EE 330
Lecture 28

• Graphical Small Signal Analysis
• Model Extensions and Simplifications
Circuit Analysis

given circuit with given sizing and given biasing

• Large signal DC analysis
  – Compute Q-point
  – Determine DC swing ranges at nodes

• Linearize to obtain small signal equivalent circuit
  – Replace each nonlinear device by its small signal model

• Small signal DC analysis
  – Apply any linear circuit analysis tools
  – Obtain DC parameters such as gain $A_v$, $R_{in}$, $R_o$
Q-point Computation

• Open all caps and short all inductors
• Assume correct region for all nonlinear devices
  – Diodes on, MOS in saturation, BJT in forward active
• Write down device models and KCL’s
  – One KCL at each ono-trivial node in nodal analysis
  – For hand calculation, use simplified device models
• Solve the simultaneous equations
  – Produce Q values for all node voltages and branch currents
• Check correctness of assumptions in 2nd step
Active Device Model Summary

What are the simplified dc equivalent models?

dc equivalent

0.6V

Review from Last Lecture
Review from Last Lecture

Small Signal Model for Active Devices

$V_S \rightarrow V_{D} \rightarrow V_G \rightarrow V_S$

$g_m = \mu C_{ox} \frac{W}{L} (V_{GS} - V_T) \quad g_o \approx \lambda I_{DQ}$

$g_r = \frac{I_{CQ}}{\beta V_t} \quad g_m = \frac{I_{CQ}}{V_t} \quad g_o = \frac{I_{CQ}}{V_{AF}}$
Review from Last Lecture

Small Signal Model for Active Devices

\[
\frac{1}{R} = g_{\text{diode}} = \frac{I_{\text{diodeQ}}}{V_t}
\]

\[
\frac{1}{R} = g_m = \frac{2I_{\text{DQ}}}{V_{GSQ} - V_{TH}}
\]

\[
\frac{1}{R} = g_m = \frac{I_{CQ}}{V_t}
\]
Determine the small signal voltage gain \( A_V = \frac{V_{OUT}}{V_{IN}} \). Assume \( M_1 \) and \( M_2 \) are operating in the saturation region and that \( \lambda = 0 \).

Small-signal MOSFET model for \( \lambda = 0 \)

By KCL

\[ g_{m1} V_{GS1} = g_{m2} V_{GS2} \]

but

\[ V_{GS1} = V_{IN} \quad -V_{GS2} = V_{OUT} \]

thus:

\[ A_V = \frac{V_{OUT}}{V_{IN}} = -\frac{g_{m1}}{g_{m2}} \]

\[ g_m = -\sqrt{2I_D \mu C_{ox}} \sqrt{\frac{W_1}{L_1}} \]

\[ A_V = -\sqrt{\frac{2I_D \mu C_{ox} \frac{W_1}{L_1}}{2I_D \mu C_{ox} \frac{W_2}{L_2}}} = -\sqrt{\frac{W_1}{W_2} \sqrt{\frac{L_2}{L_1}}} \]
Determine the small signal voltage gain $A_V = \frac{V_{OUT}}{V_{IN}}$. Assume $M_1$ and $M_2$ are operating in the saturation region and that $\lambda = 0$.

$g_{o1}$ is in parallel with $g_{m2}$, hence can be ignored.

\[ V_{OUT} = - \left( g_{m1} V_{IN} \right) \frac{1}{g_{m2}} \]

\[ A_V = \frac{V_{OUT}}{V_{IN}} = - \frac{g_{m1}}{g_{m2}} \]
By KCL

$g_{m1} v_{IN} + g_{o1} v_{x} = g_{m2} v_{GS2} + g_{o2} (v_{OUT} - v_{x}) = 0$

$v_{x} = -\frac{g_{m1}}{g_{o1}} v_{IN} \quad v_{OUT} = -\frac{g_{m2}}{g_{o1}} v_{GS2} + v_{x}$

but $v_{GS2} = -v_{x}$

$v_{OUT} = -\frac{g_{m2}}{g_{o2}} \frac{g_{m1}}{g_{o1}} v_{IN} - \frac{g_{m1}}{g_{o1}} v_{IN}$

thus:

$A_v = \frac{v_{OUT}}{v_{IN}} = -\frac{g_{m2}}{g_{o2}} \left(\frac{g_{m1}}{g_{o1}} + 1\right)$
Graphical Analysis and Interpretation

Consider Again

\[ V_{OUT} = V_{DD} - I_D R \]

\[ I_D = \frac{\mu C_{ox} W}{2L} (V_{IN} - V_{SS} - V_T)^2 \]

\[ I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{SS} + V_T)^2 \]
Graphical Analysis and Interpretation

Device Model (family of curves)

\[ I_D = \frac{\mu C_{ox} W}{2L} (V_{gs} - V_T)^2 (1 + \lambda V_{ds}) \]

Load Line

Device Model

Device Model at Operating Point

\[ V_{out} = V_{dd} - I_D R \]

\[ I_D = \frac{\mu C_{ox} W}{2L} (V_{in} - V_{ss} - V_T)^2 \]

\[ I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{ss} + V_T)^2 \]
Graphical Analysis and Interpretation

Device Model (family of curves)  
\[ I_D = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \]

Load Line

Q-Point

Load Line

\[ V_{GSQ} = -V_{SS} \]

\[ I_{DQ} \approx \frac{\mu C_{ox} W}{2L} (V_{SS} + V_T)^2 \]

\[ V_{OUT} = V_{DD} - I_D R \]

\[ I_D = \frac{\mu C_{ox} W}{2L} (V_{IN} - V_{SS} - V_T)^2 \]

Must satisfy both equations all of the time!
Graphical Analysis and Interpretation

Device Model (family of curves) \( I_d = \frac{\mu C_{ox} W}{2L} (V_{IN} - V_{SS} - V_T)^2 (1 + \lambda V_{DS}) \)

- As \( V_{IN} \) changes around Q-point, due to changes \( V_{IN} \) induces in \( V_{GS} \), the operating point must remain on the load line!
- Small sinusoidal changes of \( V_{IN} \) will be nearly symmetric around the \( V_{GSQ} \) line
- This will cause nearly symmetric changes in both \( I_D \) and \( V_{DS} \)!
- Since \( V_{SS} \) is constant, change in \( V_{DS} \) is equal to change in \( V_{OUT} \)!
Graphical Analysis and Interpretation

Device Model (family of curves) \( I_D = \frac{\mu C_{ox} W}{2L} (V_{in} - V_{ss} - V_T)^2 (1 + \lambda V_{ds}) \)

As \( V_{in} \) changes around Q-point, due to changes \( V_{in} \) induces in \( V_{gs} \), the operating point must remain on the load line!
Graphical Analysis and Interpretation

Device Model (family of curves)

\[ I_{DQ} = \frac{\mu C_{ox} W}{2L} \left( V_{GS} - V_T \right)^2 (1 + \lambda V_{DS}) \]

- Linear signal swing region smaller than saturation region
- Modest nonlinear distortion provided saturation region operation maintained
- Symmetric swing about Q-point
- Signal swing can be maximized by judicious location of Q-point
Graphical Analysis and Interpretation

Device Model (family of curves) \[ I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \]

Q-Point

Load Line

Saturation region

Very limited signal swing with non-optimal Q-point location
Graphical Analysis and Interpretation

Device Model (family of curves) 

\[ I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T)^2 \left( 1 + \lambda V_{DS} \right) \]

- Signal swing can be maximized by judicious location of Q-point
- Often selected to be at middle of load line in saturation region
Small-Signal MOSFET Model Extension

Existing 3-terminal small-signal model does not depend upon the bulk voltage!

