EE 330
Lecture 29

Small-Signal Model Extension
Comparison of MOS and BJT performance
Basic amplifier architectures
Chinese and Indian Entrepreneurs Are Eating America's Lunch

Watch out, Silicon Valley: China and India aren't just graduating bad engineers and stealing intellectual property anymore. They're fostering innovations that will shake the world.

BY VIVEK WADHWA | DECEMBER 28, 2010
over 20% of the articles on this search are now in non-English venues
Engineers educated today will be under increasing pressure to be able to communicate with, supervise, work with, and work for Asian engineers that may or may not have good English communication skills and must understand the culture of engineers around the world to be effective.

This is not something that may happen in the future but rather is something that is already occurring and WILL become increasingly critical in the next decade.
The increasing role Asia is playing in both the engineering field and the world’s economy is unlike anything we have seen in many decades.

All indicators suggest that this role will become even more significant in the future.

Both opportunities and expectations in the field will invariable show increased alignment with business and engineering in a global economy.

Understanding the culture and the environment of engineers working in Asia will offer substantial benefits for many/most engineers in the short-term and will likely be expected of many/most engineers within a decade.
Graphical Analysis and Interpretation

Device Model (family of curves)

\[ I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \]

- Linear signal swing region smaller than saturation region
- Modest nonlinear distortion provided saturation region operation maintained
- Symmetric swing about Q-point
- Signal swing can be maximized by judicious location of Q-point
Further Model Extensions

Existing model does not depend upon the bulk voltage!

Observe that changing the bulk voltage will change the electric field in the channel region!

Changing the bulk voltage will change the thickness of the inversion layer.

Changing the bulk voltage will change the threshold voltage of the device.

\[
V_T = V_{T0} + \gamma \left( \sqrt{\phi} - V_{BS} - \sqrt{\phi} \right)
\]
Review from Last Lecture

Typical Effects of Bulk on Threshold Voltage for n-channel Device

\[ V_T = V_{T0} + \gamma \left( \sqrt{\phi - V_{BS}} - \sqrt{\phi} \right) \]

\[ \gamma \approx 0.4V^{\frac{1}{2}} \quad \phi \approx 0.6V \]

Bulk-Diffusion Generally Reverse Biased \((V_{BS} < 0\) or at least less than \(0.3V\)) for n-channel

Shift in threshold voltage with bulk voltage can be substantial

Often \(V_{BS} = 0\)
Review from Last Lecture

Typical Effects of Bulk on Threshold Voltage for p-channel Device

\[ V_T = V_{T0} - \gamma \left( \sqrt{\phi + V_{BS}} - \sqrt{\phi} \right) \]

\[ \gamma \approx 0.4V^{\frac{1}{2}} \quad \phi \approx 0.6V \]

Bulk-Diffusion Generally Reverse Biased \((V_{BS} > 0 \text{ or at least greater than } -0.3V)\) for n-channel

Same functional form as for n-channel devices but \(V_{T0}\) is now negative and the magnitude of \(V_T\) still increases with the magnitude of the reverse bias
Model Extension Summary

\( I_G = 0 \)
\( I_B = 0 \)

\[
I_d = \begin{cases} 
0 & \text{if } V_{GS} \leq V_T \\
\mu C_{OX} \frac{W}{L} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & \text{if } V_{GS} \geq V_T, V_{DS} < V_{GS} - V_T \\
\mu C_{OX} \frac{W}{2L} \left( V_{GS} - V_T \right)^2 \cdot (1 + \lambda V_{DS}) & \text{if } V_{GS} \geq V_T, V_{DS} \geq V_{GS} - V_T 
\end{cases}
\]

\[ V_T = V_{T0} + \gamma \left( \sqrt{\phi - V_{BS}} - \sqrt{\phi} \right) \]

Model Parameters : \( \{ \mu, C_{OX}, V_{T0}, \phi, \gamma, \lambda \} \)

Design Parameters : \( \{ W, L \} \) but only one degree of freedom \( W/L \)
Review from Last Lecture

