad y_{12} = \left. \frac{\partial I_G}{\partial V_{DS}} \right|_{\vec{V} = \vec{V}_Q}$$

$$y_{21} = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{\vec{V} = \vec{V}_Q} \qquad y_{22} = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{\vec{V} = \vec{V}_Q}$$

Small Signal Model of MOSFET

$$I_G = 0$$

$$I_D = \mu C_{OX} \frac{W}{2L} \left(V_{GS} - V_T\right)^2 \left(1 + \lambda V_{DS}\right)$$

Small-signal model:

$$y_{11} = \left. \frac{\partial I_G}{\partial V_{GS}} \right|_{\vec{V} = \vec{V}_Q} = ?$$

$$y_{12} = \left. \frac{\partial I_G}{\partial V_{DS}} \right|_{\vec{V} = \vec{V}_Q} = ?$$

$$y_{21} = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{\vec{V} = \vec{V}_Q} = ?$$

$$y_{22} = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{\vec{V} = \vec{V}_Q} = ?$$

Recall: termed the y-parameter model

Small Signal Model of MOSFET

$$I_1 = f_1(V_1, V_2) \quad \Longleftrightarrow \quad I_G = 0$$

$$I_2 = f_2(V_1, V_2) \quad \Longleftrightarrow \quad I_D = \mu C_{OX}\frac{W}{2L}(V_{GS} - V_T)^2(1 + \lambda V_{DS})$$

Small-signal model:

$$y_{11} = \left.\frac{\partial I_G}{\partial V_{GS}}\right|_{\vec{V} = \vec{V}_Q} = 0 \qquad y_{12} = \left.\frac{\partial I_G}{\partial V_{DS}}\right|_{\vec{V} = \vec{V}_Q} = 0$$

$$y_{21} = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{\vec{V} = \vec{V}_Q} = 2\mu C_{OX}\frac{W}{2L}(V_{GS} - V_T)^1(1 + \lambda V_{DS})\Big|_{\vec{V} = \vec{V}_Q} = \mu C_{OX}\frac{W}{L}(V_{GSQ} - V_T)(1 + \lambda V_{DSQ})$$

$$y_{21} \cong \mu C_{OX}\frac{W}{L}(V_{GSQ} - V_T)$$

$$y_{22} = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{\vec{V} = \vec{V}_Q} = \mu C_{OX}\frac{W}{2L}(V_{GS} - V_T)^2\lambda\Big|_{\vec{V} = \vec{V}_Q} \cong \lambda I_{DQ}$$

Small Signal Model of MOSFET

Nonlinear model:

$$I_G = 0$$

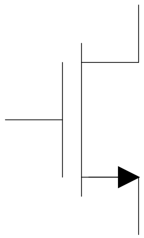
$$I_D = \mu C_{OX} \frac{W}{2L} \left(V_{GS} - V_T \right)^2 \left(1 + \lambda V_{DS} \right)$$

Small-signal model:

$$y_{11} = 0 \qquad y_{12} = 0$$

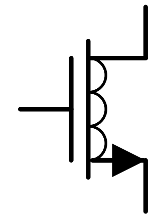
$$y_{21} \cong \mu C_{OX} \frac{W}{L} \left(V_{GSQ} - V_T \right) \qquad y_{22} \cong \lambda I_{DQ}$$

Small Signal Model of MOSFET



$$I_G = 0$$

$$I_D = \mu C_{OX} \frac{W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$



$$y_{12} = 0$$

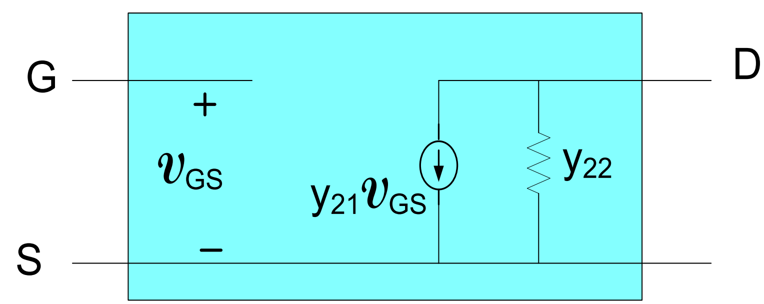
$$y_{11} = 0$$

$$y_{21} \cong \mu C_{OX} \frac{W}{L} (V_{GSQ} - V_T)$$

$$y_{22} \cong \lambda I_{DQ}$$

$$i_G = y_{11} v_{GS} + y_{12} v_{DS}$$

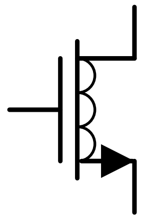
$$i_D = y_{21} v_{GS} + y_{22} v_{DS}$$



An equivalent circuit

(y-parameter model)

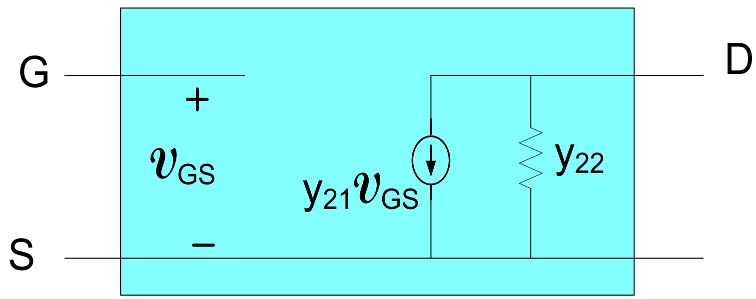
Small-Signal Model of MOSFET



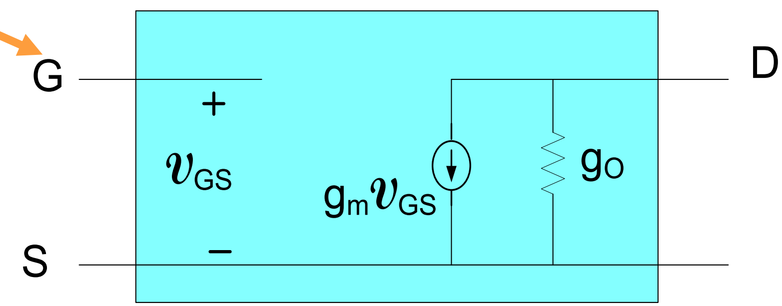
by convention, $y_{21}=g_m$, $y_{22}=g_0$

$$\therefore \qquad y_{21} \cong g_m = \mu C_{ox}\frac{W}{L}(V_{GSQ} - V_T)$$

$$y_{22} = g_o \cong \lambda I_{DQ}$$



(y-parameter model)



$$i_G = 0$$

$$i_D = g_m v_{GS} + g_o v_{DS}$$

still y-parameter model
but use "g" parameter notation

Note: g_o vanishes when $\lambda=0$

Small Signal Model of MOSFET

Saturation Region Summary

Nonlinear model:

$$\begin{cases} I_G = 0 \\ I_D = \mu C_{OX}\frac{W}{2L}\left(V_{GS}-V_T\right)^2\left(1+\lambda V_{DS}\right) \end{cases}$$

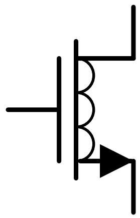
Small-signal model:

$$\begin{cases} i_G = y_{11}v_{GS} + y_{12}v_{DS} = 0 \\ i_D = y_{21}v_{GS} + y_{22}v_{DSE} \end{cases}$$

$$y_{11} = 0 \qquad y_{12} = 0$$

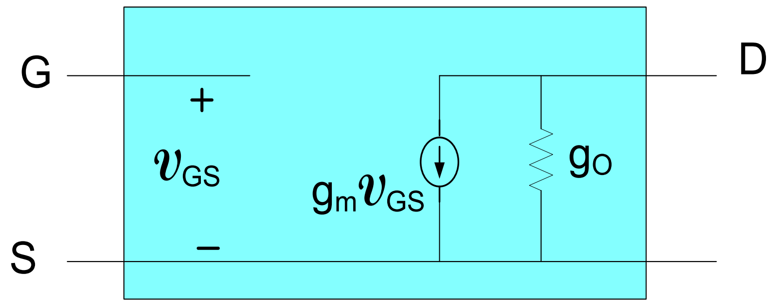
$$y_{21} = g_m \cong \mu C_{OX}\frac{W}{L}\left(V_{GSQ}-V_T\right) \qquad y_{22} = g_0 \cong \lambda I_{DQ}$$

Small-Signal Model of MOSFET



$$g_m = \mu C_{OX} \frac{W}{L}\left(V_{GSQ} - V_T\right)$$

$$g_O \cong \lambda I_{DQ}$$



Alternate equivalent expressions for g_m:

$$I_{DQ} = \mu C_{OX} \frac{W}{2L}\left(V_{GSQ} - V_T\right)^2\left(1 + \lambda V_{DSQ}\right) \cong \mu C_{OX} \frac{W}{2L}\left(V_{GSQ} - V_T\right)^2$$

$$g_m = \mu C_{OX} \frac{W}{L}\left(V_{GSQ} - V_T\right)$$

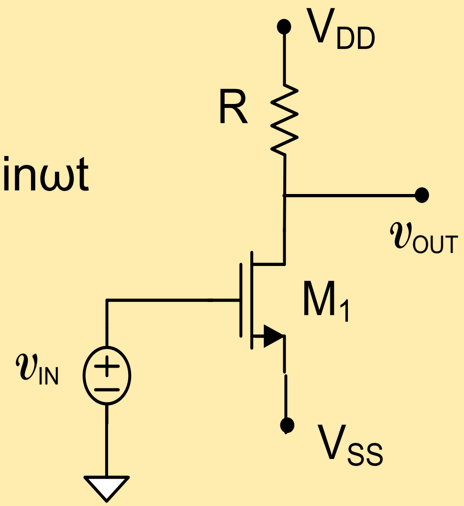
$$g_m = \sqrt{2\mu C_{OX} \frac{W}{L}} \bullet \sqrt{I_{DQ}}$$

