Small-Signal Models
Comparison of MOS and BJT performance
Basic amplifier architectures
Quiz 20

Obtain the small signal model of the following circuit. Assume MOSFET is operating in the saturation region.
And the number is ....
And the number is ....
Quiz 20

Obtain the small signal model of the following circuit. Assume MOSFET is operating in the saturation region.

Solution:

\[ V \left( g_m + g_0 \right) = I \]

\[ R_{EQ} = \frac{1}{g_m + g_0} \approx \frac{1}{g_m} \]
Graphical Analysis and Interpretation

Device Model (family of curves) \( I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \)

- Linear signal swing region smaller than saturation region
- Modest nonlinear distortion provided saturation region operation maintained
- Symmetric swing about Q-point
- Signal swing can be maximized by judicious location of Q-point

Review from Last Time
Review from Last Time

Typical Effects of Bulk on Threshold Voltage for n-channel Device

\[ V_T = V_{T0} + \gamma \left( \sqrt{\phi - V_{BS}} - \sqrt{\phi} \right) \]

\[ \gamma \approx 0.4V^{1/2} \quad \phi \approx 0.6V \]

Bulk-Diffusion Generally Reverse Biased ($V_{BS} < 0$ or at least less than 0.3V) for n-channel
Shift in threshold voltage with bulk voltage can be substantial
Often $V_{BS} = 0$
Review from Last Time

Small-Signal Model Extension

\[ I_G = 0 \]
\[ I_B = 0 \]
\[ I_D = \begin{cases} 
0 & \text{if } V_{GS} \leq V_T \\
\mu C_{ox} \frac{W}{L} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & \text{if } V_{GS} \geq V_T \land V_{DS} < V_{GS} - V_T \\
\mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2 \cdot (1 + \lambda V_{DS}) & \text{if } V_{GS} \geq V_T \land V_{DS} \geq V_{GS} - V_T
\end{cases} \]

\[ V_T = V_{T0} + \gamma \left( \sqrt{\phi - V_{BS}} - \sqrt{\phi} \right) \]

\[ y_{11} = \frac{\partial I_G}{\partial V_{GS}} \bigg|_{V = V_Q} = 0 \quad y_{12} = \frac{\partial I_G}{\partial V_{DS}} \bigg|_{V = V_Q} = 0 \quad y_{13} = \frac{\partial I_G}{\partial V_{GS}} \bigg|_{V = V_Q} = 0 \]

\[ y_{31} = \frac{\partial I_B}{\partial V_{GS}} \bigg|_{V = V_Q} = 0 \quad y_{32} = \frac{\partial I_B}{\partial V_{DS}} \bigg|_{V = V_Q} = 0 \quad y_{33} = \frac{\partial I_B}{\partial V_{GS}} \bigg|_{V = V_Q} = 0 \]

\[ y_{21} = \frac{\partial I_D}{\partial V_{GS}} \bigg|_{V = V_Q} = g_m \quad y_{12} = \frac{\partial I_D}{\partial V_{DS}} \bigg|_{V = V_Q} = g_o \quad y_{13} = \frac{\partial I_D}{\partial V_{GS}} \bigg|_{V = V_Q} = g_{mb} \]
Review from Last Time

Small Signal Model Summary

\[ g_m = \frac{\mu C_{OX} W}{L} V_{EBQ} \]

\[ g_o = \lambda I_{DQ} \]

\[ g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right) \]

\[ i_g = 0 \]

\[ i_b = 0 \]

\[ i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds} \]
Consider:

\[ i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds} \]

3 alternate equivalent expressions for \( g_m \)

\[ g_m = \frac{\mu C_{OX} W}{L} v_{EBQ} \]  
\[ g_m = \sqrt{\frac{2 \mu C_{OX} W}{L}} \sqrt{I_{DQ}} \]  
\[ g_m = \frac{2I_{DQ}}{V_{EBQ}} \]

If \( \mu C_{OX} = 100 \mu A/V^2 \), \( \lambda = .01 V^{-1} \), \( \gamma = 0.4V^{0.5} \), \( v_{EBQ} = 1V \), \( W/L = 1 \), \( v_{BSQ} = 0V \)

\[ I_{DQ} \approx \frac{\mu C_{OX} W}{2L} v_{EBQ}^2 = \frac{10^{-4} W}{2L} (1V)^2 = 5E-5 \]

\[ g_m = \frac{\mu C_{OX} W}{L} v_{EBQ} = 1E-4 \]

\[ g_o = \lambda I_{DQ} = 5E-7 \]

\[ g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{\phi - v_{BSQ}}} \right) = .26g_m \]

