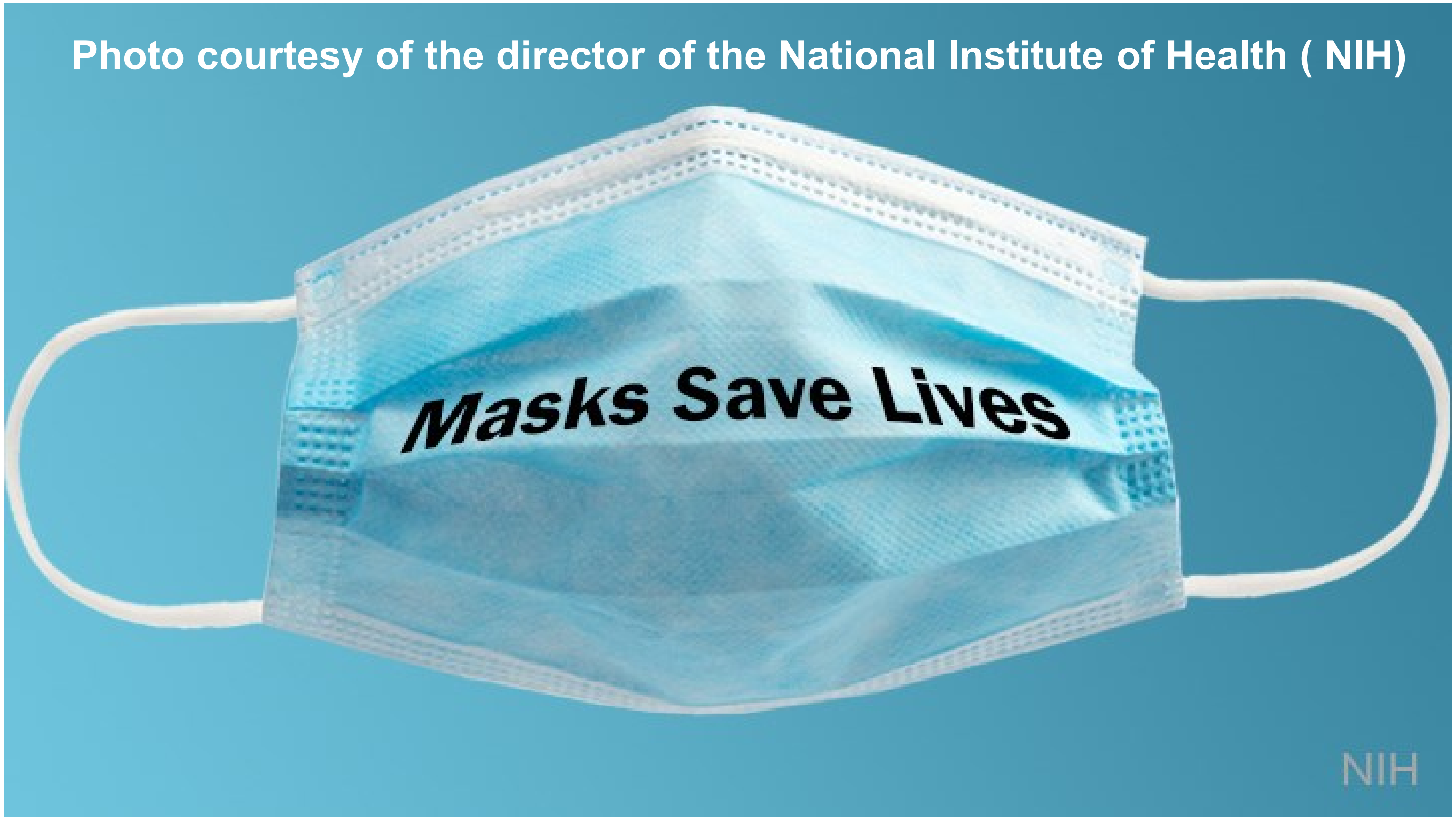
EE 330
Lecture 28

Two-Port Amplifier Models

Exam Schedule

Exam 2 will be given on Friday March 11

Exam 3 will be given on Friday April 15



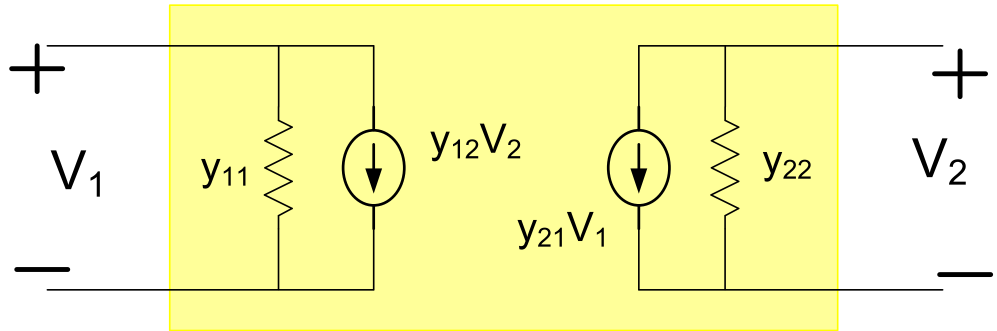
As a courtesy to fellow classmates, TAs, and the instructor

Wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status

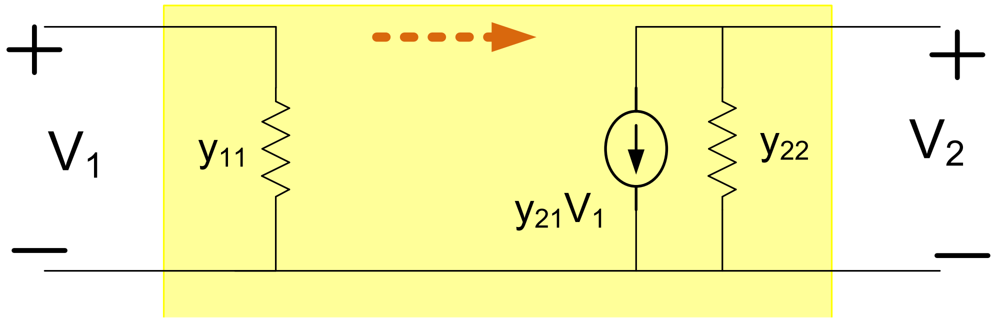
Review From Previous Lecture

Two-port representation of amplifiers

Amplifiers can be modeled as a two-port



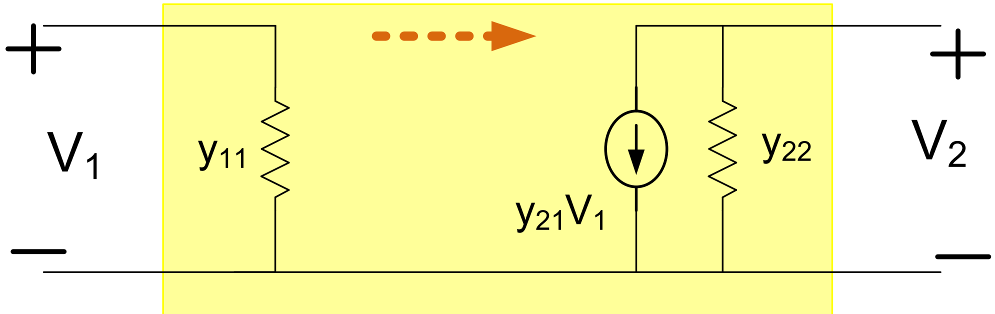
- Amplifier often **unilateral** (signal propagates in only one direction: wlog y_{12}=0)
- One terminal is often common



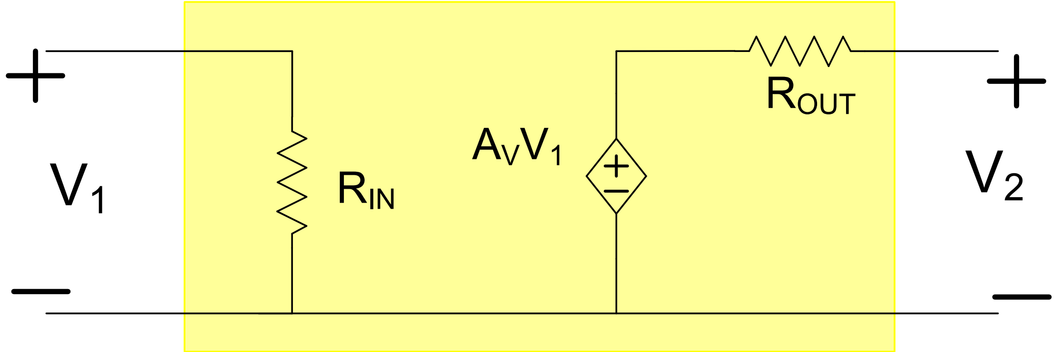
Review from Previous Lecture

Two-port representation of amplifiers

Unilateral amplifiers:



- Thevenin equivalent output port often more standard
- R_{IN}, A_V, and R_{OUT} often used to characterize the two-port of amplifiers



Unilateral amplifier in terms of "amplifier" parameters

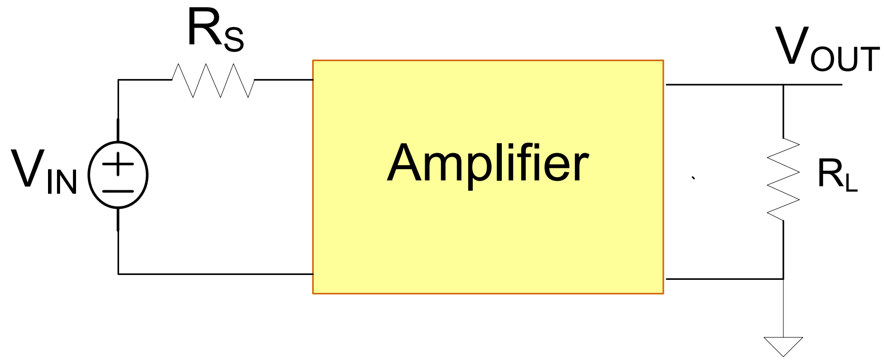
$$R_{IN} = \frac{1}{y_{11}} \qquad A_V = -\frac{y_{21}}{y_{22}} \qquad R_{OUT} = \frac{1}{y_{22}}$$

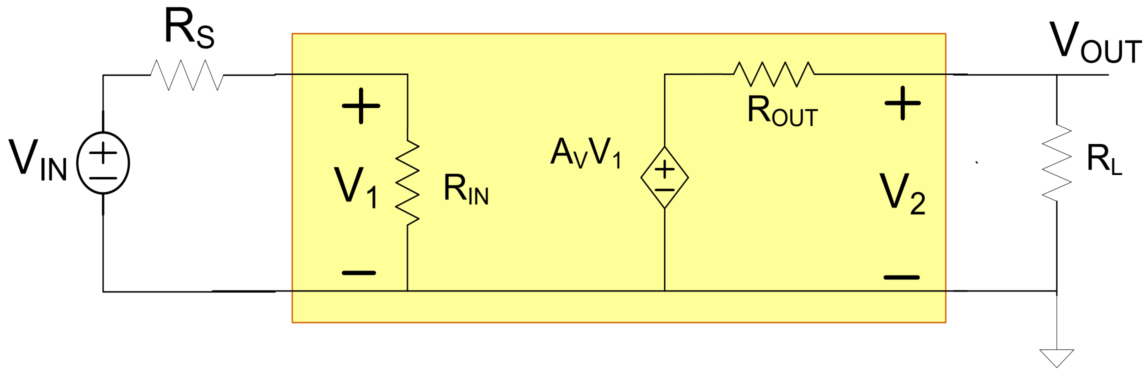
Review from Previous Lecture

Amplifier input impedance, output impedance and gain are usually of interest

Why?

