Small-Signal Models
Comparison of MOS and BJT performance
Basic amplifier architectures
Quiz 20

Obtain the small signal model of the following circuit. Assume MOSFET is operating in the saturation region.
And the number is ....
And the number is ....
Quiz 20

Obtain the small signal model of the following circuit. Assume MOSFET is operating in the saturation region

Solution:

\[ V \left( g_m + g_0 \right) = I \]

\[ R_{EQ} = \frac{1}{g_m + g_0} \approx \frac{1}{g_m} \]
Graphical Analysis and Interpretation

Device Model (family of curves):

\[ I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \]

- Linear signal swing region smaller than saturation region
- Modest nonlinear distortion provided saturation region operation maintained
- Symmetric swing about Q-point
- Signal swing can be maximized by judicious location of Q-point

Review from Last Time
Review from Last Time

Typical Effects of Bulk on Threshold Voltage for n-channel Device

$$V_T = V_{T0} + \gamma(\sqrt{\phi} - V_{BS} - \sqrt{\phi})$$

$$\gamma \approx 0.4V^{\frac{1}{2}} \quad \phi \approx 0.6V$$

Bulk-Diffusion Generally Reverse Biased ($V_{BS} < 0$ or at least less than 0.3V) for n-channel
Shift in threshold voltage with bulk voltage can be substantial
Often $V_{BS} = 0$
Small-Signal Model Extension

\[ I_G = 0 \]
\[ I_B = 0 \]
\[ I_D = \begin{cases} 
0 & \text{if} \quad V_{GS} \leq V_T \\
\mu C_{ox} \frac{W}{L} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & \text{if} \quad V_{GS} \geq V_T, \quad V_{DS} < V_{GS} - V_T \\
\mu C_{ox} \frac{W}{2L} \left( V_{GS} - V_T \right)^2 \cdot (1 + \lambda V_{DS}) & \text{if} \quad V_{GS} \geq V_T, \quad V_{DS} \geq V_{GS} - V_T 
\end{cases} \]

\[ V_T = V_{T0} + \gamma \left( \sqrt{\phi} - V_{BS} - \sqrt{\phi} \right) \]

\[ y_{11} = \left. \frac{\partial I_G}{\partial V_{GS}} \right|_{V = \tilde{V}_q} = 0 \quad y_{12} = \left. \frac{\partial I_G}{\partial V_{DS}} \right|_{V = \tilde{V}_q} = 0 \quad y_{13} = \left. \frac{\partial I_G}{\partial V_{GS}} \right|_{V = \tilde{V}_q} = 0 \]

\[ y_{31} = \left. \frac{\partial I_B}{\partial V_{GS}} \right|_{V = \tilde{V}_q} = 0 \quad y_{32} = \left. \frac{\partial I_B}{\partial V_{DS}} \right|_{V = \tilde{V}_q} = 0 \quad y_{33} = \left. \frac{\partial I_B}{\partial V_{GS}} \right|_{V = \tilde{V}_q} = 0 \]

\[ y_{21} = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V = \tilde{V}_a} = g_w \quad y_{12} = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V = \tilde{V}_a} = g_v \quad y_{13} = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V = \tilde{V}_a} = g_{wb} \]
Small Signal Model Summary

\[ i_g = 0 \]
\[ i_b = 0 \]
\[ i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds} \]

\[ g_m = \frac{\mu C_{ox} W}{L} V_{EBQ} \]
\[ g_o = \lambda I_{DQ} \]
\[ g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right) \]
Relative Magnitude of Small Signal MOS Parameters

Consider:

\[ i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds} \]

3 alternate equivalent expressions for \( g_m \):

\[ g_m = \frac{\mu C_{OX} W}{L} v_{EBQ} \]
\[ g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}} \]
\[ g_m = \frac{2I_{DQ}}{V_{EBQ}} \]

If \( \mu C_{OX} = 100 \mu A/V^2 \), \( \lambda = 0.01 V^{-1} \), \( \gamma = 0.4 V^{0.5} \), \( V_{EBQ} = 1V \), \( W/L = 1 \), \( V_{BSQ} = 0V \)

\[ I_{DQ} \approx \frac{\mu C_{OX} W}{2L} V_{EBQ}^2 = \frac{10^4 W}{2L} (1V)^2 = 5E-5 \]

\[ g_m = \frac{\mu C_{OX} W}{L} V_{EBQ} = 1E-4 \]

\[ g_o = \lambda I_{DQ} = 5E-7 \]

\[ g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right) = 0.26g_m \]

\[ g_o << g_m, g_{mb} \]

\[ g_{mb} < g_m \]

