Two-Port Amplifier Models

- Obtaining 2-port parameters
- Dependent Sources
Exam Schedule

Exam 2  Friday October 27
Exam 3  Friday November 17
How does $g_m$ vary with $I_{DQ}$?

$$g_m = \sqrt{\frac{2\mu C_{OX}W}{L}} \sqrt{I_{DQ}}$$

Varies with the square root of $I_{DQ}$

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$$

Varies linearly with $I_{DQ}$

$$g_m = \frac{\mu C_{OX}W}{L} (V_{GSQ} - V_T)$$

Doesn’t vary with $I_{DQ}$
Two-port representation of amplifiers

- Thevenin equivalent output port often more standard
- $R_{IN}$, $A_V$, and $R_{OUT}$ often used to characterize the two-port of amplifiers

Review from a Previous Lecture
Two-Port Equivalents of Interconnected Two-ports

Example:

Review from a Previous Lecture
Determination of two-port model parameters

(One method will be discussed here)

A method of obtaining $R_{in}$

\[ \begin{align*}
    i_1 &= v_1 \left( \frac{1}{R_{in}} \right) + v_2 \left( \frac{-A_{VR}}{R_{in}} \right) \\
    i_2 &= v_1 \left( \frac{-A_{V0}}{R_0} \right) + v_2 \left( \frac{1}{R_0} \right)
\end{align*} \]

\[ \begin{align*}
    v_2 &= 0 \\
    v_1 &= v_{test} \\
    i_1 &= i_{test}
\end{align*} \]

\[ R_{in} = \frac{v_{test}}{i_{test}} \]

Terminate the output in a short-circuit
Determination of two-port model parameters

A method of obtaining $A_{v0}$

Terminate the output in an open-circuit

\[
i_1 = v_1 \left( \frac{1}{R_{in}} \right) + v_2 \left( \frac{-A_{VR}}{R_{in}} \right)
\]

\[
i_2 = v_1 \left( \frac{-A_{V0}}{R_0} \right) + v_2 \left( \frac{1}{R_0} \right)
\]

\[
i_2 = 0
\]

\[
v_1 = v_{test}
\]

\[
v_2 = v_{out-test}
\]

\[
A_{V0} = \frac{v_{out-test}}{v_{test}}
\]
Determination of two-port model parameters

A method of obtaining $R_0$

Terminate the input in a short-circuit

$$i_1 = v_1 \left( \frac{1}{R_{in}} \right) + v_2 \left( -\frac{A_{VR}}{R_{in}} \right)$$

$$i_2 = v_1 \left( -\frac{A_{V0}}{R_0} \right) + v_2 \left( \frac{1}{R_0} \right)$$

$v_1 = 0$

$$R_0 = \frac{v_{test}}{i_{test}}$$
Determination of two-port model parameters

A method of obtaining $A_{VR}$

\[
\begin{align*}
i_1 &= v_1 \left( \frac{1}{R_{in}} \right) - v_2 \left( \frac{A_{VR}}{R_{in}} \right) \\
i_2 &= v_1 \left( \frac{-A_{V0}}{R_0} \right) + v_2 \left( \frac{1}{R_0} \right)
\end{align*}
\]

Terminate the input in an open-circuit

\[ A_{VR} = \frac{v_{\text{out-test}}}{v_{\text{test}}} \]
Determination of Amplifier Two-Port Parameters

• Input and output parameters are obtained in exactly the same way, only distinction is in the notation used for the ports.

• Methods given for obtaining amplifier parameters $R_{in}$, $R_{OUT}$ and $A_V$ for unilateral networks are a special case of the non-unilateral analysis by observing that $A_{VR}=0$.

• In some cases, other methods for obtaining the amplifier parameters are easier than what was just discussed.
Determine $V_{\text{OUTQ}}$ and the SS voltage gain ($A_V$), assume $\beta = 100$

($A_V$ is one of the small-signal model parameters for this circuit)
Determine $V_{\text{OUTQ}}$ and the SS voltage gain ($A_V$), assume $\beta=100$

($A_V$ is one of the small-signal model parameters for this circuit)
Examples

Determine $V_{OUTQ}$

This circuit is most practical when $I_B << I_{BB}$

With this assumption,

$$V_B = \left( \frac{R_{B2}}{R_{B1} + R_{B2}} \right) 12V$$

$$I_{CQ} = I_{EQ} = \left( \frac{V_B - 0.6V}{R_1} \right) = \frac{1.4V - 0.6V}{0.5K} = 2.8mA$$

$$V_{OUTQ} = 12V - I_{CQ} R_1 = 6.4V$$

Note: This Q-point is nearly independent of the characteristics of the nonlinear BJT!
Examples

Determine SS voltage gain

$$v_{OUT} = -g_m v_{BE} R_2$$

$$v_{IN} = v_{BE} + R_1 (v_{BE} [g_m + g_m])$$

$$A_v = \frac{-R_2 g_m v_{BE}}{v_{BE} + R_1 (v_{BE} [g_m + g_m])} = \frac{-R_2 g_m}{1 + R_1 ([g_m + g_m])}$$

$$A_v \approx \frac{-R_2 g_m}{R_1 g_m} = \frac{-R_2}{R_1}$$

This voltage gain is nearly independent of the characteristics of the nonlinear BJT!

