The three basic amplifier types for both MOS and bipolar processes:

- Common Emitter
- Common Base
- Common Collector
- Common Source
- Common Gate
- Common Drain

More general models are needed to accommodate biasing, understand performance capabilities, and include effects of loading of the basic structures.

Two-port models are useful for characterizing the basic amplifier structures.

How can the two-port parameters be obtained for these or any other linear two-port networks?
Common Source/ Common Emitter Configurations

\[ R_{in} = \frac{1}{g_{\pi}} \quad A_{V0} = -\frac{g_m}{g_0} \quad R_0 = \frac{1}{g_0} \]

\[ R_{in} = \infty \quad A_{V0} = -\frac{g_m}{g_0} \quad R_0 = \frac{1}{g_0} \]

In terms of operating point and model parameters:

\[ R_{in} = \frac{\beta V_t}{I_{CQ}} \quad A_{V0} = -\frac{V_{AF}}{V_t} \quad R_0 = \frac{V_{AF}}{I_{CQ}} \]

\[ A_{V0} = -\frac{2}{\lambda V_{EBQ}} = -2 \frac{V_{AF}}{V_{EBQ}} \]

Characteristics:

- Input impedance is mid-range (infinite for MOS)
- Voltage Gain is Large and Inverting
- Output impedance is large
- Widely used to build voltage amplifiers
Common Source/Common Emitter Configuration

In terms of operating point and model parameters:

\[
R_{\text{out}} = \frac{1}{g_0 + g_C} \approx R_C
\]

\[
A_v \approx -g_m R_C
\]

\[
R_{\text{in}} = r_\pi
\]

\[
A_v \approx -\frac{I_{\text{CQ}} R_C}{V_t}
\]

\[
R_{\text{out}} \approx R_C
\]

\[
R_{\text{in}} = \frac{\beta V_t}{I_{\text{CQ}}}
\]

Characteristics:
- Input impedance is mid-range (infinite for MOS)
- Voltage Gain is Large and Inverting
- Output impedance is mid-range
- Widely used as a voltage amplifier
Consider Common Collector/Common Drain Two-port Models

Will focus on Bipolar Circuit since MOS counterpart is a special case obtained by setting $g_{\pi}=0$
Two-port model for Common Collector Configuration

It can be readily shown that the common-collector and the common base configurations are not unilateral.

Thus a 4-parameter two-port model is needed to characterize these structures.

Or, equivalently
Two-port model for Common Collector Configuration

\[ V_{be} \quad g_{\pi} \quad g_m V_{be} \quad g_O \]

\[ v_1 \quad i_1 \quad R_{iX} \quad B \quad v_2 \quad i_2 \quad R_{0X} \quad E \]

\[ \{ R_{iX}, A_{V0}, A_{V0r} \text{ and } R_{0X} \} \]
Two-Port Models of Basic Amplifiers widely used for Analysis and Design of Amplifier Circuits

Methods of Obtaining Amplifier Two-Port Network

1. $V_{TEST} : i_{TEST}$ Method

2. Write $V_1 : V_2$ equations in standard form

   $V_1 = i_1 R_{IN} + A_{VR} V_2$

   $V_2 = i_2 R_O + A_{V0} V_1$

3. Thevenin-Norton Transformations

4. Ad Hoc Approaches
Two-port model for Common Collector Configuration

Applying KCL at the input and output node, obtain

\[ i_1 = (v_1 - v_2)g_\pi \]
\[ i_2 = (g_m + g_\pi + g_o)v_2 - (g_m + g_\pi)v_1 \]

These can be rewritten as

\[ v_1 = i_1r_\pi + v_2 \]
\[ v_2 = \left( \frac{1}{g_m + g_\pi + g_o} \right)i_2 + \left( \frac{g_m + g_\pi}{g_m + g_\pi + g_o} \right)v_1 \]

It thus follows that

\[ r_{iX} = r_\pi \quad A_{VOR} = 1 \]
\[ R_{0X} = \left( \frac{1}{g_m + g_\pi + g_o} \right) \]
\[ A_{V0} = \frac{g_m + g_\pi}{g_m + g_\pi + g_o} \]
Two-port model for Common Collector Configuration