In this example

This relationship is common

In many circuits, \( V_{BS} = 0 \) as well
Review from Last Time

Large and Small Signal Model Summary

Large Signal Model

\[
I_d = \begin{cases} 
0 & V_{GS} \leq V_T \\
\mu C_{OX} W \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} > V_T \quad V_{DS} < V_{GS} - V_T \\
\mu C_{OX} \frac{W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) & V_{GS} \geq V_T \quad V_{DS} \geq V_{GS} - V_T 
\end{cases}
\]

\[
V_T = V_{T0} + \gamma \left( \sqrt{\phi} - V_{BS} - \sqrt{\phi} \right)
\]

Small Signal Model

\[
i_g = 0 \\
i_b = 0 \\
i_d = g_m V_{gs} + g_{mb} V_{bs} + g_o V_{ds}
\]

where

\[
g_m = \frac{\mu C_{OX} W}{L} V_{EBQ} \\
g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{\phi} - V_{BSQ}} \right) \\
g_o = \lambda I_{DQ}
\]
Relative Magnitude of Small Signal BJT Parameters

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

Often the \( g_o \) term can be neglected in the small signal model because it is so small.
Relative Magnitude of Small Signal Parameters

\[ g_m = \frac{I_{CQ}}{V_t} \quad g_\pi = \frac{I_{CQ}}{\beta V_t} \quad g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[ g_m = \begin{bmatrix} \frac{I_Q}{V_t} \\ \frac{I_Q}{\beta V_t} \end{bmatrix} = \beta \]

\[ g_\pi = \begin{bmatrix} \frac{I_Q}{\beta V_t} \end{bmatrix} = \frac{V_{AF}}{\beta V_t} \approx \frac{200V}{100 \cdot 26mV} = 77 \]

\[ g_m >> g_\pi >> g_o \]

Often the go term can be neglected in the small signal model because it is so small.
Large and Small Signal Model Summary

\[ I_C = \beta I_B \left(1 + \frac{V_{CE}}{V_{AF}}\right) \quad V_{BE} > 0.4V \]
\[ I_B = \frac{J_s A_E}{\beta} e^{V_{BE}/V_t} \quad V_{BC} < 0 \]

- \( V_{BE} = 0.7V \)
- \( V_{CE} = 0.2V \)
- \( I_C = I_B = 0 \)

Small Signal Model

\[ i_b = g_{\pi} v_{be} \]
\[ i_c = g_m v_{be} + g_0 v_{ce} \]

where

\[ g_m = \frac{i_{CQ}}{V_t} \]
\[ g_{\pi} = \frac{i_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{i_{CQ}}{V_{AF}} \]
Small Signal Model Simplifications for the MOSFET and BJT

Often simplifications of the small signal model are adequate for a given application.

These simplifications will be discussed next.
Small Signal Model Summary

An equivalent circuit (4-terminal MOSFET)

\[ g_m = \frac{\mu C_{ox} W}{L} (V_{GSQ} - V_T) \]

\[ g_o = \lambda I_{DQ} \]

\[ g_{mb} = g_m \left( \frac{\gamma}{2 \sqrt{\phi - V_{BSQ}}} \right) \]

This contains absolutely no more information than the set of small-signal model equations
Small Signal Model Summary

More convenient representation
Small Signal Model Summary

Alternate equivalent representations for $g_m$

$$g_m = \frac{\mu C_{ox} W}{L} (V_{GSQ} - V_T)$$

from

$$I_D \approx \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2$$

$$g_m = \sqrt{\frac{2\mu C_{ox} W}{L}} \sqrt{I_{DQ}}$$

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$$
Small Signal Model Simplifications

Simplification that is often adequate
Small Signal Model Simplifications

Even further simplification that is often adequate
Small Signal BJT Model Summary

An equivalent circuit

This contains absolutely no more information than the set of small-signal model equations
Small Signal BJT Model Simplifications

Simplification that is often adequate
Gains for MOSFET and BJT Circuits

**BJT**

\[ A_{VB} = -\frac{I_{CQ} R_1}{V_t} \]

**MOSFET**

\[ A_{VM} = \frac{2I_{DQ} R}{[V_{SS} + V_T]} \]

\[ A_v = -g_m R \]
How does $g_m$ vary with $I_{DQ}$?

Varies with the square root of $I_{DQ}$

$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}}$$

Varies linearly with $I_{DQ}$

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$$

Doesn’t vary with $I_{DQ}$

$$g_m = \frac{\mu C_{OX} W}{L} (V_{GSQ} - V_T)$$
How does $g_m$ vary with $I_{DQ}$?