Recall that changing the bulk voltage changes the electric field in the channel region and thus the threshold voltage!
Typical Effects of Bulk on Threshold Voltage for n-channel Device

\[ V_T = V_{T0} + \gamma \left[ \sqrt{\phi - V_{BS}} - \sqrt{\phi} \right] \]

\[ \gamma \approx 0.4V^{\frac{1}{2}} \quad \phi \approx 0.6V \]

Bulk-Diffusion Generally Reverse Biased \((V_{BS} < 0\) or at least less than 0.3V) for n-channel
Shift in threshold voltage with bulk voltage can be substantial
Often \(V_{BS}=0\)
Recall: Typical Effects of Bulk on Threshold Voltage for p-channel Device

\[ V_T = V_{T0} - \gamma \left[ \sqrt{\phi + V_{BS}} - \sqrt{\phi} \right] \]

\[ \gamma \approx 0.4V^{\frac{1}{2}} \]
\[ \phi \approx 0.6V \]

Bulk-Diffusion Generally Reverse Biased (\( V_{BS} > 0 \) or at least greater than -0.3V) for n-channel

Same functional form as for n-channel devices but \( V_{T0} \) is now negative and the magnitude of \( V_T \) still increases with the magnitude of the reverse bias.
Recall:

4-terminal model extension

\[ I_G = 0 \]
\[ I_B = 0 \]

\[ I_D = \begin{cases} 
0 & \text{if } V_{GS} \leq V_T \\
\mu C_{OX} \frac{W}{L} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & \text{if } V_{GS} \geq V_T \text{ and } V_{DS} < V_{GS} - V_T \\
\mu C_{OX} \frac{W}{2L} \left( V_{GS} - V_T \right)^2 \cdot \left( 1 + \lambda V_{DS} \right) & \text{if } V_{GS} \geq V_T \text{ and } V_{DS} \geq V_{GS} - V_T 
\end{cases} \]

\[ V_T = V_{T0} + \gamma \left( \sqrt{\phi - V_{BS}} - \sqrt{\phi} \right) \]

Model Parameters: \{\mu, C_{OX}, V_{T0}, \phi, \gamma, \lambda\}

Design Parameters: \{W, L\} but only one degree of freedom \( W/L \) biasing or quiescent point
Small-Signal 4-terminal Model Extension

\[ I_G = 0 \]
\[ I_B = 0 \]
\[ I_D = \begin{cases} 
0 & V_{GS} \leq V_T \\
\mu C_{ox} \frac{W}{L} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T, V_{DS} < V_{GS} - V_T \\
\mu C_{ox} \frac{W}{2L} \left( V_{GS} - V_T \right)^2 \cdot (1 + \lambda V_{DS}) & V_{GS} \geq V_T, V_{DS} \geq V_{GS} - V_T 
\end{cases} \]

\[ V_T = V_{T0} + \gamma \left( \sqrt{\phi - V_{BS}} - \sqrt{\phi} \right) \]

\[ y_{11} = \left. \frac{\partial I_G}{\partial V_{GS}} \right|_{V=V_Q} = 0 \quad y_{12} = \left. \frac{\partial I_G}{\partial V_{DS}} \right|_{V=V_Q} = 0 \quad y_{13} = \left. \frac{\partial I_G}{\partial V_{GS}} \right|_{V=V_Q} = 0 \]

\[ y_{31} = \left. \frac{\partial I_B}{\partial V_{GS}} \right|_{V=V_Q} = 0 \quad y_{32} = \left. \frac{\partial I_B}{\partial V_{DS}} \right|_{V=V_Q} = 0 \quad y_{33} = \left. \frac{\partial I_B}{\partial V_{GS}} \right|_{V=V_Q} = 0 \]

\[ y_{21} = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V=V_Q} = g_m \quad y_{12} = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V=V_Q} = g_o \quad y_{13} = \left. \frac{\partial I_D}{\partial V_{BS}} \right|_{V=V_Q} = g_{mb} \]
Small-Signal 4-terminal Model Extension

\[ I_D = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2 \cdot (1 + \lambda V_{DS}) \]

\[ V_T = V_{T0} + \gamma \left( \sqrt{\phi} - \sqrt{V_{BS}} - \sqrt{\phi} \right) \]

\[ g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V = V_Q} = \mu C_{ox} \frac{W}{2L} 2 (V_{GS} - V_T)^{1} \cdot (1 + \lambda V_{DS}) \left|_{V = V_Q} \right. \approx \mu C_{ox} \frac{W}{L} V_{EBQ} \]