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T}$$

Consider again:

Small-signal analysis example

$v_{IN}=V_M\sin\omega t$

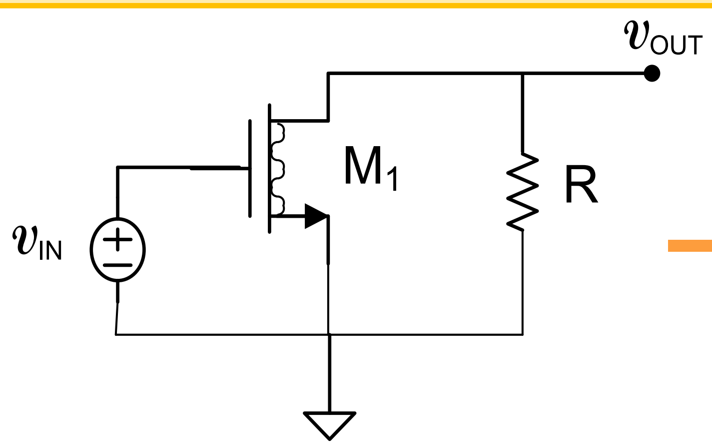


$$A_v = \frac{2I_{DQ}R}{\left[V_{SS}+V_T\right]}$$

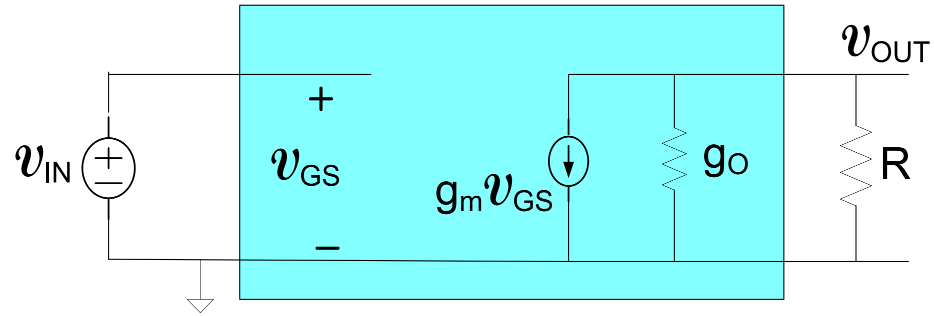
Derived for $\lambda=0$ (equivalently $g_0=0$)

$$I_D=\mu C_{OX}\frac{W}{2L}\left(V_{GS}-V_T\right)^2$$

Recall the derivation was very tedious and time consuming!

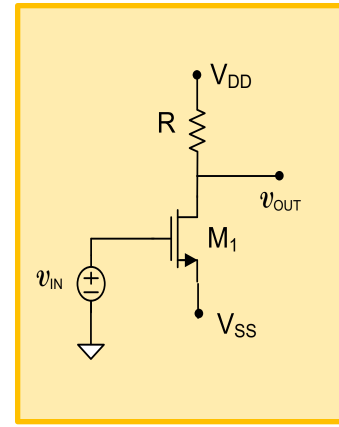


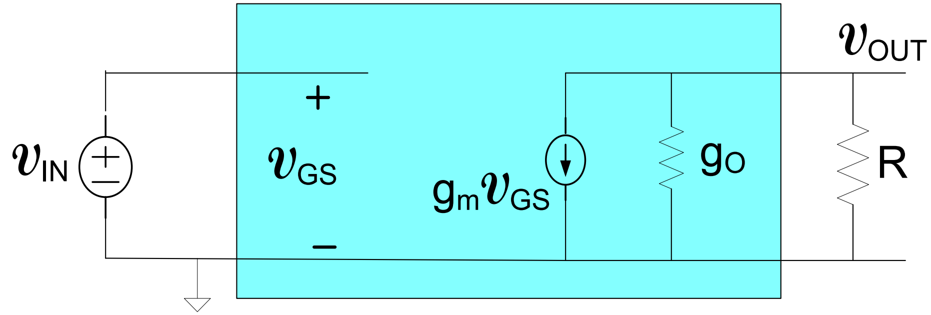
ss circuit



Consider again:

Small-signal analysis example

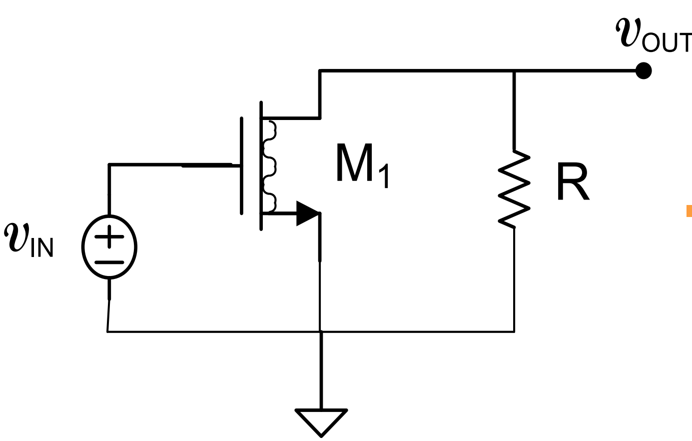




$$A_V = \frac{V_{OUT}}{V_{IN}} = -\frac{g_m}{g_o + 1/R}$$

This gain is expressed in terms of small-signal model parameters

For λ=0, $g_O = \lambda I_{DQ} = 0$



$$A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R$$

but

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T}$$

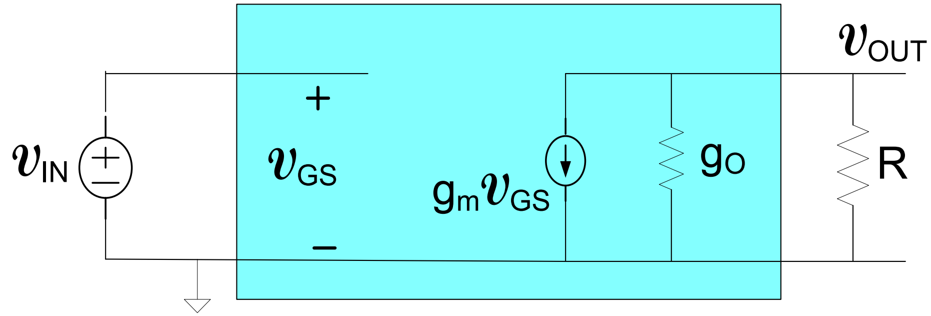
$V_{GSQ} = -V_{SS}$

thus

$$A_v = \frac{2I_{DQ} R}{[V_{SS} + V_T]}$$

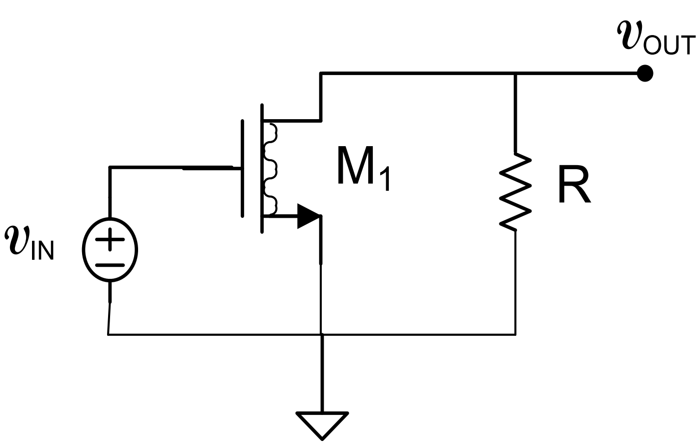
Consider again:

Small-signal analysis example



$$A_V = \frac{V_{OUT}}{V_{IN}} = -\frac{g_m}{g_O + 1/R}$$

For $\lambda=0$, $g_O = \lambda I_{DQ} = 0$

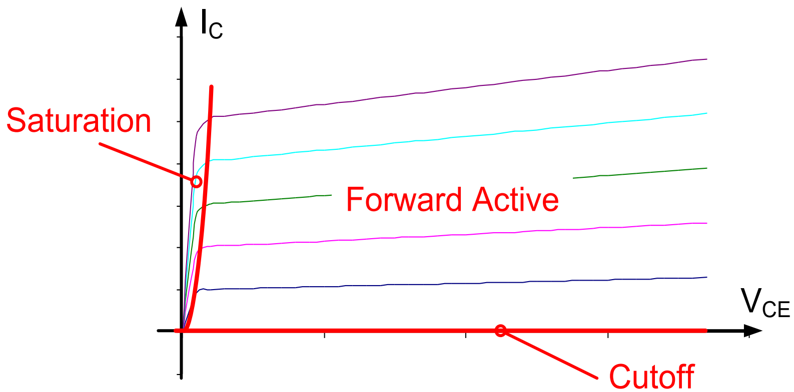


$$A_v = \frac{2I_{DQ}R}{[V_{SS} + V_T]}$$

- Same expression as derived before !
- More accurate gain can be obtained if λ effects are included and does not significantly increase complexity of small-signal analysis