In this example

\( g_o << g_m, g_{mb} \)

\( g_{mb} < g_m \)

This relationship is common

In many circuits, \( v_{BSQ} = 0V \) as well
Review from Last Time

Large and Small Signal Model Summary

Large Signal Model

$$I_D = \begin{cases} 
0 & V_{GS} \leq V_T \\
\mu C_{OX} \frac{W}{L} \left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS} & V_{GS} > V_T \\
\mu C_{OX} \frac{W}{2L} \left(V_{GS} - V_T\right)^2 \cdot (1 + \lambda V_{DS}) & V_{GS} > V_T, V_{DS} \geq V_{GS} - V_T 
\end{cases}$$

$$V_T = V_{T0} + \gamma \left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Small Signal Model

$$i_g = 0$$
$$i_b = 0$$
$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

where

$$g_m = \mu C_{OX} \frac{W}{L} V_{EBQ}$$
$$g_{mb} = g_m \left(\frac{\gamma}{2\sqrt{\phi - V_{BS}}}\right)$$
$$g_o = \lambda I_{DQ}$$
Relative Magnitude of Small Signal BJT Parameters

\[ g_m = \frac{I_{CQ}}{V_t} \quad g_\pi = \frac{I_{CQ}}{\beta V_t} \quad g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[ g_m = \begin{bmatrix} \frac{I_Q}{V_t} \\ \frac{I_Q}{\beta V_t} \end{bmatrix} \quad g_\pi = \begin{bmatrix} \frac{I_Q}{\beta V_t} \\ \frac{I_Q}{V_{AF}} \end{bmatrix} \]

\[ g_m \gg g_\pi \gg g_o \]

Often the \( g_o \) term can be neglected in the small signal model because it is so small.
Relative Magnitude of Small Signal Parameters

\[ g_m = \frac{I_{CQ}}{V_t} \quad g_{\pi} = \frac{I_{CQ}}{\beta V_t} \quad g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[ g_m = \begin{bmatrix} \frac{I_Q}{V_t} \\ \frac{I_Q}{\beta V_t} \end{bmatrix} = \beta \]

\[ g_{\pi} = \begin{bmatrix} \frac{I_Q}{\beta V_t} \end{bmatrix} = \frac{V_{AF}}{V_t} \approx \frac{200V}{100 \cdot 26mV} = 77 \]

\[ g_m >> g_{\pi} >> g_o \]

Often the go term can be neglected in the small signal model because it is so small.
Large and Small Signal Model Summary

Large Signal Model

\[ I_C = \beta I_B \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \quad V_{BE} > 0.4V \]

\[ I_B = \frac{J_S A_E}{\beta} e^{V_{BE}/V_t} \quad V_{BC} < 0 \]

Forward Active

\[ V_{BE} = 0.7V \]
\[ V_{CE} = 0.2V \]
\[ I_C < \beta I_B \]

\[ I_C = I_B = 0 \]
\[ V_{BE} < 0 \]
\[ V_{BC} < 0 \]

Small Signal Model

Forward Active

\[ i_b = g_\pi v_{be} \]
\[ i_c = g_m v_{be} + g_0 v_{ce} \]

where

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]
Small Signal Model Simplifications for the MOSFET and BJT

Often simplifications of the small signal model are adequate for a given application.

These simplifications will be discussed next.
An equivalent circuit (4-terminal MOSFET)

\[ g_m = \frac{\mu C_{OX} W}{L} (V_{GSQ} - V_T) \]

\[ g_o = \lambda l_{DQ} \]

\[ g_{mb} = g_m \left( \frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right) \]

This contains absolutely no more information than the set of small-signal model equations
Small Signal Model Summary

More convenient representation
Alternate equivalent representations for $g_m$

\[ g_m = \frac{\mu C_{OX} W}{L} (V_{GSQ} - V_T) \quad \text{from} \quad I_D \approx \mu C_{OX} \frac{W}{2L} (V_{GS} - V_T)^2 \]

\[ g_m = \sqrt{2\mu C_{OX} W} \sqrt{I_{DQ}} \]

\[ g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}} \]
Small Signal Model Simplifications

Simplification that is often adequate
Small Signal Model Simplifications

Even further simplification that is often adequate
Small Signal BJT Model Summary

An equivalent circuit

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

This contains absolutely no more information than the set of small-signal model equations
Small Signal BJT Model Simplifications

Simplification that is often adequate
Gains for MOSFET and BJT Circuits

**BJT**

\[ A_{VB} = -\frac{I_{CQ} R_1}{V_t} \]

**MOSFET**

\[ A_{VM} = \frac{2I_{DQ} R}{[V_{SS} + V_T]} \]

Gains vary linearly with small signal parameter \(g_m\)
How does $g_m$ vary with $I_{DQ}$?