Two-port Common Collector Model

$R_{ix} = r_\pi$

$A_{V0r} = 1$

$R_{0x} = \left( \frac{1}{g_m + g_\pi + g_o} \right) \approx \frac{1}{g_m}$

$A_{V0} = \left( \frac{g_m + g_\pi}{g_m + g_\pi + g_o} \right) \approx 1$
Consider the following CC application

Determine $R_{in}$, $R_0$, and $A_V$

(this is not asking for a two-port model for the CC application – $R_{in}$ and $A_V$ defined for no additional load on output, $R_0$ defined for short-circuit input)

\[ A_V = A_{V0} \frac{g_{ox}}{g_{ox} + g_E} = \frac{g_m + g_{\pi}}{g_m + g_{\pi} + g_o} \left( \frac{g_m + g_{\pi} + g_o}{g_m + g_{\pi} + g_o + g_E} \right) = \frac{g_m + g_{\pi}}{g_m + g_{\pi} + g_o + g_E} \approx \frac{g_m}{g_m + g_E} \quad \text{if } g_s >> g_E \]

\[ v_{in} = i_1 R_{ix} + A_{V0r} A_{V0} \frac{g_{0x}}{g_{0x} + g_E} v_{in} \quad \Rightarrow \quad R_{in} = \frac{r_{\pi}}{1 - \frac{g_{m} + g_{\pi}}{g_m + g_{\pi} + g_o + g_E}} \approx r_{\pi} g_m + g_{\pi} + g_o + g_E \]

\[ R_0 \approx \frac{1}{g_m + g_E + g_o + g_{\pi}} = \frac{1}{g_m + g_E} = \frac{R_E}{1 + g_m R_E} \quad \text{if } g_s >> g_E \]

\[ \approx \frac{1}{g_m} \]
Consider the following CC application

(this is not asking for a two-port model for the CC application, $-R_{in}$ and $A_V$ defined for no additional load on output, $R_o$ defined for short-circuit input)

Alternately, this circuit can also be analyzed directly

\[
\begin{align*}
V_{out} &= (g_E + g_0 + g_\pi) = V_{in} g_\pi + g_m V_1 \\
V_{in} &= V_1 + V_{out}
\end{align*}
\]
Common Collector Configuration

Consider the following CC application

(this is not asking for a two-port model for the CC application, – $R_{in}$ and $A_V$ defined for no additional load on output, $R_o$ defined for short-circuit input -)

To obtain $R_0$, set $V_{in} = 0$

\[ i_{out} = \frac{V_{out}}{R_E} \left( g_E + g_0 + g_m \right) - g_m \left( -V_{out} \right) \]

\[ R_{out} = \frac{1}{g_m + g_E} \approx \frac{1}{g_m} \]

\[ g_s \ll g_m \]
Consider the following CC application (this is not asking for a two-port model for the CC application, – $R_{in}$ and $A_V$ defined for no additional load on output, $R_o$ defined for short-circuit input -)

$$A_V = \frac{g_m + g_\pi}{g_m + g_E + g_0 + g_\pi} \approx \frac{g_m}{g_m + g_E} = \frac{I_{CQ}R_E}{I_{CQ}R_E + V_t} \approx 1$$

$$R_{in} = r_\pi \frac{g_m + g_\pi + g_0 + g_E}{g_0 + g_E} \frac{g_\pi}{g_\pi \gg g_o} = r_\pi + \beta R_E$$

$$R_{out} = \frac{1}{g_m + g_\pi + g_0 + g_E} \frac{1}{g_\pi \ll g_o} \approx \frac{1}{g_m}$$

Question: Why are these not the two-port parameters of this circuit?