Small Signal Model of BJT

3-terminal device



Forward Active Model:

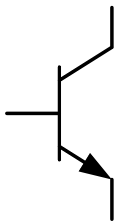
$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

$$\mathbf{I_B} = \frac{\mathbf{J_S A_E}}{\boldsymbol{\beta}} \mathbf{e}^{\frac{\mathbf{V_{BE}}}{\mathbf{V_t}}}$$

- Usually operated in Forward Active Region when small-signal model is needed
- Will develop small-signal model in Forward Active Region

Small Signal Model of BJT

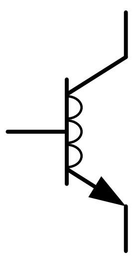
Nonlinear model:



$$I_1 = f_1(V_1, V_2) \quad \Longleftrightarrow \quad \mathbf{I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}}$$

$$I_2 = f_2(V_1, V_2) \quad \Longleftrightarrow \quad I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

Small-signal model:



$$\boldsymbol{i}_B = y_{11} \boldsymbol{v}_{BE} + y_{12} \boldsymbol{v}_{CE}$$

$$\boldsymbol{i}_C = y_{21} \boldsymbol{v}_{BE} + y_{22} \boldsymbol{v}_{CE}$$

$$y_{ij} = \left. \frac{\partial f_i(V_1, V_2)}{\partial V_j} \right|_{\vec{V} = \vec{V}_Q}$$

y-parameter model

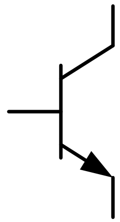
$$y_{11} = g_\pi = \left. \frac{\partial I_B}{\partial V_{BE}} \right|_{\vec{V} = \vec{V}_Q} \qquad y_{12} = \left. \frac{\partial I_B}{\partial V_{CE}} \right|_{\vec{V} = \vec{V}_Q}$$

$$y_{21} = g_m = \left. \frac{\partial I_C}{\partial V_{BE}} \right|_{\vec{V} = \vec{V}_Q} \qquad y_{22} = g_o = \left. \frac{\partial I_C}{\partial V_{CE}} \right|_{\vec{V} = \vec{V}_Q}$$

Note: g_m, g_π and g_o used for notational consistency with legacy terminology

Small Signal Model of BJT

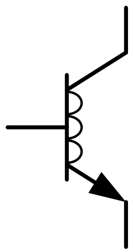
Nonlinear model:



$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

Small-signal model:



$$i_B = y_{11} v_{BE} + y_{12} v_{CE}$$

$$i_C = y_{21} v_{BE} + y_{22} v_{CE}$$

$$y_{ij} = \left. \frac{\partial f_i(V_1, V_2)}{\partial V_j} \right|_{\vec{V} = \vec{V}_Q}$$

$$y_{11} = g_\pi = \left. \frac{\partial I_B}{\partial V_{BE}} \right|_{\vec{V} = \vec{V}_Q} = ?$$

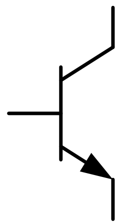
$$y_{12} = \left. \frac{\partial I_B}{\partial V_{CE}} \right|_{\vec{V} = \vec{V}_Q} = ?$$

$$y_{21} = g_m = \left. \frac{\partial I_C}{\partial V_{BE}} \right|_{\vec{V} = \vec{V}_Q} = ?$$

$$y_{22} = g_o = \left. \frac{\partial I_C}{\partial V_{CE}} \right|_{\vec{V} = \vec{V}_Q} = ?$$

Small Signal Model of BJT

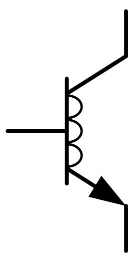
Nonlinear model:



$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

Small-signal model:



$$\boldsymbol{i}_B = y_{11} \boldsymbol{v}_{BE} + y_{12} \boldsymbol{v}_{CE}$$

$$\boldsymbol{i}_C = y_{21} \boldsymbol{v}_{BE} + y_{22} \boldsymbol{v}_{CE}$$

$$y_{11} = g_\pi = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{\vec{V}=\vec{V}_Q} = \frac{1}{V_t} \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \Bigg|_{\vec{V}=\vec{V}_Q} = \frac{I_{BQ}}{V_t} \cong \frac{I_{CQ}}{\beta V_t}$$

$$y_{21} = g_m = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{\vec{V}=\vec{V}_Q} = \frac{1}{V_t} J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)\Bigg|_{\vec{V}=\vec{V}_Q} = \frac{I_{CQ}}{V_t}$$

$$y_{12} = \left.\frac{\partial I_B}{\partial V_{CE}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

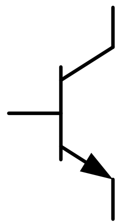
$$y_{22} = g_O = \left.\frac{\partial I_C}{\partial V_{CE}}\right|_{\vec{V}=\vec{V}_Q} = \frac{J_S A_E e^{\frac{V_{BE}}{V_t}}}{V_{AF}} \Bigg|_{\vec{V}=\vec{V}_Q} \cong \frac{I_{CQ}}{V_{AF}}$$

Note: usually prefer to express in terms of I_{CQ}

Small Signal Model of BJT

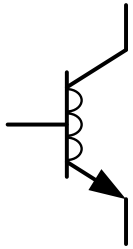
Forward Active Region Summary

Nonlinear model:



$$\begin{cases} I_B = \dfrac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \\ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \dfrac{V_{CE}}{V_{AF}}\right) \end{cases}$$

Small-signal model:



$$\begin{cases} i_B = y_{11} v_{BE} + y_{12} v_{CE} \\ i_C = y_{21} v_{BE} + y_{22} v_{CE} \end{cases}$$

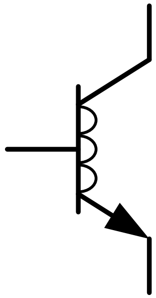
$$y_{11} = g_\pi \cong \frac{I_{CQ}}{\beta V_t}$$

$$y_{21} = g_m = \frac{I_{CQ}}{V_t}$$

$$y_{12} = 0$$

$$y_{22} = g_o \cong \frac{I_{CQ}}{V_{AF}}$$

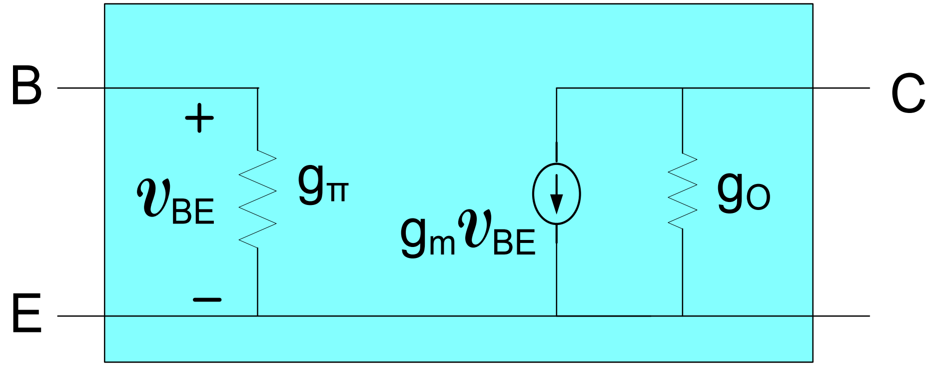
Small Signal Model of BJT



$$i_B = y_{11} v_{BE} + y_{12} v_{CE}$$
$$i_C = y_{21} v_{BE} + y_{22} v_{CE}$$

$$i_B = g_\pi v_{BE}$$
$$i_C = g_m v_{BE} + g_O v_{CE}$$

$$g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_m = \frac{I_{CQ}}{V_t} \qquad g_O = \frac{I_{CQ}}{V_{AF}}$$

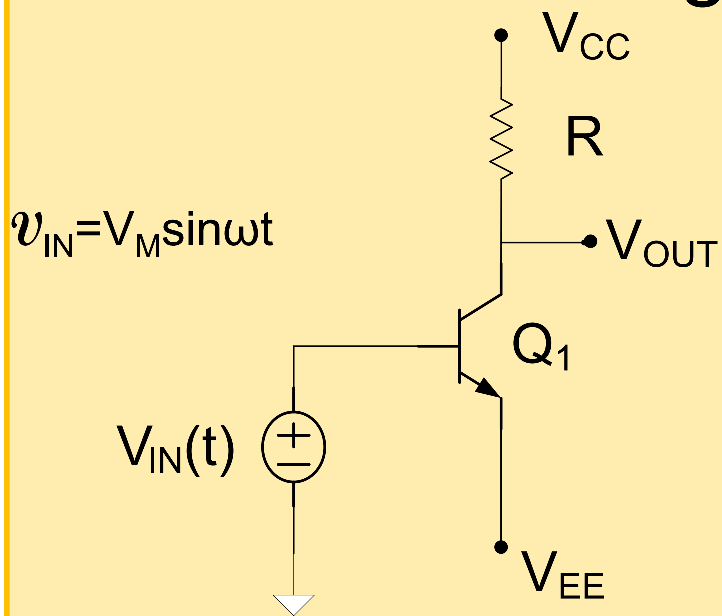


An equivalent circuit

y-parameter model using “g” parameter notation

Consider again:

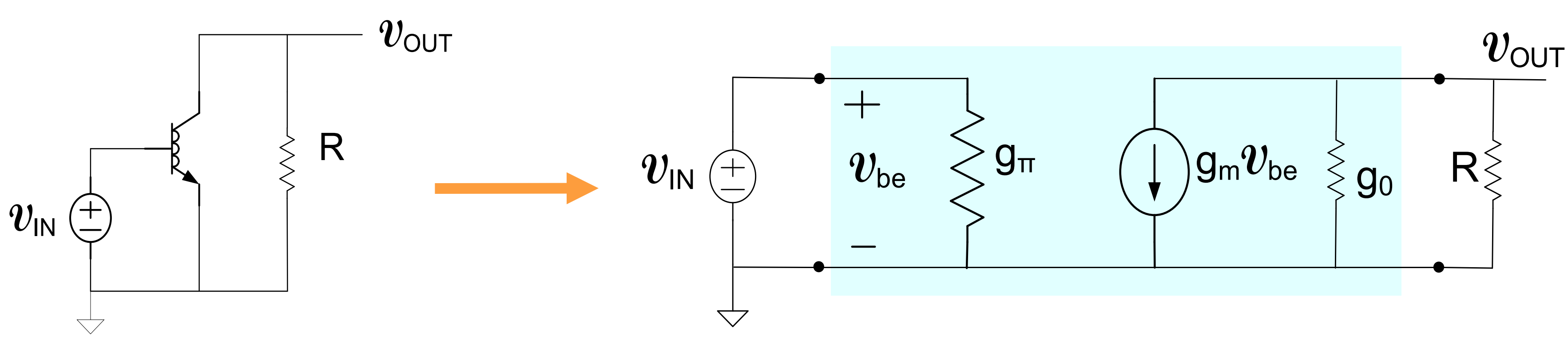
Small signal analysis example



$$A_{VB} = -\frac{I_{CQ} R}{V_t}$$

Derived for $V_{AF}=0$ (equivalently $g_o=0$)

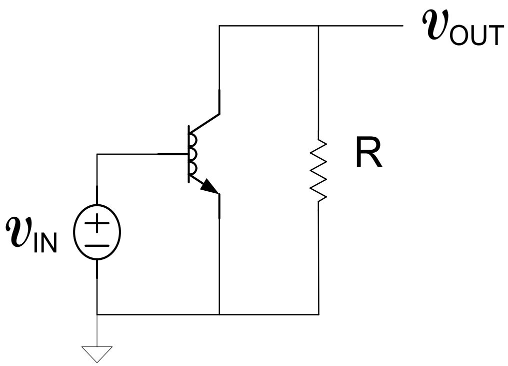
Recall the derivation was very tedious and time consuming!

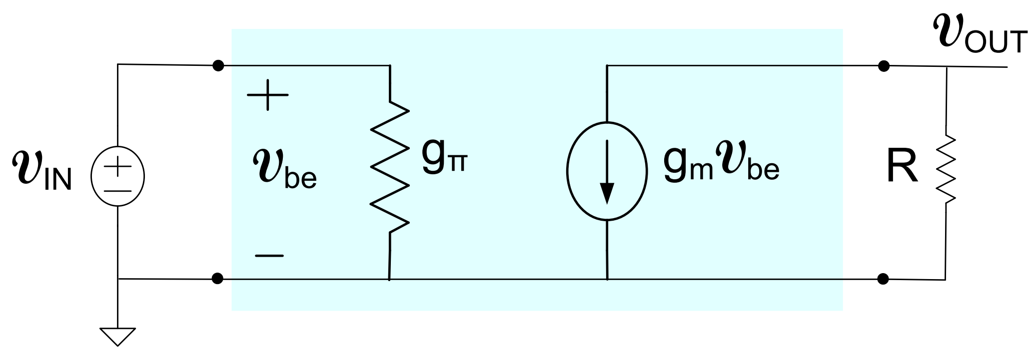


ss circuit

Neglect V_{AF} effects (i.e. $V_{AF}=\infty$) to be consistent with earlier analysis

$$g_o = \frac{I_{CQ}}{V_{AF}} \underset{V_{AF}=\infty}{=} 0$$





$$\left.\begin{aligned} v_{OUT} &= -g_m R v_{BE} \\ v_{IN} &= v_{BE} \end{aligned}\right\}$$

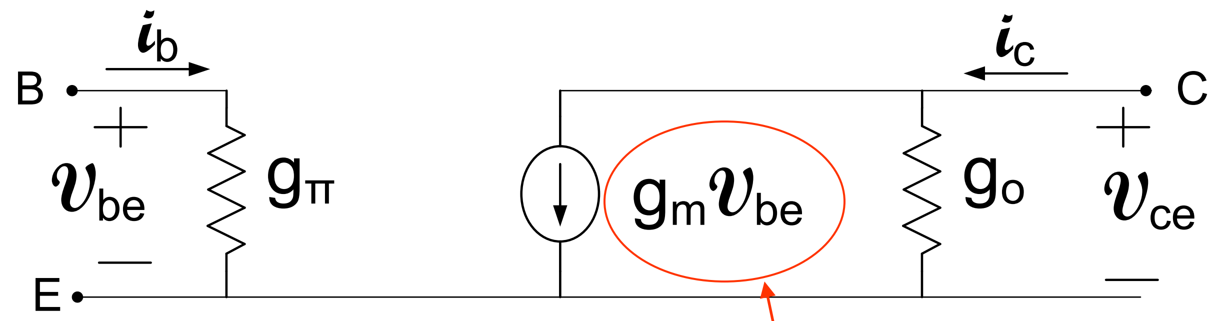
$$A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R$$

$$g_m = \frac{I_{CQ}}{V_t}$$

$$A_V = -\frac{I_{CQ} R}{V_t}$$

Note this is identical to what was obtained with the direct nonlinear analysis

Small Signal BJT Model – alternate representation



$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Observe :

$$g_\pi v_{be} = i_b$$

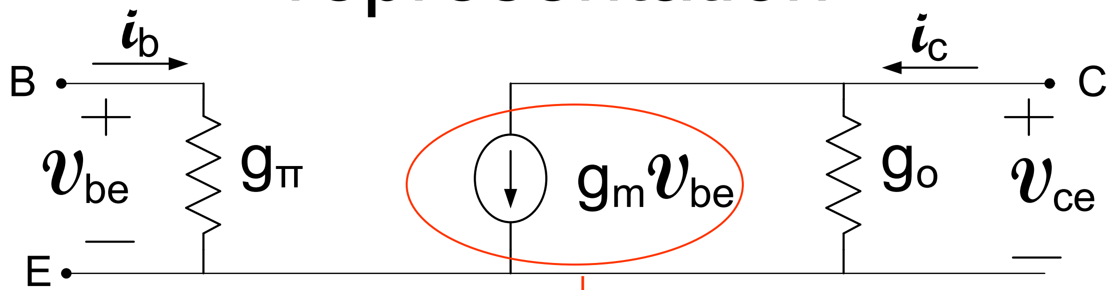
$$g_m v_{be} = i_b \frac{g_m}{g_\pi}$$

$$\frac{g_m}{g_\pi} = \frac{\left[\frac{I_Q}{V_t}\right]}{\left[\frac{I_Q}{\beta V_t}\right]} = \beta$$

$$g_m v_{be} = \beta i_b$$

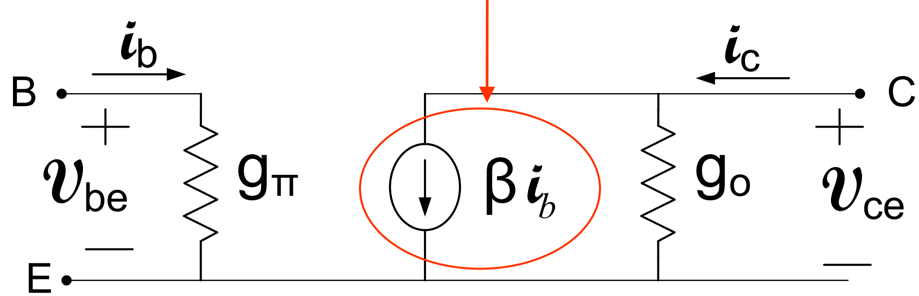
Can replace the voltage dependent current source with a current dependent current source