All of the above are true – but with qualification

g$_m$ is a function of more than one variable ($I_{DQ}$) and how it varies depends upon how the remaining variables are constrained
Comparison of BJT and MOSFET

How do the small signal models of the MOSFET and BJT compare?
Comparison of MOSFET and BJT

The transconductance of the BJT is typically much larger than that of the MOSFET (and larger is better!)
This is due to the exponential rather than quadratic output/input relationship.

BJT

\[ g_m = \frac{I_{CQ}}{V_t} \]

\[ \frac{g_{mBJT}}{g_{mMOS}} = \frac{I_{CQ}}{V_t} \cdot \frac{2I_{DQ}}{2I_{DQ}} \cdot \frac{V_{EBQ}}{V_{EB}} \]

MOSFET

\[ g_m = \frac{\mu C_{OX} W}{L} V_{EB} \]

\[ g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \cdot \sqrt{I_{DQ}} \]

\[ g_m = \frac{2I_{DQ}}{V_{EBQ}} \]
Comparison of MOSFET and BJT

BJT

\[ g_m = \frac{I_{CQ}}{V_t} \]

MOSFET

\[ g_m = \frac{\mu C_{ox} W}{L} V_{EB} \]

\[ g_m = \sqrt{\frac{2\mu C_{ox} W}{L}} \sqrt{I_{DQ}} \]

\[ g_m = \frac{2I_{DQ}}{V_{EBQ}} \]

The transconductance of the BJT is typically much larger than that of the MOSFET (and larger is better)
This is due to the exponential rather than quadratic output/input relationship
Comparison of MOSFET and BJT

BJT

\[ g_o = \lambda I_{DQ} \]

MOSFET

\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[
\frac{g_{o\text{BJT}}}{g_{o\text{MOS}}} = \frac{I_{CQ}}{V_{AF}} = \frac{1}{\lambda I_{DQ}} \approx \frac{1}{\lambda V_{AF}} \cdot \frac{0.01V^{-1}}{200V} = 0.5
\]

The output conductances are comparable but that of the BJT is usually modestly smaller (and smaller is better!)
Comparison of MOSFET and BJT

BJT

$$g_{\pi} = \frac{I_{CQ}}{\beta V_t}$$

MOSFET

$$g_{\pi} = 0$$

g_{\pi} is the reciprocal of the input impedance

g_{\pi} of a MOSFET is much smaller than that of a BJT (and smaller is better!)
Standard Approach to small-signal analysis of nonlinear networks

Nonlinear Network

dc Equivalent Network

Q-point

Values for small-signal parameters

Small-signal equivalent network

Small-signal output

Total output
(good approximation)
Systematic Approach to Small-Signal Circuit Analysis

• Obtain dc equivalent circuit by replacing all elements with large-signal (dc) equivalent circuits

• Obtain dc operating points (Q-point)

• Obtain ac equivalent circuit by replacing all elements with small-signal equivalent circuits

• Analyze linear small-signal equivalent circuit
Recall

### Dc and small-signal equivalent elements

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<tr>
<th>Element</th>
<th>ss equivalent</th>
<th>dc equivalent</th>
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<td>Simplified</td>
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</table>
The simplified large signal models for the MOSFET and the BJE

Simplified large-signal models (sometimes termed dc equivalent models) are usually adequate for determining operating point in practical MOS and Bipolar circuits.

Can create circuits where the simplified models are not adequate but these are often not practical circuits.

Will discuss only for npn and n-channel but similar models for pnp and p-channel devices.
Square-Law Model

\( I_G = 0 \)
\( I_B = 0 \)

\[
I_D = \begin{cases} 
0 & V_{GS} \leq V_T \\
\mu C_{ox} \frac{W}{L} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T, V_{DS} < V_{GS} - V_T \\
\mu C_{ox} \frac{W}{2L} \left( V_{GS} - V_T \right)^2 \left( 1 + \lambda V_{DS} \right) & V_{GS} \geq V_T, V_{DS} \geq V_{GS} - V_T 
\end{cases}
\]

\[ V_T = V_{T0} + \gamma \left( \sqrt{\phi - V_{BS}} - \sqrt{\phi} \right) \]
Simplified MOS Model for Q-point Analysis

\[ I_G = 0 \]
\[ I_B = 0 \]
\[ I_D = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2 \]

Simplified dc equivalent circuit
dc BJT model

\[ I_C = \beta I_B \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ V_t = \frac{kT}{q} \]

\[ V_{BE} > 0.4V \]
\[ V_{BC} < 0 \]