- $R_{in}$ defined for open-circuit on output instead of short-circuit (see previous slide: -2 slides)
- $A_{V0r} \neq 0$
Common Collector Configuration

For this CC application

(this is not a two-port model for this CC application)

Small signal parameter domain

\[ A_V = \frac{g_\pi + g_m}{g_m + g_E + g_0 + g_\pi} \approx 1 \]

if \( g_a >> g_e \)

\[ R_{in} \approx r_\pi + \beta R_E \]

\[ R_0 \approx \frac{R_E}{1 + g_m R_E} \approx \frac{1}{g_m} \]

Operating point and model parameter domain

\[ A_V \approx \frac{I_{CQ} R_E}{I_{CQ} R_E + V_t} \approx 1 \]

if \( I_{CQ} R_E >> V_t \)

\[ R_{in} \approx \beta R_E \]

\[ R_0 \approx \frac{I_{CQ} R_E >> V_t}{V_t} \approx \frac{V_t}{I_{CQ}} \]

Characteristics:

- Output impedance is low
- \( A_{V0} \) is positive and near 1
- Input impedance is very large
- Widely used as a buffer
- Not completely unilateral but output-input transconductance (or \( A_{Vr} \)) is small and effects are generally negligible though magnitude same as \( A_V \)
Common Collector/Common Drain Configurations

For these CC/CD applications (not two-port models for these applications)

Common Collector

\[ A_V = \frac{g_m + g_{mE}}{g_m + g_E + g_0 + g_{1\pi}} \approx 1 \]

\[ R_{in} \approx r_{1\piE} + \beta R_E \]

\[ R_0 \approx \frac{R_E}{1+g_mR_E} \approx \frac{1}{g_m} \]

In terms of operating point and model parameters:

\[ A_V \approx \frac{I_{cqE}}{I_{cqE}+V_t} \approx 1 \]

\[ R_{in} \approx \beta R_E \]

- Output impedance is low
- \( A_{V0} \) is positive and near 1
- Input impedance is very large

Common Drain

\[ A_V = \frac{g_m}{g_m + g_S + g_0} \approx \frac{1}{1} \]

\[ R_{in} = \infty \]

\[ R_0 \approx \frac{R_S}{1+g_mR_S} \approx \frac{1}{g_m} \]

\[ \frac{V_{EBQ}}{V_{EBQ} + 2I_{DQ}R_S} \approx \frac{1}{2I_{DQ}} \]

- Widely used as a buffer
- Not completely unilateral but output-input transconductance is small
Consider Common Base/Common Gate Two-port Models

Will focus on Bipolar Circuit since MOS counterpart is a special case obtained by setting $g_{m}=0$. 
Two-port model for Common Base Configuration

\[ \begin{align*}
\nu_{be} & : \text{g}_m \nu_{be} \\
\text{g}_\pi & : \text{g}_O
\end{align*} \]

\[ \begin{align*}
\{ R_{ix}, A_{V0}, A_{V0r} \text{ and } R_{0X} \} 
\end{align*} \]
Methods of Obtaining Amplifier Two-Port Network

1. $v_{\text{TEST}} : i_{\text{TEST}}$ Method
2. Write $v_1 : v_2$ equations in standard form
   
   $v_1 = i_1 R_{\text{IN}} + A_{\text{VR}} v_2$
   
   $v_2 = i_2 R_O + A_{\text{V0}} v_1$

3. Thevenin-Norton Transformations

4. Ad Hoc Approaches
Two-port model for Common Base Configuration

From KCL

\[ i_1 = v_1 g_m + (v_1 - v_2) g_0 + g_m v_1 \]

\[ i_2 = (v_2 - v_1) g_0 - g_m v_1 \]

These can be rewritten as

\[ v_1 = \frac{1}{g_m + g_\pi + g_0} i_1 + \left( \frac{g_0}{g_m + g_\pi + g_0} \right) v_2 \]

\[ v_2 = \left( \frac{1}{g_0} \right) i_2 + \left( 1 + \frac{g_m}{g_0} \right) v_1 \]