Example 1: Assume amplifier is unilateral





$$V_{OUT} = \left(\frac{R_L}{R_L + R_{OUT}}\right) A_V \left(\frac{R_{IN}}{R_S + R_{IN}}\right) V_{IN} \qquad A_{VAMP} = \frac{V_{OUT}}{V_{IN}} = \left(\frac{R_L}{R_L + R_{OUT}}\right)\left(\frac{R_{IN}}{R_S + R_{IN}}\right) A_V$$

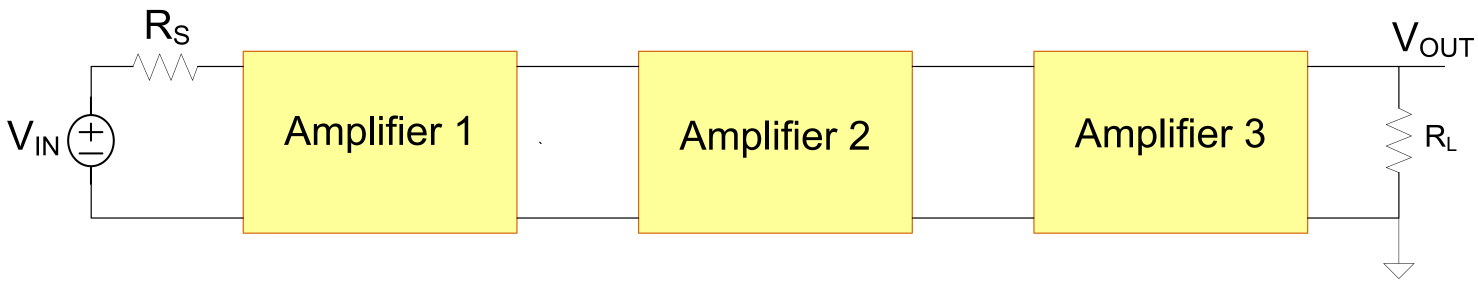
- Can get gain without reconsidering details about components internal to the Amplifier !!!
  - Analysis more involved when not unilateral

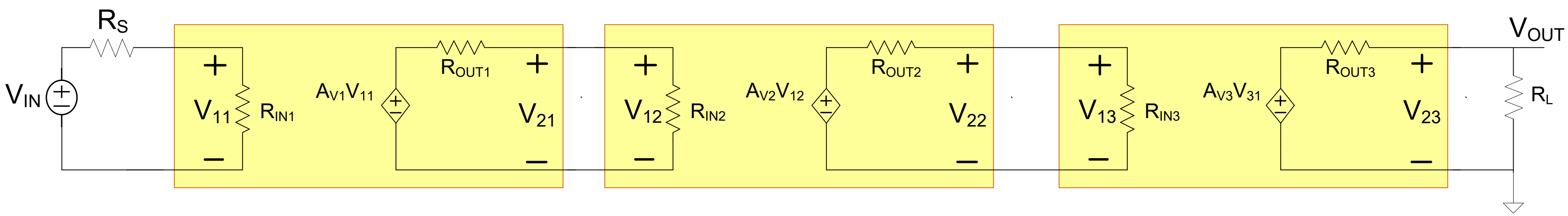
Review from Previous Lecture

Amplifier input impedance, output impedance and gain are usually of interest

Why?

Example 2: Assume amplifiers are unilateral





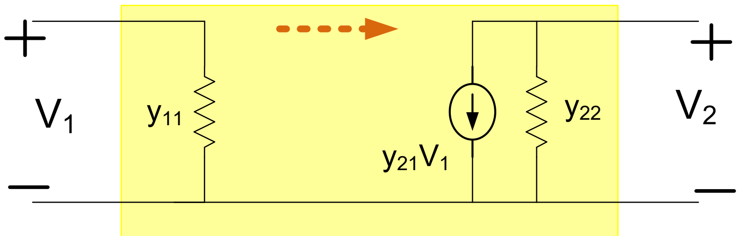
$$V_{OUT} = \left(\frac{R_L}{R_L + R_{OUT3}}\right) A_{V3} \left(\frac{R_{IN3}}{R_{OUT2} + R_{IN3}}\right) A_{V2} \left(\frac{R_{IN2}}{R_{OUT1} + R_{IN2}}\right) A_{V1} \left(\frac{R_{IN1}}{R_S + R_{IN1}}\right) V_{IN}$$

$$A_{VAMP} = \frac{V_{OUT}}{V_{IN}} = \left(\frac{R_L}{R_L + R_{OUT3}}\right) A_{V3} \left(\frac{R_{IN3}}{R_{OUT2} + R_{IN3}}\right) A_{V2} \left(\frac{R_{IN2}}{R_{OUT1} + R_{IN2}}\right) A_{V1} \left(\frac{R_{IN1}}{R_S + R_{IN1}}\right)$$

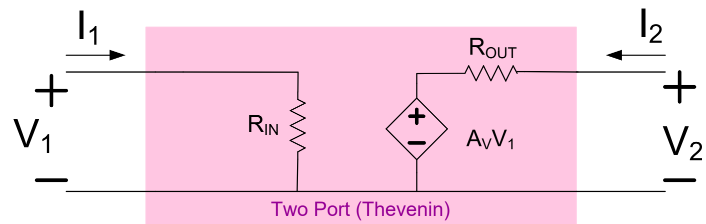
- Can get gain without reconsidering details about components internal to the Amplifier !!!
- Analysis more involved when not unilateral

Two-port representation of amplifiers

- Amplifier often **unilateral** (signal propagates in only one direction: wlog $y_{12}=0$)
- One terminal is often common
- "Amplifier" parameters often used

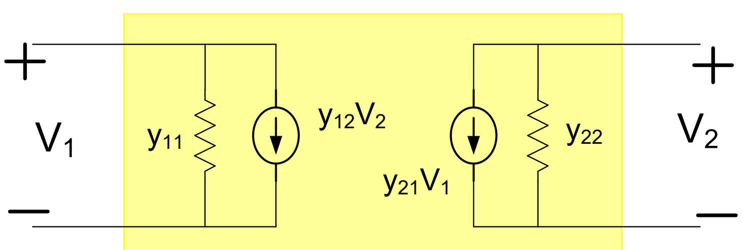


y parameters

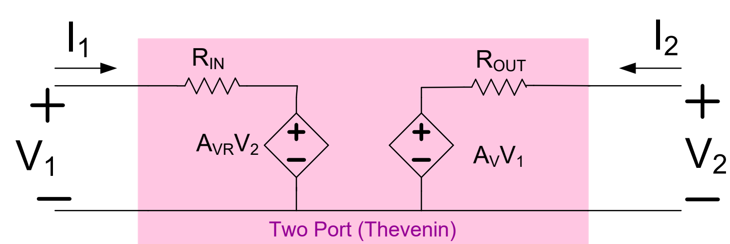


Amplifier parameters

- Amplifier parameters can also be used if not **unilateral**
- One terminal is often common

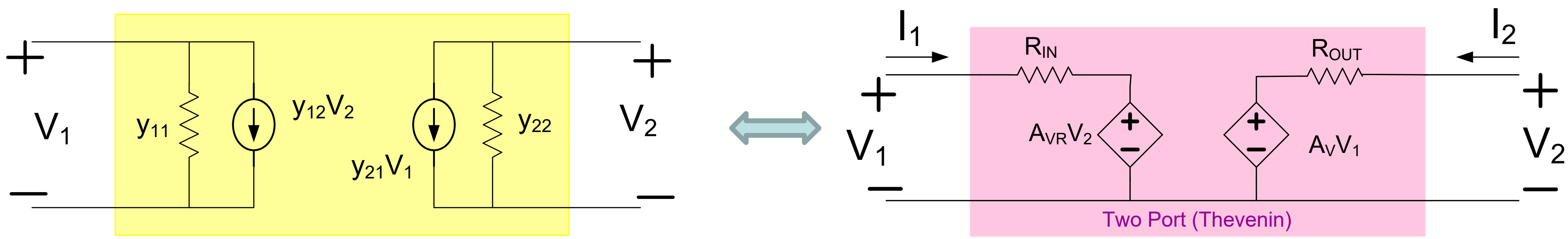


y parameters



Amplifier parameters

Determination of small-signal model parameters:



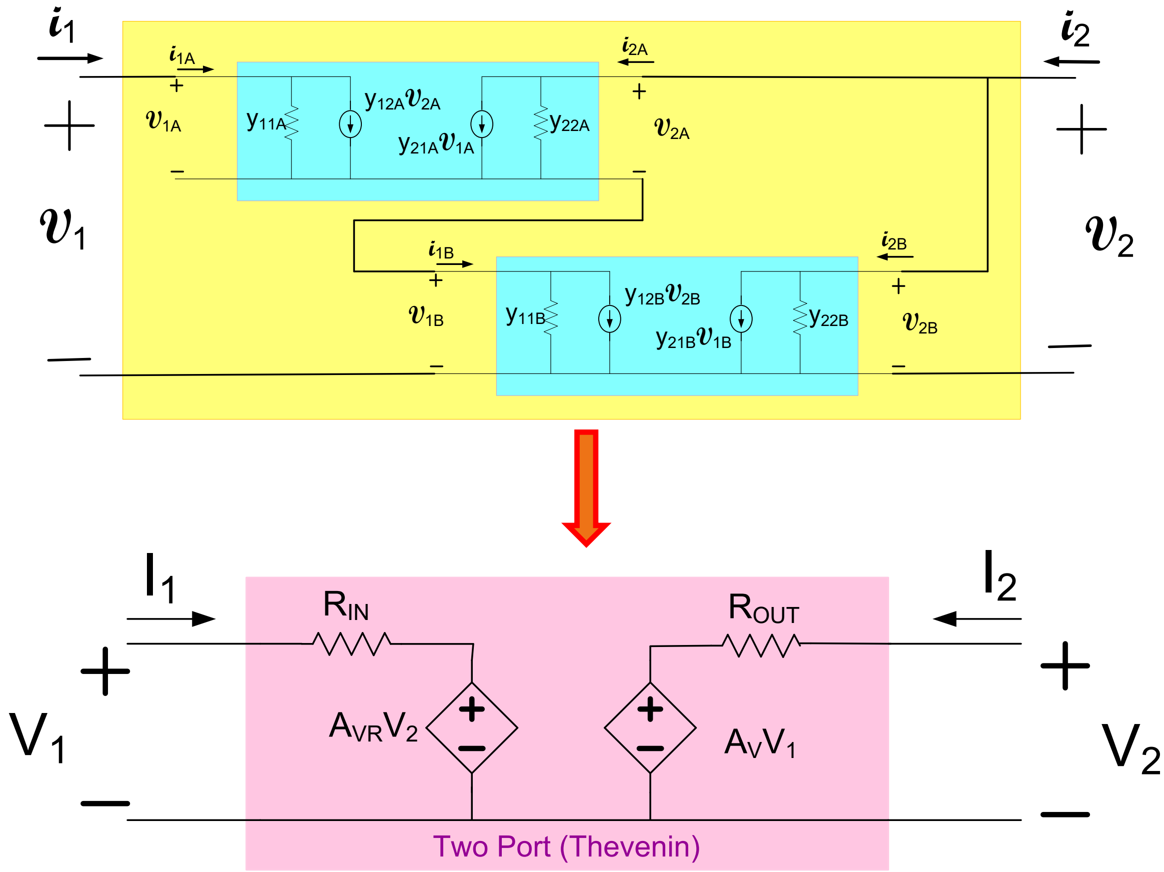
In the past, we have determined small-signal model parameters of electronic devices from the nonlinear port characteristics

$$\left.\begin{array}{l} \mathbf{I_1} = \mathbf{f_1}(\mathbf{V_1},\mathbf{V_2}) \\ \mathbf{I_2} = \mathbf{f_2}(\mathbf{V_1},\mathbf{V_2}) \end{array}\right\} \qquad \mathbf{y_{ij}} = \left.\frac{\partial \mathbf{f_i}(\mathbf{V_1},\mathbf{V_2})}{\partial \mathbf{V_j}}\right|_{\vec{\mathbf{V}}=\vec{\mathbf{V}}_{\mathbf{Q}}}$$

- Will now determine small-signal model parameters for two-port comprised of linear networks (instead of just electronic devices)
- Could go back to the nonlinear models and analyze as we did for electronic devices
- Will follow a different approach (results are identical) that is often much easier