Forward Active

\[ V_{BE} = 0.7V \]
\[ V_{CE} = 0.2V \]

Saturation

\[ I_C < \beta I_B \]

Cutoff

\[ I_C = I_B = 0 \]
\[ V_{BE} < 0 \]
\[ V_{BC} < 0 \]

A small portion of the operating region is missed with this model but seldom operate in the missing region.
Simplified dc BJT model for Q-point Analysis

\[
\begin{align*}
I_C &= \beta I_B \\
I_B &= \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}
\end{align*}
\]

\[
I_C = \beta I_B \\
V_{BE} = 0.6V
\]

Simplified dc equivalent circuit
Examples

Not convenient to have multiple dc power supplies
$V_{\text{OUT}}$ very sensitive to $V_{\text{EE}}$
Examples

Not convenient to have multiple dc power supplies
V_{OUTQ} very sensitive to V_{EE}

Compare the small-signal equivalent circuits of these two structures
Examples

Compare the small-signal equivalent circuits of these two structures

Since Thevenin equivalent circuit in red circle is $V_{IN}$, both circuits have same voltage gain
Examples

Determine $V_{OUTQ}$, $A_V$, $R_{IN}$
Examples

Determine $v_{OUT}$ and $V_{OUT}(t)$ if $v_{in} = 0.002 \sin(400t)$
Amplifier input impedance, output impedance and gain are usually of interest

Why?

Amplifiers can be modeled as a two-port

Amplifier usually unilateral and thevenin equivalent output port often more standard
Determination of Amplifier Parameter $A_v$
Determination of Amplifier Parameter $R_{IN}$
Determination of Amplifier Parameter $R_{OUT}$
Examples

Several different biasing circuits can be used

Biasing Circuit

$V_{CC} = 12V$

$R_B = 500K$

$C = 1uF$

$V_{IN}(t)$

$R_1 = 2K$

$V_{OUT}$

$Q_1$
Examples

Determine \( V_{\text{OUTQ}} \) and the SS voltage gain, assume \( \beta = 100 \)
Examples

Determine $V_{OUTQ}$

**dc equivalent circuit**

\[
I_{CQ} = \beta I_{BQ} = 100 \left( \frac{12V - 0.6V}{500K} \right) = 2.3mA
\]

\[
V_{OUTQ} = 12V - I_{CQ} R_1 = 12V - 2.3mA \times 2K = 7.4V
\]
Examples

Determine the SS voltage gain

\[ \beta = 100 \]

This basic amplifier structure is widely used and repeated analysis serves no useful purpose.

Have seen this circuit before but will repeat for review purposes.
Determine $V_{OUTQ}$ and the SS voltage gain, assume $\beta=100$
Examples

- DC equivalent circuit
  - $V_{CC} = 12V$
  - $R_{B1} = 50K$
  - $R_{B2} = 10K$
  - $R_1 = 0.5K$
  - $R_2 = 2K$
  - $C = 1\mu F$
  - $\beta = 100$

Determine $V_{OUTQ}$

This circuit is most practical when $I_B << I_{BB}$

With this assumption,

1. $V_B = \left( \frac{R_{B2}}{R_{B1} + R_{B2}} \right) 12V$
2. $I_{CQ} = I_{EQ} = \left( \frac{V_B - 0.6V}{R_1} \right) = \frac{1.4V}{0.5K} = 2.8mA$
3. $V_{OUTQ} = 12V - I_{CQ}R_1 = 6.4V$
Examples

Determine SS voltage gain

\[ V_{OUT} = -g_m V_{BE} R_2 \]

\[ V_{IN} = V_{BE} + R_1 (V_{BE} [g_\pi + g_m]) \]

\[ A_v = \frac{\frac{-R_2 g_m V_{BE}}{V_{BE} + R_1 (V_{BE} [g_\pi + g_m])}}{1 + R_1 ([g_\pi + g_m])} = \frac{-R_2 g_m}{R_1 g_m} = \frac{-R_2}{R_1} \]

Note: This voltage gain is nearly independent of the characteristics of the nonlinear BJT!
Examples

Determine \( V_{\text{OUTQ}} \) and the SS voltage gain, assume \( \beta = 100 \).
Examples

Determine $V_{OUTQ}$

This is the same as the previous circuit!

$V_{OUTQ} = 6.4V$

The dc equivalent circuit
Examples

Determine the SS voltage gain

This is the same as the previous-previous circuit!

\[ A_V \approx -g_m R_2 \]

\[ A_V \approx -\frac{I_{CQ} R_2}{V_t} \]

The SS equivalent circuit