Small Signal Model Summary

\[ i_g = 0 \]

\[ i_b = 0 \]

\[ i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds} \]

\[ g_m = \frac{\mu C_{OX} W}{L} V_{EBQ} \]

\[ g_o = \lambda I_{DQ} \]

\[ g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right) \]
Large Signal Model

\[ I_D = \begin{cases} 
0 & \text{if } V_{GS} \leq V_T \\
\frac{\mu C_{ox} W}{L} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & \text{if } V_{GS} \geq V_T \text{ and } V_{DS} < V_{GS} - V_T \\
\frac{\mu C_{ox} W}{2L} \left( V_{GS} - V_T \right)^2 \cdot (1 + \lambda V_{DS}) & \text{if } V_{GS} \geq V_T \text{ and } V_{DS} \geq V_{GS} - V_T
\end{cases} \]

\[ V_T = V_{T0} + \gamma \left( \sqrt{\phi - V_{BS}} - \sqrt{\phi} \right) \]

Small Signal Model

\[ i_g = 0 \]
\[ i_b = 0 \]
\[ i_d = g_m v_g + g_{mb} v_{bs} + g_o v_{ds} \]

where

\[ g_m = \frac{\mu C_{ox} W}{L} V_{EBQ} \]
\[ g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right) \]
\[ g_o = \lambda I_{DQ} \]
\[ g_{mb} < g_m \quad g_o \ll g_m, g_{mb} \]
Review from Last Time

Example: Obtain the small signal model of the following circuit. Assume MOSFET is operating in the saturation region.
Obtain the small signal model of the following circuit. Assume MOSFET is operating in the saturation region.

Solution:

\[ V \left( g_m + g_0 \right) = I \]

\[ R_{EQ} = \frac{1}{g_m + g_0} \approx \frac{1}{g_m} \]
Relative Magnitude of Small Signal Parameters

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[ g_m = \begin{bmatrix} \frac{I_Q}{V_t} \end{bmatrix} = \beta \]
\[ g_\pi = \begin{bmatrix} \frac{I_Q}{\beta V_t} \end{bmatrix} = \frac{V_{AF}}{V_t} \approx \frac{200V}{100 \cdot 26mV} = 77 \]

\[ g_m \gg g_\pi \gg g_o \]

Often the \( g_\pi \) term can be neglected in the small signal model because it is so small.
Small Signal Model Simplifications for the MOSFET and BJT

Often simplifications of the small signal model are adequate for a given application. These simplifications will be discussed next.
Small Signal MOSFET Model Summary

An equivalent Circuit:

\[ g_m = \frac{\mu C_{OX} W}{L} (V_{GSQ} - V_T) \]

\[ g_o = \lambda I_{DQ} \]

\[ g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right) \]

Alternate equivalent representations for \( g_m \)

\[ g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}} \]

\[ g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}} \]

\[ g_{mb} < g_m \]

\[ g_o << g_m, g_{mb} \]
Small Signal Model Simplifications

Simplification that is often adequate
Small Signal Model Simplifications

Even further simplification that is often adequate
Small Signal BJT Model Summary

An equivalent circuit

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[ g_m >> g_\pi >> g_o \]

This contains absolutely no more information than the set of small-signal model equations
Small Signal BJT Model Simplifications

Simplification that is often adequate
Gains for MOSFET and BJT Circuits

**BJT**

\[ A_{VB} = -\frac{I_{CQ} R_1}{V_t} \]

**MOSFET**

\[ A_{VM} = \frac{2I_{DQ} R}{V_{SS} + V_T} \]

For both circuits

\[ A_v = -g_m R \]

Gains vary linearly with small signal parameter \( g_m \)

Power is often a key resource in the design of an integrated circuit

In both circuits, power is proportional to \( I_{CQ}, I_{DQ} \)
How does \( g_m \) vary with \( I_{DQ} \)?

\[
g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}}
\]

Varies with the square root of \( I_{DQ} \)

\[
g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}
\]

Varies linearly with \( I_{DQ} \)

\[
g_m = \frac{\mu C_{OX} W}{L} (V_{GSQ} - V_T)
\]

Doesn’t vary with \( I_{DQ} \)
How does $g_m$ vary with $I_{DQ}$?

All of the above are true – but with qualification

$g_m$ is a function of more than one variable ($I_{DQ}$) and how it varies depends upon how the remaining variables are constrained.
Comparison of BJT and MOSFET

How do the small signal models of the MOSFET and BJT compare?
Comparison of MOSFET and BJT

The transconductance of the BJT is typically much larger than that of the MOSFET (and larger is better!) This is due to the exponential rather than quadratic output/input relationship.
Comparison of MOSFET and BJT

The output conductances are comparable but that of the BJT is usually modestly smaller (and smaller is better!)

\[ g_o = \lambda I_{DQ} \]