Two-Port Equivalents of Interconnected Two-ports

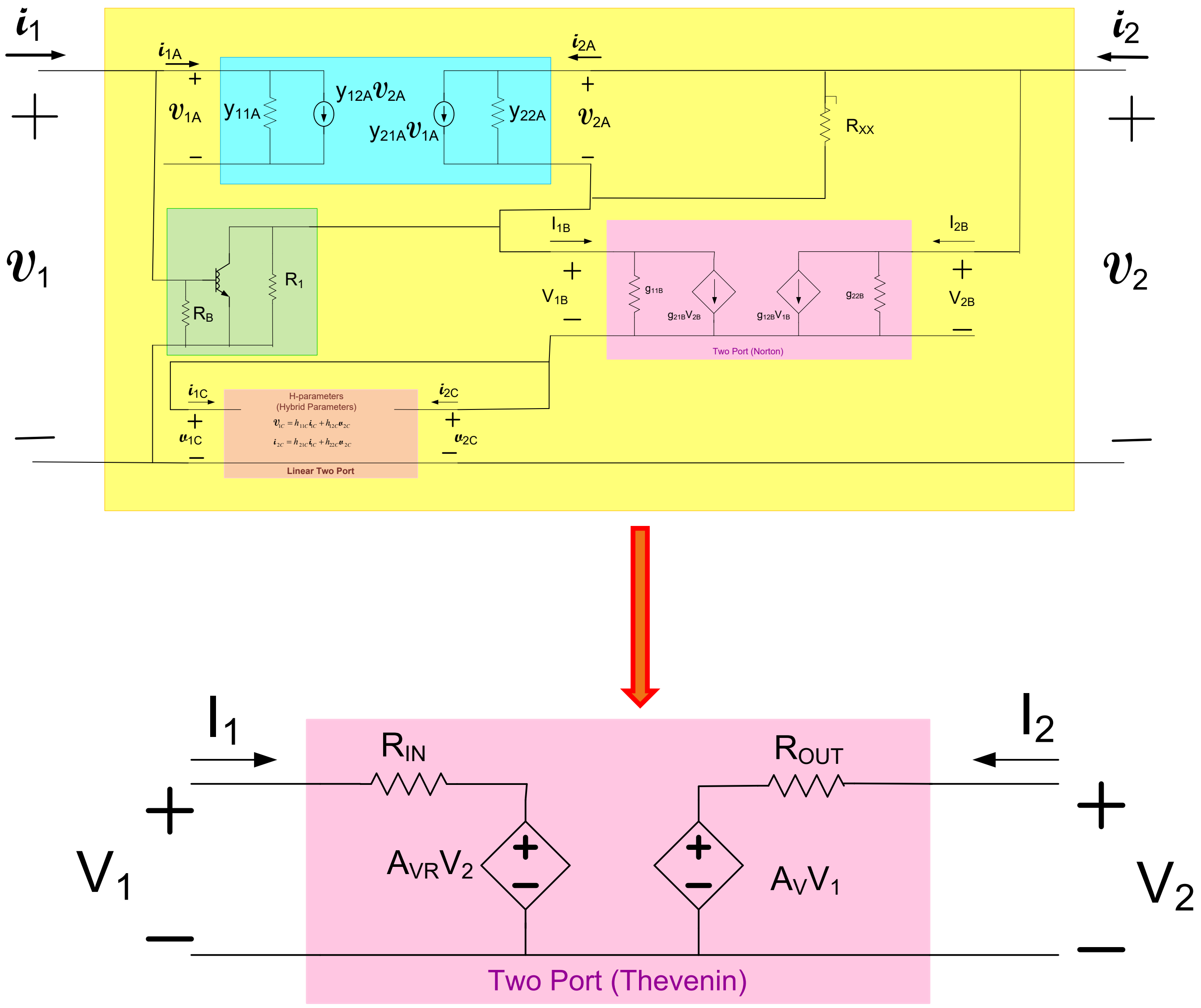
Example:



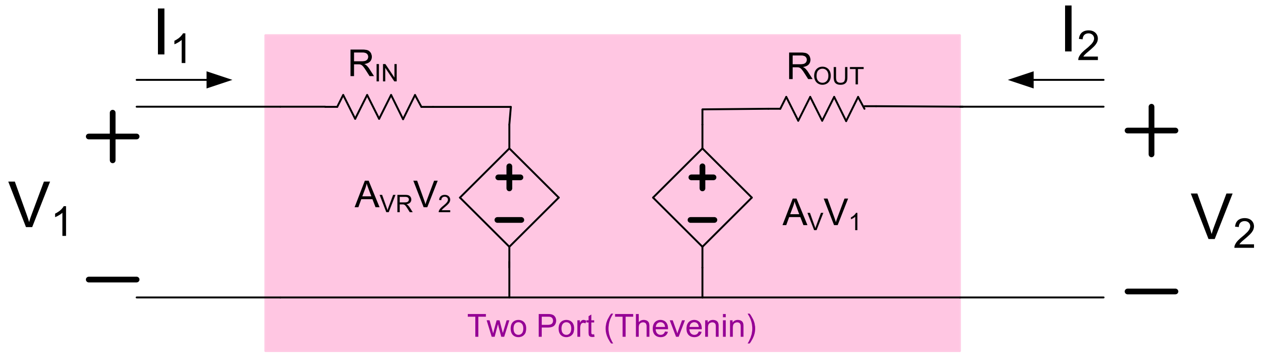
- could obtain two-port in any form
- often obtain equivalent circuit w/o identifying independent variables
- Unilateral iff $A_{VR}=0$ (or if $A_V=0$ though would probably relabel ports)
- Thevenin-Norton transformations can be made on either or both ports

Two-Port Equivalents of Interconnected Two-ports

Example:



Two-Port Equivalents of Interconnected Two-ports



$$v_1 = i_1 R_{in} + A_{VR} v_2$$

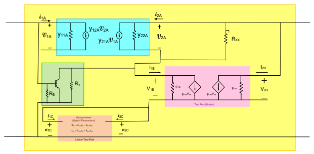
$$v_2 = i_2 R_0 + A_{V0} v_1$$

Or equivalently in form where port voltages are the independent variables

$$i_1 = v_1\left(\frac{1}{R_{in}}\right) + v_2\left(\frac{-A_{VR}}{R_{in}}\right)$$

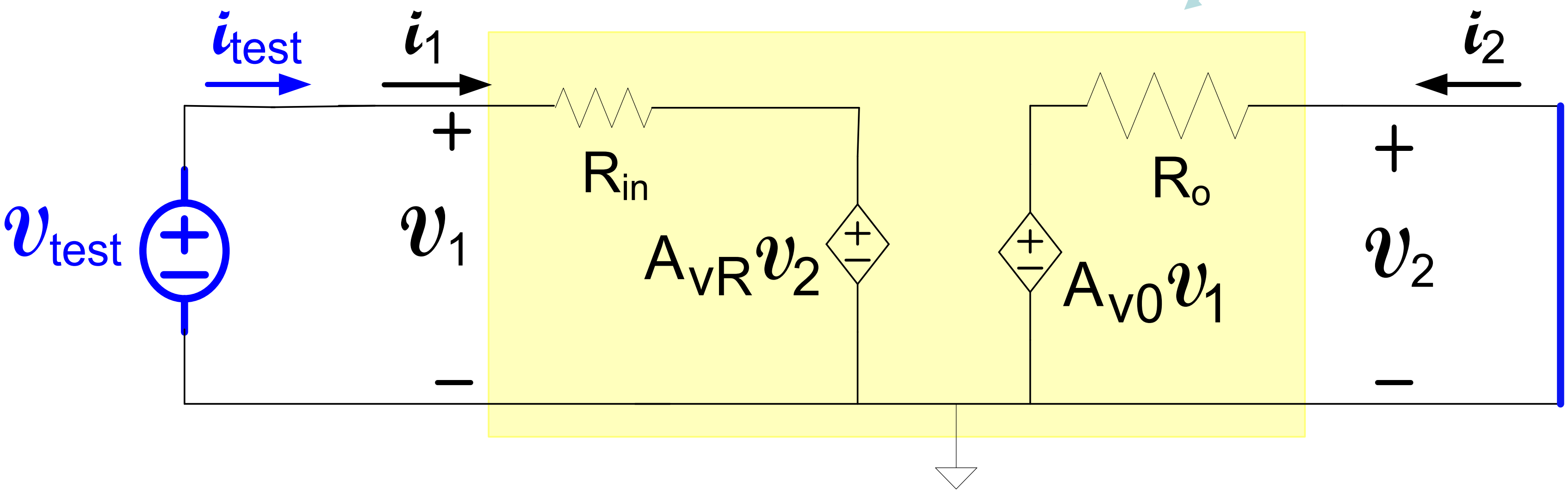
$$i_2 = v_1\left(\frac{-A_{V0}}{R_0}\right) + v_2\left(\frac{1}{R_0}\right)$$

Determination of two-port small-signal model parameters



(One method will be discussed here)

A method of obtaining R_{in}



Terminate the output in a (small signal) short-circuit

$$i_1 = v_1\left(\frac{1}{R_{in}}\right) + v_2\left(\frac{-A_{VR}}{R_{in}}\right)$$

$$i_2 = v_1\left(\frac{-A_{V0}}{R_0}\right) + v_2\left(\frac{1}{R_0}\right)$$

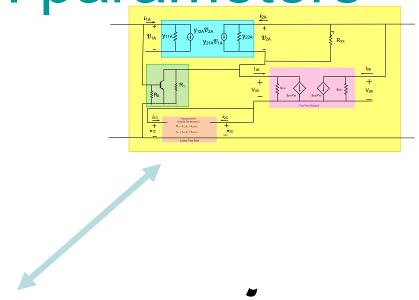
$v_2 = 0$

$v_1 = v_{test}$

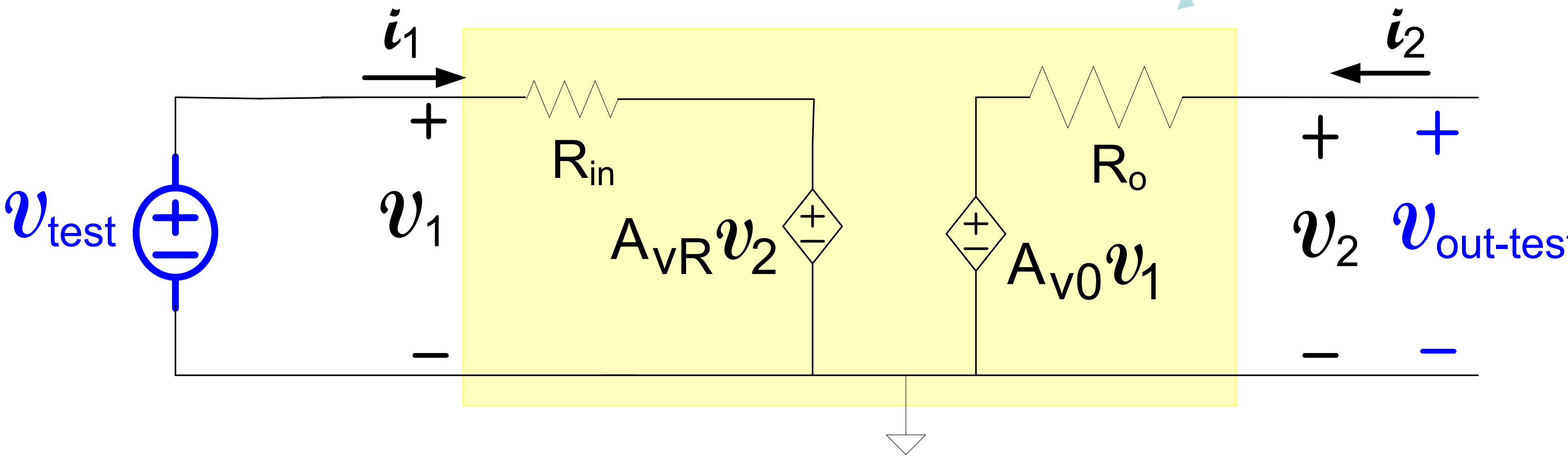
$i_1 = i_{test}$

$$R_{in} = \frac{v_{test}}{i_{test}}$$

Determination of two-port small-signal model parameters



A method of obtaining A_{V0}



Terminate the output in a (small signal) open-circuit

$$i_1 = v_1\left(\frac{1}{R_{in}}\right) + v_2\left(\frac{-A_{VR}}{R_{in}}\right)$$