$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}}$

- Varies with the square root of $I_{DQ}$

$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$

- Varies linearly with $I_{DQ}$

$g_m = \frac{\mu C_{OX} W}{L} (V_{GSQ} - V_T)$

- Doesn’t vary with $I_{DQ}$
How does $g_m$ vary with $I_{DQ}$?

All of the above are true – but with qualification

g_m is a function of more than one variable ($I_{DQ}$) and how it varies depends upon how the remaining variables are constrained
Comparison of BJT and MOSFET

How do the small signal models of the MOSFET and BJT compare?
The transconductance of the BJT is typically much larger than that of the MOSFET (and larger is better!)
This is due to the exponential rather than quadratic output/input relationship
Comparison of MOSFET and BJT

**BJT**

\[ g_o = \lambda I_{DQ} \]

**MOSFET**

\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[
\frac{g_{oBJT}}{g_{oMOS}} = \frac{I_{CQ}}{\lambda I_{DQ}} = \frac{1}{\lambda V_{AF}} \approx \frac{1}{0.01V^{-1} \times 200V} = 0.5
\]

The output conductances are comparable but that of the BJT is usually modestly smaller (and smaller is better!)
## Comparison of MOSFET and BJT

<table>
<thead>
<tr>
<th>BJT</th>
<th>MOSFET</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_\pi = \frac{I_{CQ}}{\beta V_t}$</td>
<td>$g_\pi = 0$</td>
</tr>
</tbody>
</table>

$g_\pi$ is the reciprocal of the input impedance

$g_\pi$ of a MOSFET is much smaller than that of a BJT (and smaller is better!)
Standard Approach to small-signal analysis of nonlinear networks

Nonlinear Network

dc Equivalent Network

Q-point

Values for small-signal parameters

Small-signal equivalent network

Small-signal output

Total output
(good approximation)
Systematic Approach to Small-Signal Circuit Analysis

• Obtain dc equivalent circuit by replacing all elements with large-signal (dc) equivalent circuits

• Obtain dc operating points (Q-point)

• Obtain ac equivalent circuit by replacing all elements with small-signal equivalent circuits

• Analyze linear small-signal equivalent circuit
## Dc and small-signal equivalent elements

<table>
<thead>
<tr>
<th>Element</th>
<th>ss equivalent</th>
<th>dc equivalent</th>
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<tbody>
<tr>
<td><strong>MOS Transistors</strong></td>
<td></td>
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<tr>
<td>![MOS symbol]</td>
<td>![MOS ss equivalent]</td>
<td>![MOS dc equivalent]</td>
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</table>
The simplified large signal models for the MOSFET and the BJT

Simplified large-signal models (sometimes termed dc equivalent models) are usually adequate for determining operating point in practical MOS and Bipolar circuits.

Can create circuits where the simplified models are not adequate but these are often not practical circuits.

Will discuss only for npn and n-channel but similar models for pnp and p-channel devices.
Square-Law Model

\[ I_G = 0 \]

\[ I_B = 0 \]

\[ I_d = \begin{cases} 
0 & V_{GS} \leq V_T \\
\mu C_{ox} \frac{W}{L} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T \quad V_{DS} < V_{GS} - V_T \\
\mu C_{ox} \frac{W}{2L} \left( V_{GS} - V_T \right)^2 \cdot (1 + \lambda V_{DS}) & V_{GS} \geq V_T \quad V_{DS} \geq V_{GS} - V_T 
\end{cases} \]

\[ V_T = V_{T0} + \gamma \left( \sqrt{\phi - V_{BS}} - \sqrt{\phi} \right) \]

\[ g_m = \frac{2}{\tau} \left( \phi - V_{BS} \right)^{0.5} \]
Simplified MOS Model for Q-point Analysis

\[ I_G = 0 \]

\[ I_B = 0 \]

\[ I_D = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2 \]