\[ g_o = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V = V_Q} = \mu C_{ox} \frac{W}{2L} 2 (V_{GS} - V_T)^{2} \cdot \lambda \left|_{V = V_Q} \right. \approx \lambda I_{DQ} \text{ Same as 3-term} \]

\[ g_{mb} = \left. \frac{\partial I_D}{\partial V_{BS}} \right|_{V = V_Q} = \mu C_{ox} \frac{W}{2L} 2 (V_{GS} - V_T)^{1} \cdot \left( - \frac{\partial V_T}{\partial V_{BS}} \right) \cdot (1 + \lambda V_{DS}) \left|_{V = V_Q} \right. \]

\[ g_{mb} = \left. \frac{\partial I_D}{\partial V_{BS}} \right|_{V = V_Q} \approx \mu C_{ox} \frac{W}{L} V_{EBQ} \cdot \left( - \frac{\partial V_T}{\partial V_{BS}} \right) \left|_{V = V_Q} \right. = \left( \mu C_{ox} \frac{W}{L} V_{EBQ} \right) (-1) \frac{1}{2} (\phi - V_{BS})^{-\frac{1}{2}} \left|_{V = V_Q} \right. (-1) \]

\[ g_{mb} \approx g_m \frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \]
**Small Signal Model Summary**

\[ i_g = 0 \]

\[ i_b = 0 \]

\[ i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds} \]

\[ g_m = \frac{\mu C_{OX} W}{L} V_{EBQ} \]

\[ g_o = \lambda I_{DQ} \]

\[ g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right) \]
Relative Magnitude of Small Signal MOS Parameters

Consider:

\[ i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds} \]

3 alternate equivalent expressions for \( g_m \)

\[ g_m = \frac{\mu C_{ox} W}{L} v_{EBQ} \quad g_m = \sqrt{\frac{2\mu C_{ox} W}{L}} \sqrt{I_{DQ}} \quad g_m = \frac{2I_{DQ}}{V_{EBQ}} \]

If \( \mu C_{ox} = 100\mu A/V^2 \), \( \lambda = .01 V^{-1} \), \( \gamma = 0.4V^{0.5} \), \( V_{EBQ} = 1V \), \( W/L = 1 \), \( V_{BSQ} = 0V \)

\[ I_{DQ} \approx \frac{\mu C_{ox} W}{2L} V_{EBQ}^2 = \frac{10^{-4} W}{2L} (1V)^2 = 5E-5 \]

\[ g_m = \frac{\mu C_{ox} W}{L} v_{EBQ} = 1E-4 \]

\[ g_o = \lambda I_{DQ} = 5E-7 \]

\[ g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right) = .26g_m \]

- Often the \( g_o \) term can be neglected in the small signal model because it is so small
- Be careful about neglecting \( g_o \) prior to obtaining a final expression
Large and Small Signal Model Summary

**Large Signal Model**

\[ I_0 = \begin{cases} 
0 & V_{GS} \leq V_T \\
\mu C_{OX} \frac{W}{L} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T, V_{DS} < V_{GS} - V_T \\
\mu C_{OX} \frac{W}{2L} \left( V_{GS} - V_T \right)^2 \cdot (1 + \lambda V_{DS}) & V_{GS} \geq V_T, V_{DS} \geq V_{GS} - V_T 
\end{cases} \]

\[ V_T = V_{T0} + \gamma \left( \sqrt{\phi - V_{BS}} - \sqrt{\phi} \right) \]

**Small Signal Model**

\[ i_g = 0 \]
\[ i_b = 0 \]
\[ i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds} \]

where

\[ g_m = \mu C_{OX} \frac{W}{L} V_{EBQ} \]
\[ g_{mb} = g_m \left( \frac{\gamma}{2 \sqrt{\phi - V_{BSQ}}} \right) \]
\[ g_o = \lambda I_{DQ} \]
Large and Small Signal Model Summary