Small Signal BJT Model – alternate representation



$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

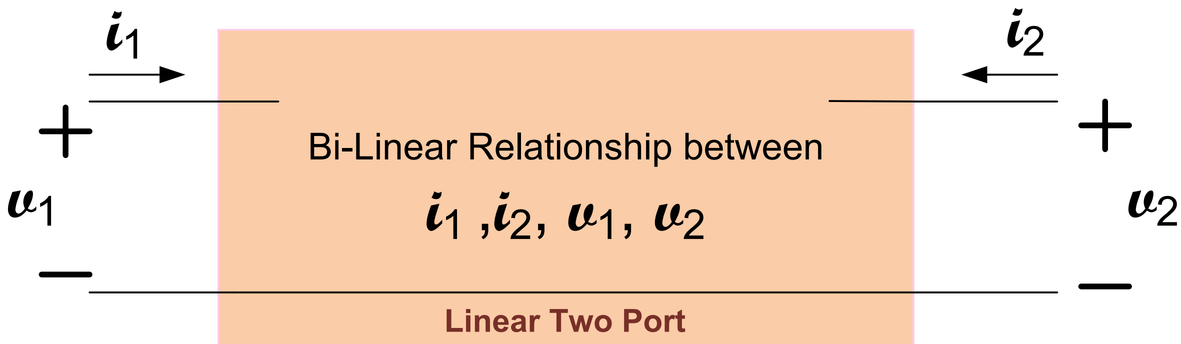
Alternate equivalent small signal model



$$g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

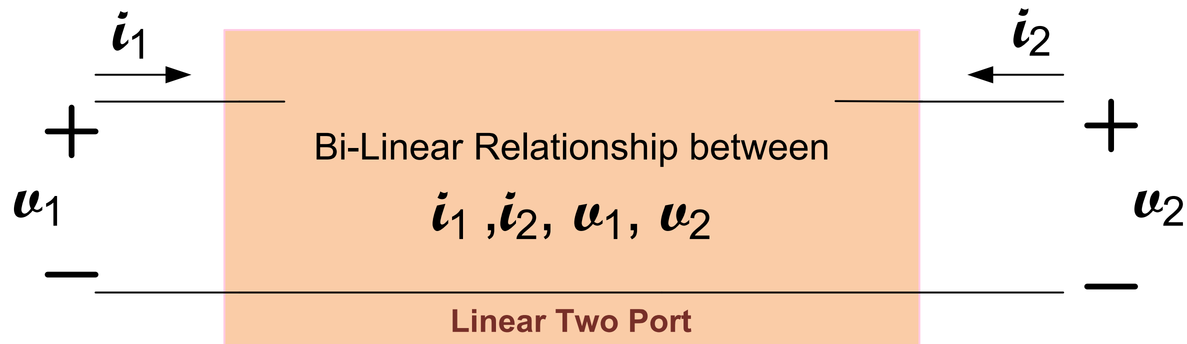
Small-Signal Model Representations

(3-terminal network – also relevant with 4-terminal networks)



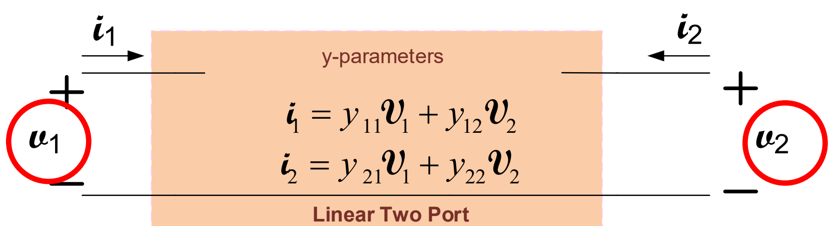
- Have developed small-signal models for the MOSFET and BJT
- Models have been based upon arbitrary assumption that v_1, v_2 are independent variables
- Models are y-parameter models expressed in terms of "g" parameters
- Have already seen some alternatives for "parameter" definitions in these models
- Alternative representations are sometimes used

Small-Signal Model Representations

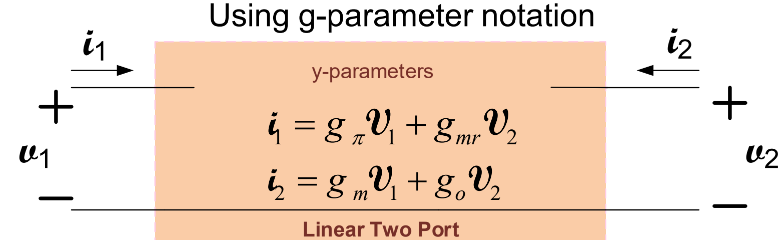


The good, the bad, and the unnecessary !!

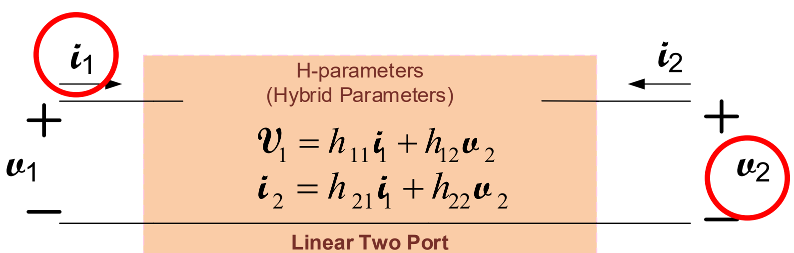
what we have developed:



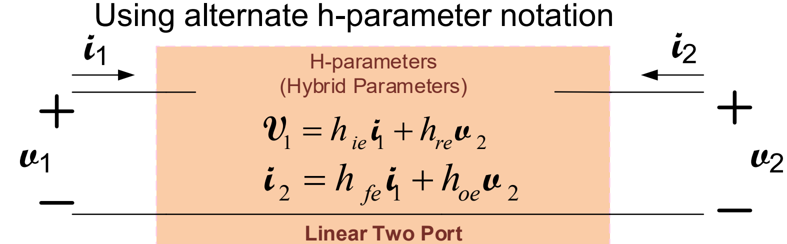
Using g-parameter notation



The hybrid parameters:

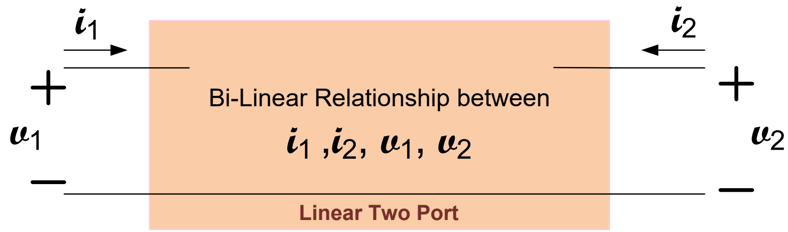


Using alternate h-parameter notation

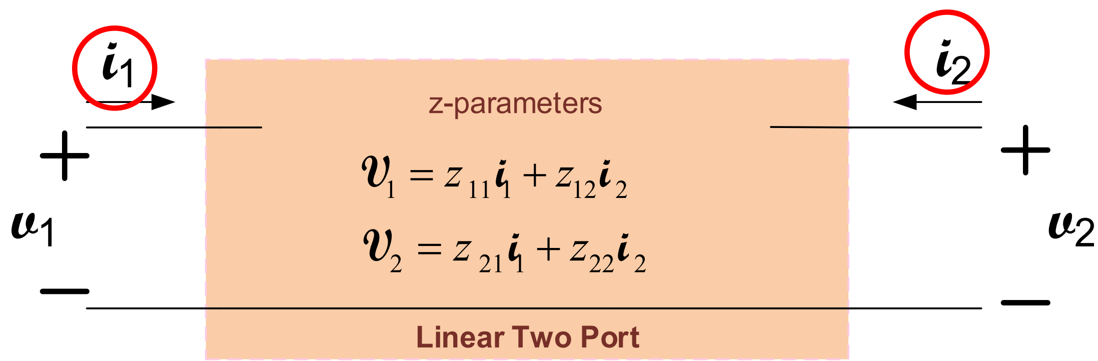


Independent parameters

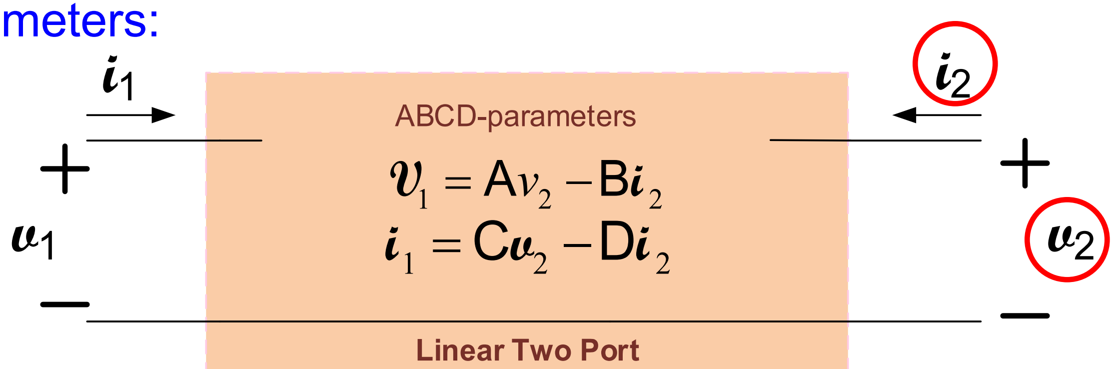
Small-Signal Model Representations



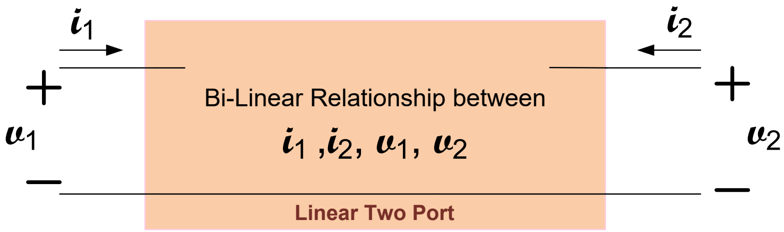
The z-parameters



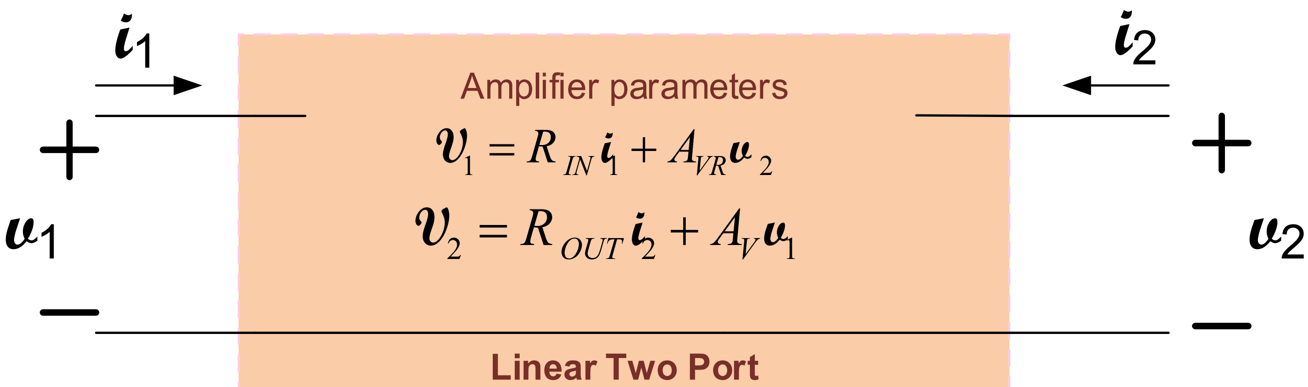
The ABCD parameters:



Small-Signal Model Representations

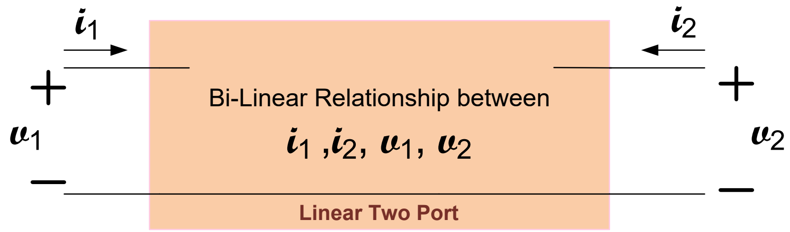


Amplifier parameters

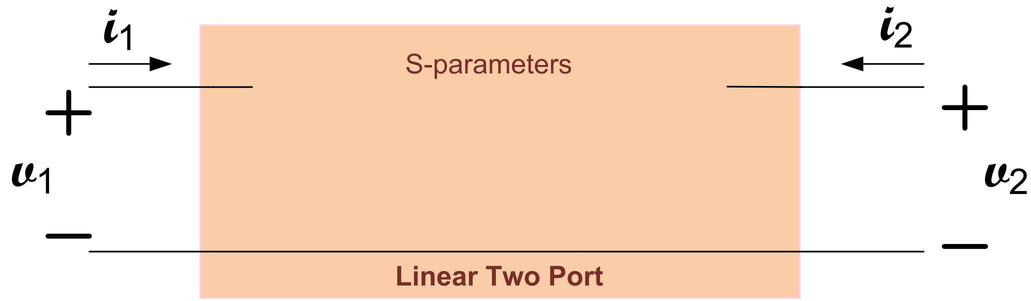


- Alternate two-port characterization but not expressed in terms of independent and dependent parameters
- Widely used notation when designing amplifiers

Small-Signal Model Representations

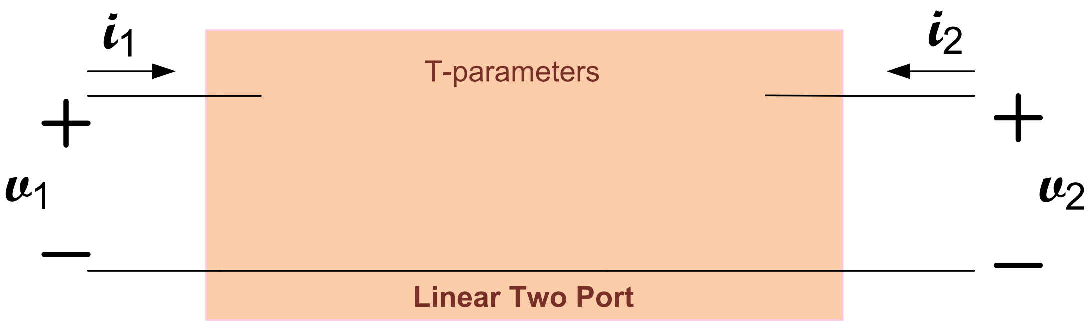


The S-parameters



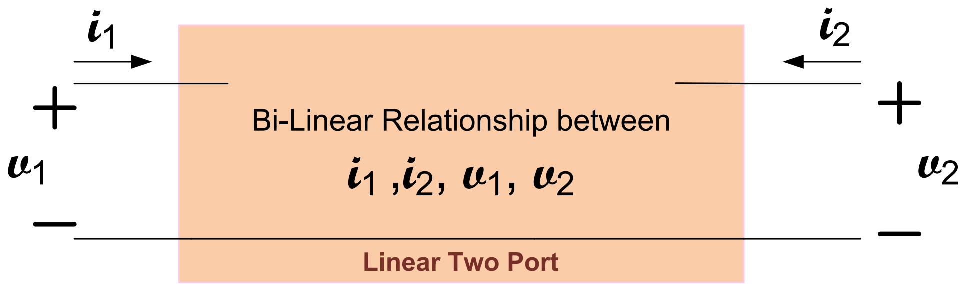
(embedded with source and load impedances)

The T parameters:

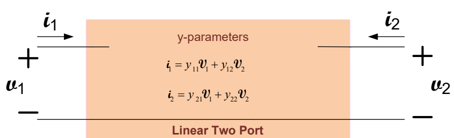


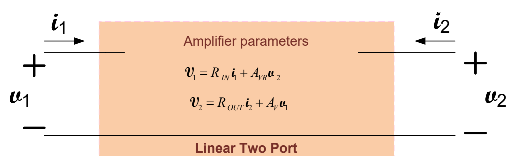
(embedded with source and load impedances)

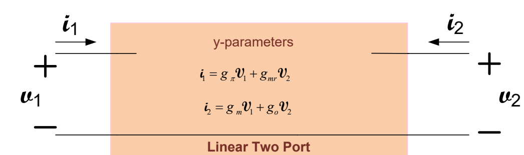
Small-Signal Model Representations

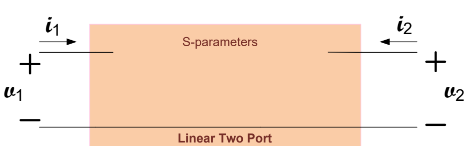


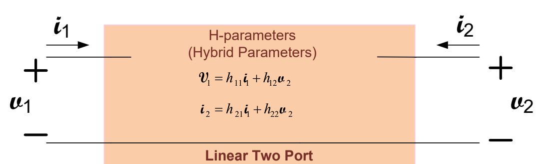
The good, the bad, and the **unnecessary** !!

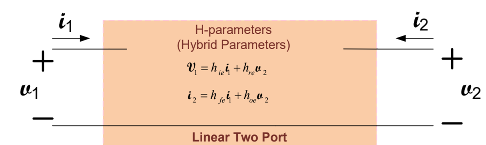


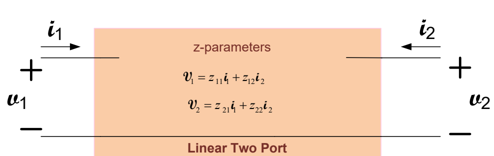


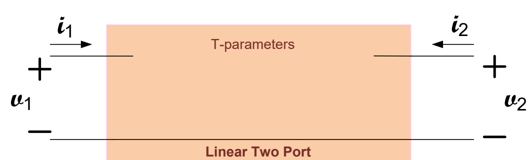


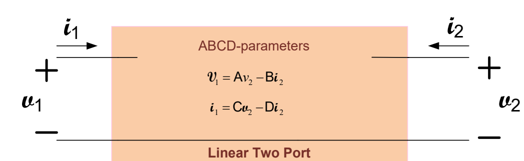






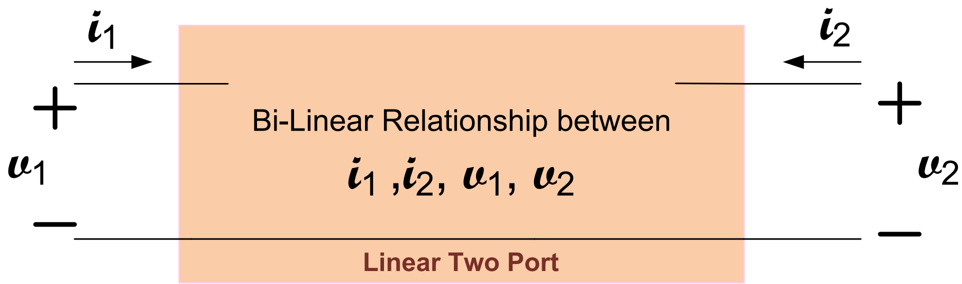






- Equivalent circuits often given for each representation
- All provide identical characterization
- Easy to move from any one to another

Small-Signal Model Representations



The good, the bad, and the **unnecessary** !!

IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 42, NO. 2, FEBRUARY 1994

Conversions Between *S*, *Z*, *Y*, *h*, *ABCD*, and *T* Parameters which are Valid for Complex Source and Load Impedances

Dean A. Frickey, *Member, IEEE*

Conversions **between S**, **Z**, **Y**, H, **ABCD**, and T parameters which are valid for complex source and load impedances

DA Frickey - IEEE Transactions on microwave theory and …, 1994 - ieeexplore.ieee.org

… 2. FEBRUARY 1994 TABLE m EQUATIONS FOR THE **CONVERSION BETWEEN s** PARAMEIERS AND NORMALIZED 2, **Y** , h … V. CONCLUSION This paper developed the equations for **c(**
between the various common 2-port parameters, **Z**, **Y**, h, **ABCD**, **S**, and T …

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Comments on" **Conversions between S**, **Z**, **Y**, h, **ABCD**, and T parameters which are valid for complex source and load impedances"[with reply]

…, DF Williams, DA **Frickey** - IEEE Transactions on …, 1995 - ieeexplore.ieee.org

In his recent paper, Frickey presents formulas for conversions between various network matrices. Four of these matrices (Z, Y, h, and ABCD) relate voltages and currents at the ports: the other two (S and T) relate wave quantities. These relationships depend on the …

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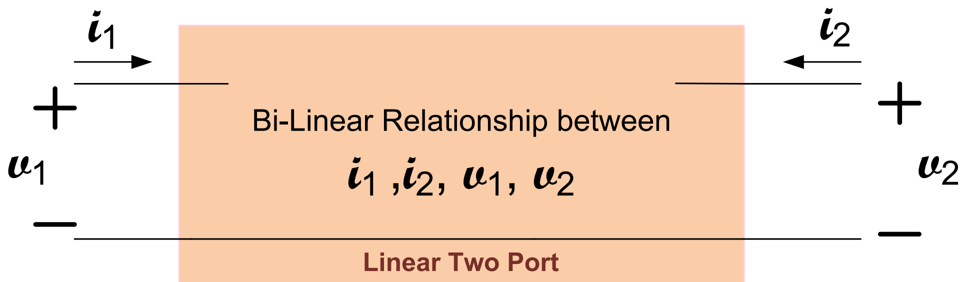
Small-Signal Model Representations



The good, the bad, and the **unnecessary** !!

IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 42, NO. 2, FEBRUARY 1994

Conversions Between S, Z, Y, h, $ABCD$, and T Parameters which are Valid for Complex Source and Load Impedances

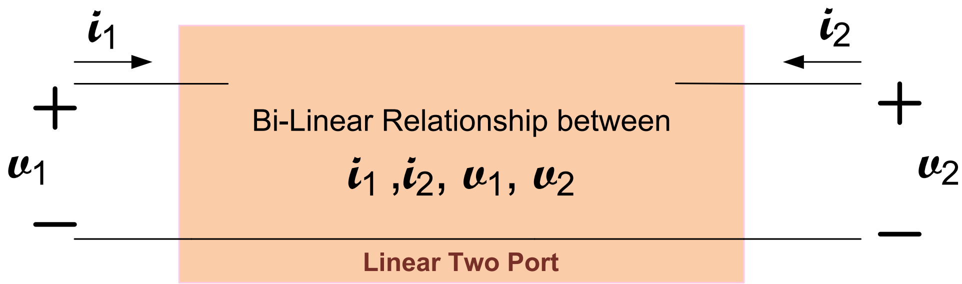
Conversions **between** S, Z, Y, H, ABCD, and T parameters which are valid for complex source and load impedances

DA **Frickey** - IEEE Transactions on microwave theory and …, 1994 - ieeexplore.ieee.org

This paper provides tables which contain the conversion between the various common two-port parameters, Z, Y, H, ABCD, S, and T. The conversions are valid for complex normalizing impedances. An example is provided which verifies the conversions to and from S

As of Mar 6, 2018

Small-Signal Model Representations



The good, the bad, and the **unnecessary** !!

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Conversions Between S, Z, Y, h, $ABCD$, and T Parameters which are Valid for Complex Source and Load Impedances

Dean A. Frickey, *Member, IEEE*

Conversions between S, Z, Y, H, ABCD, and T parameters which are valid for complex source and load impedances
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…, DF Williams, DA **Frickey** - Microwave Theory and …, 1995 - ieeexplore.ieee.org
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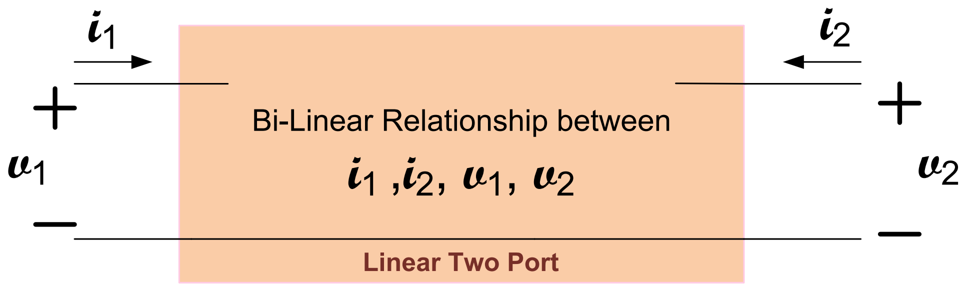
Small-Signal Model Representations



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Conversions Between S, Z, Y, h, $ABCD$, and T Parameters which are Valid for Complex Source and Load Impedances

Dean A. Frickey, *Member, IEEE*

Conversions between S, Z, Y, H, ABCD, and T parameters which are valid for complex source and load impedances
DA **Frickey** - ... theory and techniques, IEEE Transactions on, 1994 - ieeexplore.ieee.org
Abstract This paper provides tables which contain the **conversion between** the various common two-port parameters, Z, Y, H, ABCD, S, and T. The **conversions** are valid for complex normalizing impedances. An example is provided which verifies the **conversions** ...
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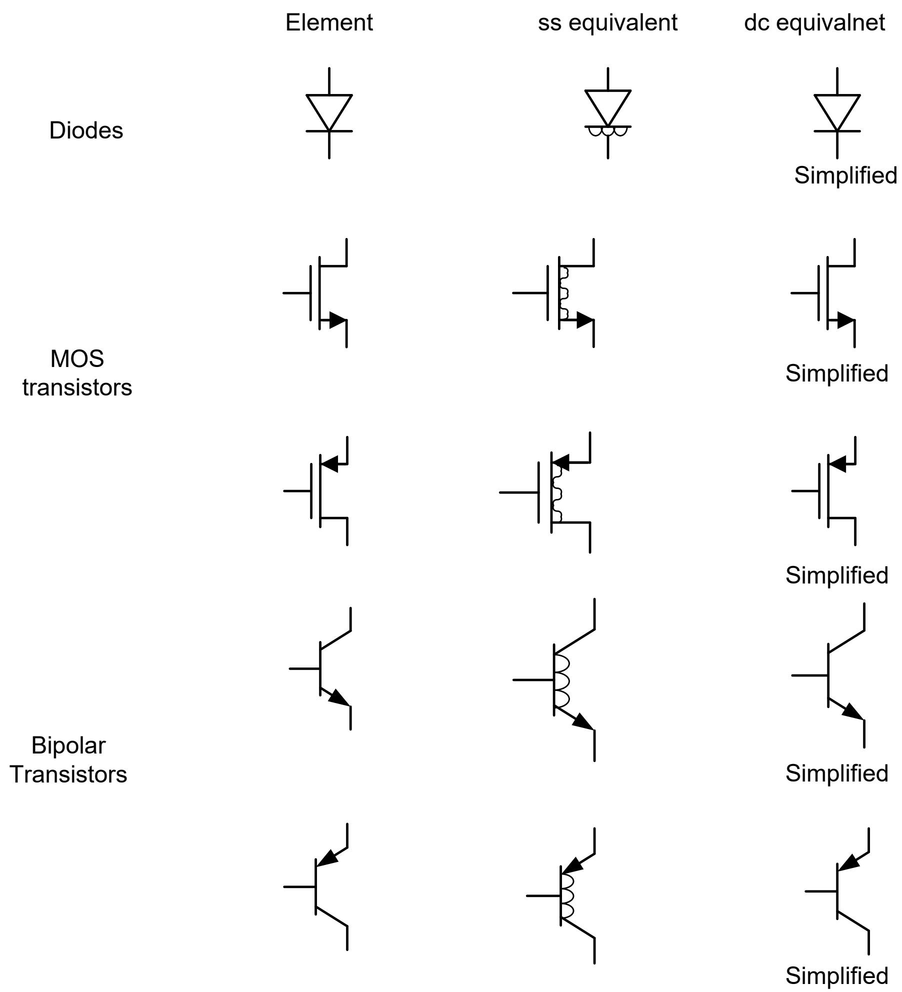
Active Device Model Summary