Simplified dc equivalent circuit

\[ \left( \frac{\mu C_{ox} W}{2L} \right) (V_{gs} - V_T)^2 \]
dc BJT model

\[ I_C = \beta I_B \left(1 + \frac{V_{CE}}{V_{AF}}\right) \]
\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{v_{BE}}{V_t}} \]
\[ V_t = \frac{kT}{q} \]

- \( V_{BE} > 0.4 \text{V} \)
- \( V_{BC} < 0 \)
  - Forward Active

- \( V_{BE} = 0.7 \text{V} \)
- \( V_{CE} = 0.2 \text{V} \)
  - \( I_C < \beta I_B \)
  - Saturation

- \( I_C = I_B = 0 \)
- \( V_{BE} < 0 \)
- \( V_{BC} < 0 \)
  - Cutoff

A small portion of the operating region is missed with this model but seldom operate in the missing region.
Simplified dc BJT model for Q-point Analysis

\[ I_C = \beta I_B \]
\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]
\[ V_{BE} = 0.6V \]

Simplified dc equivalent circuit
Examples

Not convenient to have multiple dc power supplies $V_{OUTQ}$ very sensitive to $V_{EE}$
Examples

Not convenient to have multiple dc power supplies
$V_{OUTQ}$ very sensitive to $V_{EE}$

Compare the small-signal equivalent circuits of these two structures
Examples

Compare the small-signal equivalent circuits of these two structures

Since Thevenin equivalent circuit in red circle is $V_{IN}$, both circuits have same voltage gain
Examples

Determine $V_{OUTQ}$, $A_V$, $R_{IN}$
Examples

Determine $v_{\text{OUT}}$ and $V_{\text{OUT}}(t)$ if $v_{\text{IN}} = 0.02\sin(400t)$

\[ V_{\text{CC}} = 12\text{V} \]
\[ V_{\text{in}} \]
\[ R_{1} = 2K \]
\[ R_{B} = 500K \]
\[ C_{1} = 1\text{uF} \]

Diagram:
- Input voltage $V_{\text{in}}$
- Feedback resistor $R_{B} = 500K$
- Load resistor $R_{1} = 2K$
- Capacitor $C_{1} = 1\text{uF}$
- Transistor $Q_{1}$
- Output voltage $V_{\text{out}}$
Examples

Several different biasing circuits can be used
Examples

Biasing Circuit

Determine $V_{OUTQ}$ and the SS voltage gain, assume $\beta=100$
**Examples**

Determine $V_{OUTQ}$

$$I_{CQ} = \beta I_{BQ} = 100 \left( \frac{12V - 0.6V}{500K} \right) = 2.3mA$$

$$V_{OUTQ} = 12V - I_{CQ} R_1 = 12V - 2.3mA \times 2K = 7.4V$$
Examples

Determine the SS voltage gain

\[ \beta = 100 \]

\[ V_{CC} = 12V \]

\[ R_B = 500K \]

\[ C = 1\mu F \]

\[ V_{IN(t)} \]

\[ V_{OUT} \]

\[ R_1 = 2K \]

\[ V_{OUT} \]

\[ V_{IN} \]

\[ V_{BE} \]

\[ g_m \]

\[ V_{BE} \]

\[ i_B \]

\[ g_\pi \]

\[ R_1 \]

This basic amplifier structure is widely used and repeated analysis serves no useful purpose.

Have seen this circuit before but will repeat for review purposes.
Examples

Determine the $R_{\text{IN}}$

$$R_{\text{in}} = \frac{V_{\text{IN}}}{i_{\text{IN}}}$$

$$R_{\text{in}} = R_B \parallel r_\pi$$

Usually $R_B \gg r_\pi$

$$R_{\text{in}} = R_B \parallel r_\pi \approx r_\pi$$

$$R_{\text{in}} \approx r_\pi = \frac{I_{CQ}}{\beta V_t}$$
Examples

Determine $V_{\text{OUT}}$ and $V_{\text{OUT}}(t)$ if $V_{\text{IN}} = 0.002\sin(400t)$

\[ V_{\text{OUT}} \approx V_{\text{OUTQ}} + A_V V_{\text{IN}} \]

\[ V_{\text{OUT}} \approx 7.4V - 1.77 \cdot 0.002 \cdot \sin(400t) \]

\[ V_{\text{OUT}} \approx 7.4V - 0.35 \cdot \sin(400t) \]