$$i_2 = v_1\left(\frac{-A_{V0}}{R_0}\right) + v_2\left(\frac{1}{R_0}\right)$$

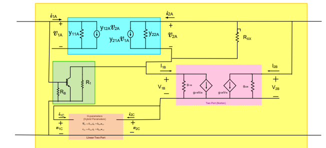
$i_2 = 0$

$v_1 = v_{test}$

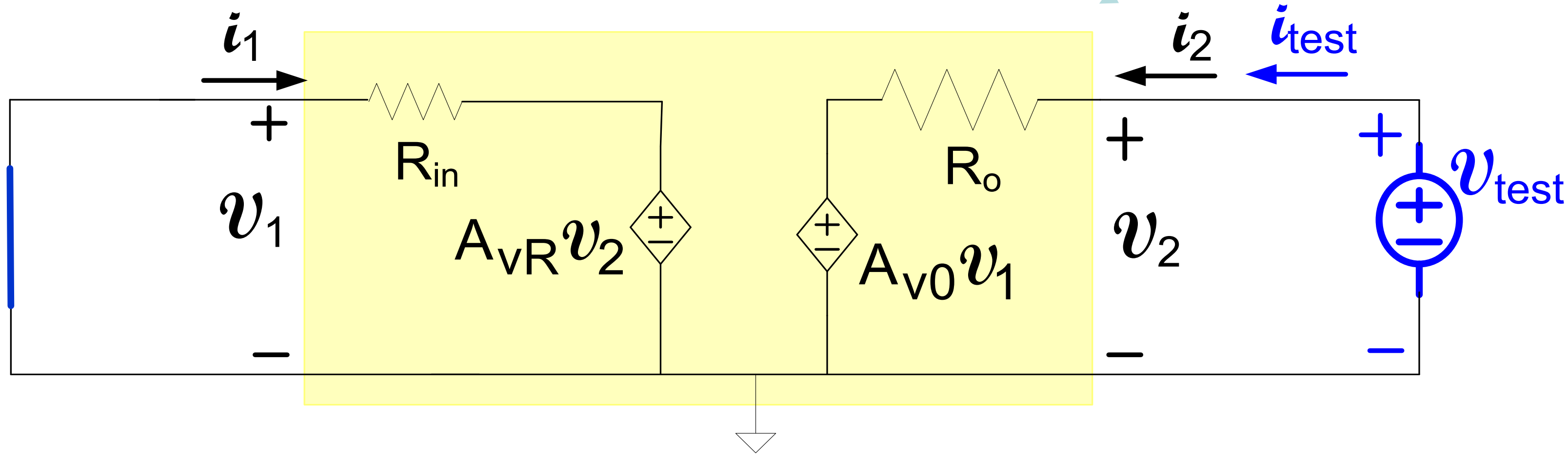
$v_2 = v_{out\text{-}test}$

$$A_{V0} = \frac{v_{out\text{-}test}}{v_{test}}$$

Determination of two-port small-signal model parameters



A method of obtaining R_0

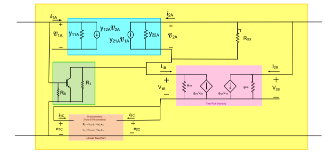


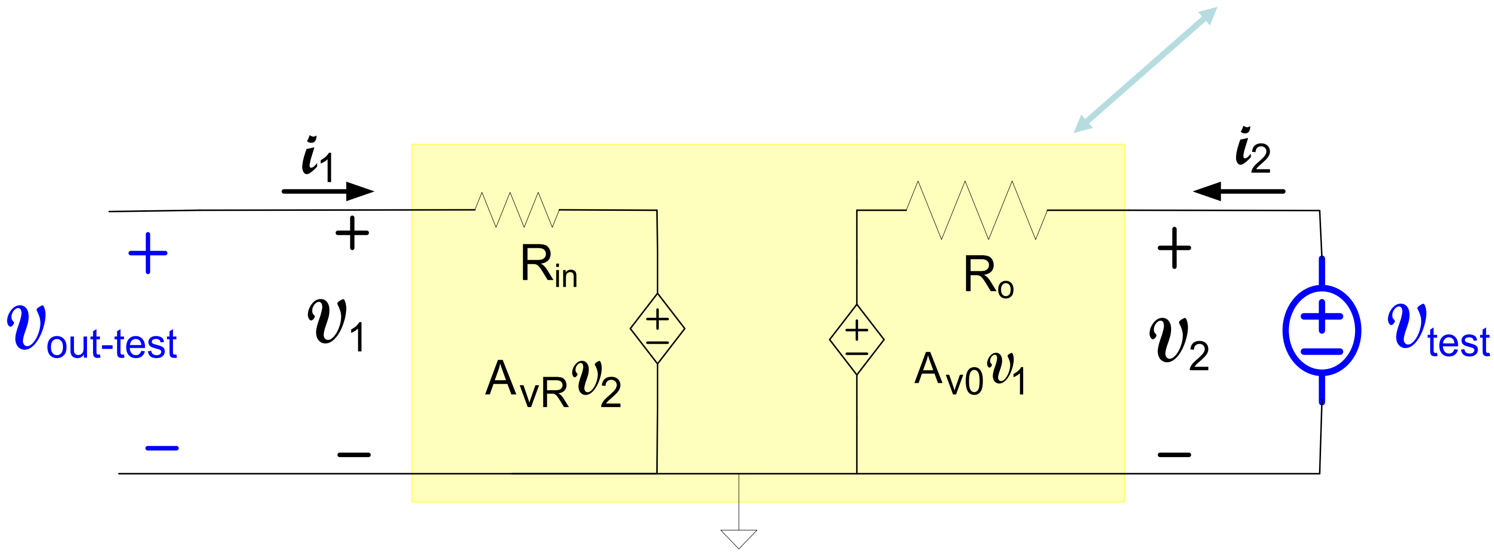
Terminate the input in a (small-signal) short-circuit

$$
\left.\begin{aligned}
i_1 &= v_1\left(\frac{1}{R_{in}}\right) + v_2\left(\frac{-A_{VR}}{R_{in}}\right) \\
i_2 &= v_1\left(\frac{-A_{V0}}{R_0}\right) + v_2\left(\frac{1}{R_0}\right)
\end{aligned}\right\}
\xrightarrow{v_1 = 0}
R_0 = \frac{v_{test}}{i_{test}}
$$

Determination of two-port small-signal model parameters

A method of obtaining A_{VR}





Terminate the <u>input</u> in a (small-signal) open-circuit

$$i_1 = v_1\left(\frac{1}{R_{in}}\right) - v_2\left(\frac{A_{VR}}{R_{in}}\right)$$

$$i_2 = v_1\left(\frac{-A_{V0}}{R_0}\right) + v_2\left(\frac{1}{R_0}\right)$$

$$\xrightarrow{i_1 = 0} \quad A_{VR} = \frac{v_{\text{out-test}}}{v_{\text{test}}}$$

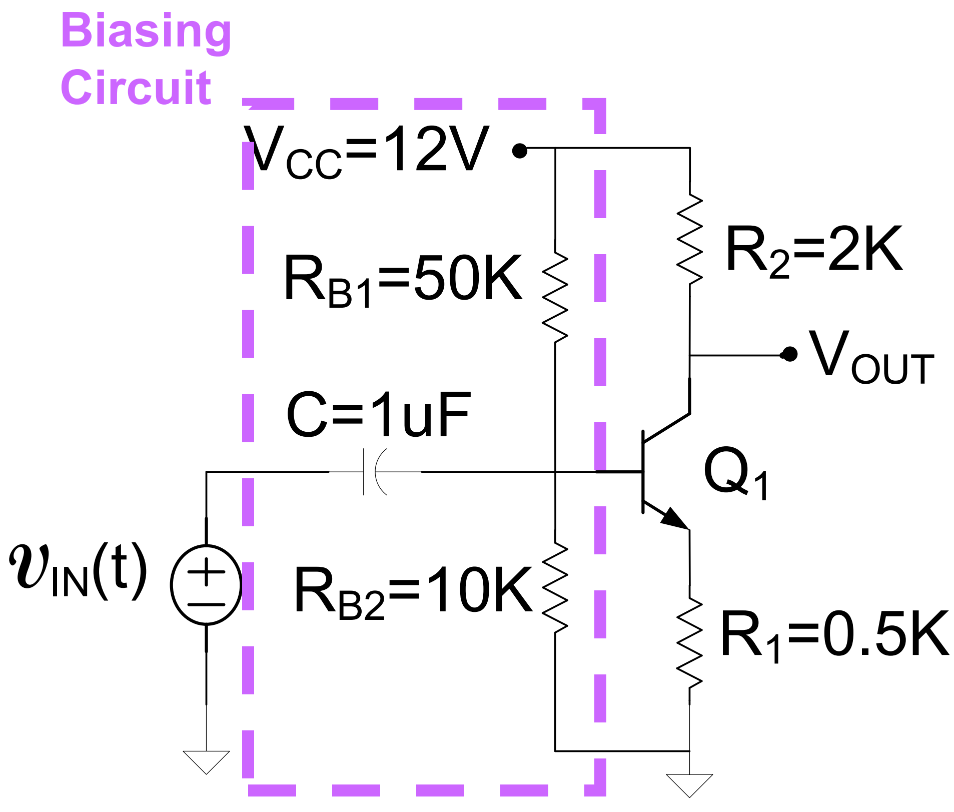
Determination of Amplifier Two-Port Parameters

- Input and output parameters are obtained in exactly the same way, only distinction is in the notation used for the ports.

- Methods given for obtaining amplifier parameters R_{in}, R_{OUT} and A_V for unilateral networks are a special case of the non-unilateral analysis by observing that A_{VR}=0.

- In some cases, other methods for obtaining the amplifier parameters are easier than the "V_{TEST} : I_{TEST}" method that was just discussed

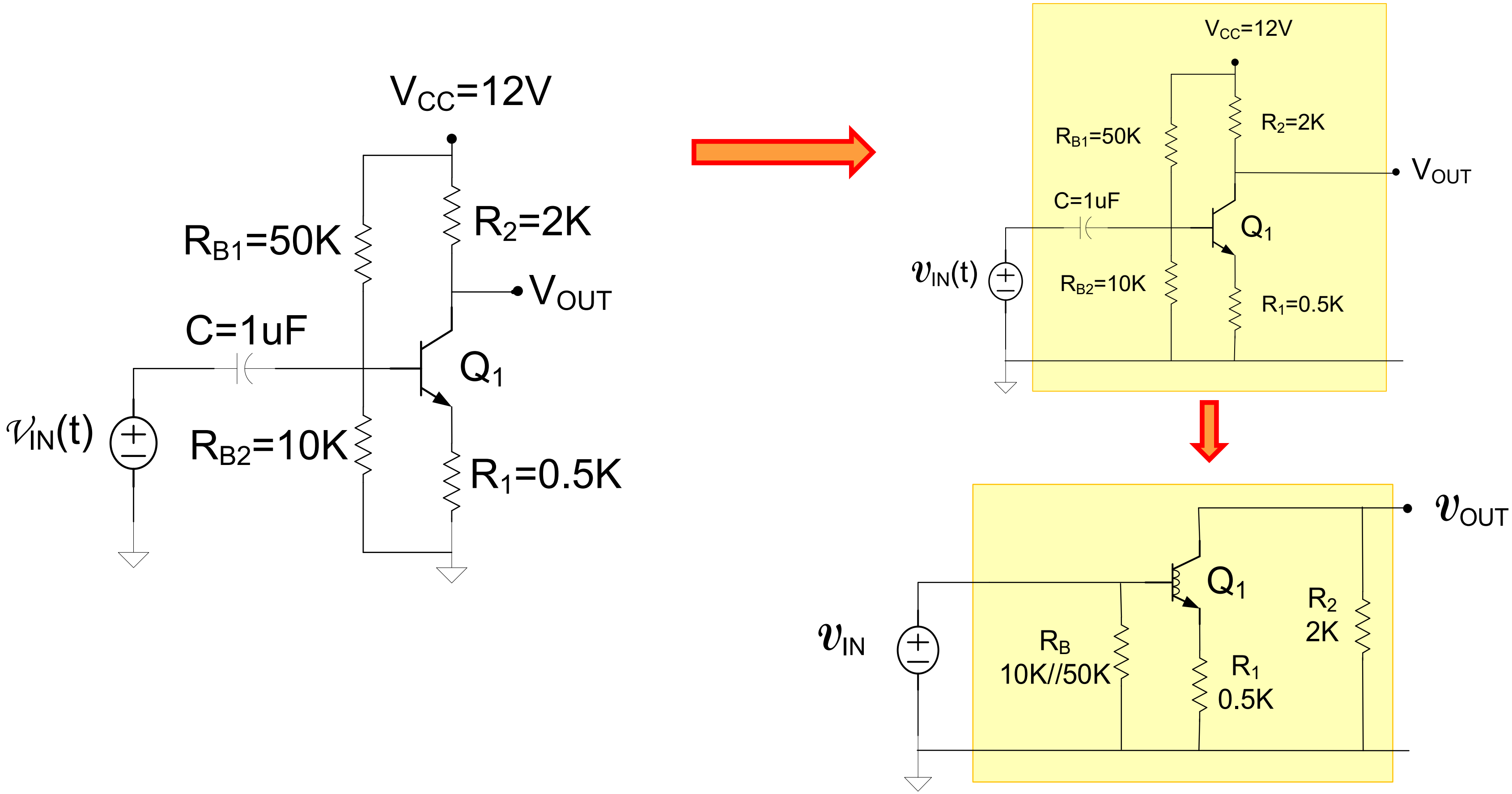
Examples



Determine V_{OUTQ} and the SS voltage gain (A_V), assume β=100

(A_V is one of the small-signal model parameters for this circuit)