It thus follows that:

\[ R_{iX} = \frac{1}{g_m + g_\pi + g_0} \approx \frac{1}{g_m} \]

\[ A_{\text{VOR}} = \frac{g_0}{g_m + g_\pi + g_0} \]

\[ A_{\text{VO}} = 1 + \frac{g_m}{g_0} \approx \frac{g_m}{g_0} \]

\[ R_{oX} = \frac{1}{g_0} \]
Two-port model for Common Base Configuration

Two-port Common Base Model

\[ R_{iX} = \frac{1}{g_m + g_\pi + g_0} \approx \frac{1}{g_m} \]

\[ A_{V0r} = \frac{g_0}{g_m + g_\pi + g_0} \approx \frac{g_0}{g_m} \]

\[ A_{V0} = 1 + \frac{g_m}{g_0} \approx \frac{g_m}{g_0} \]

\[ R_{oX} = \frac{1}{g_0} \]
Consider the following CB application

(this is not asking for a two-port model for this CB application – \( R_{\text{in}} \) and \( A_v \) defined for no load on output, \( R_o \) defined for short-circuit input)

\[
A_v = A_{v0} \frac{R_C}{R_C + R_{0X}} = \left( \frac{g_m + g_0}{g_0} \right) \left( \frac{g_0}{g_C + g_0} \right) = \frac{g_m + g_0}{g_C + g_0} \approx g_m R_C
\]

\[
R_{\text{in}} = \frac{v_{\text{in}}}{i_1} = \frac{i_1 R_{iX} + A_{v0} v_{\text{out}}}{i_1} = \frac{R_{iX}}{1 - A_{v0} A_v}
\]

\[
R_{\text{out}} = R_C // R_{0X} = \frac{R_C}{1 + g_0 R_C}
\]
Common Base Configuration

Consider the following CB application

\[ V_{DD} \]
\[ R_C \]
\[ V_{out} \]
\[ V_{in} \]
\[ V_{BB} \]

\[ V_{be} \]
\[ g_m V_{be} \]
\[ g_O \]

Alternately, this circuit can also be analyzed directly

By KCL at the output node, obtain

\[ (g_C + g_0) V_0 = (g_m + g_0) V_{in} \]

\[ A_V = \frac{g_m + g_0}{g_C + g_0} \approx g_m R_C \]

By KCL at the emitter node, obtain

\[ i_1 = (g_m + g_\pi + g_0) V_{in} - g_0 V_{out} \]

\[ R_{in} = \frac{g_0 + g_C}{g_C (g_m + g_\pi + g_0) + g_\pi g_0} \approx \frac{1}{g_m} \]

\[ R_{out} = R_C // r_0 \]

\[ R_{out} = \frac{R_C}{1 + g_0 R_C} \approx R_C \]
Common Base Application

(this is not a two-port model for this CB application)

\[ A_V \approx g_m R_C \]
\[ R_{in} \approx \frac{1}{g_m} \]
\[ R_{c} \ll r_0 \]
\[ R_{out} \approx R_C \]

Characteristics:

- Output impedance is mid-range
- \( A_{V0} \) is large and positive (equal in mag to that to CE)
- Input impedance is very low
- Not completely unilateral but output-input transconductance is small
In terms of operating point and model parameters:

\[ A_V \approx g_m R_C \quad R_{in} \approx \frac{1}{g_m} \quad R_{out} \approx R_C \]

\[ A_V \approx g_m R_D \quad R_{in} \approx \frac{1}{g_m} \quad R_{out} \approx R_D \]

Characteristics:

- Output impedance is mid-range
- \( A_{V_0} \) is large and positive (equal in mag to that to CE)
- Input impedance is very low
- Not completely unilateral but output-input transconductance is small
By KCL at two non-grounded nodes

\[ \begin{align*}
    v_{out} (g_C + g_0) + (v_{in} - v_E) g_m &= g_0 v_E \\
    v_E (g_E + g_0 + g_\pi) - (v_{in} - v_E) g_m &= g_0 v_{out} + g_\pi v_{in}
\end{align*} \]

\[ A_V = \frac{v_{out}}{v_{in}} = \frac{-g_m g_E + g_0 g_\pi}{g_C g_m + g_C (g_0 + g_\pi + g_E) + g_0 (g_\pi + g_E)} \approx -\frac{R_C}{R_E} \]
Common Emitter with Emitter Resistor Configuration

\[ V_{DD} \]
\[ V_{in} \]
\[ V_{out} \]
\[ V_{E} \]
\[ V_{EE} \]
\[ R_C \]
\[ R_E \]

\[ A_V \approx -\frac{R_C}{R_E} \]