This is a fundamentally different amplifier structure

It can be shown that this is slightly non-unilaterial
Determine $V_{\text{OUTQ}}$, $R_{\text{IN}}$, $R_{\text{OUT}}$, and the SS voltage gain, and $A_{\text{VR}}$ assume $\beta=100$
Examples

Determine $V_{OUTQ}$, $R_{IN}$, $R_{OUT}$, and the SS voltage gain, and $A_{VR}$; assume $\beta = 100$

($A_V$, $R_{IN}$, $R_{OUT}$, and $A_{VR}$ are the small-signal model parameters for this circuit)
Examples

Determine $V_{OUTQ}$

This is the same as the previous circuit!

$$V_{OUTQ} = 6.4V$$

$$I_{CQ} = \frac{5.6V}{2K} = 2.8mA$$

Note: This Q-point is nearly independent of the characteristics of the nonlinear BJT!
Examples

Determine the SS voltage gain

\[ A_v \approx -g_m R_2 \]

\[ A_v \approx -\frac{I_{CQ} R_2}{V_t} \]

\[ A_v \approx -\frac{5.6V}{26mV} = -215 \]

Note: This Gain is nearly independent of the characteristics of the nonlinear BJT!
Examples

Determination of $R_{IN}$

The SS equivalent circuit

$R_{IN} = R_{B1} // R_{B2} // r_\pi \cong r_\pi$

$r_\pi = \frac{I_{CQ}}{\beta V_t} = \left( \frac{2.8 \text{mA}}{100 \cdot 26 \text{mV}} \right)^{-1} = 928 \Omega$

$R_{IN} = R_{B1} // R_{B2} // r_\pi \cong r_\pi = 930 \Omega$
Determination of $R_{\text{OUT}}$

The SS equivalent circuit

$$R_{\text{OUT}} = R_2 // r = V_{\text{TEST}} / i_{\text{TEST}}$$

$$r_o = \left( \frac{I_{CQ}}{V_{AF}} \right)^{-1} = \left( \frac{2.8 \text{mA}}{200 \text{V}} \right)^{-1} = \left( 1.4 \times 10^{-5} \right)^{-1} = 71 \text{K\Omega}$$

$$R_{\text{OUT}} = R_2 // r_o \approx R_2 = 2K$$
Examples

Determine $A_{VR}$

The SS equivalent circuit

$V_{OUT TEST} = 0$

$A_{VR} = 0$
Determination of small-signal two-port representation

\( V_{IN}(t) \)

\( V_{CC} = 12 \text{V} \)

\( R_{B1} = 50 \text{K} \)

\( R_{B2} = 10 \text{K} \)

\( C_1 = 1 \text{uF} \)

\( C_2 = 100 \text{uF} \)

\( R_1 = 0.5 \text{K} \)

\( R_2 = 2 \text{K} \)

\( V_{OUT} \)

\( R_{OUT} \)

\( A_V \approx -215 \)

\( R_{IN} \approx r_\pi = 930 \Omega \)

\( R_{OUT} \approx R_2 = 2K \)

This is the same basic amplifier that was considered many times.
Relationship with Dependent Sources?

Dependent sources from EE 201

Example showing two dependent sources
Relationship with Dependent Sources?

\[ V_s = \mu V_x \]
\[ I_s = \alpha V_x \]
\[ V_s = \rho I_x \]
\[ I_s = \beta I_x \]

Dependent sources from EE 201

Two Port (Thevenin)
Relationship with Dependent Sources?

It follows that

\[ V_s = \mu V_x \]

Voltage dependent voltage source is a unilateral floating two-port voltage amplifier with \( R_{IN} = \infty \) and \( R_{OUT} = 0 \)
Relationship with Dependent Sources?

It follows that

\[ V_s = \rho I_x \]

Current dependent voltage source is a unilateral floating two-port transresistance amplifier with \( R_{\text{IN}} = 0 \) and \( R_{\text{OUT}} = 0 \)
Relationship with Dependent Sources?

It follows that

\[ I_S = \beta I_x \]

Current dependent current source is a floating unilateral two-port current amplifier with \( R_{IN} = 0 \) and \( R_{OUT} = \infty \)
Relationship with Dependent Sources?

It follows that

\[ I_s = \alpha V_x \]

Voltage dependent current source is a floating unilateral two-port transconductance amplifier with \( R_{IN} = \infty \) and \( R_{OUT} = \infty \)
Dependent Sources

Dependent sources are unilateral two-port amplifiers with ideal input and output impedances.

Dependent sources do not exist as basic circuit elements but amplifiers can be designed to perform approximately like a dependent source.

- Practical dependent sources typically are not floating on input or output.
- One terminal is usually grounded.
- Input and output impedances of realistic structures are usually not ideal.

Why were “dependent sources” introduced as basic circuit elements instead of two-port amplifiers???
End of Lecture 27