\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[ \frac{g_{oBJT}}{g_{oMOS}} = \frac{\frac{I_{CQ}}{V_{AF}}}{\frac{1}{\lambda I_{DQ}}} = \frac{1}{\lambda V_{AF}} \approx \frac{1}{.01V^{-1} 200V} = 0.5 \]
Comparison of MOSFET and BJT

\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \quad \text{BJT} \quad \quad \quad \quad g_\pi = 0 \quad \text{MOSFET} \]

\( g_\pi \) is the reciprocal of the input impedance

\( g_\pi \) of a MOSFET is much smaller than that of a BJT (and smaller is better!)
Examples

Not convenient to have multiple dc power supplies
$V_{OUTQ}$ very sensitive to $V_{EE}$
Examples

Not convenient to have multiple dc power supplies
$V_{OUTQ}$ very sensitive to $V_{EE}$

Compare the small-signal equivalent circuits of these two structures
Examples

Compare the small-signal equivalent circuits of these two structures

Since Thevenin equivalent circuit in red circle is $V_{IN}$, both circuits have same voltage gain.
Examples

Determine $V_{\text{OUTQ}}$, $A_V$, $R_{\text{IN}}$

$V_{\text{CC}}=12\,\text{V}$
$V_{\text{in}}$
$R_B=500\,\text{K}\Omega$
$C_1=1\,\mu\text{F}$
$R_1=2\,\text{K}\Omega$
$V_{\text{out}}$

$Q_1$
Examples

Determine $v_{out}$ and $V_{OUT}(t)$ if $v_{in} = .002\sin(400t)$
Examples

Several different biasing circuits can be used

(biasing components: \( C, R_B, V_{CC} \) in this case, all disappear in small-signal gain circuit)
Determine $V_{OUTQ}$ and the SS voltage gain, assume $\beta=100$
Examples

Determine $V_{OUTQ}$

$$I_{CQ} = \beta I_{BQ} = 100 \left( \frac{12V - 0.6V}{500K} \right) = 2.3mA$$

$$V_{OUTQ} = 12V - I_{CQ} R_1 = 12V - 2.3mA \cdot 2K = 7.4V$$
Examples

Determine the SS voltage gain

$\beta = 100$

This basic amplifier structure is widely used and repeated analysis serves no useful purpose

Have seen this circuit before but will repeat for review purposes
Examples

\[ V_{CC} = 12V \]
\[ R_B = 500K \]
\[ R_1 = 2K \]
\[ C = 1 \mu F \]
\[ V_{IN} = \] \[
\begin{align*}
\text{Determine the } R_{IN} \\
R_{in} &= \frac{V_{IN}}{i_{IN}} \\
R_{in} &= R_B \parallel r_\pi \\
\text{Usually } R_B &>> r_\pi \\
R_{in} &= R_B \parallel r_\pi \simeq r_\pi \\
R_{in} &\simeq r_\pi = \frac{I_CQ}{\beta V_t}
\end{align*}
\]
Examples

Determine $V_{OUT}$ and $V_{OUT}(t)$ if $V_{IN}=0.002\sin(400t)$

\[ V_{OUT} \approx V_{OUTQ} + A_V V_{IN} \]

\[ V_{OUT} \approx 7.4V - 177 \cdot 0.002 \cdot \sin(400t) \]

\[ V_{OUT} \approx 7.4V - 0.35 \cdot \sin(400t) \]
Amplifier input impedance, output impedance and gain are usually of interest

Why?

Amplifiers can be modeled as a two-port

Amplifier usually unilateral and thevenin equivalent output port often more standard
(unilateral means signal propagates in only one direction: $y_{12}=0$)
Amplifier input impedance, output impedance and gain are usually of interest. Why?

Examples: Assume amplifiers are unilateral

\[ V_{\text{OUT}} = \left( \frac{R_L}{R_L + R_{\text{OUT}}} \right) A_V \left( \frac{R_{\text{IN}}}{R_S + R_{\text{IN}}} \right) V_{\text{IN}} \]

\[ A_{\text{VAMP}} = \frac{V_{\text{OUT}}}{V_{\text{IN}}} = \left( \frac{R_L}{R_L + R_{\text{OUT}}} \right) \left( \frac{R_{\text{IN}}}{R_S + R_{\text{IN}}} \right) A_V \]

Can get gain without reconsidering details about components internal to the Amplifier
Amplifier input impedance, output impedance and gain are usually of interest. Why?

Examples: Assume amplifiers are unilateral

Can get gain without reconsdering details about components internal to the Amplifier
End of Lecture 29