Examples



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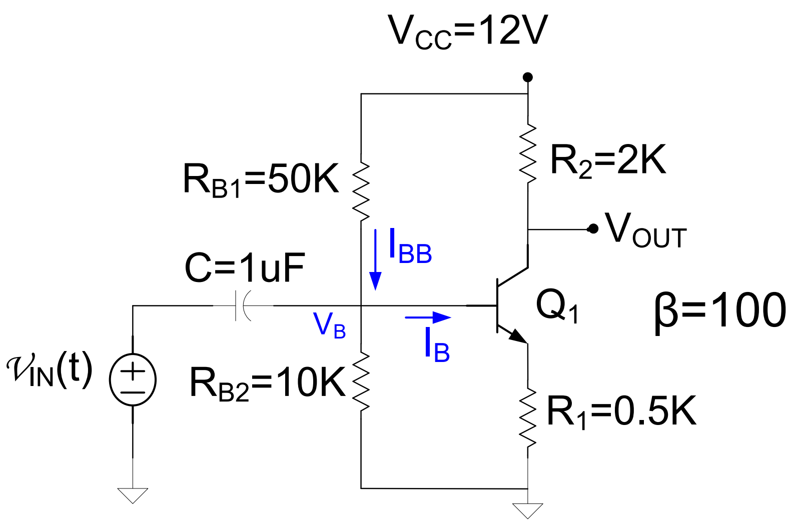
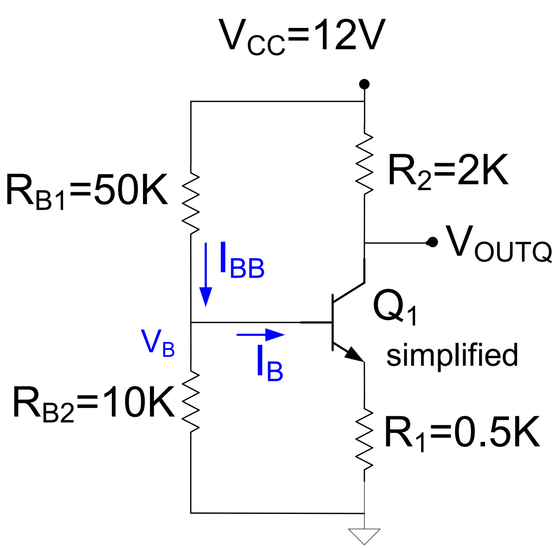
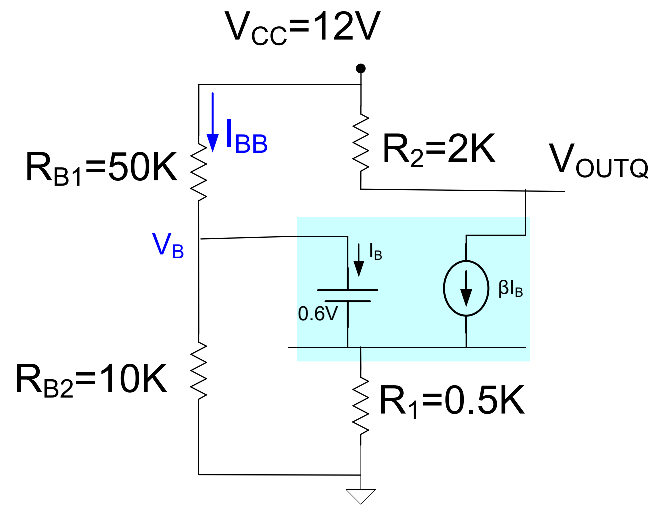
Examples





dc equivalent circuit

Determine V_{OUTQ}



dc equivalent circuit

This circuit is most practical when $I_B << I_{BB}$

With this assumption,

$$V_B = \left(\frac{R_{B2}}{R_{B1}+R_{B2}}\right)12V$$

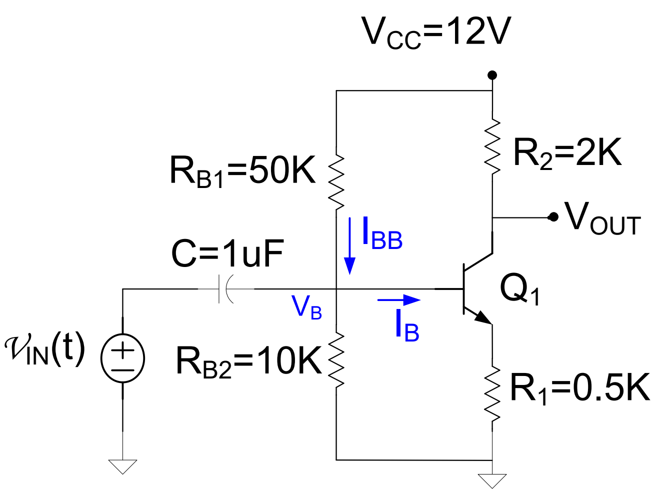
$$I_{CQ} = I_{EQ} = \left(\frac{V_B - 0.6V}{R_1}\right) = \frac{1.4V}{.5K} = 2.8mA$$

$$V_{OUTQ} = 12V - I_{CQ}R_1 = 6.4V$$

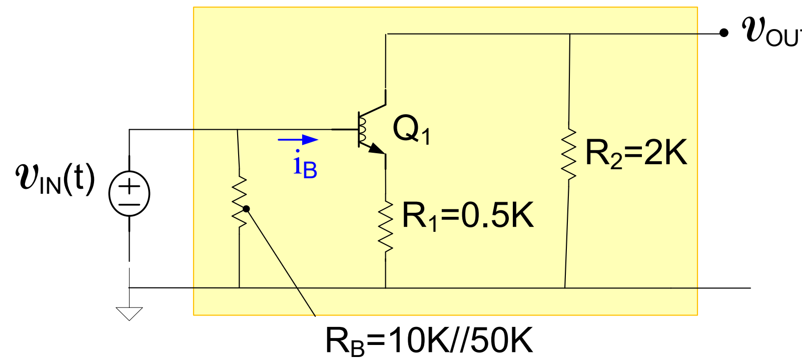
Note: This Q-point is nearly independent of the characteristics of the nonlinear BJT !

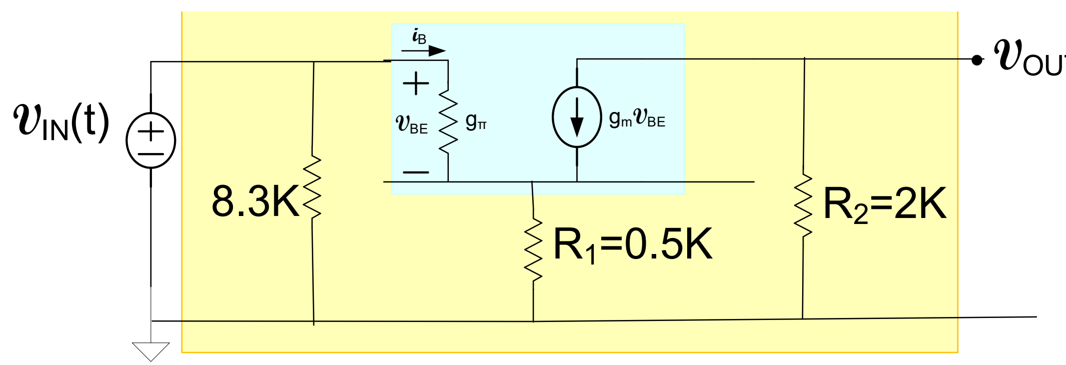
Examples

Determine SS voltage gain



$\beta=100$





$$\left.\begin{aligned} v_{OUT} &= -g_m v_{BE} R_2 \\ v_{IN} &= v_{BE} + R_1\left(v_{BE}\left[g_\pi + g_m\right]\right) \end{aligned}\right\}$$

$$A_V = \frac{-R_2 g_m v_{BE}}{v_{BE} + R_1\left(v_{BE}\left[g_\pi + g_m\right]\right)} = \frac{-R_2 g_m}{1 + R_1\left(\left[g_\pi + g_m\right]\right)}$$

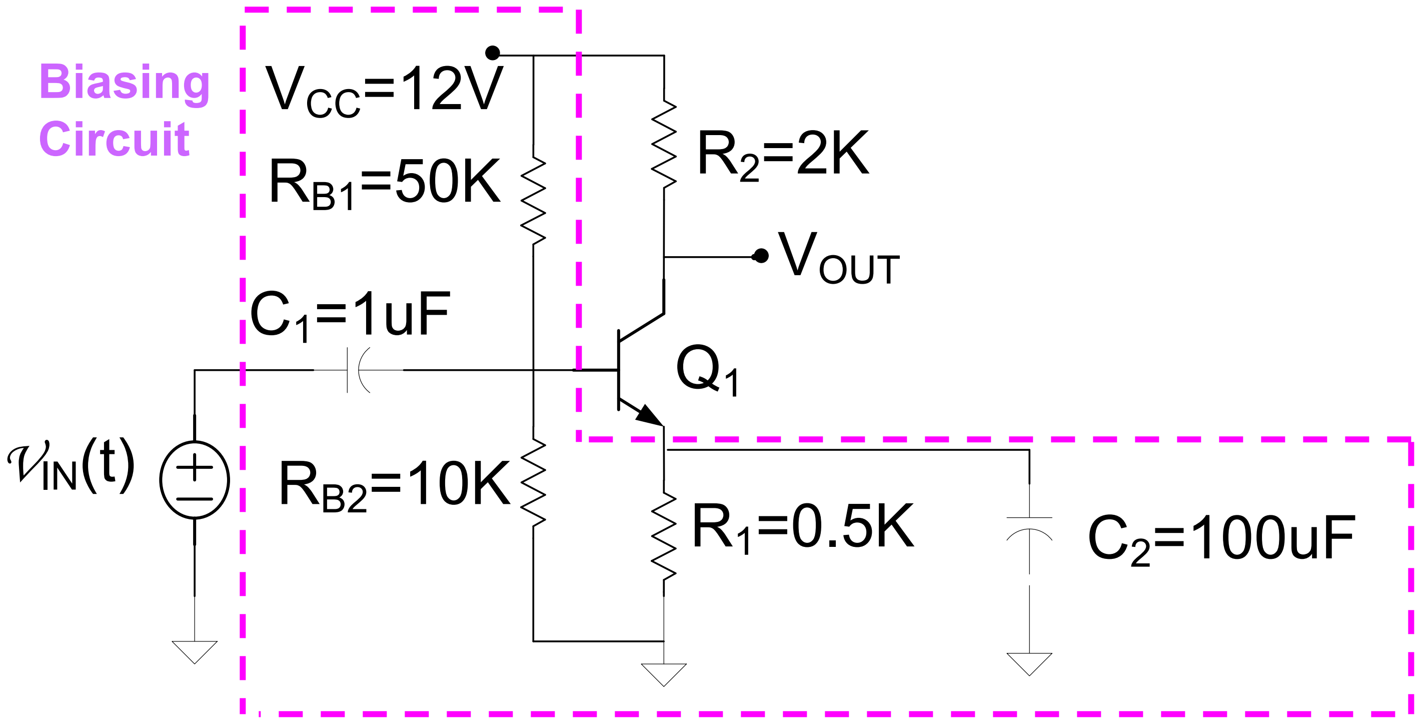
$$A_V \cong \frac{-R_2 g_m}{R_1 g_m} = \frac{-R_2}{R_1} = -4$$

This voltage gain is nearly independent of the characteristics of the nonlinear BJT !

This is a fundamentally different amplifier structure

It can be shown that this is slightly non-unilateral

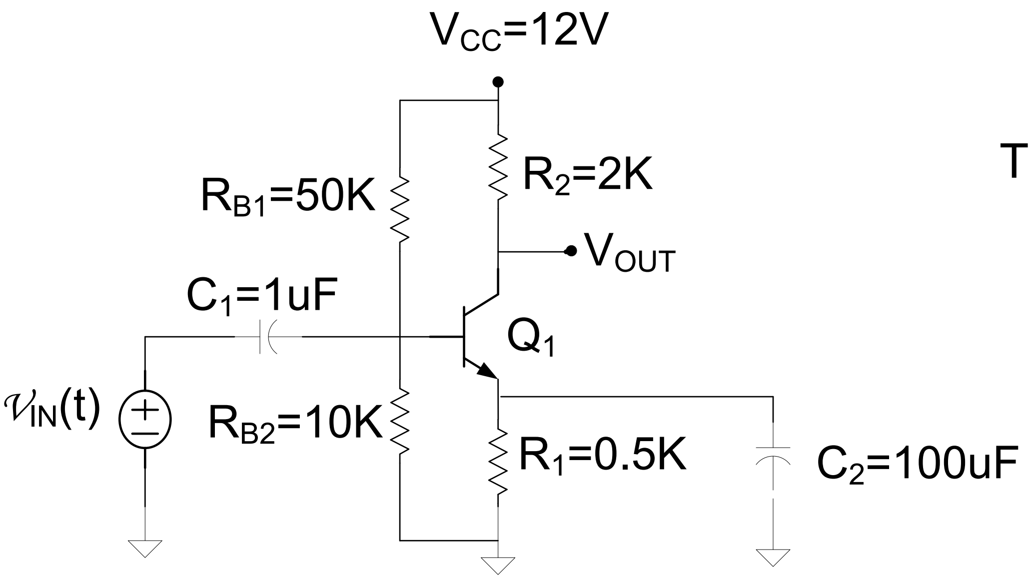
Examples



Determine V_{OUTQ} , R_{IN}, R_{OUT},and the SS voltage gain, and A_{VR} assume β=100