It can also be shown that

\[ R_{in} \approx r_{\pi} + \beta R_E \]
\[ R_{out} \approx R_C \]

Nearly unilateral (is unilateral if \( g_o = 0 \))
Common Emitter with Emitter Resistor Configuration

\[ V_{DD} \]
\[ R_C \]
\[ \nu_{out} \]
\[ \nu_{in} \]
\[ V_{EE} \]

\[ V_{out} \]
\[ R_C \]
\[ R_E \]

\[ A_V \approx - \frac{R_C}{R_E} \]
\[ R_{in} \approx r_T + \beta R_E \]
\[ R_{out} \approx R_C \]

(ten this is not a two-port model)

Characteristics:
- Analysis would simplify if \( g_0 \) were set to 0 in model
- Gain can be accurately controlled with resistor ratios
- Useful for reasonably accurate low gains
- Input impedance is high
## Basic Amplifier Gain Table

<table>
<thead>
<tr>
<th>Structure</th>
<th>CE/CS</th>
<th>CC/CD</th>
<th>CB/CG</th>
<th>CEwRE/CSwRS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BJT</strong></td>
<td><img src="image" alt="BJT CE/CS" /></td>
<td><img src="image" alt="BJT CC/CD" /></td>
<td><img src="image" alt="BJT CB/CG" /></td>
<td><img src="image" alt="BJT CEwRE/CSwRS" /></td>
</tr>
<tr>
<td><strong>MOS</strong></td>
<td><img src="image" alt="MOS CE/CS" /></td>
<td><img src="image" alt="MOS CC/CD" /></td>
<td><img src="image" alt="MOS CB/CG" /></td>
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</tr>
</tbody>
</table>

### Gain ($A_V$)

<table>
<thead>
<tr>
<th>Structure</th>
<th>CE/CS</th>
<th>CC/CD</th>
<th>CB/CG</th>
<th>CEwRE/CSwRS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BJT</strong></td>
<td>$-g_mR_C$</td>
<td>$\frac{g_m}{g_m + g_E}$</td>
<td>$g_mR_C$</td>
<td>$-\frac{R_C}{R_E}$</td>
</tr>
<tr>
<td><strong>MOS</strong></td>
<td>$\frac{I_CQ R_C}{V_t}$</td>
<td>$\frac{I_CQ R_E}{I_CQ R_E + V_t}$</td>
<td>$\frac{2I_DQ R_E}{2I_DQ R_E + V_EB}$</td>
<td>$\frac{I_CQ R_C}{V_t}$</td>
</tr>
</tbody>
</table>

### Input Resistance ($R_{in}$)

<table>
<thead>
<tr>
<th>Structure</th>
<th>CE/CS</th>
<th>CC/CD</th>
<th>CB/CG</th>
<th>CEwRE/CSwRS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BJT</strong></td>
<td>$\infty$</td>
<td>$r_T + \beta R_E$</td>
<td>$\infty$</td>
<td>$r_T + \beta R_E$</td>
</tr>
<tr>
<td><strong>MOS</strong></td>
<td>$\frac{V_t}{I_CQ}$</td>
<td>$\beta \left( \frac{V_t}{I_CQ} + R_E \right)$</td>
<td>$\frac{V_t}{I_CQ}$</td>
<td>$\beta \left( \frac{V_t}{I_CQ} + R_E \right)$</td>
</tr>
</tbody>
</table>

### Output Resistance ($R_{out}$)

<table>
<thead>
<tr>
<th>Structure</th>
<th>CE/CS</th>
<th>CC/CD</th>
<th>CB/CG</th>
<th>CEwRE/CSwRS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BJT</strong></td>
<td>$R_C$</td>
<td>$g_m$</td>
<td>$R_C$</td>
<td>$R_C$</td>
</tr>
<tr>
<td><strong>MOS</strong></td>
<td>$\frac{V_t}{I_CQ}$</td>
<td>$\frac{V_EB}{2I_DQ}$</td>
<td>$R_C$</td>
<td>$R_C$</td>
</tr>
</tbody>
</table>

*(not two-port models for the four basic structures)*
Can use these equations only when small signal circuit is EXACTLY like that shown!!
End of Lecture 31