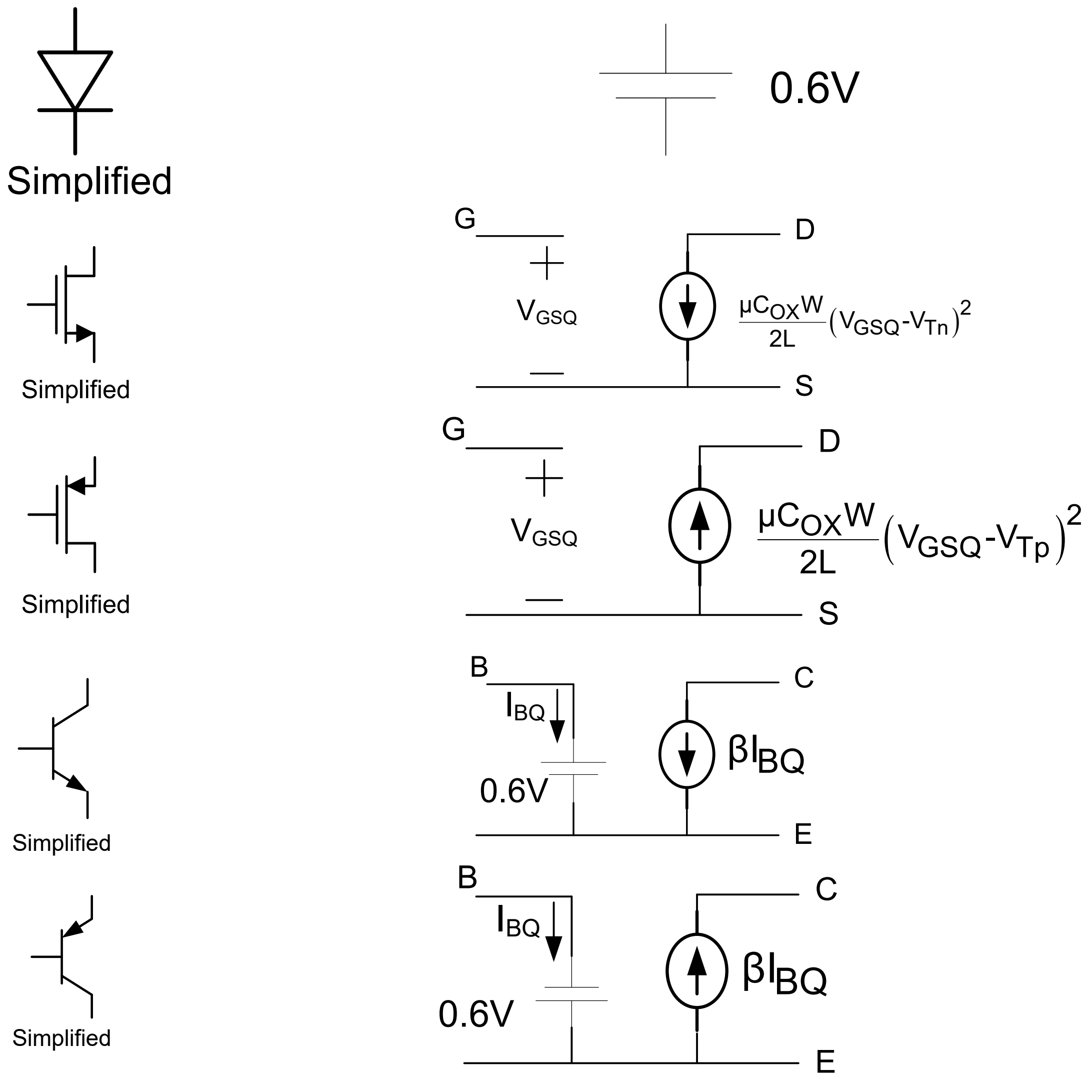


What are the simplified dc equivalent models?

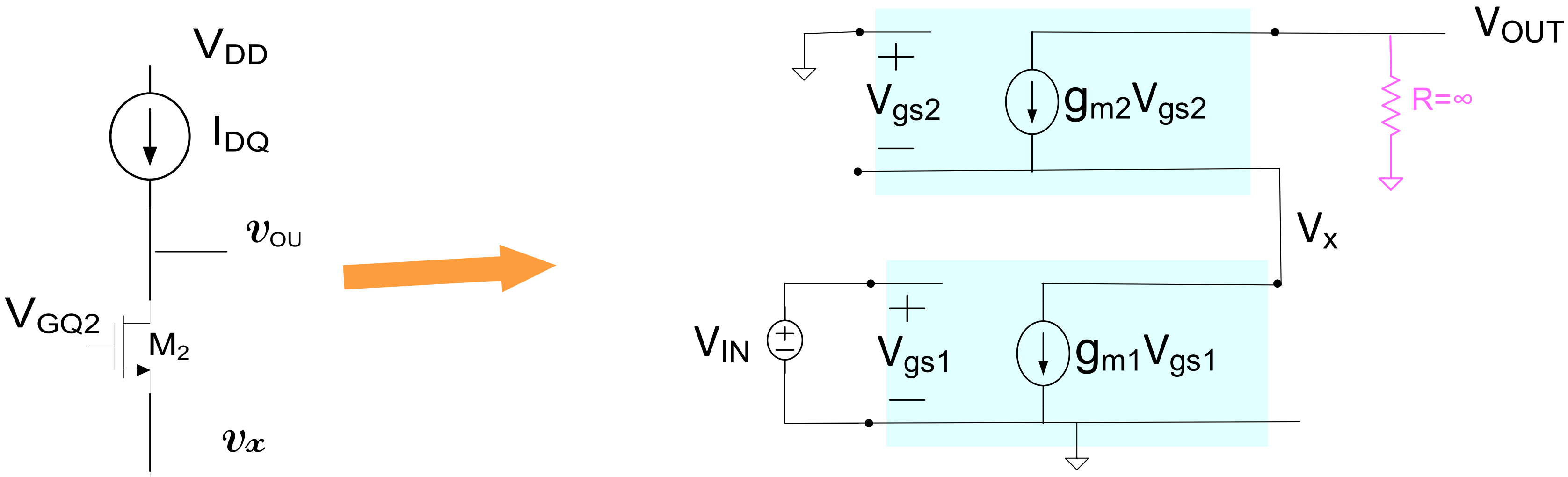
Active Device Model Summary

What are the simplified dc equivalent models?

dc equivalent



Example: Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that $\lambda=0$



By KCL

$$g_{m1}v_{IN} = g_{m2}v_{gs2}$$

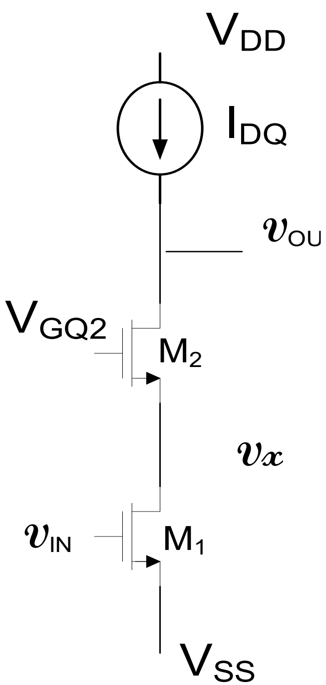
$$g_{m2}v_{gs2}R = -v_{OUT}$$

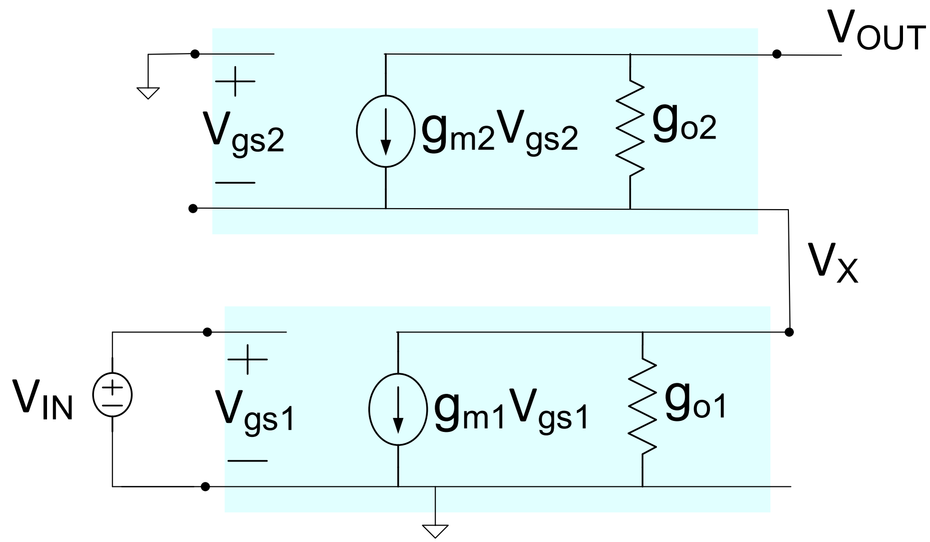
Solving obtain:

$$A_V = \frac{v_{OUT}}{v_{IN}} = -g_{m1}R \xrightarrow[R=\infty]{} \infty$$

Unexpectedly large, need better device models!

Example: Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that $\lambda \neq 0$





By KCL

$$\left.\begin{aligned} & g_{m1}v_{IN} + g_{o1}v_X = g_{m2}v_{GS2} + g_{o2}(v_{OUT} - v_X) \\ & v_{GS2} = -v_X \\ & (v_{OUT} - v_X)g_{o2} + g_{m2}v_{GS2} = 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} & g_{m1}v_{IN} + (g_{m2} + g_{o1} + g_{o2})v_X = g_{o2}v_{OUT} \\ & v_{OUT}g_{o2} = (g_{m2} + g_{o2})v_X \end{aligned}\right\}$$

thus:

$$A_V = \frac{v_{OUT}}{v_{IN}} = -\frac{g_{m1}g_{m2} + g_{m1}g_{o2}}{g_{o1}g_{o2}} \cong -\frac{g_{m1}}{g_{o1}}\frac{g_{m2}}{g_{o2}}$$

- Analysis is straightforward but a bit tedious
- A_V is very large and would go to ∞ if g_{01} and g_{02} were both 0
- Will look at how big this gain really is later

Stay Safe and Stay Healthy !

End of Lecture 26