Examples

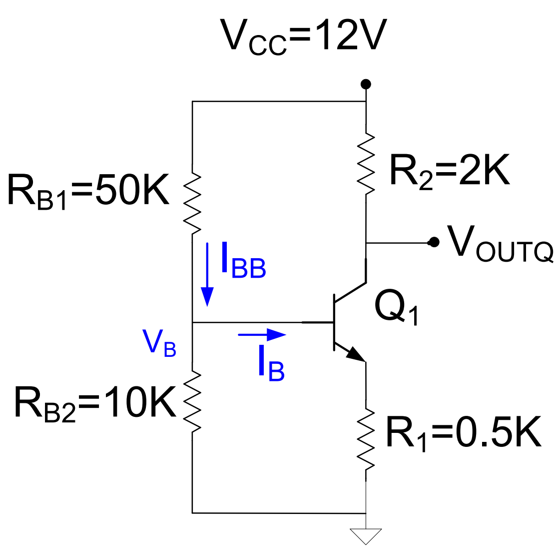
Determine V_{OUTQ} ✓



This is the same as the previous circuit !

$$V_{OUTQ} = 6.4V$$

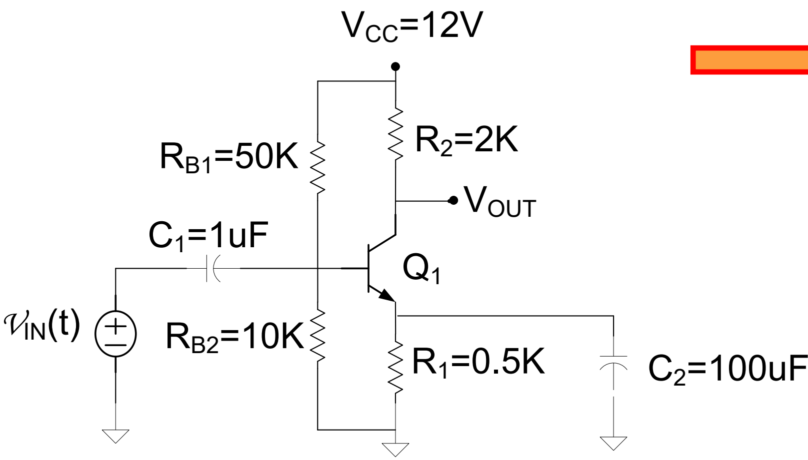
$$I_{CQ} = \frac{5.6V}{2K} = 2.8mA$$

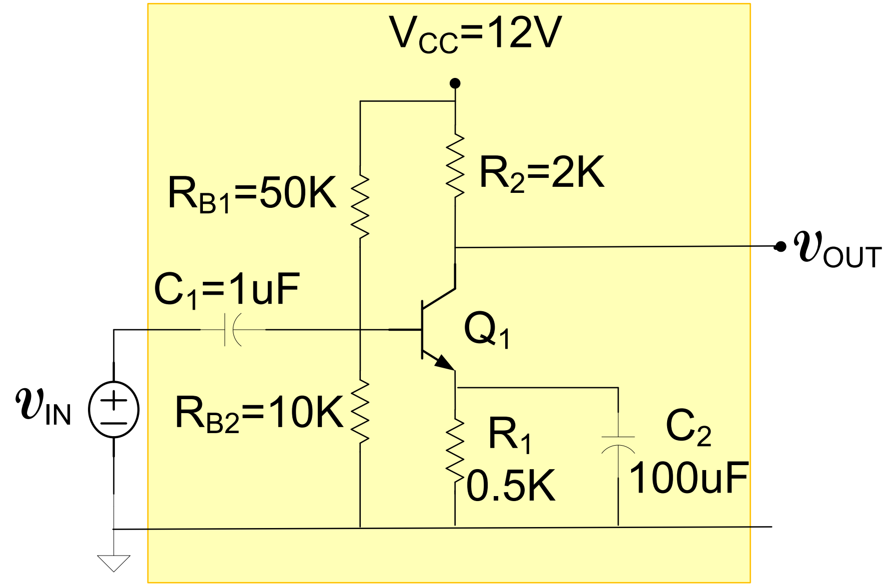


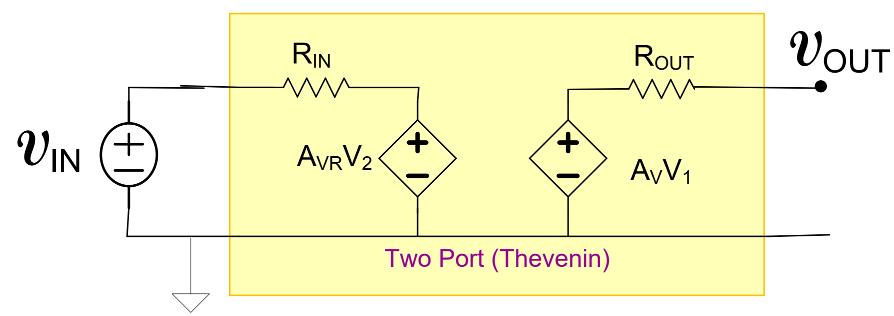
The dc equivalent circuit

Note: This Q-point is nearly independent of the characteristics of the nonlinear BJT !

Examples





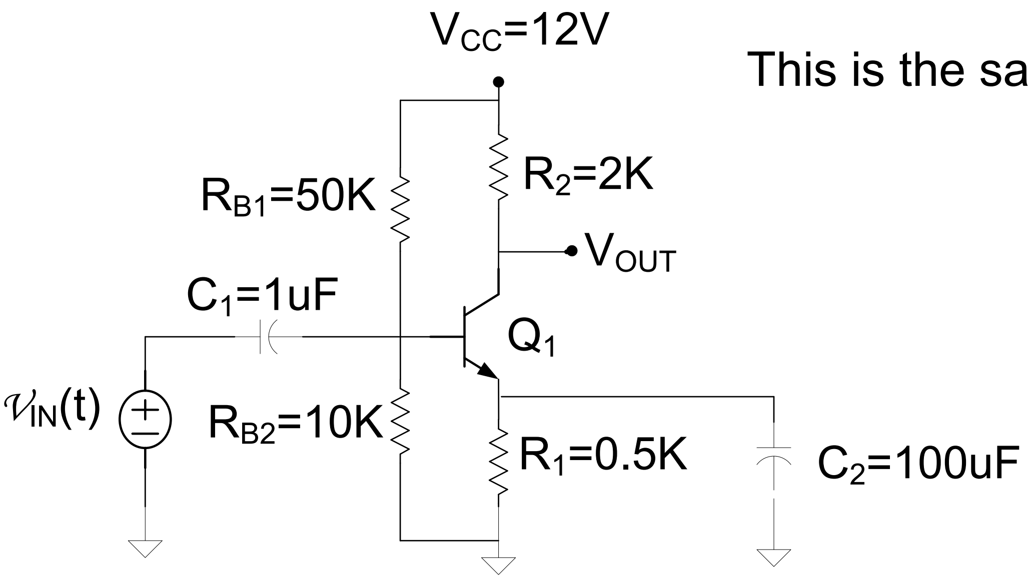


Determine V_{OUTQ} , R_{IN}, R_{OUT}, A_V, and A_{VR}; assume β=100

(A_V , R_{IN}, R_{OUT} , and A_{VR} are the small-signal model parameters for this circuit)

Examples

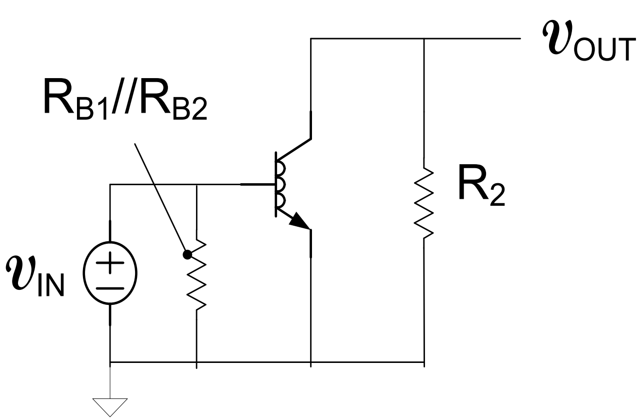
Determine the SS voltage gain A_V



This is the same as another previous-previous circuit !

$$A_V \cong -g_m R_2$$

$$A_V \cong -\frac{I_{CQ} R_2}{V_t}$$



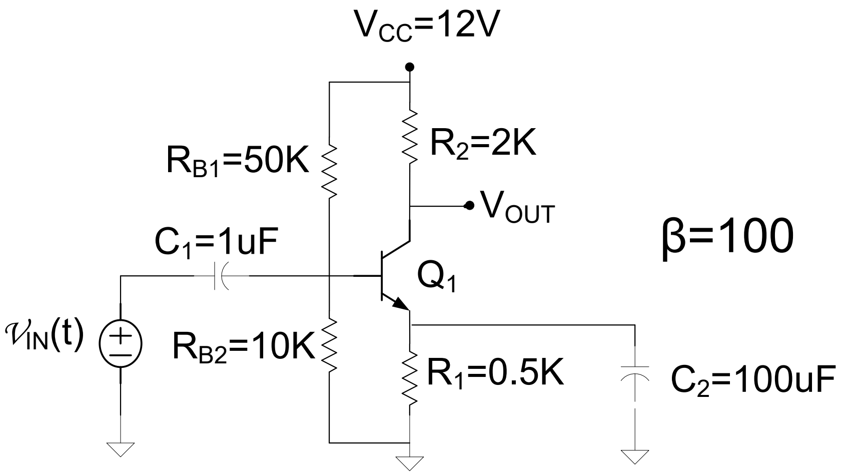
The SS equivalent circuit

$$A_V \cong -\frac{5.6V}{26mV} = -215$$

Note: This Gain is nearly independent of the characteristics of the nonlinear BJT !

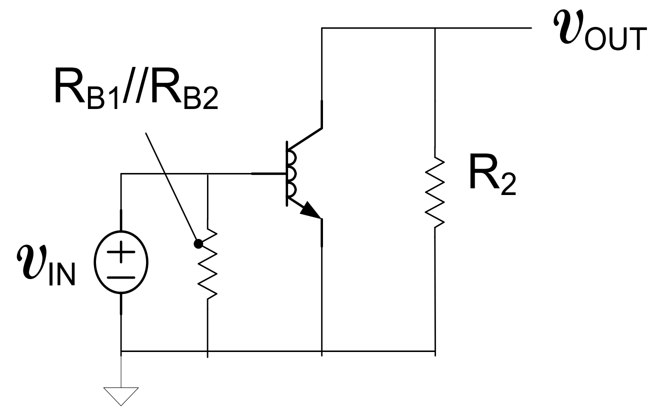
Examples

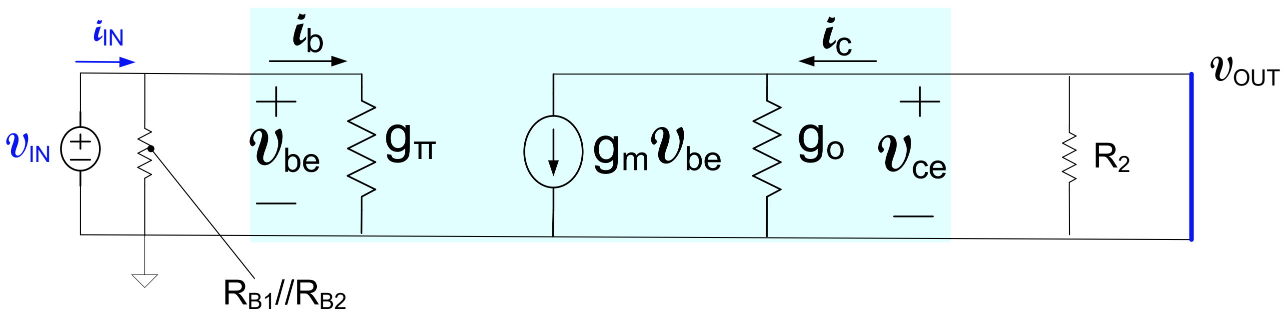
Determination of R_{IN}



β=100

The SS equivalent circuit





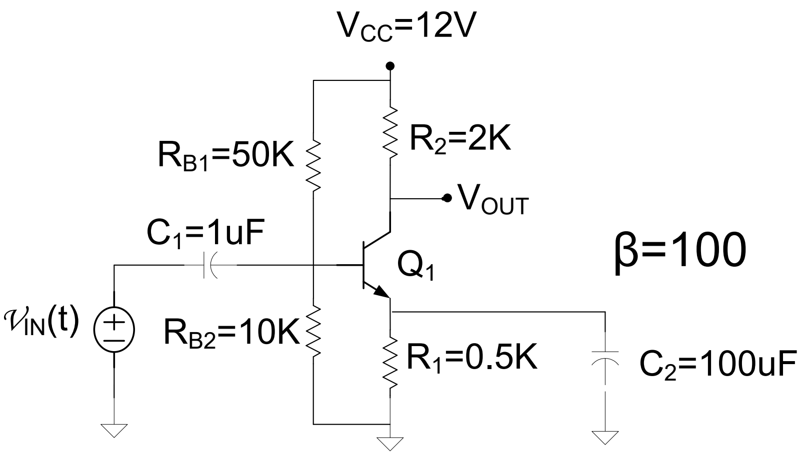
$$R_{IN} = R_{B1}//R_{B2}//r_\pi \cong r_\pi$$

$$r_\pi = \left(\frac{I_{CQ}}{\beta V_t}\right)^{-1} = \left(\frac{2.8\text{mA}}{100 \bullet 26\text{mV}}\right)^{-1} = 928\Omega$$