Large Signal Model

\[ I_C = \beta I_B \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

Forward Active

- \( V_{BE} > 0.4 \text{V} \)
- \( V_{BC} < 0 \)

Small Signal Model

Forward Active

\[ i_b = g_\pi v_{be} \]
\[ i_c = g_m v_{be} + g_0 v_{ce} \]

where

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

- \( I_C = I_B = 0 \)
- \( V_{BE} < 0 \)
- \( V_{BC} < 0 \)

- \( V_{BE} = 0.7 \text{V} \)
- \( V_{CE} = 0.2 \text{V} \)
Relative Magnitude of Small Signal BJT Parameters

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[ g_m = \begin{bmatrix} \frac{I_0}{V_t} \\ \frac{I_0}{\beta V_t} \end{bmatrix} \]
\[ g_\pi = \begin{bmatrix} \frac{I_0}{\beta V_t} \end{bmatrix} \]
\[ g_o = \begin{bmatrix} \frac{I_0}{V_{AF}} \end{bmatrix} \]

\[ g_m >> g_\pi >> g_o \]

Often the \( g_o \) term can be neglected in the small signal model because it is so small.
Relative Magnitude of Small Signal Parameters

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[ \frac{g_m}{g_\pi} = \begin{bmatrix} \frac{I_Q}{V_t} \\ \frac{I_Q}{\beta V_t} \end{bmatrix} = \beta \]

\[ \frac{g_\pi}{g_o} = \begin{bmatrix} \frac{I_Q}{\beta V_t} \\ \frac{I_Q}{V_{AF}} \end{bmatrix} = \frac{V_{AF}}{\beta V_t} \approx \frac{200V}{100 \cdot 26mV} = 77 \]

\( g_m \gg g_\pi \gg g_o \)

- Often the \( g_o \) term can be neglected in the small signal model because it is so small
- Be careful about neglecting \( g_o \) prior to obtaining a final expression
Small Signal Model Simplifications for the MOSFET and BJT

Often simplifications of the small signal model are adequate for a given application.

These simplifications will be discussed next.
Small Signal MOSFET Model Summary

An equivalent Circuit:

\[ g_m = \frac{\mu C_{OX} W}{L} (V_{GSQ} - V_T) \]

\[ g_o = \lambda I_{DQ} \]

\[ g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right) \]

Alternate equivalent representations for \( g_m \) from \( I_D \approx \mu C_{OX} \frac{W}{2L} (V_{GS} - V_T)^2 \)

\[ g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}} \]

\[ g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}} \]

\( g_{mb} < g_m \)

\( g_0 << g_m, g_{mb} \)
Small Signal Model Simplifications

Simplification that is often adequate
Small Signal Model Simplifications

Even further simplification that is often adequate
Small Signal BJT Model Summary

An equivalent circuit

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[ g_m >> g_\pi >> g_o \]

This contains absolutely no more information than the set of small-signal model equations
Small Signal BJT Model Simplifications

Simplification that is often adequate
Gains for MOSFET and BJT Circuits

**BJT**

\[ A_{VB} = -\frac{I_{CQ} R_1}{V_t} \]

**MOSFET**

\[ A_{VM} = \frac{2I_{DQ} R}{[V_{SS} + V_T]} \]

For both circuits

\[ A_v = -g_m R \]

Gains vary linearly with small signal parameter \( g_m \)

Power is often a key resource in the design of an integrated circuit.

In both circuits, power is proportional to \( I_{CQ}, I_{DQ} \)
How does $g_m$ vary with $I_{DQ}$?

\[ g_m = \sqrt{\frac{2\mu C_{Ox} W}{L}} \sqrt{I_{DQ}} \]

Varies with the square root of $I_{DQ}$

\[ g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}} \]

Varies linearly with $I_{DQ}$

\[ g_m = \frac{\mu C_{Ox} W}{L} (V_{GSQ} - V_T) \]

Doesn't vary with $I_{DQ}$
How does $g_m$ vary with $I_{DQ}$?

All of the above are true – but with qualification

g_m is a function of more than one variable ($I_{DQ}$) and how it varies depends upon how the remaining variables are constrained
End of Lecture 28