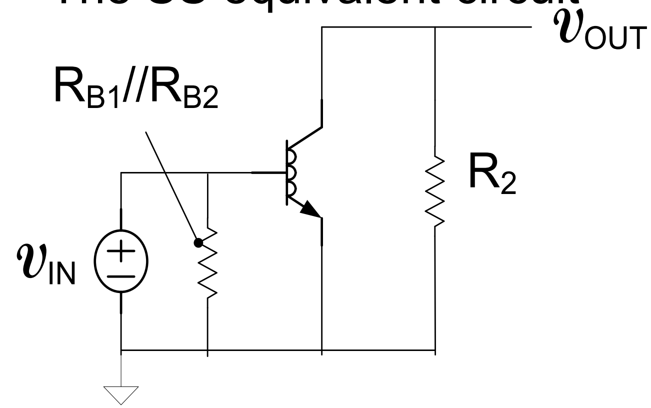
$$R_{IN} = R_{B1}//R_{B2}//r_\pi \cong r_\pi = 930\Omega$$

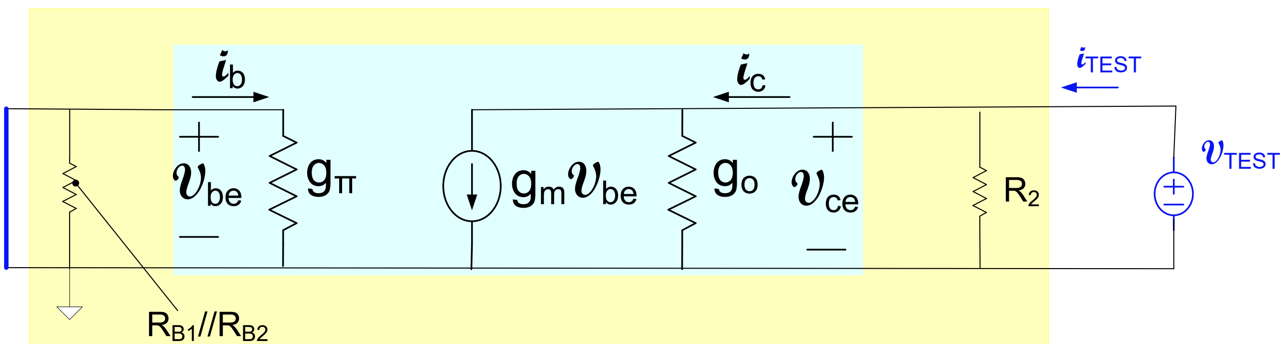
Examples

Determination of R_{OUT}



The SS equivalent circuit



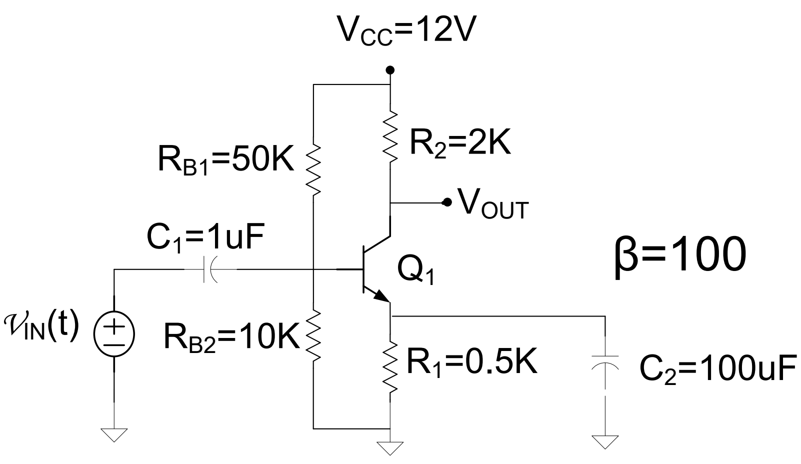


$$R_{OUT} = \frac{v_{TEST}}{i_{TEST}} = R_2 // r_o$$

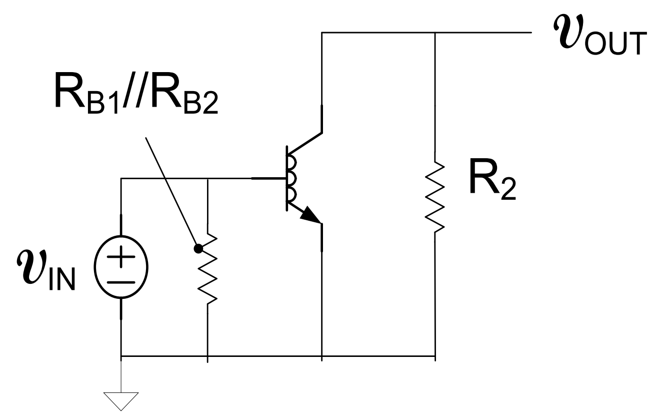
$$r_o = \left(\frac{I_{CQ}}{V_{AF}}\right)^{-1} = \left(\frac{2.8mA}{200V}\right)^{-1} = (1.4E\text{-}5)^{-1} = 71K\Omega$$

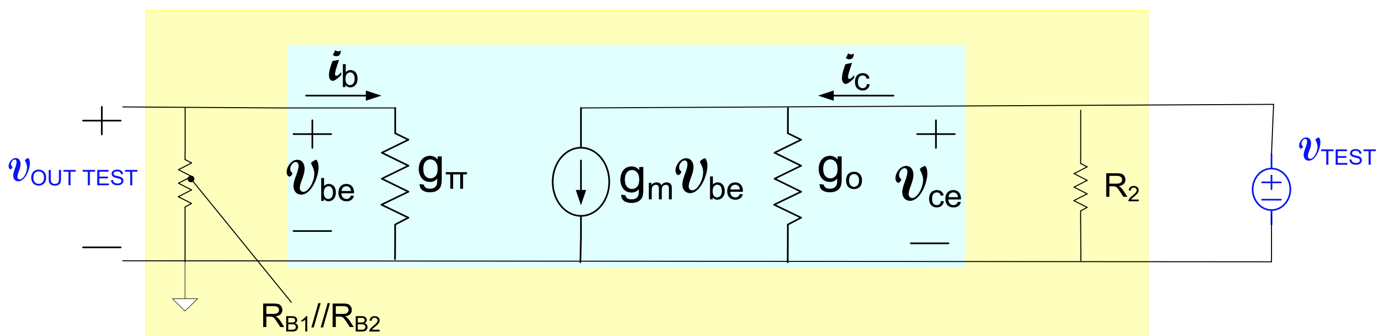
$$R_{OUT} = R_2 // r_o \cong R_2 = 2K$$

Examples Determine A_{VR}



The SS equivalent circuit

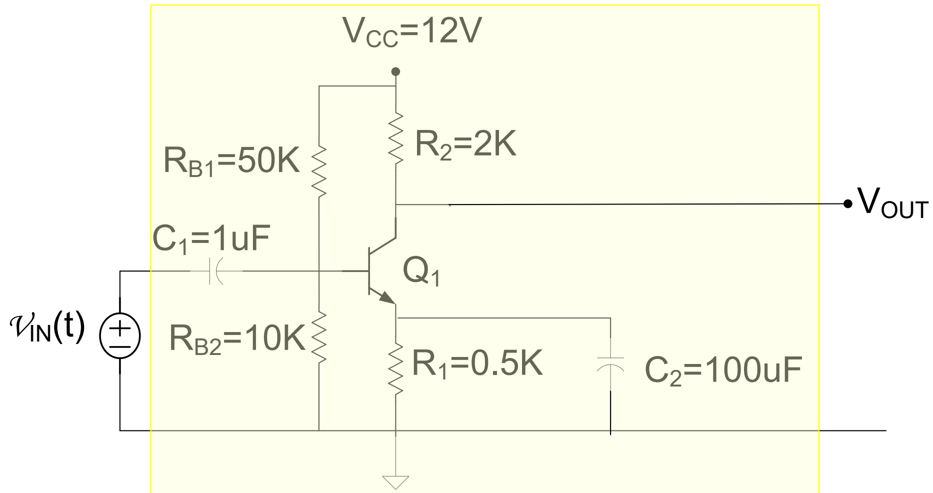


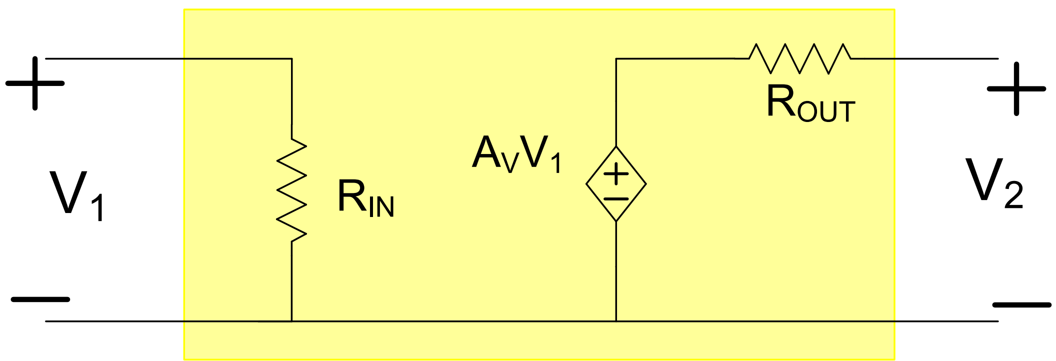


$$v_{OUT\ TEST}=0$$

$$A_{VR}=0$$

Determination of small-signal two-port representation

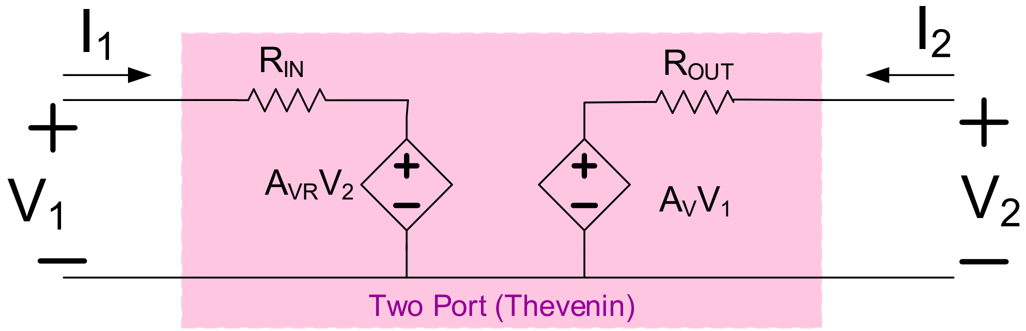




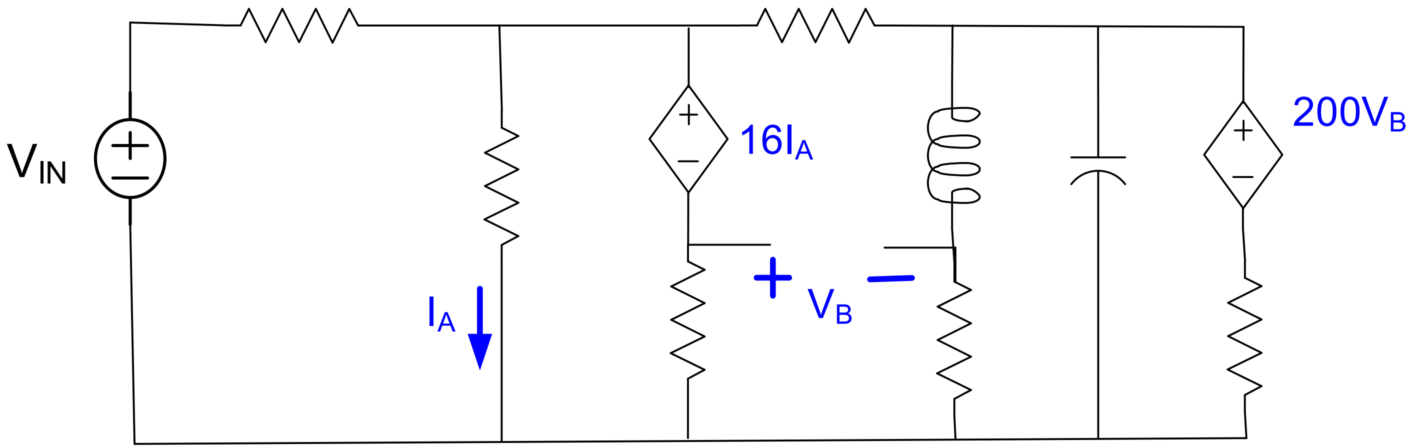
$A_V \cong -215$ $\quad$ $R_{IN} \cong r_\pi = 930\Omega$ $\quad$ $R_{OUT} \cong R_2 = 2K$

This is the same basic amplifier that was considered many times

Relationship with Dependent Sources ?

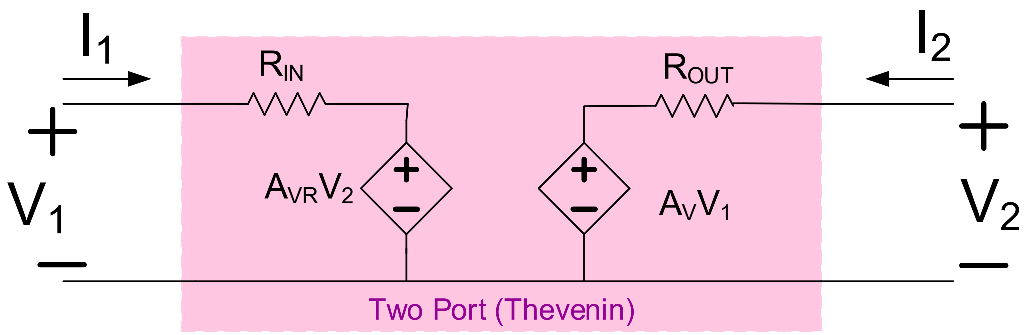


Dependent sources from EE 201



Example showing two dependent sources

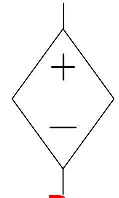
Relationship with Dependent Sources ?



Dependent sources from EE 201

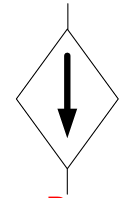
Voltage Amplifier

$\mathcal{V}_s = \mu \mathcal{V}_x$



Voltage Dependent Voltage Source

$I_s = \alpha \mathcal{V}_x$

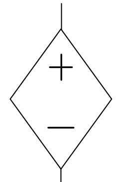


Voltage Dependent Current Source

Transconductance Amplifier

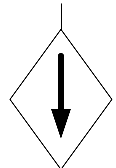
Transresistance Amplifier

$\mathcal{V}_s = \rho I_x$



Current Dependent Voltage Source

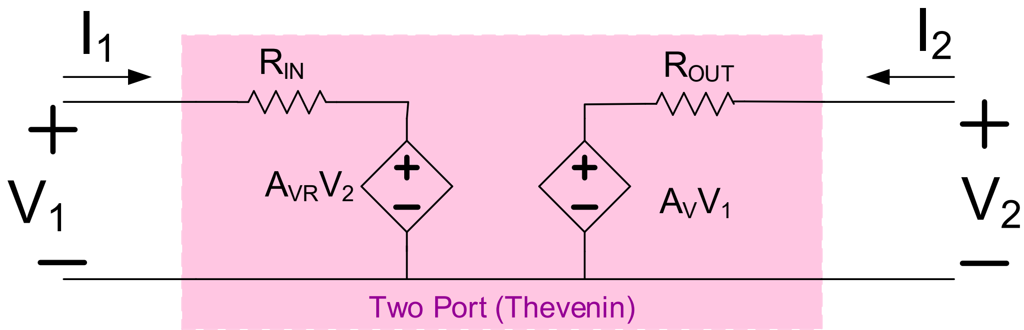
$I_s = \beta I_x$



Current Dependent Current Source

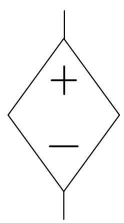
Current Amplifier

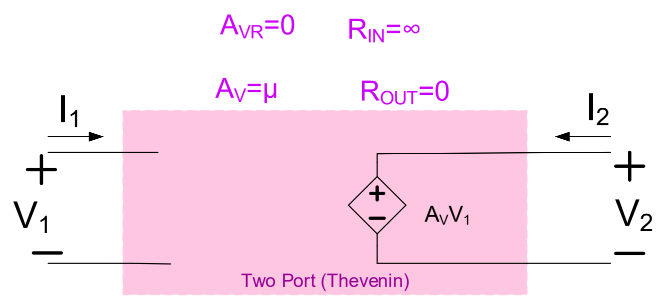
Relationship with Dependent Sources ?



It follows that

$\mathcal{V}_s=\mu\mathcal{V}_x$

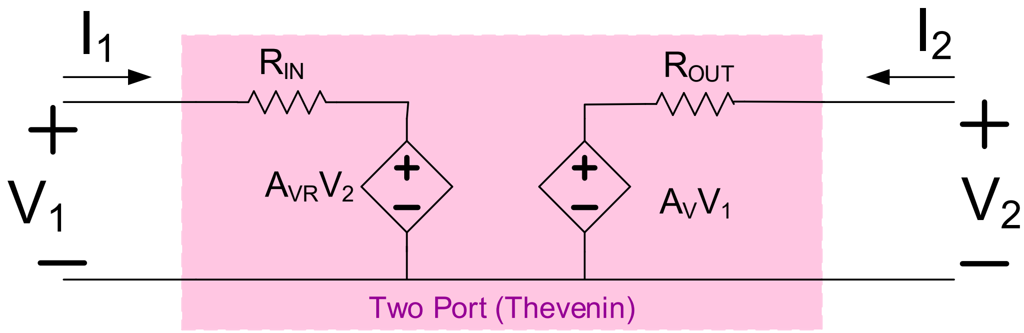




$$V_2=A_V V_1$$

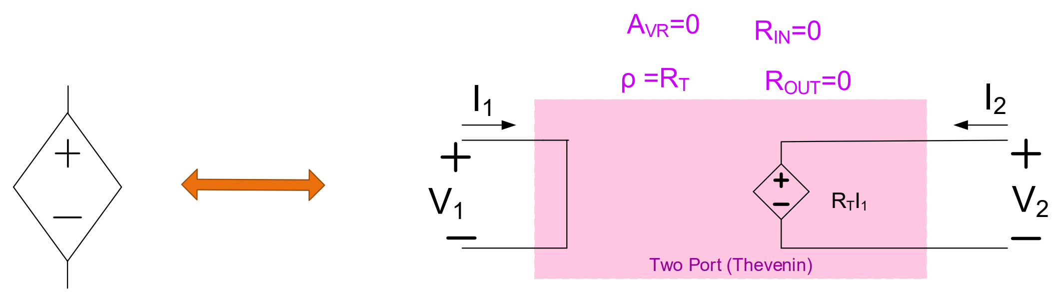
Voltage dependent voltage source is a unilateral floating two-port voltage amplifier with $R_{IN}=\infty$ and $R_{OUT}=0$

Relationship with Dependent Sources ?



It follows that

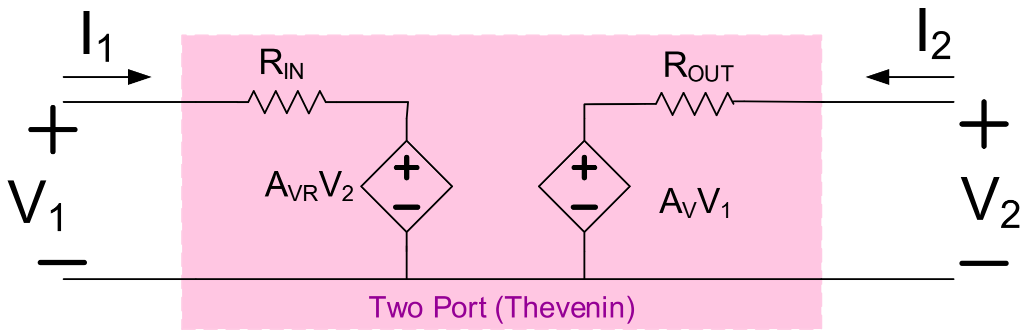
$\mathcal{v}_s = \rho I_x$



$V_2 = R_T I_1$

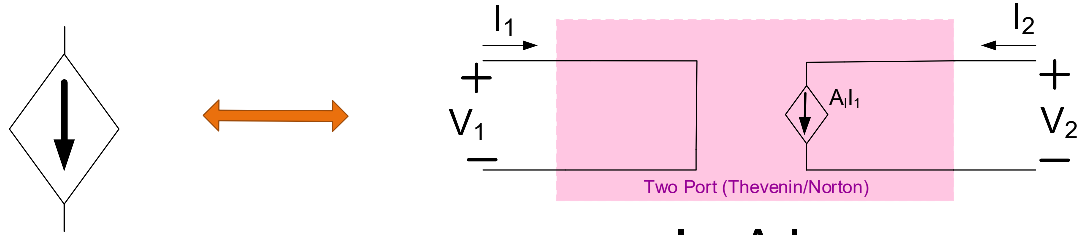
Current dependent voltage source is a unilateral floating two-port transresistance amplifier with $R_{IN}=0$ and $R_{OUT}=0$

Relationship with Dependent Sources ?



It follows that

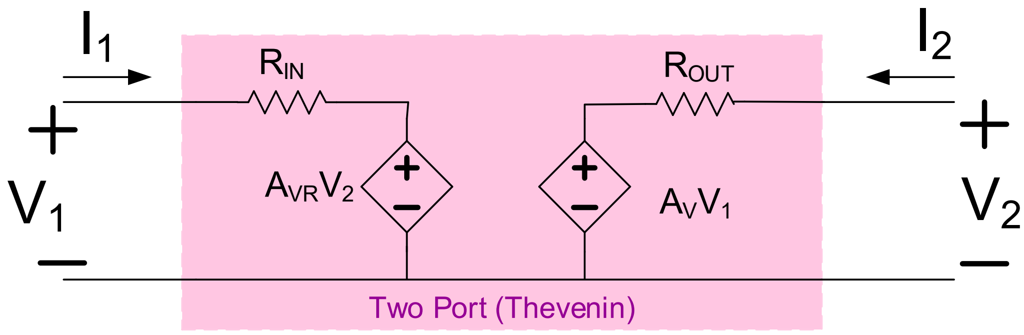
$I_S = \beta I_x$



$I_2 = A_I I_1$

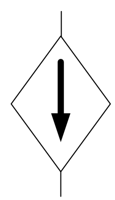
Current dependent current source is a floating unilateral two-port current amplifier with $R_{IN}=0$ and $R_{OUT}=\infty$

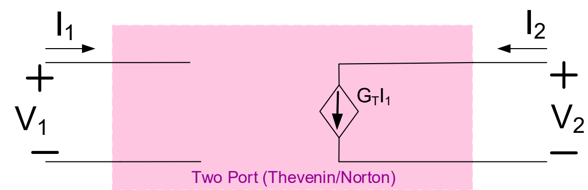
Relationship with Dependent Sources ?



It follows that

$I_s = \alpha v_x$

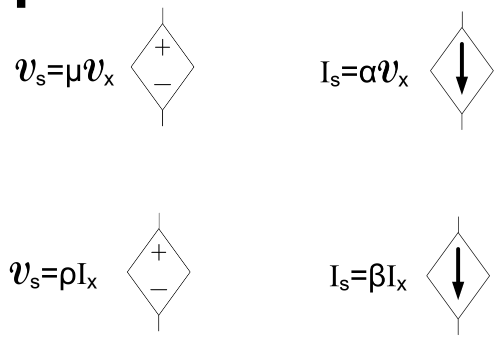




$I_2 = G_T V_1$

Voltage dependent current source is a floating unilateral two-port transconductance amplifier with $R_{IN}=\infty$ and $R_{OUT}=\infty$

Dependent Sources



Dependent sources are unilateral two-port amplifiers with ideal input and output impedances

Dependent sources do not exist as basic circuit elements but amplifiers can be designed to perform approximately like a dependent source

- Practical dependent sources typically are not floating on input or output
- One terminal is usually grounded
- Input and output impedances of realistic structures are usually not ideal

Why were “dependent sources” introduced as basic circuit elements instead of two-port amplifiers in the basic circuits courses???

Why was the concept of “dependent sources” not discussed in the basic electronics courses???

Stay Safe and Stay Healthy !

End of Lecture 28